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U.S. Army Force Structure Optimization and Sufficiency Analysis

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U.S. ARMY FORCE STRUCTURE OPTIMIZATION AND SUFFICIENCY ANALYSIS

THESIS

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Abstract

The United States Army perpetually deploys rotational forces across the globe in support of the National Security Strategy. These forces meet a set of discrete mission demands over an extended time period before redeploying, modernizing, and preparing for the next deployment. The U.S. Army now utilizes the Regionally Aligned Readiness and Modernization Model to execute these cyclical stages for unit deployments. Specific emphasis is placed on aligning forces against a Geographic Combatant Command, which allows units to build readiness and lethality oriented towards the same series of threats, physical terrain, and civilian considerations. This research provides an Integer Programming model that offers the U.S. Army an optimal solution outlining how many units by Modification Table of Organizational Equipment, Active or Reserve Component Status, and Geographic Combatant Command location alignment, needed to meet every mission demand, for a prescribed set of time periods, at the battalion level echelon.

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U.S. ARMY FORCE STRUCTURE OPTIMIZATION AND SUFFICIENCY ANALYSIS

I. Introduction

This chapter discusses the problem background, the research's problem statement, a brief summary of contributions, and outlines the remaining structure for this thesis.

1.1 Problem Background

Having the correct type and quantity of workers is a problem faced by organizations from every facet of society, whether it is discussing business, academia, or government, etc. Having too few workers forces an organization to overwork its individuals, provide a lesser product, lose profit, or a combination of the three different effects, along with a number of other negative by-products. Having too many workers may lead to an individual loss of productivity and an increase in operational cost to an organization. Finally, having the wrong type of workers may lead to a less than optimal product and a loss of expertise gained. In order to have an effective and efficient workforce, an organization must address the composition of its personnel.

This same dilemma applies to the United States Army (U.S. Army). The U.S. Army needs to consider the quantity and type of Soldiers not only at the individual level but also in terms of units at each echelon as well. In doing so, the U.S. Army seeks to maintain the correct type and quantities of units in order to fulfill its mission and serve the American people.

The U.S. Army has begun implementation of its new deployment model, the Regionally Aligned Readiness and Modernization Model (ReARMM). Under ReARMM, the U.S. Army will align individual units against certain missions in specific geographic areas or Combatant Commands (COCOM). These aligned units will not only receive and utilize equipment according to their aligned mission, but their training will also be designed to prepare the unit for that specific mission. ReARMM also allows the U.S. Army to prioritize its modernization and fielding efforts towards units aligned against specific theaters or specific threats. For example, the current situation in Europe may dictate that the U.S. Army provides units aligned with a European Command (EUCOM) mission the newest equipment, conversely, before a unit aligned with Southern Command (SOUTHCOM). ReARMM contrasts with the previous deployment model, Army Force Generation (ARFORGEN) [1].

Under ARFORGEN, similarly structured units received the same equipment, the same training, and mostly deployed to the same geographic region, Central Command (CENTCOM). This model was adequate while the Army focused on counterinsurgency operations in Afghanistan and Iraq, but ARFORGEN does not adequately prepare the force to face peer or near peer adversaries [1]. Formerly, under the ARFORGEN model, Department of Defense (DoD) and U.S. Army planners took a set of mission demands along with a set of available units and evaluated which missions could be supported. Under ARFORGEN, not every mission received support; conversely, this research applies ReARMM to answer how every mission can be supported and at what cost. The desire is to minimize the number of U.S. Army units that can support all mission demand while ensuring that the correct type of unit is assigned to missions for which it is structurally built and aligned and also ensuring that units have enough time between missions to adequately modernize and train.

Importantly, ReARRM also allows for a more predictable schedule for units and Soldiers, with a decreased operational tempo (OPTEMPO). The model calls for cyclical periods for units to execute missions, modernize, and conduct training. This allows for greater readiness and also decreases the stress placed on Soldiers and their families [2].

By regionally aligning units against missions in a specific Geographic COCOM, the U.S. Army has greater ability to allocate the correct and sufficient equipment and supplies in each of the regionally based Army Prepositioned Stocks (APS). For example, if no Armor Brigade Combat Team (ABCT) is aligned with a CENTCOM mission, equipment specifically and strictly used by an ABCT can be moved from an APS supporting CENTCOM to an APS supporting EUCOM or Indo-Pacific Command (INDOPACOM). This reallocation of equipment and supplies increases readiness for the gaining COCOM and its rotational forces [3].

Most importantly, ReARMM allows units to establish "habitual relationships to missions and theaters" [4]. Establishing a specialized focus increases unit preparedness since units are perpetually preparing for a specific threat. Additionally, if units are cyclically training to combat a specific threat and subsequently and repeatedly deploying to regions to deter or confront that threat, the unit's institutional knowledge of the regional environment and potential enemy will increase over time. This once again allows for increased readiness [4].

1.2 Problem Statement

This research seeks to support ongoing work at Army Future Command's The Research and Analysis Center - Fort Leavenworth (TRAC-FLVN). The primary goal of this research is to answer the question: How many U.S. Army units, by Modification Table of Organizational Equipment (MTOE) type, Component status, and Geographic COCOM location alignment, are required to meet all missions over a planning horizon given a set of forecasted mission demands, cost parameters, and operating guidelines? In doing so, this research will also address the degree to which the

U.S. Army will be able to adhere to its desired goals in accordance with the ReARMM framework for unit deployments. With the desire of obtaining high-quality solutions in a timely manner, this research will also explore the suitability of an integer programming (IP) model to help inform U.S. Army force planning.

1.3 Summary of Contributions

First, this research formulates an integer programming model that minimizes the number of units required by MTOE, Component status, and designated Geographic COCOM location alignment over a desired time period while adhering to ReARMM force planning guidelines. Second, this research focuses on Warfighting Function (WfF) specific mission demands individually, and solve the model to determine the number of units required by WfF. Combining the solutions of each individual WfF instance then informs the overall U.S. Army force structure required to meet mission demands. Each instance of the model solves to optimality, with the largest and most complex set of mission demands solving to optimality within 24.04 seconds. Sensitivity analysis is also performed to evaluate the effect of changing specific parameters. Additionally, and perhaps most importantly, the framework of this model can be utilized to perform sufficiency analysis for any problem involving layered workforce attributes with a desired set of job demands, each correlating with a set of preferred worker attributes.

1.4 Thesis Structure

In addition to this introductory chapter, the structure of this thesis is as follows. Chapter II provides an overview of how and why the U.S. Army deploys its forces under ReARMM. The literature review in this chapter also provides an overview of relevant academic literature regarding methodologies that examine workforce scheduling and workforce sufficiency. This thesis also examines problem types with similarities to the focus of this research, including: assignment problems, set cover problems, goal programming, and multi-objective optimization. Chapter III provides the formulation of the integer programming model used to address the problem statement of this research. In Chapter IV, the IP model is solved for a variety of test instances, and provides a sensitivity analysis examining the effects of changing the model's parameters. Finally, Chapter V summarizes this research's key findings as well as discussing avenues for future research.

II. Background and Literature Review

This chapter is separated into three distinct sections. The first section discusses U.S. National Security Strategy (NSS) and the employment of U.S. Army Forces. The second section examines academic literature that concerns the use of military forces. The third and final section explores academic literature that is directly relevant to this research.

2.1 Topic Literature

2.1.1 National Security Strategy / DoD Doctrine

The 2021 Interim National Security Strategic Guidance discusses three key components: the first component, "Defend and nurture the underlying sources of American strength, including our people, our economy, our national defense, and our democracy at home"; the second component, "Promote a favorable distribution of power to deter and prevent adversaries from directly threatening the United States and our allies, inhibiting access to the global commons, or dominating key regions"; and the final component, "Lead and sustain a stable and open international system, underwritten by strong democratic alliances, partnerships, multilateral institutions, and rules" [5]. The Interim NSS Guidance then discusses the need to strengthen our alliances and partnerships with other nations from around the world. The guidance specifically addresses the North Atlantic Treaty Organization (NATO) and key U.S. allies and partners in Asia, the Middle East, Africa, and the Americas [5].

The Department of Defense, and specifically the U.S. Army, plays a significant role in National Security. Not only in addressing each of the three key components of the Interim NSS Guidance but also in the continued development of partnerships with other nations all over the world. This is done through Security Force Assistance

(SFA). The DoD policy regarding SFA reads, "The Department of Defense shall develop and maintain the capability within DoD general purpose forces (GPF), special operations forces (SOF), and the civilian expeditionary workforce (CEW) to conduct SFA activities in support of U.S. policy and in coordination with the relevant U.S. Government (USG) departments or agencies". U.S. Armed Forces are rotationally assigned to meet required SFA demands and support the U.S. National Security Strategy [6].

Forces are apportioned, allocated, and assigned to each COCOM. Apportionment of forces occurs when each military department within the DoD, in coordination with the Chairman of the Joint Chiefs of Staff, determines the quantity of forces by type available for a rotational allocation to any COCOM. Dependent on each COCOM's operational or strategic needs, these forces are then allocated to a COCOM. Finally, the COCOM Commanders assign forces to each mission demand. The assignment of forces begins with the Preferred Force Identification and Contingency Sourcing [7]. This occurs when the COCOM identifies which units *can* accomplish a given mission and which units *should* accomplish a given mission. To complete the assignment process, the COCOM Commander then approves the assignment of forces to mission through Execution Sourcing [7].

2.1.2 Assignment or Tasking of Military Personnel

Assignment or tasking of military personnel can take place at any echelon, from the departmental level, down to the individual service member. This can be done to assign missions or tasks, or for force structure maintenance or creation.

The problem of assigning or tasking defense personnel is not specific to the U.S. Army; it is something encountered by the other services within the DoD and other nations' armed forces as well. Holder [8] discusses the assignment of U.S. Navy Sailors.

His original model utilizes binary decision variables, and the model is then solved using an interior point algorithm on the relaxed linear program (LP). The author uses an assignment model designed to minimize the cost associated with pairing a Sailor to an assigned duty. Each cost of assigning a Sailor to an assigned duty is a combination of four different binary sub-costs. These sub-costs account for Sailor training, assignment location, Navy priority, and the Sailor's geographical preference. This model is designed to assign the approximately 90,000 Sailors that are reassigned each year; however, computational results are not included in the article [8].

Fauske [9] discusses troops to task analysis within the context of a project scheduling problem. This article is primarily about Norwegian army forces, specifically at the brigade level. The troops to task analysis in this work takes the form of an integer program, with the solution providing an answer to what activity is accomplished at which time, by what unit, and with what resources. Their model includes a range of unit types executing missions in conjunction. This model is unique in that it seeks to minimize the total time required to complete all tasks associated with the given project. One important fact conveyed in their work is that computational resources (e.g., equipment) available to model and solve the problem can limit the size of problem that can be solved within a reasonable amount of time [9].

Hannan et al. [10] discusses the use of both the U.S. Air Force Active component personnel and units, as well as the U.S. Air Force Reserve component personnel and units. They present a decision support system, titled "The Grey Space", designed to allow policy makers to understand how many aircraft, and their accompanying Airmen, should be assigned to the Active or Reserve components of the U.S. Air Force [10]. It describes five key elements, or "drivers" that determine the mix of Active to Reserve component personnel and units, "(1) wartime demand and requirements for weapon systems and manpower, (2) weapon system inventory, (3) manpower inventory, (4) force employment policies, and (5) costs" [10]. It's model outputs this mix when considering three different phases of conflict: routine peacetime (nonsurge), major combat operations (surge), and stability operations (postsurge) [10].

Checco et al. [11] proposes their methods to optimize the U.S. Army force size prior to the inception of ReARRM. The article's model examines a force structure down to the personnel level. The formulation accounts for Component status, deployment status, unit types, and costs. Their model also deals with the uncertainty of demand, resulting in an objective function that seeks to minimize expected cost [11]. Although this research's problem of interest, optimizing U.S. Army force structure, closely aligns to that discussed in Checco et al. [11], there are differences in how the model is formulated. Notably, Checco et al. [11] deals with stochastic mission demands, all of which do not need to be met; and constraints that limit the number of Soldiers and units employed. This research deals with known mission demands, all of which must be met, with no limitation on the quantity or type of units employed.

2.2 Methodology Literature

2.2.1 Assignment Problems

Assignment Problems entail assigning a specific worker to a specific job or set of jobs. This can also be accomplished repetitively over time in order to construct a work schedule. Typically, the objectives in these problems involve minimizing a cost of paying workers to accomplish a job, minimizing total time to accomplish all jobs, or maximizing utility gained by assigning workers to jobs.

Balachandran [12] presents a formulation designed to maximize value gained from completing jobs. Its model specifically looks at assigning computers in a network to process a job created within that network. The work modifies a transportation problem with integer constraints to construct its model [12].

Fisher et al. [13] initially presents the Generalized Assignment Problem (GAP) formulation:

$$Max \quad \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} c_{ij}x_{ij}, \tag{1}$$

$$S.T. \quad \sum_{i\in\mathcal{I}} x_{ij} = 1, \qquad \forall\, j \in \mathcal{J}, \tag{2}$$

$$\sum_{j\in\mathcal{J}} a_{ij}x_{ij} \leq b_i, \qquad \forall\, i \in \mathcal{I}, \tag{3}$$

$$x_{ij} \in \{0,1\} \quad \forall\, i \in \mathcal{I},\ \forall\, j \in \mathcal{J}. \tag{4}$$

In the above model, c_{ij} represents a value gained from assigning agent i to perform job j, with the binary decision variable $x_{ij} = 1$ if agent i performs job j. The Objective function (1) for the GAP seeks to maximize the total value gained from completing all jobs. The first constraint (2) ensures that a single agent completes each job, while the second constraint (3) ensures that the resources required by each agent to complete a job, a_{ij}, is no greater then each agent's resources, b_i. The article continues by discussing how the GAP can be solved by utilizing a Lagrangian Relaxation of the first constraint, which in turn allows the formulation to be converted into a series of Knapsack Problems [13].

Krumke et al. [14] discusses how the GAP is a generalized form of the Knapsack Problem and the Bin Packing Problem. Its model specifically examines Bin Packing, with the additional constraint of ensuring bins are packed to a minimum quantity if a bin is used at all. The objective function for its model seeks to maximize profit [14].

Phillips et al. [15] modifies the GAP to assign classrooms to courses. Its formulation incorporates an additional constraint to ensure that the assignment of a classroom to a course exist in a feasible pattern of classroom to courses. The objective function for its model seeks to maximize the quality of assignments [15].

Premkumar et al. [16] discusses the Locomotive Assignment Problem (LAP). This version of an assignment problem seeks to assign locomotives to sets of rail-cars to execute railway operations over a period of time. The objective function minimizes cost. This formulation also incorporates a penalty cost for using a non-preferred locomotive for a given railway operation as well as the incorporation of time and geographic space [16].

2.2.2 Workforce Allocation Problems

There are a number of formulations that can be placed under the umbrella of "Workforce Allocation". These problems generally seek to minimize the total number of workers employed or the total cost of employing a workforce. Workers must fill a given input demand, such as a job or a shift. Workers can also be represented by "agents" or "vehicles", or even a specific position such as "nurse" or a "postal worker" depending on the nature of the problem. As it relates to this research, U.S. Army units might represent workers, with missions representing different jobs that need to be completed. Various attributes can be applied to a worker, such as "full-time" or "part-time", even "skilled", "multi-skilled", or "non-skilled".

An example of a workforce allocation problem is discussed in Dantzig [17], which proposes a solution to the optimal number of toll booth operators necessary during a given work day, provided a given number of required operators during discrete time periods. This formulation is given below:

$$Min \quad \sum_{j=0}^{n} x_j, \tag{5}$$

$$S.T. \quad \sum_{j=0}^{n} a_{tj} x_j \geq b_t, \quad (t = 1, 2, ..., n + L), \tag{6}$$

$$x_j \geq 0, \qquad \forall\, j \, \in \, \mathcal{J}. \tag{7}$$

In the model above, Dantzig [17] seeks to minimize the total number of operators needed over a given day by summing the number of operators that begin work at time j, the value of which is represented by the decision variable x_j. The value n represents the potential work shift start time intervals. The right hand side of his constraint represents the required number of operators at time t, shown as b_t. The model uses a binary parameter a_{tj} to authorize the crediting of workers beginning at all shifts with their ability to meet the required demand at a specific shift t. This is determined by the approved shift length L [17].

Maier-Rothe et al. [18] discusses the allocation of nurses to shifts over an extended period of time. The work's focus is to provide a formulation that adequately covers all shifts while minimizing the employment cost of nurses for a given hospital. Interestingly, this model accounts for different qualification levels of nurses employed. Importantly, the problem also describes the desire to preferentially keep individual nurses in the same department over time instead of a rotation, "because of the competence and expertise they are able to develop in a particular unit" [18].

Baker et al. [19] describes the need of organizations to allocate workers to meet demand, specifically when the demand occurs cyclically. The article provides the allocation of baggage handlers as an example. Over a 24 hour period, baggage handlers work shifts of eight continuous hours with shifts beginning every four hours. The cyclical demand for baggage handlers is also provided in four hour increments. The following integer programming formulation is provided to minimize the total number of baggage handlers needed:

$$Min \quad \sum_{i=1}^{n} x_i, \tag{8}$$

$$S.T. \quad x_n + x_1 \geq b_1, \tag{9}$$

$$x_{i-1} + x_i \geq b_i, \quad (i = 2, ..., n), \tag{10}$$

$$x_i \in \{0, \mathbb{Z}^+\}, \qquad \forall\ i\ \in\ \mathcal{I}. \tag{11}$$

The decision variable x_i represents the number of baggage handlers that begin work at shift start time i. As stated previously, the objective function (8) minimizes the total number of baggage handlers that start during periods 1 through n. Constraints (9) and (10) represent the same thing, Constraint (9) simply referencing the previous 24 hour periods last starting shift as n. Importantly, this formulation involves the use of workers to meet demand occurring in the time period they begin working or one time period after to their shift begins, as shown in Constraint (10) [19].

Trivedi [20] also examines the scheduling and allocation of nurses for a given hospital with an integer programming model that accounts for different qualification levels of nurses employed, while also examining the use of both full-time and part-time nurses. This model also incorporates the use of overtime scheduling. The objective function for the model is established in the form of a goal programming objective function, where weights are applied, and can therefore be changed, to each aspect of the objective function. The model seeks to minimize the nurse staffing budget deficit, minimize nurse under-staffing at each skill level, and minimize the number of part-time nurses. Additionally, this model's objective functions direction can be reversed, choosing instead to maximize budget surplus, maximize nurse staffing at each skill level, and maximize the number of full-time nurses. Due to the application of weights to this goal programming model, a sensitivity analysis was then performed [20]. Additional goal programming models are discussed in a subsequent section of this research.

Bard et al. [21] discusses an integer programming model designed to minimize cost associated with employing both full-time and part-time employees at a given processing and distribution center. Its model examines the scheduling of postal workers over a period of 24 hours a day over a week. This model incorporates various United States Postal Service (USPS) union rules, specifically half hour lunch breaks, full time employees required two days off per week, and a limiting ratio of part-time employees to full time employees. Importantly, the model implements the series of constraints shown below:

$$w_f \geq x_{fd}, \quad (f = 1, ..., n^f), (d = 1, ..., 7), \tag{12}$$

$$v_p \geq y_{pd}, \quad (p = 1, ..., n^p), (d = 1, ..., 7). \tag{13}$$

The decision variable w_f and v_p represent the total number of full-time postal workers needed during shift f and total number of part-time postal workers needed during shift p respectively. The constraints shown above ensure that the value for each of these variables is at least as great as the maximum number of postal workers needed during that shift (f or p) during each day d, represented by the decision variables x_{fd} and y_{pd}. The decision variables x_{fd} and y_{pd} are both determined through additional constraints, not depicted here. However because of these two minimax constraints, the variables, w_f and v_p can then used in the model's objective function [21].

Wu, P. et al. [22] gives an approach to solving the rental fleet-sizing problem

(RFS), specifically as it applies to the truck rental industry. Its model utilizes a linear program formulation. To solve the model, it utilizes both Benders Decomposition as well as Lagrangian Relaxation to determine the optimal size and composition of a truck rental company's fleet, minimizing the total operation and maintenance cost for the company's fleet. A fleet is made of heterogeneous assets; trucks of varying sizes and attributes. This model incorporates preferred truck options to meet demand; if a preferred truck is not available, a different truck may be used and additional cost then applied [22].

Cuevas et al. [23] proposes a formulation designed to maximize worker utility while deducting penalty cost from that utility if a worker preforms an activity they are under or over qualified for given the worker's skill set. This modeling approach differs from those discussed previously because the actual allocation of workers by skill-type is a pre-determined parameter. The optimal solution for the model results in the placement of those workers into a specific schedule. A given organization utilizes this model after already accounting for employment costs, and instead seeks to maximize its effectiveness and efficiency [23].

Similarly, Shao et al. [24] utilizes Benders Decomposition to assist in solving a mixed integer programming problem. The work focuses on an airline scheduling problem broken into four stages: Schedule Planning, Fleet Assignment, Aircraft Routing, and Crew Pairing. The model, as outlined, incorporates the latter three phases into a single framework. Here, the first component of their model, Schedule Planning, seeks to maximize profit. Given a predetermined allocation of aircraft and aircrew, the resulting model seeks to maximize profit through the optimal scheduling of these assets against a given set of demands [24].

Xie et al. [25] discusses the scheduling of skilled workers through the use of a Workforce Scheduling and Routing Problem (WSRP). This problem utilizes time windows as well as skill requirements for required jobs. The workers in this problem have a set of skills that only allow specific workers to execute specific jobs. The presented model also incorporates task location. Additionally, this model penalizes unexecuted tasks by outsourcing those tasks to third party contractors. This problem once again seeks to minimize cost, the operational cost of sending workers to complete jobs and the additional penalty cost associated with workers executing jobs they are not intended for [25].

Wickert et al. [26] discusses building physician schedules for a hospital. The article refers to the Physician Rostering Problem (PRP). This work's PRP incorporates constraints that account for limitations of a physician's workload, the number of successive shifts a physician is assigned to, the number of physicians required by the hospital, and the physicians' qualifications, among many others. The objective function seeks to minimize the weighted value of violating the various constraints in place [26].

This research does not rely on a singular variant of the workforce allocation problem as the inspiration for the model formulation in Chapter III. However, many elements of various workforce allocation type problems have similarities both in structure and in the intent behind as the model presented in this research.

2.2.3 Set Cover Problems / Location Problems

Although determining the number and type of U.S. Army units that can fulfill specific missions may be more easily compared to a Workforce Allocation Problem with regard to terminology; another field of integer programming can also be examined: Set Cover Problems. The Set Cover Problem (SCP) or Minimum Set Cover Problem involves the placement of facilities with the geographic ability and capacity to service a given set of demands by location. The SCP involves minimizing the number of facilities placed to satisfy all demand. In terms of this research, U.S. Army units by MTOE, Component status, and aligned COCOM can be viewed as potential candidate facilities. Similarly, mission demand can be viewed as the demand locations.

Instead of a geographic servicing range, this research instead examines required mission demands over time. In terms of capacity, a unit can only fulfill one mission demand per time period but can meet mission demand over a series of time periods. Additionally, because this research examines the placement of units against mission demands over time, the formulation provided in Chapter III can be viewed as a minimizing the maximum (Minimax) number of units by type required for a series of Set Cover Problems over time.

Once again, this research does not rely solely on any of the models discussed below regarding Set Cover Problems. However, there are many parallels between the formulation presented in Chapter III and these problems.

Toregas et al. [27] proposes an integer programming formulation that seeks to minimize the total number of Emergency Service Facilities needed to satisfy demand at all locations. The presented formulation is shown below:

$$Min \quad \sum_{j=0}^{n} x_j, \tag{14}$$

$$S.T. \quad \sum_{j \in \mathcal{N}_i} x_j \geq 1, \quad (i = 1, 2, ..., n), \tag{15}$$

$$x_j \in \{0, 1\}, \quad (j = 1, 2, ..., n). \tag{16}$$

This model ensures that at least one facility j from the set $\mathcal{J}$ satisfies or "covers" the demand at location i. Here the constructed set $j \in \mathcal{N}_i$ accounts for the candidate facility locations j that can cover each demand location i [27].

Huag [28] presents a model for selecting facility locations in order to maximize a company's profits. This model does not adhere to the formulation of a Set Cover Problem, as the model does not require all demand to be met. This model is more closely aligned with a maximal covering location problem (MCLP). However, what is interesting and applicable to this research is the author's incorporation of time. Here the author defines the decision variable x_{ijk} as the number of products shipped from facility i to customer j in year k. While also defining the binary decision variable y_{it} as 1 if facility i is opened in year t and 0 otherwise. The formulation then includes the following linking constraint:

$$\sum_{j \in \mathcal{J}} x_{ijk} \leq \sum_{t=1}^{k} q_{itk} y_{it}, \quad \forall\, i \,\in\, \mathcal{I},\, \forall\, k \,\in\, \mathcal{K}. \tag{17}$$

Here the parameter q_{itk} represents the facility's i manufacturing capacity for year k if the facility opened in year t. This constraint ensures that the only product that can satisfy customer demand in a given year is produced in facilities that have the capability to produce product in that same given year [28].

Wu, L.Y. et al. [29] presents an extension to the Capacitated Facility Location Problem (CFLP). This extension and the associated CFLP are Set Cover Problems, where facilities must be placed in order to cover demand from all locations. Additionally, both the CFLP and the presented extension also involve a capacity on the number of demands each facility can cover. Here, the objective function seeks to minimize the cost associated with both the establishment of facilities and the added cost of a facility covering a specific location [29].

Zhang et al. [30] discusses uncertainty when dealing with the Set Cover Problem. The Uncertain Location Set Covering Problem (ULSCP) does not have known response times, and therefore the set of facilities that can cover specific demand locations is uncertain. The model still seeks to minimize the total number of facilities used [30].

2.2.4 Goal Programming

Goal Programming is a form of mathematical programming that incorporates weighted coefficient values into the objective function that correspond with the significance of how important meeting each of the model's constraints are. Often, these constraints cannot be met outright, in which case a slack variable or elastic variable would be necessary and subsequently penalized. Other times, a model will seek to reward the value of the slack or elastic variable when a model is not limited by a constraint. Typically organizations use goal programming when it has multiple objectives it wishes to accomplish. These objectives are given weights or priorities, which are then reflected in a goal programming model's objective function.

Price et al. [31] discusses the use of a goal programming model in terms of manpower planning, specifically with regards to officer accession, attrition, and promotion for the Canadian Armed Forces. The work provides the basic goal programming formulation in standard form:

$$Min \quad c^{+}r^{'} + c^{-}s', \tag{18}$$

$$S.T. \quad Ax^{'} + r^{'} - s' = b'. \tag{19}$$

Here $r^{'}$ and $s^{'}$ represent the slack variables necessary to achieve a constraint's equality condition, and c^{+} and c^{-} represent the weights associated with each goal. The article then presents a goal programming model that seeks to minimize the weighted sum of penalties incurred from failing to adhere to each of the constraints which are prioritized and used to assign weights with the appropriate magnitude. This model incorporates financial, manning, and promotion constraints, among others [31].

Charnes et al. [32] uses goal programming to address the U.S. Coast Guard's Marine Environmental Protection (MEP) program. It uses a similarly structured formulation:

$$Min \quad \sum_{i=1}^{m}(w_i^+ d_i^+ + w_i^- d_i^-), \tag{20}$$

$$S.T. \quad \sum_{i=1}^{m} a_{ri}x_i \leq \quad b_r, \qquad (r = 1, 2, ..., R), \tag{21}$$

$$x_i - d_i^+ + d_i^- = g_i, \qquad (i = 1, 2, ..., m), \tag{22}$$

$$x_i, d_i^+, d_i^- \geq 0, \qquad (i = 1, 2, ..., m). \tag{23}$$

The variables d_i^+ and d_i^- represent deviations from the stated goal g_i, w_i^+ and w_i^- represent the weighted values assigned to the deviation from each goal. The goals of the U.S Coast Guard MEP that Charnes et al. [32] address include pollution, specifically: preventing pollution, enforcing environmental laws, surveillance operations to detect pollution, and responding to pollution events. The formulation's decision variable x_i represents the number of U.S. Coast Guard patrols aligned against each of the MEP goals; while a_{ri} represents the manpower hours required for each patrol in terms of resource, r, availability, and b_r equates to the limitation on each resource. The article subsequently discusses a method for transforming its goal programming formulation into a goal interval programming formulation. Here, deviations are only penalized when a solution fails to meet the lower bounds or upper bounds of a stated goal [32].

Tingley et al. [33] uses a goal programming model to examine the U.S. Department of Agriculture's allocation of resources for the Women Infant and Children (WIC) program. The article outlines the legally defined priority of groups, whom receive resources and funds. Although the priority of each goal is definitive, and therefore a goal programming model's weighted values would have a corresponding order, the actual values of each weight are not defined. Here the article discusses the importance of sensitivity analysis and how the objective function may be more appropriately defined as a "subjective function" [33].

2.2.5 Multi-Objective Optimization

Multi-Objective Optimization involves mathematical programming with more then one objective function. The solution to a multi-objective model is the set of solutions to each of the individual objective functions. The direction of optimization does not need to be universal for each of the objective functions. For example, a bi-objective optimization model may involve maximizing profit as one objective, and minimizing time as the second objective. In comparison to goal programming, these objective functions do not need to be weighted. Instead, each set of optimal solutions for a multi-objective model exist along a Pareto or efficient frontier; where a single set of solutions for each objective function contains at least one solution that is not dominated by any other feasible solution to the same objective function within a different set.

Filho et al. [34] presents the generic bi-objective model formulation:

$$Min \quad z(x) = (z_1(x), z_2(x))^T, \tag{24}$$

$$S.T. \quad Ax = b, \tag{25}$$

$$x \in \mathbb{Z}^n_+. \tag{26}$$

The work then presents different scalarization methods used to define the Pareto

Frontier. Specifically the it addresses the Tchebycheff scalarization method (THC), a modified THC method, a modified Benson method, and the normal constraints method. The article then present a new scalarizaton method, the Multiple Reference Vectors Scalarization method (MRV). The article then examines each of these methods while solving the bi-objective one-dimensional cutting stock problem (BCSP) [34].

Wei et al. [35] proposes a multi-objective mixed integer linear programming (MOMILP) model to examine the optimization of aircraft arrival and departure scheduling using multiple runways. The model the article examines is a bi-objective function. The first objective function seeks to minimize total delay time for flights, while the second objective function seeks to minimize total runway idle time. Although not always the case for all multi-objective optimization models, the article discusses its results showing a directly inverse relationship between both objectives. If the number of runways remains the same and total idle time for runways is decreased then total delay time for flights is increased. Conversely if the number of runways is increased and total delay time for flights is decreased, total idle time for runways is increased [35].

Foroozandeh et al. [36] proposes a multi-objective model to optimize energy management in smart buildings. The work's presented formulation uses two objective functions; the first seeks to minimize total energy consumption costs while the second seeks to minimize the peak energy load. The article then discusses the use of the Pascoletti-Serafini scalarization approach to create a singular objective function that can then be used to determine all of the points along a Pareto Frontier [36].

Guerriero et al. [37] examines a multi-objective optimization model used to optimize a workforce schedule under COVID-19 conditions. The first objective function seeks to maximize employee work on-site, and the second objective function seeks to maximize employee remote work. The article then discusses how a feasible region may not always exists using this bi-objective model with hard constraints. The articles then presents a series of mono-objective models, with soft constraints that can be used in place of the original bi-objective model [37].

III. Methodology

Preamble

The problem this research examines concerns U.S. Army units and deployments in support of Geographic COCOMs. The Geographic COCOMs are: Africa Command (USAFRICOM), Central Command (USCENTCOM), European Command (USEUCOM), Indo-Pacific Command (USINDOPACOM), Northern Command (USNORTHCOM), Southern Command (USSOUTHCOM), and Space Command (USSPACECOM). For the sake of brevity, the 'US' is removed from each Geographic COCOM's abbreviation. Due to the unique mission set of SPACECOM, this COCOM is left out of this study and left for future work.

The model seeks to determine how many U.S. Army units, by MTOE type, Component status, and Geographic COCOM alignment are required to meet all missions given a forecast of mission demand. However, this model is useful for any organization seeking to align specific types of workers against a series of jobs or tasks and is therefore generalizable beyond the military context. The remainder of this chapter restates the goals of the research sponsor, key assumptions, and an integer programming formulation followed by a detailed discussion.

3.1 Problem Description

3.1.1 Sponsor Goals

The objective is to determine the minimal number of units by type (defined by MTOE type, Component status, and Geographic COCOM location alignment) required to meet all mission demand. The number of units required by each type is defined as a unit *mix*. Specifically, the objective is to minimize the cost associated with employing each unit and the cost incurred by using a unit not explicitly designed to complete a specific mission, defined as *non-preferred* unit.

Additionally, the model should be solve-able in a reasonable amount of time. Regardless of the method used, the model formulation should be easy to modify and allow quick changes to input parameters to investigate and compare the resulting unit *mixes*.

3.1.2 Initial Assumptions

There are a number of important assumptions to discuss before moving on to the model formulation. These assumptions involve the measurement of time, a unit's ability to both surge and extend, and the unit of measurement for a unit's employment costs. There are also assumptions regarding the feasibility of assigning a unit to meet mission demand given the demand's geographic location, and separately the feasibility of assigning units to meet mission demand given a unit's Warfighting Function.

First, this formulation uniformly operates with time measured in months. This assists the formulation for two reasons. It brings the model closer to the reality of scheduling unit deployments, and it also significantly reduces the computational complexity that would have resulted from a smaller unit of measurement for time periods.

Surges and extensions are discussed in much greater detail later in this chapter. However, in developing this model it is necessary to assume that an individual unit has both the ability to surge in order to meet a mission demand earlier then its scheduled deployment or to extend its deployment to meet additional mission demand.

Unit employment costs are also discussed later in this chapter. Employment costs are not measured monetarily; rather they are defined in terms of the MTOE assigned Soldiers for a specific unit. This is important because the actual monetary cost incurred by the DoD might not be proportional to the number of Soldiers in a unit. For example, smaller units might require expensive equipment which would increase their cost monetarily.

When assigning units to missions, there is a potential for a specific unit to be assigned a series of consecutive, non-identical missions. This is not inherently an issue, but it may result in unusual and potentially impracticable solutions for deployment schedules. For example, over three consecutive time periods, a unit could be assigned missions in the following locations: EUCOM, CENTCOM, EUCOM. The actual cost in terms of travel time and money are not accounted for in this formulation; however a series of events such as this would incur at least one penalty cost. This adds a layer of deterrence designed to limit unit mission assignments from continuously jumping between different COCOMs and is further discussed in Chapter V.

Finally, due to the nature of the given demand signals, not every unit can execute every mission demand. For example, a maneuver unit cannot accomplish a mission designed for a sustainment unit. This allows the overall problem to be formulated as a series of models solved separately for each unit MTOE type according to its WfF. This does not have an effect on the solution when considering the U.S. Army as a whole, and it is actually advantageous as it allows individual model formulations for each WfF instance which are smaller in size and solve more efficiently.

3.2 Model

3.2.1 Sets

A number of sets and indexed sets play an important role in this model. These sets are first presented mathematically and defined briefly. We then subsequently elaborate on and further explain each of these sets in greater detail.

Sets:

$\mathcal{U}$: The set of unit MTOE types, indexed by u, $u \in \{1, ..., U\}$.

$\mathcal{C}$: The set of unit Component statuses, indexed by c, $c \in \{1, ..., C\}$.

$\mathcal{L}$: The set of unit Geographic COCOM location alignments, indexed by l, $l \in \{1, ..., L\}$.

$\mathcal{A}$: The set of valid units by MTOE type, Component status, and Geographic COCOM location alignment, indexed by (u, c, l), $u, c, l \in \{(u, c, l) : u \in \mathcal{U}, c \in \mathcal{C}, l \in \mathcal{L}\}$

$\mathcal{T}$: The set of time periods, indexed by t, $t \in \{1, ..., T\}$.

$\mathcal{J}$: The set of mission demands, indexed by j, $j \in \{1, ..., J\}$.

$\mathcal{K}$: The set of possible unit extensions or surges; for example a unit's *1st* extension or a unit's *5th* surge, indexed by k, $k \in \{1, ..., K\}$.

Indexed Sets:

$\mathcal{J}_t$: The set of mission demands $j \in \mathcal{J}$ that must be met in time period $t \in \mathcal{T}$.

$\mathcal{A}_j$: The set of units $(u, c, l) \in \mathcal{A}$ that can meet a mission demand $j \in \mathcal{J}$.

$\mathcal{J}_{ucl}$: The set of mission demands $j \in \mathcal{J}$ that can be met by unit $(u, c, l) \in \mathcal{A}$.

The set of unit MTOE types, $\mathcal{U}$, refers to a unit's MTOE by WfF. Maneuver is an example of a WfF. For example, in this research, the Maneuver WfF unit types are: Maneuver-1, Maneuver-2, Maneuver-3, Maneuver-4, Maneuver-5. The Fires WfF Unit Types are: Fires-1, Fires-2, Fires-3. This naming convention is used for purely theoretical purposes. In reality, a Maneuver-1 unit type may represent a Light Infantry Battalion while a Maneuver-2 unit type may represent a Combined Arms Battalion. Similarly, a Fires-1 unit type may represent a M777A2 155mm Medium Towed Howitzer Battalion while a Fires-2 unit type may represent a M142 High Mobility Rocket Artillery System (HIMARS) Battalion. The unit types contained in each set $\mathcal{U}$ are not directly indicative of any actual U.S. Army MTOE unit types but are merely a notational representation for mathematical purposes. Naming conventions for the remaining WfF Unit Types follow suit.

The set of unit Component statuses, $\mathcal{C}$, can further distinguish units. A unit's Component status specifically indicates whether a unit is within the Active or Reserve U.S. Army Components.

The set of unit Geographic COCOM location alignments, $\mathcal{L}$, describes the physical location within which a specific unit is intended or designed to operate. Under ReARMM, U.S. Army units align with specific geographic locations, mission sets, or threats. The mission demand input data used to build this model specifically discusses unit alignments with Geographic COCOMs, for example EUCOM or CENTCOM.

The set valid units by MTOE type, Component status, and Geographic COCOM location alignment, $\mathcal{A}$, exists to define valid combinations of unit MTOE types, by Component status, and by location alignment. Together, (u, c, l) act as a 3-tuple index. For example a Maneuver-1 Type unit, that is within the Active Component, and ReARMM aligned with AFRICOM would be given the index of $(Maneuver - 1, \mathit{Active}, AFRICOM)$, with $(Maneuver - 1, \mathit{Active}, AFRICOM) \in \mathcal{A}$. However units identified with every possible combination of each index do not necessarily exist due to U.S. Army policy, these units can therefore never be employed. For example, if policy dictates that there are no Maneuver-2 Active status units, regardless of location alignment; then $(Maneuver - 2, \mathit{Active}, l \in \mathcal{L}) \notin \mathcal{A}$ for the Maneuver WfF formulation.

The set of time periods, $\mathcal{T}$, refers to the month(s) that a mission can occur within. This formulation assumes the set $\mathcal{T}$ contains a sequence of integer values $\{1, 2, \ldots, T\}$ representing each month of the planning horizon, with T the final month examined. It is also important to note that this formulation does not observe time in a cyclical manner even if missions are assigned or units can be completely scheduled in a cyclical manner. For example; if solving to meet mission demand over the course of 5 years, even if every year's schedule can be constructed cyclically, $\mathcal{T}$ must still be organized as $t \in \{1, 2, \ldots, 60\}$. This accounts for a unit's active and inactive length of time periods in a unit cycle, to be discussed further in this chapter. If scheduling were instead completed in a cyclical manner all units would essentially be granted an automatic reset at each time T, allowing a given unit to potentially deploy for a longer duration or deploy much earlier than its modernization and training inactive length of time periods should allow for.

The set of mission demands, $\mathcal{J}$, refer to the set of demands that must be met. Each mission demand, given as input data, has a defined unit by type and status that is determined to be best suited to meet that mission demand, with that demand also occurring within a Geographic COCOM area of operation. The unit by type, status, and Geographic COCOM alignment that is best suited to meet a given mission demand is said to be the preferred unit. However mission demand can still often be met by a non-preferred unit; this is discussed further when examining the indexed sets $\mathcal{A}_j$ and $\mathcal{J}_{ucl}$, and the parameter α_j^{ucl}.

The final independent set, the set of possible unit extensions or surges, $\mathcal{K}$, allows for individual units to meet additional mission demands. For this research, it is possible to extend a unit's deployment past its typical re-deployment window, or even surge a unit into a deployment earlier then typically scheduled, thus cutting into that unit's modernization and training periods. The set $\mathcal{K}$ therefore contains elements corresponding to the number of months a unit can surge (extend) and meet a mission demand before (after) the start (end) of its "routine" deployment time.

This formulation also utilizes three indexed sets. The set of mission demands $j \in \mathcal{J}$ that must be met in time period $t \in \mathcal{T}$, represented as $\mathcal{J}_t$, is a subset of missions in $\mathcal{J}$ that occur during a time period t. Not every mission occurs during every time period, with the set $\mathcal{J}_t$ containing only "active" missions during a selected time period. Similarly $\mathcal{A}_j$ represents the indexed set of units $(u, c, l) \in \mathcal{A}$ that *can* meet a specific mission demand $j \in \mathcal{J}$. Since a unit may not be able to meet a specific mission demand, the set $\mathcal{A}_j$ identifies the correct unit types (preferred or non-preferred) that could be assigned to meet a specific mission demand. The final indexed set, $\mathcal{J}_{ucl}$, is similar to the set $\mathcal{A}_j$ except it contains the set of mission demands $j \in \mathcal{J}$ that *can* be met by unit $(u, c, l) \in \mathcal{A}$, which corresponds to a subset of mission demands that a given unit is capable of meeting. These final two indexed sets can be thought of as reciprocals of one another; however, both are subsequently used independently to allow the model to function correctly and efficiently.

3.2.2 Define Parameters

The parameters used in this formulation can be broken into two distinct categories: parameters that account for incurred cost and parameters that limit the duration and frequency of unit deployments. These parameters, defined briefly below, are subsequently explained in the following paragraphs.

Parameters / Data:

α_j^{ucl}: Cost for unit (u, c, l) to meet mission demand j.

β^{ucl}: Cost for unit (u, c, l) to operate under extension conditions to meet a mission demand.

γ^{ucl}: Cost for unit (u, c, l) to operate under surge conditions to meet a mission demand.

Ω^{ucl}: Employment cost for unit (u, c, l).

λ^c: Cycle limit for unit with Component status c.

μ^c: Length of time periods a unit is active for (w/o extensions or surges) for unit with Component status c (deployment length).

g^c: Maximum number of surges for unit with Component status c.

e^c: Maximum number of extensions for unit with Component status c.

Of the cost parameters, the first, cost for unit (u, c, l) to meet mission demand j, α_j^{ucl}, serves to penalize a non-preferred unit if it meets a mission demand it can meet but was not designed to. The actual incurred cost is dependent on the nature of the mission demand and the description of the unit based on the given mission demand's preferred unit type, status, and geographic location, and the unit assigned to meet the given mission demand. The penalty cost is zero if the unit assigned to meet a mission demand is perfectly suited to meet that specific mission demand. Conversely, the penalty cost is then raised reflecting the unsuitability of the assigned unit across each of the three indexed factors. The unsuitability therefore depends on whether or not the assigned unit is the preferred unit type, the preferred Component status, and geographically aligned with the mission demand location. If there is a mismatch in any of these components then a cost is incurred. The cost of the unsuitability can also account for the degree of the unsuitability. Perhaps there is a range of unit types that can meet a given mission demand, and although a single unit type is most preferred, there can still be a variable range of preferences among the non-preferred unit types. Additionally, there may not be a range of preferences and instead simply a number of units that are equally non-preferred; this then involves the binary choice of a preferred unit or a non-preferred unit. This formulation uses both methodologies for incurring a non-preferred unit penalty cost. When examining a unit's suitability to meet a given mission demand, there is a binary preference choice with regard to unit Component status and Unit Geographic COCOM location alignment. However, the MTOE type of U.S. Army unit used is dependent on the degree of unsuitability.

The Example Mission Preference with Unit Suitability (Figure 1) illustrates how

Unit Type Meeting Mission Demand	Mission Demand Unit Type Preference				
	Maneuver - 1	Maneuver - 2	Maneuver - 3	Maneuver - 4	Maneuver - 5
Maneuver - 1	1	6	3	5	8
Maneuver - 2	10	1	6	3	7
Maneuver - 3	4	5	1	5	6
Maneuver - 4	9	2	3	1	7
Maneuver - 5	7	7	7	5	1

Figure 1: Example Mission Preference with Unit Suitability

unit types can hold various degrees of suitability for a given mission demand. For example, when examining mission demand intended for units identified as Maneuver-1; naturally, Maneuver-1 type units are most preferred for Maneuver-1 missions. In terms of unit types that are not fully suitable for a Maneuver-1 mission, the most suitable to least suitable unit types range from Maneuver-3, Maneuver-5, Maneuver-4, and finally Maneuver-2. Independent of unit Component status or Geographic COCOM location alignment, the associated penalty cost with using each of these four non-preferred units would need to account for these differences in degree of suitability for a Maneuver-1 mission.

Two additional cost parameters account for the cost of extending or surging a unit to meet a mission demand outside of that unit's active time window; the cost for unit (u, c, l) to operate under extension conditions to meet a mission demand, β^{ucl}, and the cost for unit (u, c, l) to operate under surge conditions to meet a mission demand, γ^{ucl} respectively. There are two important notes concerning these cost parameters. First, while the type, Component status, and Geographic COCOM location alignment of a unit chosen to surge or extend do factor into the cost to meet a mission demand, the specific mission demand for which that unit is extending or surging to meet does not matter. These penalty cost are independent. A penalty cost for the mission demand being met is completely captured by the previously defined α_j^{ucl} parameter. These extension and surge cost parameters are only meant to replicate the incurred cost of breaking the U.S. Army's outlined policy regarding deployment lengths. The second important note for these parameters is that it is a one-time incurred cost if a unit is extended and a one-time incurred cost if a unit is surged, regardless of the number of time periods a unit is extended or surged for, within a single active cycle. For example, when applying the cost of extending a unit, the cost is only applied to the first extension and not any additional extensions. Due to this, it is important to weight the parameter cost accordingly to justify any unit's repeated extensions or surges.

The final cost parameter, employment cost for unit (u, c, l), Ω^{ucl}, accounts for the cost of maintaining that unit within the U.S. Army. This can be thought as a unit's cost to pay Soldier salaries, purchase all unit equipment, and pay for all ongoing operational cost. This cost is incurred if an individual unit is ever utilized throughout the complete time horizon to meet any mission demand.

Unit Types	Active Component Personnel (PAX)	Reserve Component Personnel (PAX)
Maneuver – 1	525	525
Maneuver – 2	630	630
Maneuver – 3	525	525
Maneuver – 4	420	420
Maneuver – 5	525	525

Figure 2: Maneuver Unit Employment Cost in terms of Personnel

Figure 2 provides a sample of one way for accounting for the employment cost of an unit, Ω^{ucl}, specifically for Maneuver Type units of Active and Reserve Component statuses. Here the unit of measurement for cost is the number of personnel (PAX) in each unit by type, and Component status. The data captured in this table does not provide unit employment cost with regards to a unit's Geographic COCOM location alignment; however, the formulation presented in this research retains the ability to further differentiate cost based on this information, if necessary.

For the parameters that limit the duration and frequency of unit deployments, the first parameter is the cycle limit for unit with Component status c, λ^c. This parameter represents the total length of a cycle, consisting of both an active and inactive length of time periods. This parameter ensures a unit's deployment/modernization/training balance is maintained and incorporates different cycle limits dependent on a unit's Component status. For this research, both Active and Reserve component units can deploy for the same length of time periods without extensions or surges; that is, the lengths of active time periods are equal. However, the length of inactive time periods is longer for Reserve component units, and therefore the cycle for Reserve components units is also longer. This allows Active component units to deploy more frequently than Reserve component units.

The next parameter is the length of time periods a unit is active for (w/o extensions or surges) for unit with Component status c, defined as the deployment length and represented by μ^c. This parameter limits the length of active time periods when a unit can meet mission demands based on that unit's Component status. For example, this parameter has the potential to limit a Reserve Component unit's length of active time periods, causing the unit to initiate their inactive length of time periods earlier, regardless of that unit's given cycle limit.

The final two parameters are the maximum number of surges for unit with Component status c, g^c, and the maximum number of extensions for unit by Component status c, e^c. The first parameter limits the length of time periods a unit can surge to meet mission demand prior to its regularly scheduled active length of time periods. These surges activate a unit prior to the beginning of its cycle. The second parameter similarly limits the length of time a unit can extend to meet mission demand after its active length of time periods. Surges cut into a unit's inactive number of time periods during its last cycle, while extensions cut into a unit's initial inactive number of time periods during its current cycle. These parameters prevent the U.S. Army from over-utilizing a specific unit even if it may be cost effective to do so.

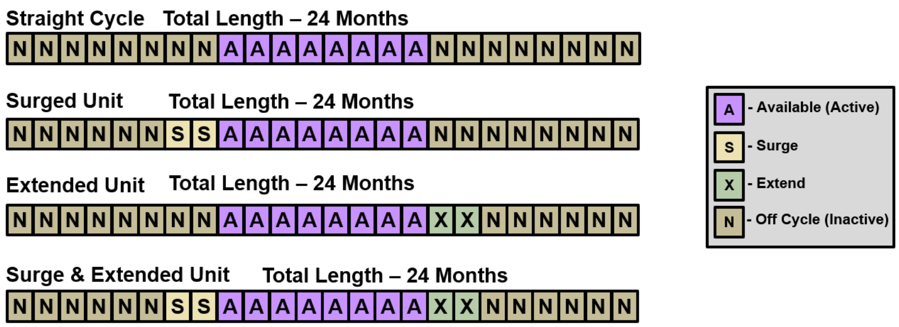


Figure 3: Example 24 Month Unit Cycles

Figure 3 illustrates an example cycle for each of the different surge/extend possibilities a given unit may be tasked with. Here, the cycle limit $\lambda^c = 24$, and the deployment duration $\mu^c = 8$. For this example, the maximum number of surges equals the maximum number of extensions and $g^c = e^c = 2$.

3.2.3 Define Decision Variables

<u>**Decision Variables:**</u>

$x_{tj}^{ucl} = 1$ if a unit designated (u, c, l) is assigned to meet mission demand j during time period t; $(u, c, l) \in \mathcal{A},\ t \in \mathcal{T},\ j \in \mathcal{J}_t$.

$w_{tjk}^{ucl} = 1$ if a unit designated (u, c, l) is assigned to meet mission demand j during time period t as the k^{th} extension; $(u, c, l) \in \mathcal{A},\ t \in \mathcal{T},\ j \in \mathcal{J}_t,\ k \in \mathcal{K}$.

$s_{tjk}^{ucl} = 1$ if a unit designated (u, c, l) is assigned to meet mission demand j during time period t as the k^{th} surge; $(u, c, l) \in \mathcal{A},\ t \in \mathcal{T},\ j \in \mathcal{J}_t,\ k \in \mathcal{K}$.

y_t^{ucl}: Non-Negative Integer; number of units designated (u, c, l) beginning active length of time periods at time period t; $(u, c, l) \in \mathcal{A},\ t \in \mathcal{T}$.

v^{ucl}: Non-Negative Integer; number of units (u, c, l) needed to meet all mission demand over the complete time horizon; $(u, c, l) \in \mathcal{A}$.

Of the five different sets of decision variables (DV), three are binary: x_{tj}^{ucl}, w_{tjk}^{ucl}, and s_{tjk}^{ucl}. Each of these variables serve a similar purpose. They describe the unit designated by type, Component status, and Geographic COCOM location alignment that meets a mission demand for every time period a given mission demand exist. The first binary set of DV, x_{tj}^{ucl}, represents units meeting mission demand during that unit's active length of time periods, its normal deployment. The second set, w_{tjk}^{ucl}, represents units meeting mission demand during that unit's authorized extension length of time periods within that unit's cycle. Finally, s_{tjk}^{ucl}, represents units meeting mission demand during that unit's authorized surge length of time periods prior to the beginning of that unit's cycle.

Each of the remaining two sets of decision variables, y_t^{ucl} and v^{ucl}, are non-negative integers. The first integer DV, y_t^{ucl}, represents the number of units by type, Component status, and Geographic COCOM location alignment that begin an active length of time periods (and therefore a cycle) at a given time period. The second integer DV, v^{ucl}, represents the complete number of units by type, Component status, and Geographic COCOM location alignment ever used to meet any mission demand over the complete time horizon. Each of these DV are further examined when reviewing the formulations constraints.

3.2.4 Model Formulation

Min

$$\sum_{(u,c,l)\in\mathcal{A}} \Omega^{ucl} \upsilon^{ucl} \tag{1}$$

$$+$$

$$\sum_{(u,c,l)\in\mathcal{A}} \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{J}} \alpha_j^{ucl}(x_{tj}^{ucl}) \tag{2}$$

$$+$$

$$\sum_{(u,c,l)\in\mathcal{A}} \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} \alpha_j^{ucl}(w_{tjk}^{ucl} + s_{tjk}^{ucl}) \tag{3}$$

$$+$$

$$\sum_{(u,c,l)\in\mathcal{A}} \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{J}} \beta^{ucl} w_{(t,j,1)}^{ucl} \tag{4}$$

$$+$$

$$\sum_{(u,c,l)\in\mathcal{A}} \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{J}} \gamma^{ucl} s_{(t,j,1)}^{ucl} \tag{5}$$

Subject To:

$$\sum_{(u,c,l)\in\mathcal{A}} \sum_{t=1}^{\mu^c} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} w_{tjk}^{ucl} = 0 \tag{6}$$

$$\sum_{(u,c,l)\in\mathcal{A}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} s_{T,j,k}^{ucl} = 0 \tag{7}$$

$$\sum_{(u,c,l)\in\mathcal{A}_j} \sum_{k\in\mathcal{K}} (x_{tj}^{ucl} + w_{tjk}^{ucl} + s_{tjk}^{ucl}) = 1 \ \forall \ j \ \in \ \mathcal{J}_t, \ \forall \ t \ \in \ \mathcal{T} \tag{8}$$

$$\sum_{t'=t}^{t+(\lambda^c-1)} y_{t'}^{ucl} \leq v^{ucl} \; \forall \; (u,c,l) \in \mathcal{A}, \; \forall \; t \; \in \; \mathcal{T} \tag{9}$$

$$\sum_{t'=t-\mu^c+1}^{t} y_{t'}^{ucl} \geq \sum_{j \in \{\mathcal{J}_t \cap \mathcal{J}_{ucl}\}} x_{tj}^{ucl} \; \forall \; (u,c,l) \in \mathcal{A}, \; \forall \; t \; \in \; \mathcal{T} \tag{10}$$

$$y_t^{ucl} \geq \sum_{j \in \{\mathcal{J}_t \cap \mathcal{J}_{ucl}\}} w_{(t+\mu^c),j,1}^{ucl} \; \forall \; (u,c,l) \in \mathcal{A}; \; t = 1, ..., (T - \mu^c - 1) \tag{11}$$

$$\sum_{j \in \{\mathcal{J}_t \cap \mathcal{J}_{ucl}\}} w_{tjk}^{ucl} \geq \sum_{j \in \{\mathcal{J}_t \cap \mathcal{J}_{ucl}\}} w_{t+1,j,k+1}^{ucl} \; \forall \; (u,c,l) \in \mathcal{A}, \; t = 1, ..., (T-1); \; k = 1, ..., e^c - 1 \tag{12}$$

$$y_t^{ucl} \geq \sum_{j \in \{\mathcal{J}_t \cap \mathcal{J}_{ucl}\}} s_{(t-1),j,1}^{ucl} \; \forall \; (u,c,l) \in \mathcal{A}; \; t = 2, ..., T \tag{13}$$

$$\sum_{j \in \{\mathcal{J}_t \cap \mathcal{J}_{ucl}\}} s_{tjk}^{ucl} \geq \sum_{j \in \{\mathcal{J}_t \cap \mathcal{J}_{ucl}\}} s_{t-1,j,k+1}^{ucl} \; \forall \; (u,c,l) \in \mathcal{A}; \; t = 2, ..., T; \; k = 1, ..., g^c - 1 \tag{14}$$

$$x_{tj}^{ucl} \; \in \{0,1\} \; \forall \; (u,c,l) \in \mathcal{A}, \; \forall \; t \; \in \; \mathcal{T}, \; \forall \; j \; \in \; \mathcal{J} \tag{15}$$

$$w_{tjk}^{ucl} \; \in \{0,1\} \; \forall \; (u,c,l) \in \mathcal{A}, \; \forall \; t \; \in \; \mathcal{T}, \; \forall \; j \; \in \; \mathcal{J}, \; \forall \; k \; \in \; \mathcal{K} \tag{16}$$

$$s_{tjk}^{ucl} \; \in \{0,1\} \; \forall \; (u,c,l) \in \mathcal{A}, \; \forall \; t \; \in \; \mathcal{T}, \; \forall \; j \; \in \; \mathcal{J}, \; \forall \; k \; \in \; \mathcal{K} \tag{17}$$

$$v^{ucl} \; \in \{0, \mathbb{Z}^+\} \;\; \forall \; (u,c,l) \in \mathcal{A} \tag{18}$$

$$y_t^{ucl} \; \in \{0, \mathbb{Z}^+\} \;\; \forall \; (u,c,l) \in \mathcal{A}, \; \forall \; t \; \in \; \mathcal{T} \tag{19}$$

3.2.5 Review of Objective Function

First, the Objective Function of this formulation seeks to minimize overall cost, separated into components **(1)**-**(5)** for the sake of clarity and discussion.

The first component of the Objective Function, **(1)**, accounts for the cost associated with employing each unit needed to meet any mission demand throughout the duration of the time horizon. Component **(2)** applies a penalty cost for each instance that a non-preferred unit meets a mission demand that the unit is not fully suited for, but does meet during its active length of time periods. Similarly, component **(3)** also applies a penalty cost for each instance that a non-preferred unit meets a mission demand, but this part of the Objective Function accounts for units operating under extension or surge conditions. Examining components **(2)** and **(3)**, it can be seen that the same penalty cost parameter is applied to both. Component **(4)** individually penalizes a unit for operating under extension conditions, meeting a mission demand after its authorized active length of time periods. This penalty is incurred by a unit only once per cycle if it extends, regardless of the length of time periods it is extended for. This is why only extension variables with an indexes of $k = 1$ are penalized. Finally, component **(5)** also individually penalizes a unit for operating under surge conditions, meeting a mission demand before its authorized active length of time periods begins. Similarly to component **(4)**, the penalty cost in component **(5)**, is only incurred by a unit once per cycle if that unit surges, regardless of the length of time periods it surges for.

3.2.6 Review of Constraints

This model satisfies a set of mission demands over a finite number of time periods; because time is continuous but this model is finite, there is a degree of ambiguity on either side of the time horizon. Constraints **(6)** and **(7)** partially account for this ambiguity. Constraint **(6)** eliminates all variables representing a unit's first extension before that unit could even meet a mission demand under extension conditions because the mission demand occurs within the time span of $1 \leq t \leq u^c$. During that time span every unit, specifically by Component status, is still operating within its active length of time periods and therefore cannot exercise an extension. Constraint **(7)** similarly eliminates all variables representing a unit's last surge if that surge occurs during the final time period of the time horizon. Although both of these constraints only eliminate variables associated with the first extension and the last surge, additional unit extensions or surges occurring within the beginning or end of the finite time horizon are also subsequently eliminated by a series of precedent constraints outlined below, Constraints **(12)** and **(14)**.

Constraint **(8)**, can be described as the "Meet Mission Demand" constraint. This constraint ensures that every mission demand that occurs in a time period is met and applies to every time period, regardless of whether that mission demand is met by a unit operating before (surge), during (active), or after (extension) its given active length of time periods.

Constraint **(9)** is the minimax constraint that provides the objective function the value of each v^{ucl} DV by unit type, Component status, and Geographic COCOM location alignment. This constraint accounts for every unit within a cycle regardless of whether or not that unit is within its active or inactive length of time periods for every time period. This constraint ensures that each v^{ucl} is the maximum number of units that are employed during any time period, while the part of the objective function, **(1)**, seeks to minimize this number.

Constraint **(10)** is a linking constraint. This constraint ensures that the total number of units assigned to meet a mission demand during a given time period either started their deployment in a prior time period and remain in the active state for the the current time period, or started their deployment in the current time period. That is, if a unit starts deployment in time period t then it can meet mission demand during month t, $t+1$, ..., $t+u^c-1$, but at month $t+u^c$ that unit is scheduled to start its length of inactive time periods. Units must be active in order to meet a mission demand outside of utilizing an extension or surge. Constraint **(11)** is also a linking constraint. This constraint is similar to **(10)**, but this constraint ensures that a unit meeting a mission demand as a first extension began its active length of time periods during the correct time period; a unit that started is deployment in time period t can now meet mission demand during time period $t+u^c$, as an initial extension. Constraint **(13)**, once again is also a linking constraint. This constraint ensures a unit meeting a mission demand as its last surge will begin its active length of time periods during the immediately following time period. Here a unit that is scheduled to start its deployment at time period t can actually meet mission demand during time period $t-1$, as its last or only surge.

As mentioned previously, Constraints **(12)** and **(14)** both act as precedent constraints. Constraint **(12)** ensures that a subsequent extension cannot occur if the previous extension did not occur. Constraint **(14)** ensures that a previous surge cannot occur if a subsequent surge will not occur.

Constraints **(15)-(19)** limit the allowable values for all of the variables used in this formulation.

3.3 Application to generic problem

As stated previously, this formulation seeks to be universally applicable and not strictly tied to U.S. Army units and unit deployments. Within the defined sets, parameters, and decision variables listed above, the word *unit* could be easily replaced by *element*, but it could just as easily be replaced by the words *worker*, *employee*, *company*, *agency*, etc.; or even *ship*, *aircraft*, *task force*, etc.

Upon further examination of the set $\mathcal{C}$, this set can be thought of as a prioritization ranking based on the utility of a unit, or universally an element using such distinguishers as *primary*, *secondary*, *tertiary*, and so forth. Within the context of a company, employee *status* could be: *Full-Time*, *Part-Time*, *Hourly* or even *Contract*. Additionally a cycle limit could potentially be different for a full-time employee when compared to a part-time employee.

The set $\mathcal{L}$ can also be generically applied to any problem set involving physical locations. Instead of focusing on Geographic COCOMs, a formulation could simply use: *Area A*, *Area B*, *Area C*, etc. Location could even be specifically defined by a number of different terms such as, *political boundaries*, *Zip Codes*, *Neighborhoods*, *Distribution Routes*, or *Security Patrols*.

When examining the sets $\mathcal{U}$, $\mathcal{C}$, $\mathcal{L}$, and $\mathcal{A}$, it can also be understood that these three factors focusing on type, Component status, and Geographic COCOM location alignment are simply the three designated factors that this research is focused on. A unit could be designated with three different factors, or a single factor, or infinite factors, and this formulation would be applicable.

Time periods $\mathcal{T}$, refer to discrete instances of a single unit measurement of time. *Day* #1, *Week* #4, *Month* #2, are all applicable values within the set ($\mathcal{T}$), if the given unit of a time period is measured in Days, Weeks, or Months respectively. The unit of measurement for time does not matter as long as it is consistent throughout the formulation.

The set of mission demands $\mathcal{J}$, could instead be viewed as a set of *Tasks*, *Jobs*, *Assignments*. Mission demand could even be altered to simply read generically: *demands*.

Universally, the set $\mathcal{K}$, allows for elements to work outside of an organizations' typical business rules, union regulations, or defined policies, to a finite degree. A prevalent example of this would be *Over-time Hours*. Employees can often volunteer or be asked to work earlier or later then their normal work hours. However, the number of these consecutive over-time hours is still typically limited, in a similar way that the U.S. Army limits total deployment length.

IV. Results and Analysis

Preamble

This chapter begins with a discussion on the information related to the various sets used in each instance of the model presented in Chapter III. This chapter then outlines the parameters used for all baseline instances. Each baseline instance corresponds to the specific mission demand input data for an associated WfF. Next, results are presented for each of the WfF instances supported by an investigation on unique aspects of the results. This chapter then concludes with a series of sensitivity analysis instances that demonstrate the flexibility of this model, key takeaways, and an alternative approach to this model.

4.1 Explanation of Sets

Of the sets previously discussed in this research, there are sets that are unique to each WfF instance, as well as sets that are common to all WfF instances. Each of the unique sets will be explained later in this chapter, while discussing individual WfF instances. The following list of sets are unique to a specific instance and therefore not universally applicable to every WfF:

$\mathcal{U}$ (unit MTOE types)

$\mathcal{A}$ (valid units by type, Component status, and location alignment)

$\mathcal{J}$ (mission demands)

$\mathcal{J}_t$ (mission demands in time period t)

$\mathcal{A}_j$ (valid units that can meet mission demand j)

$\mathcal{J}_{ucl}$ (mission demands that can be met by unit (u, c, l))

However, it is prudent to discuss the universally applicable sets now, as well as common parameters that are applied to each of the WfF test instances. The common sets applicable to every instance include:

$\mathcal{C}$: The set of unit Component statuses, indexed by c, $c \in \{Active,\ Reserve\}$.

$\mathcal{L}$: The set of unit Geographic COCOM location alignments, indexed by l, $l \in \{AFRICOM,\ CENTCOM,\ EUCOM,\ INDOPACOM,\ NORTHCOM,\ SOUTHCOM\}$

$\mathcal{T}$: The set of time periods, indexed by t, $t \in \{1, 2, ..., 60\}$.

$\mathcal{K}$: The set of possible unit extensions or surges, indexed by k, $k \in \{1, 2\}$.

Each of the WfF instances examined in this chapter are unrestricted in terms of employing units regardless of Geographic COCOM location alignment, $\mathcal{L}$. That is to say, an instance is never explicitly prevented from assigning a unit to meet a specific mission demand or even employing a specific unit based on that unit's Geographic COCOM location alignment. Regardless, the solution will never employ a unit that is aligned with a Geographic COCOM, in which the location is not represented within the given mission demand input data, as this would require an unnecessary penalty cost incursion, and the Geographic COCOM location alignment of a unit does not impact a unit's employment cost.

4.2 Parameter Construction

As discussed in the previous chapter, there are two distinct types of parameters; those that limit the duration and frequency of unit deployments and those that account for incurred cost (either employment or penalty cost). The temporal limiting parameters included in the model have been explicitly or implicitly derived from business rules and other key considerations associated with deploying U.S. Army units. The unit employment costs are notional to represent a variety of Army unit sizes, and the penalty cost parameters are derived to avoid under or over incentivizing surges, extensions, or the use of non-preferred units.

4.2.1 Temporal Limiting Parameters

It is important to note that each of the values used for the temporal limiting parameters are common to all baseline WfF instances. First examining the parameter for the cycle limit for a unit with status c, λ^c, the values of which are listed below:

$$\lambda^{Active} = 24$$

$$\lambda^{Reserve} = 32$$

These values translate to a total cycle, or the combined active and inactive periods of time, of 24 months for an *Active* status unit and 32 months for a *Reserve* status unit.

The next limiting parameter to be discussed is μ^c, the length of time periods for which a unit with Component status c is actively deployed, without considering extensions or surges. This parameter can be more colloquially described as a deployment length. The values for this parameter are listed below:

$$\mu^{Active} = 8$$

$$\mu^{Reserve} = 8$$

The use of identical values show that *Active* and *Reserve* status units can both deploy and remain in an active state for eight consecutive months. With these values selected for λ^c and μ^c, the ratio of active to inactive length of time for *Active* units is 1:2, and for *Reserve* units the ratio is 1:3. The current DoD policy limiting active to inactive time ratio or "deployment to dwell ratio" is provided in [38].

Finally the temporal limiting parameters g^c and e^c, the maximum number of surges or extensions, respectively, for a unit with Component status c, are examined. These values are provided below:

$$g^{Active} = e^{Active} = 2$$

$$g^{Reserve} = e^{Reserve} = 1$$

These values equate to an *Active* status unit being granted the ability to surge and meet mission demand up to two time periods prior to the normal start of its assigned active length of time, as well as extend and meet demand up to two time periods after its normal length of active time has ended. While a *Reserve* status unit is only granted the ability to surge for a single time period and likewise extend for a single time period.

Combining the ability of a unit to both surge and extend, an *Active* status unit can meet mission demand occurring over a duration of 12 consecutive time periods, at most, before it is forced to enter a period of rest and inactivity. Similarly, a *Reserve* status unit can meet mission demand occurring over a duration of at most 10 consecutive time periods.

Independent of the cost parameters, these temporal limiting parameters provide *Active* status units with a higher degree of utility than their *Reserve* status counterparts. An *Active* unit can meet more consecutive mission demands, while also requiring less time to conduct training and modernization. This allows *Active* units to deploy more frequently over the course of the planning horizon for each WfF instance.

4.2.2 Cost Parameters

Cost parameters can be distinguished as either a unit employment cost, a penalty cost for using a non-preferred unit to meet a mission demand, or a penalty cost for using an extension or surge to meet a mission demand. Since the penalty cost parameters are all derived from unit employment costs to prevent over or under incentivizing surges, extensions, or utilizing non-preferred units, the employment cost Ω^{ucl} parameter is examined first. Figure 4 contains the information used to construct this parameter.

Unit Types	Active Component Personnel (PAX)	Reserve Component Personnel (PAX)
Fires – 1	420	NA
Fires – 2	840	NA
Fires – 3	630	NA
Intelligence – 1	NA	210
Intelligence – 2	315	315
Maneuver – 1	525	525
Maneuver – 2	630	630
Maneuver – 3	525	525
Maneuver – 4	420	420
Maneuver – 5	525	525
Mission Command – 1	735	735
Mission Command – 2	315	315
Protection – 1	735	840
Protection – 2	735	840
Special Operations – 1	840	735
Special Operations – 2	630	525
Sustainment – 1	630	630
Sustainment – 2	735	735

Figure 4: Unit by Type and Component Status Personnel Count

The information conveyed in Figure 4 is then directly translated to produce the Ω^{ucl} employment cost parameters used for each of the WfF instances. The unit of measurement for this parameter correlates to “Personnel” or “Personnel Cost”. This information provides a cost in terms of each WfF defined by unit type and Component status. It is important to note that unit employment costs are independent of

Geographic COCOM location alignment (l), and therefore the location alignment of a unit does not factor into the employment cost. A brief example is provided for the Special Operations WfF below:

$$\Omega^{(Special\ Operations-1,Active,l)} = 840\ \forall\ l \in\ \mathcal{L}$$

$$\Omega^{(Special\ Operations-1,Reserve,l)} = 735\ \forall\ l \in\ \mathcal{L}$$

$$\Omega^{(Special\ Operations-2,Active,l)} = 630\ \forall\ l \in\ \mathcal{L}$$

$$\Omega^{(Special\ Operations-2,Reserve,l)} = 525\ \forall\ l \in\ \mathcal{L}$$

It is also important to recognize that two of the WfF possess units without employment cost. Both the Fires and Intelligence WfF contain units with a value of "NA" for the unit employment cost. This indicates that these units do not exist for employment in their respective WfF instances, which is accounted for through the construction of set $\mathcal{A}$; these are not valid units and are discussed further when reviewing the WfF instances.

The next cost parameters concern the cost associated with extending or surging a unit, β^{ucl} and γ^{ucl}, respectively. The values for these parameters are derived from the employment cost of a unit, Ω^{ucl}. Starting with the cost to surge a unit, this cost is defined by:

$$\gamma^{ucl} = \frac{g^c}{\mu^c}(\Omega^{ucl}),\ \forall\ u\ \in\ \mathcal{U},\ c\ \in\ \mathcal{C},\ l\ \in \mathcal{L}. \tag{1}$$

Similarly, the calculation used to determine the cost to extend a unit is:

$$\beta^{ucl} = \frac{e^c}{\mu^c}(\Omega^{ucl}), \forall\ u\ \in\ \mathcal{U},\ c\ \in\ \mathcal{C},\ l\ \in \mathcal{L}. \tag{2}$$

Substituting the previously defined values for g^c, or s^c, with μ^c equates to $\gamma^{u,Active,l} = \beta^{u,Active,l} = \frac{1}{4}(\Omega^{u,Active,l})$ for all *Active* status units and $\gamma^{u,Reserve,l} = \beta^{u,Reserve,l} = \frac{1}{8}(\Omega^{u,Reserve,l})$ for all *Reserve* status units. The logic for this calculation is that if an *Active* status unit can conduct a routine deployment for eight consecutive months and that same unit can also surge earlier for two months to meet a mission demand, then the cost of surging should be equivalent to employing two-eighths of an additional, yet identical, unit to meet the demand. A similar argument applies to the cost of extending a unit to meet a mission demand after the normal active time period. Since *Reserve* status units can only surge (or extend) for a single month, the logic follows that the formulation should view a *Reserve* unit surging (or extending) to be proportionally equivalent to separately employing one-eighth of a different, yet identical, unit.

There are a number of alternatives in defining values for the cost of surging or extending a unit. However, the relationship defined above is intuitive for the WfF baseline instances, as any deviation from the calculations used to construct the surge and extend cost would artificially encourage or discourage the use of unit surges or extensions. If the cost to extend or surge the original unit is less than the equivalent proportional cost of employing another identical unit from meeting the mission demand(s) in question without extensions or surges then the model will extend or surge units more frequently. Likewise if the proportional cost is greater than employing another identical unit, then the solution will simply employ more units. While the baseline WfF instances consider a proportional cost, it is possible to use non-proportional extension or surge penalty costs if the user values these actions differently, the impact of changing the surge/extensions cost is investigated in the sensitivity analysis section later in this chapter.

The final cost parameter is α_j^{ucl}, the cost for unit (u, c, l) to meet mission demand j. This cost is applied to penalize the use of non-preferred units meeting a mission demand, while accounting for degree of suitability. This cost must account for the suitability of a unit by MTOE type, Component status, and Geographic COCOM location alignment when penalizing a unit that is meeting mission demand. The construction of each α_j^{ucl} for a given WfF instance can be completed in a series of steps.

Every mission demand, j, has a preferred unit by type, Component status, and occurs within a specific Geographic COCOM. Let u' represent the most preferred unit MTOE type for a given mission demand, let c' represent the most preferred unit Component status for a given mission demand, and let l' represent the Geographic COCOM theater of operation that a given mission occurs within. The non-preferred unit penalty cost can now be calculated as follows.

Step 1: Weight Suitability Factors

A unit's suitability for a given mission is dependent on three different factors; the suitability of unit type u meeting mission demand with preferred unit type u', the suitability of unit Component status c meeting mission demand with preferred unit Component status c', and the suitability of unit with a Geographic COCOM location alignment l meeting mission demand occurring within COCOM l'. The discussion that follows outlines how each of these factors weight into the overall penalty cost when using a non-preferred unit to meet a given mission demand.

A normalized factor of $\frac{5}{12} \approx 0.417$, is selected as the weight associated with unit type, while $\frac{3}{12} = 0.25$ and $\frac{4}{12} \approx 0.333$, are selected as the weights associated with unit Component status and Geographic COCOM location alignment, respectively.

These weights represent a hierarchy of unit preference for all mission demands, with decreasing levels of preference by type, location alignment, and Component status. The relative magnitude in these weights is also small enough to prevent any one factor from significantly dominating the other factors.

Step 2: Evaluate Suitability by Unit Type

Suitability by unit type is non-binary, and instead scaled and defined in terms of degree of suitability. A cross-functional chart describing the degree of preference for each unit is shown in Figure 5.

	Mission Demand Unit Type Preference (u')																	
Unit Type Meeting Mission Demand (u)	Fires - 1	Fires - 2	Fires - 3	Intelligence - 1	Intelligence - 2	Maneuver - 1	Maneuver - 2	Maneuver - 3	Maneuver - 4	Maneuver - 5	Mission Command - 1	Mission Command - 2	Protection - 1	Protection - 2	Special Operations - 1	Special Operations - 2	Sustainment - 1	Sustainment - 2
Fires - 1	1	2	2															
Fires - 2	2	1	2															
Fires - 3	5	5	1															
Intelligence - 1				1	2													
Intelligence - 2				2	1													
Maneuver - 1						1	6	3	5	8								
Maneuver - 2						10	1	6	3	7								
Maneuver - 3						4	5	1	5	6								
Maneuver - 4						9	2	3	1	7								
Maneuver - 5						7	7	7	5	1								
Mission Command - 1											1	6						
Mission Command - 2											4	1						
Protection - 1													1	4				
Protection - 2													2	1				
Special Operations - 1															1	2		
Special Operations - 2															3	1		
Sustainment - 1																	1	5
Sustainment - 2																	9	1

Figure 5: Unit by Type Degree of Suitability (Suitability Scores)

A unit u that is perfectly suitable to meet a specific mission demand most preferring unit type u' receives a non-normalized score of "1". This occurs when $u = u'$. The non-normalized suitability scores then ranges from 1-10, with a score of "10" indicating that unit type u is least suitable to meet a mission demand preferring unit u'. For example, this occurs when a $Maneuver - 2$ unit meets a demand that prefers a $Maneuver-1$ type unit. These unit type suitability scores can then be normalized as shown in Figure 6.

Original Suitability Score	Normalized Suitability Score
1	0
2	0.111111111
3	0.222222222
4	0.333333333
5	0.444444444
6	0.555555556
7	0.666666667
8	0.777777778
9	0.888888889
10	1

Figure 6: Normalized Unit Type Suitability Scores

Now, let $F^{u}_{u'}$ represent the normalized suitability score for a unit type u meeting a mission demand most preferring unit type u'. For example, if a $Fires-1$ type unit meets mission demand intended for a $Fires-2$ type unit then: $F^{Fires-1}_{Fires-2} = 0.111111$.

Step 3: Evaluate Suitability by Unit Component status

Suitability by unit Component status is binary due to the fact that unit statuses are limited to *Active* or *Reserve*. The calculation for this factor is then very straightforward, $F^{c}_{c'} = 0$ if $c = c'$, 1 otherwise.

Step 4: Evaluate Suitability by Unit Geographic COCOM Location Alignment

Suitability by unit Geographic COCOM Location Alignment is also binary. The preferred unit's Geographic COCOM location alignment perfectly matches the location of the mission demand, while units with other location alignments are equally non-preferred. Similar to the unit Component status, the calculation of this factor is very straightforward where $F^{l}_{l'} = 0$ if $l = l'$, 1 otherwise.

Step 5: Create Comprehensive Suitability Factor

Each of the three suitability factors and their associated weights can then be combined to create a comprehensive suitability factor, $F^{ucl}_{u'c'l'}$.

$$F^{ucl}_{u'c'l'} = (\frac{5}{12})F^{u}_{u'} + (\frac{3}{12})F^{c}_{c'} + (\frac{4}{12})F^{l}_{l'}, \forall\ u, u'\ \in\ \mathcal{U},\ \forall\ c, c'\ \in\ \mathcal{C},\ \forall\ l, l'\ \in\ \mathcal{L}. \quad (3)$$

Step 6: Apply Comprehensive Suitability Factor to Mission Demand

At this point, $F^{ucl}_{u'c'l'}$ is simply a value between 0 and 1, with 0 representing a factor applied to the most suitable and most preferred unit and 1 representing a factor applied to the least suitable and least preferred unit for a given mission demand. To apply this factor and create each α^{ucl}_{j}, the following calculation is completed,

$$\alpha^{ucl}_{j} = \frac{1}{\mu^{c'}}(\Omega^{u'c'l'})(F^{ucl}_{u'c'l'}), \forall\ u\ \in\ \mathcal{U},\ \forall\ c\ \in\ \mathcal{C},\ \forall\ l\ \in\ \mathcal{C},\ \forall\ j\ \in\ \mathcal{J}. \quad (4)$$

The value of $\frac{1}{\mu^{c'}}(\Omega^{u'c'l'})$ is used to represent the proportional cost incurred for not using the most preferred unit over a single time period. This is due to the fact that the α^{ucl}_{j} penalty cost is incurred for every time period a non-preferred unit meets a mission demand instead of a single one-time cost.

4.3 Computational Notes

All instances discussed are constructed using Python, specifically the Pyomo package, and subsequently solved using Gurobi v9.5.2. The device used to implement this software was a HP Laptop 15-dy2xxx, with a 11^{th} Gen Intel® Core^{TM} i7-1165G7 2.80 GHz processor, and 8.00 GB of RAM.

4.4 Baseline WfF Instances

Seven separate baseline instances are examined, with each baseline instance corresponding to a unique WfF. Each WfF instance is introduced first by explaining the unique aspects of the input data that corresponds to a WfF. Any unique or interesting aspects of a WfF instance's input data are briefing discussed as well. The results of each instance are then presented, with a discussion and analysis on the resulting solution. Often the discussion of an instance's input data and results are compared and contrasted with other WfFs' input data and/or results.

There are two key insights that are repeatedly discussed throughout this section and chapter. The first insight is that the solution and resulting units employed in each WfF instance are heavily influenced by the structure (e.g., timing, duration, number, etc.,) of mission demands, regardless of the penalty costs that are incurred. The duration and frequency of indexed mission demands both play a large role in shaping the solution, in terms of how many units must be employed, when units must initiate a deployment, and how many unique mission demands a unit can meet while deployed.

The second key insight is that the penalty costs influence the number and type of units employed, regardless of the mission demand requirements. For some of the WfF instances, the difference in unit employment cost between one unit and another can be notably different. When this information is then combined with the individual unit suitability scores while meeting mission demands for units that are not preferred, a noticeable event occurs: some units will never be employed. Given the unit employment cost and the incurred penalty costs, some non-preferred units are always more cost effective to be employed and utilized to meet mission demand then the preferred units.

With the selected the mission demand input data and the parameters used in the construction of these WfF instances, recommendations and insights can then be provided to the primary stakeholder of this research, the U.S. Army. Each WfF instance results section concludes by providing these specific insights to the U.S. Army.

4.4.1 Explanation of Tables and Figures

The explanation of each WfF instance requires the use of a series of different figures and tables. While each of these figures and tables are unique to the WfF instance, they are all used to present similar information. Prior to the evaluating the results of each WfF instance, it is helpful to first provide a generic explanation of the various figures and tables used by theme.

WfF Mission Demand Figure. These figures provide a list of individually indexed mission demands, j, for each WfF. The mission demand preferred unit type, u' is provided for each mission demand. Additionally, the preferred unit Component status, c' is also provided. It is important to note that the preferred Component status c', may read 'Active Only' or 'Reserve Only'; in this situation *Active* or *Reserve* Component status units, respectively, are not simply preferred but are the only *suitable* units. These figures also provide the physical location, by Geographic COCOM, l', of each mission demand. Finally, these figures provide the initial month t for each indexed mission demand, along with the duration (consecutive months) in which the mission occurs, and the interval between the end of an iteration of an indexed mission demand and the beginning of the next iteration of that indexed mission demand. With the duration and frequency of mission demands provided in these tables, the set $\mathcal{J}_t$ can be constructed to provide the set of unique mission demands that must be met in each month.

WfF Consolidated Mission Demand Over Time Figure. These figures provide the total number of unique mission demands that occur during each month, $\mathcal{J}_t$, in the form of a bar chart. These figures provide an illustrative context to understanding how many mission demands occur each month, but more importantly they illustrate the difference in total mission demands that may occur from month to month. Within this context, the intervals between individual mission demands becomes clearer, as does the spacing over time among clusters of mission demands. The duration and frequency of mission demands plays a significant part in determining the optimal solution to an instance, discussed further in the results of each WfF instance.

WfF Instance Computational Results. These tables provides two key outputs from a WfF instance. First, these tables provide the Objective Function value, z, for an instance. The second key output is the optimality gap for an instance. Additionally, these tables provide the computational size of an instance's constraint matrix, in terms of columns and rows. Finally, these tables also provide the processing time, in seconds, that are required to build and solve the model.

WfF Instance Unit Employment Mix. These tables provide the information most useful in answering this research's problem statement. These tables provide the integer solution to the number of units, by MTOE type, u; by Component status, c; and by Geographic COCOM location alignment l; that the U.S. Army should employ to meet all mission demand over time, while minimizing the total unit employment costs and any penalty costs incurred.

WfF Instance Surges and Extensions. These tables provide the total number of unique mission demands that occur over the time horizon. Each of these tables then provides the numerical break-down of how these mission demands are met, in terms of units operating under surge or extension conditions. A mission demand met by a unit not surging or extending is counted as a Routine Deployment. A mission demand met by a surging unit is counted as a Surge, and a mission demand met by an extending unit is counted as an Extension. These tables provide insight into the proportionality of how often units surge or extend.

WfF Instance Use of Non-Preferred Units. The primary purpose of these tables are to provide the number of unique mission demands met using a *non-preferred* unit, further separated into non-preferred units meeting mission demand during routine deployments, surges, and extensions. The total percentage of unique mission demands met using non-preferred units is also provided.

WfF - AMPU. The Average Number of Mission Demands Met Per Unit Employed, or AMPU, is meant to provide a sense of total utilization per unit employed. For context, the absolute minimum AMPU that could ever be observed is 1. This would only occur if the resulting solution utilized each unit one time, to meet a single mission demand in a single time period. This could also theoretically occur if every mission demand occurs in the same time period. Conversely, the maximum AMPU that can be observed is 30. This value would only occur if the optimal unit employment and the occurrences of unique mission demands allow only *Active* Component status units to perpetually deploy and meet mission demand for 12 months then redeploy for 12 months, and repeat for the duration of the time horizon. This would involve two month surges prior to a routine deployment and two month extensions after a routine deployment. Although very specific input data could allow either of these extreme values for AMPU, this would be a unique and rare occurrence. This range of values provides insight for the individual WfF instance AMPUs, within the context of the time horizon, deployment length, and cycle lengths used in this research. An AMPU table is not presented for each of the WfF baseline instances, however, an AMPU value is still discussed in each of the WfF instance results. A consolidated table of AMPU values for each WfF is presented in the Consolidated

Results section of this chapter.

WfF - Unit Utilization. These tables help explain the utilization of units employed for each WfF instance, and also tie together the associated instance's AMPU with the Consolidated Mission Demand Over Time - Sufficiency Results figures. The calculations used to produce these tables only account for units that are deployed or could be deployed under surge or extension conditions. A unit that is deployed but not meeting a mission demand is 'under utilized', while a unit that is not deployed and therefore cannot meet a mission demand is not. These tables first provide the number of months that all deployed units are utilized to meet mission demand (100% Utilization), both in terms of routine deployment utilization (R) and a separate entry for the utilization for any unit that would be available if it were to surge or extend to meet mission demand, or is available through a routine deployment (RSE). These tables then provide the number of remaining months when at least one deployed unit is not utilized ($<$ 100% Utilization), and therefore available to meet a mission demand if needed. The Average Units Not Utilized per month is the average under utilization of deployed units per month. This number represents the average number of units that are actively deployed, but not meeting a mission demand. The Total Unit-Months Not Utilized is the sum of non-utilized units over the course of the time horizon. Finally the Unit-Months Not Utilized per Unit Employed equates to the non-utilized unit-months per unit the instance employs; this is the average number of months a deployed unit could be meeting mission demand but isn't over the course of the time horizon. These tables provide an indication of how many, and how often, mission demands could be added to an instance's baseline input data with little to no change to the instance's solution. The larger the non-utilization value, the greater the number of additional mission demands could be met by the same number of units.

WfF Consolidated Mission Demand Over Time - Sufficiency Results.

Each of these figures consist of three separate sub-charts. The uppermost sub-chart provides a graphical representation of the sufficiency analysis when only utilizing units that can execute routine deployments to meet mission demand or meet a mission demand under surge conditions. Similarly to the WfF Consolidated Mission Demand Over Time Figures, the blue vertical bars represent the consolidated mission demand that must be met during a given month. The additions to these sub-chart consist of black dots, representing the number of units available to meet mission demand in a given month, under routine deployment conditions. The red line represents the number of units available to meet mission demand if units are allowed to surge. The middle sub-chart is similar to the uppermost sub-chart, in this sub-chart the red line is replaced by a green line. The green line represents the total number of units available to meet mission demand if units are allowed to extend their deployments. The final sub-chart, the lower sub-chart represents a combination of the previous sub-charts. This sub-chart once again depicts the total number of unique mission demands that must be met each month and the number of units available to meet mission demands under routine deployment conditions. The orange line in this sub-chart represents the total number of units available to meet mission demand if surges and extensions are both authorized. It is important to note that this final sub-chart always presents a very specific property: while a WfF's number of units available, each month, without surging or extending (black dots) may not rise above the total number of unique mission demands occurring in a given month (blue bars), the number of mission demands will NEVER rise above the total number of units available to meet mission demand if surges and extensions are authorized (orange line). These charts show the significance of allowing surges and extensions, but most importantly, the final sub-chart graphically shows that a WfF instance's solution meets every mission demand.

WfF Unit Employment Over Time. Each WfF instance produces a collection of these figures. Each of these figures illustrate Constraint **9**, the minimax constraint that equates to the number of units by MTOE type, Component status, and Geographic COCOM location alignment, v^{ucl}, during each month, t. Each figure corresponds to a specific v^{ucl}. Within each figure the stacked bar charts contain a light blue bar, representing the number of units actively deployed. This does not directly infer that all of these units are actively meeting unique mission demands, but it does infer that they have started their active length of time in their deployment cycle and are capable of meeting mission demand. Above the light blue bars within the stacked bars is a gray bar. This gray bar represents the number of units that have redeployed and are currently in their inactive length of time periods within their unit cycle. Due to the nature of unit surges and extensions, a unit executing a surge or extension during their scheduled inactive period are still included within the gray bars. Additionally a final dark blue bar is also potentially included within each stacked bar. This dark blue bar represents the sum of units available to deploy again but have not, equating to a unit that has completed its inactive length of time periods but isn't needed yet and therefore hasn't yet started a new active deployment cycle. Although each of these figures is unique, they all follow a similar pattern: units initially deploy to meet mission demand, these units exhaust their active length of time periods (deployment length, μ^c) and return from a deployment to complete a period of inactivity (modernization and training), and at the conclusion of their inactive length of time periods (also the conclusion of the unit's cycle, λ^c) these unit can immediately deploy again or remain available for a future mission demand. Each figure results in a depiction of unit employment growth, where the number of specific units employed begins at zero, grows as mission demand peaks, and then remains at a plateau where additional units are not needed but previously deployed units can be reused. It also must be noted that in reality the U.S. Army would not simply begin to employ a unit during a specific month, graphically this would appear instead as a series of bars representing inactive deployable units always rising to the plateau level for the complete length of the time horizon. This was omitted in order to show the 'growth' towards the plateau level, as well as the precise calculation of a v^{ucl} in every month of the time horizon.

4.4.2 Fires WfF Input Data

There are three unique MTOE unit types in the Fires WfF instance, resulting in $\mathcal{U} = \{Fires-1, Fires-2, Fires-3\}$. The mission demand corresponding to the Fires WfF instances is presented in Figures 7-9. The Fires WfF mission demand is unique in that only *Active* Component status units may meet any mission demand. This is consistent with Figure 4, as there are no employment costs for Fires *Reserve* units. These units are not valid and cannot be employed. Figures 7-9 also indicate that the mission demands occur over a variety of locations, specifically $AFRICOM$, $EUCOM$, and $SOUTHCOM$. This input data allows for the construction of the sets $\mathcal{A}$, $\mathcal{A}_j$, $\mathcal{J}$, $\mathcal{J}_t$, and $\mathcal{J}_{ucl}$ as described in Section 3.2.1.

The Fires WfF instance contains 128 uniquely indexed mission demands. Figure 10 presents the total number of mission demands that must be met each month. This instance does not involve any continuous mission demands. Each of the mission demands last one time period but are then cyclically repeated at varying intervals throughout the set $\mathcal{T}$, as depicted in Figure 10. Additionally, there are 640 individual missions demands that must be met over the course of the time horizon, that is $\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}|\mathcal{J}_t| = 640$.

Mission Demand (j)	Preferred Unit Type (u)	Preferred Component (c)	Location (l)	Initial Month (t)	Duration (Months)	Interval Between Mission Demands (Months)
1	Fires - 1	Active Only	AFRICOM	4	1	12
2	Fires - 1	Active Only	AFRICOM	4	1	12
3	Fires - 1	Active Only	AFRICOM	4	1	12
4	Fires - 1	Active Only	AFRICOM	4	1	12
5	Fires - 1	Active Only	AFRICOM	4	1	12
6	Fires - 1	Active Only	AFRICOM	4	1	12
7	Fires - 1	Active Only	AFRICOM	4	1	12
8	Fires - 1	Active Only	AFRICOM	4	1	12
9	Fires - 1	Active Only	AFRICOM	5	1	12
10	Fires - 1	Active Only	AFRICOM	5	1	12
11	Fires - 1	Active Only	AFRICOM	5	1	12
12	Fires - 1	Active Only	AFRICOM	5	1	12
13	Fires - 1	Active Only	AFRICOM	5	1	12
14	Fires - 1	Active Only	AFRICOM	5	1	12
15	Fires - 1	Active Only	AFRICOM	5	1	12
16	Fires - 1	Active Only	AFRICOM	5	1	12
17	Fires - 1	Active Only	AFRICOM	5	1	12
18	Fires - 1	Active Only	AFRICOM	5	1	12
19	Fires - 1	Active Only	AFRICOM	5	1	12
20	Fires - 1	Active Only	AFRICOM	5	1	12
21	Fires - 1	Active Only	AFRICOM	6	1	12
22	Fires - 1	Active Only	AFRICOM	6	1	12
23	Fires - 1	Active Only	AFRICOM	6	1	12
24	Fires - 1	Active Only	AFRICOM	6	1	12
25	Fires - 1	Active Only	AFRICOM	6	1	12
26	Fires - 1	Active Only	AFRICOM	6	1	12
27	Fires - 1	Active Only	AFRICOM	6	1	12
28	Fires - 1	Active Only	AFRICOM	6	1	12
29	Fires - 1	Active Only	AFRICOM	6	1	12
30	Fires - 1	Active Only	AFRICOM	6	1	12
31	Fires - 1	Active Only	AFRICOM	6	1	12
32	Fires - 1	Active Only	AFRICOM	6	1	12
33	Fires - 1	Active Only	AFRICOM	6	1	12
34	Fires - 1	Active Only	AFRICOM	6	1	12
35	Fires - 1	Active Only	AFRICOM	6	1	12
36	Fires - 1	Active Only	AFRICOM	6	1	12
37	Fires - 1	Active Only	AFRICOM	6	1	12
38	Fires - 1	Active Only	AFRICOM	6	1	12
39	Fires - 1	Active Only	AFRICOM	6	1	12
40	Fires - 1	Active Only	AFRICOM	6	1	12
41	Fires - 1	Active Only	AFRICOM	6	1	12
42	Fires - 1	Active Only	AFRICOM	6	1	12
43	Fires - 1	Active Only	AFRICOM	7	1	12
44	Fires - 1	Active Only	AFRICOM	7	1	12
45	Fires - 1	Active Only	AFRICOM	7	1	12
46	Fires - 1	Active Only	AFRICOM	7	1	12
47	Fires - 1	Active Only	AFRICOM	7	1	12
48	Fires - 1	Active Only	AFRICOM	7	1	12
49	Fires - 1	Active Only	AFRICOM	7	1	12
50	Fires - 1	Active Only	AFRICOM	7	1	12

Figure 7: Fires WfF Mission Demand (1 of 3)

Mission Demand (j)	Preferred Unit Type (u)	Preferred Component (c)	Location (l)	Initial Month (t)	Duration (Months)	Interval Between Mission Demands (Months)
51	Fires - 1	Active Only	AFRICOM	7	1	12
52	Fires - 1	Active Only	AFRICOM	7	1	12
53	Fires - 1	Active Only	AFRICOM	4	1	12
54	Fires - 1	Active Only	AFRICOM	4	1	12
55	Fires - 1	Active Only	AFRICOM	4	1	12
56	Fires - 1	Active Only	AFRICOM	4	1	12
57	Fires - 1	Active Only	AFRICOM	4	1	12
58	Fires - 1	Active Only	AFRICOM	4	1	12
59	Fires - 1	Active Only	AFRICOM	4	1	12
60	Fires - 1	Active Only	AFRICOM	4	1	12
61	Fires - 1	Active Only	AFRICOM	5	1	12
62	Fires - 1	Active Only	AFRICOM	5	1	12
63	Fires - 1	Active Only	AFRICOM	5	1	12
64	Fires - 1	Active Only	AFRICOM	5	1	12
65	Fires - 1	Active Only	AFRICOM	5	1	12
66	Fires - 1	Active Only	AFRICOM	5	1	12
67	Fires - 1	Active Only	AFRICOM	5	1	12
68	Fires - 1	Active Only	AFRICOM	5	1	12
69	Fires - 1	Active Only	AFRICOM	5	1	12
70	Fires - 1	Active Only	AFRICOM	5	1	12
71	Fires - 1	Active Only	AFRICOM	5	1	12
72	Fires - 1	Active Only	AFRICOM	5	1	12
73	Fires - 1	Active Only	AFRICOM	6	1	12
74	Fires - 1	Active Only	AFRICOM	6	1	12
75	Fires - 1	Active Only	AFRICOM	6	1	12
76	Fires - 1	Active Only	AFRICOM	6	1	12
77	Fires - 1	Active Only	AFRICOM	6	1	12
78	Fires - 1	Active Only	AFRICOM	6	1	12
79	Fires - 1	Active Only	AFRICOM	6	1	12
80	Fires - 1	Active Only	AFRICOM	6	1	12
81	Fires - 1	Active Only	AFRICOM	6	1	12
82	Fires - 1	Active Only	AFRICOM	6	1	12
83	Fires - 1	Active Only	AFRICOM	6	1	12
84	Fires - 1	Active Only	AFRICOM	6	1	12
85	Fires - 1	Active Only	AFRICOM	6	1	12
86	Fires - 1	Active Only	AFRICOM	6	1	12
87	Fires - 1	Active Only	AFRICOM	6	1	12
88	Fires - 1	Active Only	AFRICOM	6	1	12
89	Fires - 1	Active Only	AFRICOM	6	1	12
90	Fires - 1	Active Only	AFRICOM	6	1	12
91	Fires - 1	Active Only	AFRICOM	6	1	12
92	Fires - 1	Active Only	AFRICOM	6	1	12
93	Fires - 1	Active Only	AFRICOM	6	1	12
94	Fires - 1	Active Only	AFRICOM	6	1	12
95	Fires - 1	Active Only	AFRICOM	7	1	12
96	Fires - 1	Active Only	AFRICOM	7	1	12
97	Fires - 1	Active Only	AFRICOM	7	1	12
98	Fires - 1	Active Only	AFRICOM	7	1	12
99	Fires - 1	Active Only	AFRICOM	7	1	12
100	Fires - 1	Active Only	AFRICOM	7	1	12

Figure 8: Fires WfF Mission Demand (2 of 3)

Mission Demand (*j*)	Preferred Unit Type (*u*)	Preferred Component (*c*)	Location (*l*)	Initial Month (*t*)	Duration (Months)	Interval Between Mission Demands (Months)
101	Fires - 1	Active Only	AFRICOM	7	1	12
102	Fires - 1	Active Only	AFRICOM	7	1	12
103	Fires - 1	Active Only	AFRICOM	7	1	12
104	Fires - 1	Active Only	AFRICOM	7	1	12
105	Fires - 2	Active Only	SOUTHCOM	8	1	11
106	Fires - 2	Active Only	SOUTHCOM	8	1	11
107	Fires - 2	Active Only	SOUTHCOM	8	1	11
108	Fires - 2	Active Only	SOUTHCOM	8	1	11
109	Fires - 2	Active Only	SOUTHCOM	9	1	11
110	Fires - 2	Active Only	SOUTHCOM	9	1	11
111	Fires - 2	Active Only	SOUTHCOM	9	1	11
112	Fires - 2	Active Only	SOUTHCOM	9	1	11
113	Fires - 2	Active Only	SOUTHCOM	8	1	11
114	Fires - 2	Active Only	SOUTHCOM	8	1	11
115	Fires - 2	Active Only	SOUTHCOM	8	1	11
116	Fires - 2	Active Only	SOUTHCOM	8	1	11
117	Fires - 2	Active Only	SOUTHCOM	9	1	11
118	Fires - 2	Active Only	SOUTHCOM	9	1	11
119	Fires - 2	Active Only	SOUTHCOM	9	1	11
120	Fires - 2	Active Only	SOUTHCOM	9	1	11
121	Fires - 3	Active Only	EUCOM	7	1	11
122	Fires - 3	Active Only	EUCOM	7	1	11
123	Fires - 3	Active Only	EUCOM	7	1	11
124	Fires - 3	Active Only	EUCOM	7	1	11
125	Fires - 3	Active Only	EUCOM	7	1	11
126	Fires - 3	Active Only	EUCOM	7	1	11
127	Fires - 3	Active Only	EUCOM	7	1	11
128	Fires - 3	Active Only	EUCOM	7	1	11

Figure 9: Fires WfF Mission Demand (3 of 3)

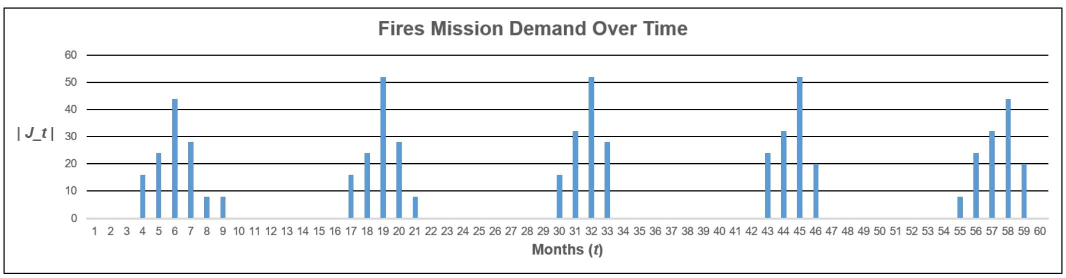


Figure 10: Fires WfF Consolidated Mission Demand Over Time

4.4.3 Fires WfF Results and Analysis

Analyzing the results of the Fires WfF instance in Table 1 shows that this instance solves to optimality in 1.41 seconds.

Table 1: Fires WfF Instance Computational Results

OBJ Value (z)	**# Cols (D.V.)**	**# Rows (Constraints)**	**Processing Time (s)**	**OPT Gap**
44,914.72	692,298	13,966	1.41	0.00%

The solution to the Fires WfF instance produces the unit mix outlined in Table 2. Interestingly despite 24 of the instance's mission demands preferring either $Fires-2$ or $Fires-3$ type units, none of these units are subsequently employed in the optimal solution. The number of mission demands preferring $Fires-1$ type units (104) completely dominates the number of mission demands for which $Fires-1$ units are non-preferred. The instance employs 16 units with a Geographic COCOM location alignment to $SOUTHCOM$, matching the number of mission demands located in $SOUTHCOM$; however it also employs 16 units with a location alignment to $EUCOM$ despite having only 8 mission demands occurring there. The instance results in the employment of 72 $(Fires-1, Active, AFRICOM)$ units for the 104 indexed mission demands preferring this unit. A further analysis of the exact mission demands that are met during each month reveals, among other insights, that some of the $(Fires-1, Active, AFRICOM)$ units are meeting mission demands where they are non-preferred; specifically non-preferred by unit type and Geographic COCOM location alignment. Similarly all $(Fires-1, Active, SOUTHCOM)$ and $(Fires-1, Active, EUCOM)$ units are meeting mission demands where they are non-preferred by unit type, while some of these units are also meeting mission demand where they are non-preferred by Geographic COCOM location alignment. The fact that no $Fires-2$ or $Fires-3$ units are employed highlights the significance to the solution in terms of the weighting of the penalty cost for using of non-preferred units. In total 25% of unique mission demands are met using non-preferred units, as illustrated in Table 3.

Table 2: Fires WfF Instance Unit Employment Mix

Unit Type Employed	**# Units Employed (v^{ucl})**
$(Fires-1, Active, AFRICOM)$	72
$(Fires-1, Active, EUCOM)$	16
$(Fires-1, Active, SOUTHCOM)$	16

Table 3: Fires WfF Use of Non-Preferred Units

Total Mission Demands	640
Routine Deployments with Non-Preferred Unit	160
Surges with Non-Preferred Unit	0
Extensions with Non-Preferred Unit	0
% of Mission Demands with Non-Preferred Units	25.00%

Table 4 also shows that no unit ever meets a mission demand under surge or extension conditions. The average number of mission demands met over the course of the 60 month time horizon by each unit employed is approximately 6.15 mission demands per unit employed (AMPU). This number, and the lack of extensions or surges, can be primarily attributed to two different factors. First, there are no individual mission demands that occur continuously over several consecutive time periods. Second, despite the lack of continuous mission demands, each of the intervals between unique mission demand occurrences have 11 or 12 month gaps, with indexed mission demands first occurring between months 4 and 9. This results in a clustering of unique mission demand occurrences throughout the time horizon, ultimately leading to units only meeting four to five mission demands during each deployment cycle instead of the maximum eight mission demands, during a routine deployment. This can be seen in both Figures 10 and 11.

Table 4: Fires WfF Instance Surges and Extensions

Total Mission Demands	640
Routine Deployments	640
Surges	0
Extensions	0

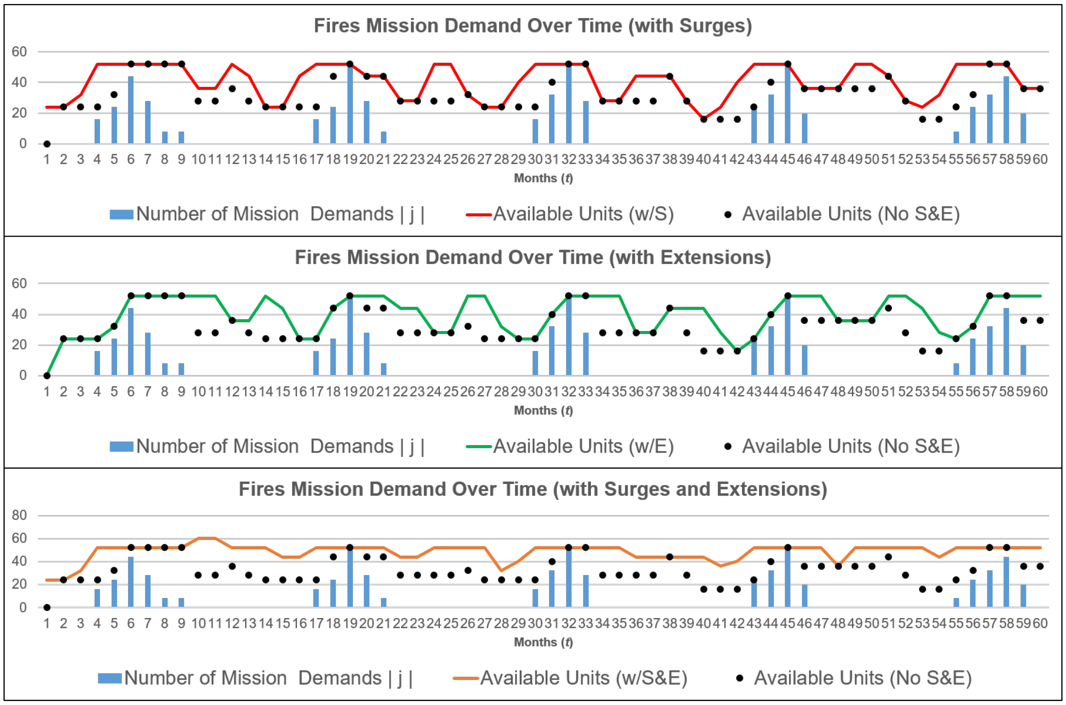


Figure 11: Fires WfF Consolidated Mission Demand Over Time - Sufficiency Results

Figure 11 also helps show why the solution never requires a unit to surge or extend. Here, the number of mission demands during any month, represented as a black dot, never surpasses the number of units available without surging or extending, the value of which is represented by the orange line.

The Fires WfF instance results in a high degree of unit under-utilization as depicted in Table 5. When only examining units on routine deployments, there are 55 out of 60 months where at least one unit is deployed but not used to meet a mission demand. Even more significant is that the average number of units under-utilized each month is 22.00 units. These values can be attributed to the clustering of unique mission demands throughout the time horizon, which can also be seen in the differences between the available units (black dots) and number of mission demands (blue bar) in Figure 11.

Table 5: Fires WfF - Unit Utilization

Metric	**R**	**RSE**
Months with 100% Utilization	5	3
Months with $<$ 100% Utilization	55	57
Average Units Not Utilized per Month	22.00	37.73
Total Unit-Months Not Utilized	1320	2264
Unit-Months Not Utilized per Unit Employed	12.70	21.77

Figures 12, 13, and 14 all seek to illustrate the employment of units over time. Each of these stacked bar charts attempt to visually illustrate Constraint **9** of the model. Initially the instance requires units to meet mission demand, and these units are then actively deployed. Eventually, when the unit reaches its deployment length it must then redeploy to conduct modernization and training. Upon redeployment, these units remain inactive and unavailable until the duration of its cycle limit. Then, the unit can either immediately deploy again to meet additional mission demand, or remain inactive but available for a deployment. Each of these charts, as well as the employment over time charts discussed in the other WfF instances plateau at the value of v^{ucl} the figure represents.

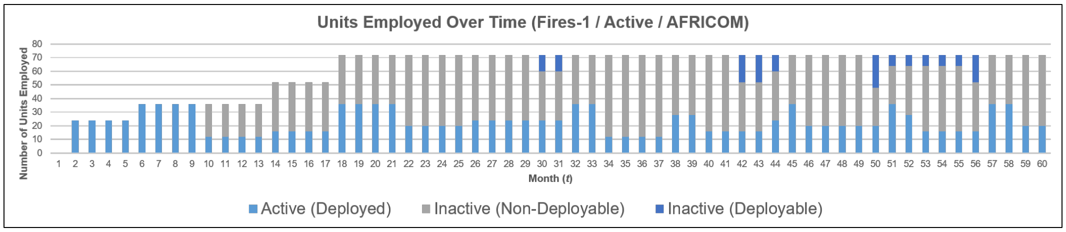


Figure 12: Fires WfF Unit Employment Over Time - Part 1 of 3

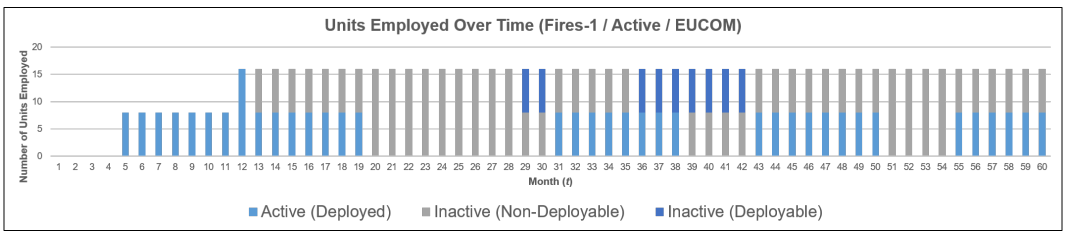


Figure 13: Fires WfF Unit Employment Over Time - Part 2 of 3

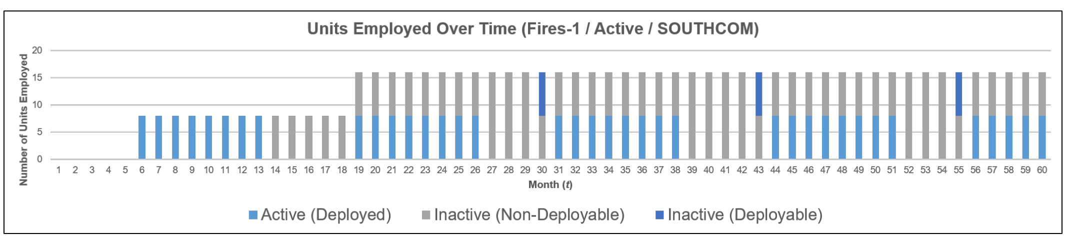


Figure 14: Fires WfF Unit Employment Over Time - Part 3 of 3

From the perspective of the U.S. Army, the results of this instance, given the parameters used, demonstrate that $Fires-1$ type units are a much more cost effective unit to employ rather then $Fires-2$ or $Fires-3$ type units. Neither of the latter two units are employed in this instance. This can be directly attributed to the difference in cost associated with employing a $Fires-1$ unit instead of a $Fires-2$ or $Fires-3$

unit, as well as the suitability score given to a $Fires - 1$ unit meeting a mission demand preferring a $Fires - 2$ or $Fires - 3$ unit. As shown in Figure 4, the cost of employing a single $Fires - 2$ *Active* unit is a 100% increase over the cost of employing a single $Fires - 1$ *Active* unit, while the cost of employing a single $Fires - 3$ *Active* unit is a 50% increase over the cost of employing a single $Fires - 1$ *Active* unit. Furthermore, a $Fires - 1$ type unit receives a non-normalized suitability score of 2 to meet a mission demand preferring a $Fires - 2$ or $Fires - 3$ type unit. From this insight, the incurred penalty cost for utilizing a $Fires - 1$ type unit when a $Fires - 2$ or $Fires - 3$ unit is preferred is not a relatively large cost. Additionally, and more perhaps more important, a suitability score of 2 signifies that a $Fires - 1$ unit is almost fully suited for meeting these $Fires - 2$ or $Fires - 3$ mission demand unit preferences. Although this instance utilizes non-preferred units to meet mission demand throughout the time horizon, these results are far from surprising.

4.4.4 Intelligence WfF Input Data

There are two unique MTOE unit types in the Intelligence WfF instance; $\mathcal{U} = \{Intellience-1, Intelligence-2\}$. $Intelligence-1$ *Active* units are not valid units; these units do not exist, as shown in Figure 4, and therefore cannot be employed to meet mission demand. As such, units with this type and Component status are omitted from the set $\mathcal{A}$. This instance is further constrained when evaluating the sets $\mathcal{A}_j$ and $\mathcal{J}_{ucl}$. First, *Active* Component status units, regardless of unit type may not meet any mission demand for $j \in \{21, ..., 42\}$, as seen in Figure 15. Additionally, this Intelligence WfF instance is unique in that every mission demand is located within the $NORTHCOM$ area of operations. Along with the construction of the $\mathcal{A}$, $\mathcal{A}_j$, and $\mathcal{J}_{ucl}$ sets, Figure 15 also provides the necessary information to construct the sets $\mathcal{J}$ and $\mathcal{J}_t$, as described in Section 3.2.1.

The Intelligence WfF instance involves 42 indexed mission demands. Figure 16 presents the total number of mission demands that must be met each month. Unlike the Fires WfF instance, this instance contains a set of continuous mission demands over the length of the entire time period, where 20 out of the 42 indexed mission demands occur during every month of the time horizon. The remaining mission demands have a duration of one time period but are then cyclically repeated at uniform 12 month intervals throughout the time horizon, as depicted in Figure 16. Additionally, there are 1,310 individual missions demands that must be met over the course of the instance's time horizon, that is; $\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} |\mathcal{J}_t| = 1,310$.

Mission Demand (j)	Preferred Unit Type (u)	Preferred Component (c)	Location (l)	Initial Month (t)	Duration (Months)	Interval Between Mission Demands (Months)
1	Intelligence - 2	Reserve	NORTHCOM	1	Continuous	N/A
2	Intelligence - 2	Reserve	NORTHCOM	1	Continuous	N/A
3	Intelligence - 2	Reserve	NORTHCOM	1	Continuous	N/A
4	Intelligence - 2	Reserve	NORTHCOM	1	Continuous	N/A
5	Intelligence - 2	Reserve	NORTHCOM	1	Continuous	N/A
6	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
7	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
8	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
9	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
10	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
11	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
12	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
13	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
14	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
15	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
16	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
17	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
18	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
19	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
20	Intelligence - 2	Active	NORTHCOM	1	Continuous	N/A
21	Intelligence - 1	Reserve Only	NORTHCOM	1	1	12
22	Intelligence - 1	Reserve Only	NORTHCOM	1	1	12
23	Intelligence - 1	Reserve Only	NORTHCOM	1	1	12
24	Intelligence - 1	Reserve Only	NORTHCOM	1	1	12
25	Intelligence - 1	Reserve Only	NORTHCOM	2	1	12
26	Intelligence - 1	Reserve Only	NORTHCOM	2	1	12
27	Intelligence - 1	Reserve Only	NORTHCOM	2	1	12
28	Intelligence - 1	Reserve Only	NORTHCOM	2	1	12
29	Intelligence - 1	Reserve Only	NORTHCOM	3	1	12
30	Intelligence - 1	Reserve Only	NORTHCOM	3	1	12
31	Intelligence - 1	Reserve Only	NORTHCOM	3	1	12
32	Intelligence - 1	Reserve Only	NORTHCOM	1	1	12
33	Intelligence - 1	Reserve Only	NORTHCOM	1	1	12
34	Intelligence - 1	Reserve Only	NORTHCOM	1	1	12
35	Intelligence - 1	Reserve Only	NORTHCOM	1	1	12
36	Intelligence - 1	Reserve Only	NORTHCOM	2	1	12
37	Intelligence - 1	Reserve Only	NORTHCOM	2	1	12
38	Intelligence - 1	Reserve Only	NORTHCOM	2	1	12
39	Intelligence - 1	Reserve Only	NORTHCOM	2	1	12
40	Intelligence - 1	Reserve Only	NORTHCOM	3	1	12
41	Intelligence - 1	Reserve Only	NORTHCOM	3	1	12
42	Intelligence - 1	Reserve Only	NORTHCOM	3	1	12

Figure 15: Intelligence WfF Mission Demand

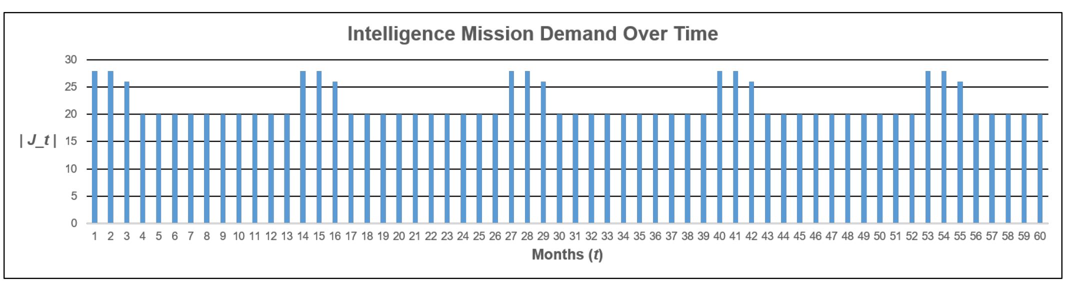


Figure 16: Intelligence WfF Consolidated Mission Demand Over Time

4.4.5 Intelligence WfF Results and Analysis

The Intelligence WfF instance solves to optimality in 2.85 seconds, as shown in Table 6.

Table 6: Intelligence WfF Instance Computational Results

OBJ Value (z)	**# Cols (D.V.)**	**# Rows (Constraints)**	**Processing Time (s)**	**OPT Gap**
22,418.23	227,898	8,814	2.08	0.00%

The solution to the Intelligence WfF instance produces the unit mix outlined in Table 7. The instances requires the employment of both $(Intelligence - 1, Reserve, NORTHCOM)$ and $(Intelligence - 2, Active, NORTHCOM)$ units, however not represented here are $(Intelligence - 2, Reserve, NORTHCOM)$ units that are preferred for five of the indexed mission demands. These mission demands are primarily met using the $(Intelligence - 1, Reserve, NORTHCOM)$ units. In total approximately 25.27% of unique mission demands are met using non-preferred units, as presented in Table 8.

Table 7: Intelligence WfF Instance Unit Employment Mix

Unit Type Employed	**# Units Employed (v^{ucl})**
$(Intelligence - 1, Reserve, NORTHCOM)$	34
$(Intelligence - 2, Active, NORTHCOM)$	45

Table 8: Intelligence WfF Use of Non-Preferred Units

Total Mission Demands	1,310
Routine Deployments with Non-Preferred Unit	331
Surges with Non-Preferred Unit	0
Extensions with Non-Preferred Unit	0
% of Mission Demands with Non-Preferred Units	25.27%

For this instance, the average number of mission demands met over the course of the 60 month time horizon by each unit employed is approximately 16.58 mission demands per unit employed (AMPU). This AMPU is the largest average number of mission demands met per unit employed by all WfF instances. This is attributed to the fact that the vast majority of unique mission demands occur continuously. Consecutively occurring mission demands allow a unit to meet a number of mission demands that more closely aligns with the unit's deployment length, fully utilizing a unit's potential for meeting mission demand. This can be seen when examining Figure 17. As shown, the number of units available without extending or surging is consistently at or just above the number of mission demands occurring during each month.

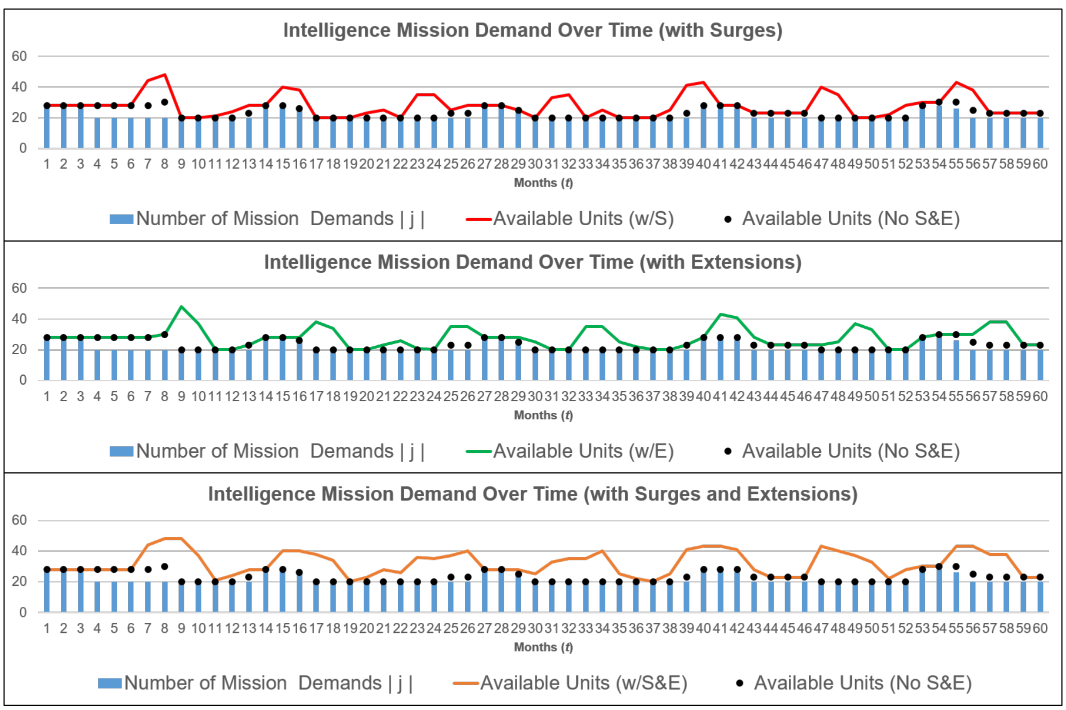


Figure 17: Intelligence WfF Consolidated Mission Demand Over Time - Sufficiency Results

Figure 17 also displays the lack of need for surges or extensions for this instance's time horizon. The exception is admittedly hard to see in this figure. For clarity, month 29 contains 26 mission demands, however there are only 25 units available to meet this demand without extending or surging; hence the need for the single unit extension as shown in Table 9.

Table 9: Intelligence WfF Instance Surges and Extensions

Total Mission Demands	1,310
Routine Deployments	1,309
Surges	0
Extensions	1

Coinciding with the high AMPU, this instance results in a high degree of unit utilization. As shown in Table 10, more than half of all months (38 of 60) fully utilize every unit on a routine deployment to meet mission demand. With an average of only 1.53 units on routine deployments not utilized each month. Continuous mission demands allow for greater unit utilization.

Table 10: Intelligence WfF - Unit Utilization

Metric	R	RSE
Months with 100% Utilization	38	7
Months with < 100% Utilization	22	53
Average Units Not Utilized per Month	1.53	10.22
Total Unit-Months Not Utilized	92	613
Unit-Months Not Utilized per Unit Employed	1.16	7.76

Figures 18 and 19 show the unit employment plateau level for each set of units employed. These plateaus represent the same value as those presented in Table 7.

Of the 331 unique mission demands met by a non-preferred unit, 300 of these mission demands correspond to the mission demands $j = (1, ..., 5)$. These particular mission demands prefer $(Intelligence - 2, Reserve, NORTHCOM)$ units;

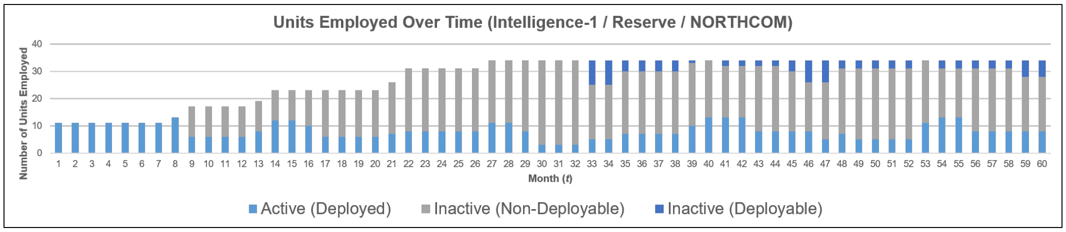


Figure 18: Intelligence WfF Unit Employment Over Time - Part 1 of 2

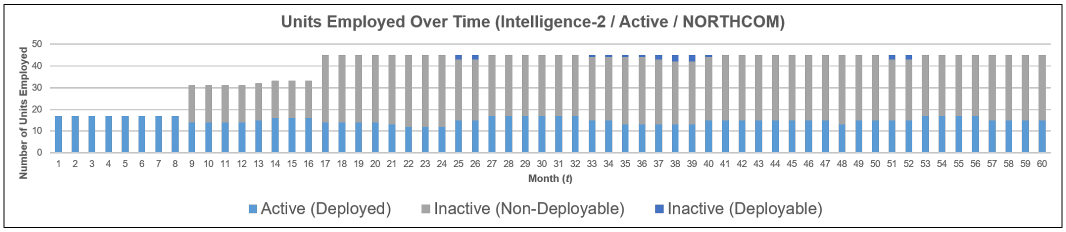


Figure 19: Intelligence WfF Unit Employment Over Time - Part 2 of 2

these units are not employed in this instance's solution. The vast majority (279 mission demands) of these unique mission demands are met by $(Intelligence - 1, Reserve, NORTHCOM)$ units. These units are the preferred units for the cyclical one month mission demands, $j = (21, ..., 42)$, that occur throughout the time horizon. Given this situation, these results inform the U.S. Army that it should allow the $(Intelligence - 1, Reserve, NORTHCOM)$ units to meet the mission demand preferring $(Intelligence - 2, Reserve, NORTHCOM)$ units, this allows $Intelligence - 1$ type units to not only meet the mission demands where they are preferred but also continue to be utilized during their deployments to meet additional continuously occurring mission demands. It is simply more cost effective to use $(Intelligence - 1, Reserve, NORTHCOM)$ units to meet mission demand intended for $(Intelligence - 2, Reserve, NORTHCOM)$ units than vice versa.

4.4.6 Maneuver WfF Input Data

The cardinality among the sets and combination of sets for the Maneuver WfF Instance is greater than any of the other baseline instance tested. There are five unique unit MTOE types that can be attributed to units created regardless of the units Component status or Geographic COCOM location alignment; $\mathcal{U} = \{Maneuver-1, Maneuver-2, Maneuver-3, Maneuver-4, Maneuver-5\}$. Every possible type of unit by type, Component status, and location alignment can be employed and utilized to meet at least one or more mission demands. However, four of the mission demands, $j = \{97, 98, 105, 106\}$, restrict suitability to only $Reserve$ status units. Outside of these four mission demands, any employed unit can meet a mission demand. This mission demand input data is shown in Figures 20-22. The location for each mission demand is also shown, with mission demands located within $NORTHCOM$, $EUCOM$, $SOUTHCOM$, $CENTCOM$, and $INDOPACOM$. The information provided in Figures 20-22 also allows for the construction of the sets $\mathcal{A}$, $\mathcal{A}_j$, $\mathcal{J}$, $\mathcal{J}_t$, and $\mathcal{J}_{ucl}$ as described in Section 3.2.1.

The cardinality of $\mathcal{J}$ for the Maneuver WfF instance is larger than each of the other WfF baseline instances, involving 148 uniquely indexed mission demands. Figure 23 presents the total number of mission demands that must be met each month. These mission demands occur both singularly, with mission demands lasting one month prior to cyclically repeating, as well as semi-continuously, with some mission demands lasting up to six months and then cyclically repeating. The intervals or gaps between mission demand occurrences is also not uniform. The interval between mission demand occurrences ranges from a single month to 12 months. The variability in frequency and duration of individual mission demands can be seen in Figure 23. This instance involves 1,460 individual missions demands that must be met over the course of our time horizon; that is, $\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}|\mathcal{J}_t| = 1,460$.

Mission Demand (j)	Preferred Unit Type (u)	Preferred Component (c)	Location (l)	Initial Month (t)	Duration (Months)	Interval Between Mission Demands (Months)
1	Maneuver - 3	Active	NORTHCOM	5	1	12
2	Maneuver - 3	Reserve	NORTHCOM	5	1	12
3	Maneuver - 3	Active	NORTHCOM	5	1	12
4	Maneuver - 3	Reserve	NORTHCOM	5	1	12
5	Maneuver - 3	Active	NORTHCOM	5	1	12
6	Maneuver - 3	Reserve	NORTHCOM	5	1	12
7	Maneuver - 3	Active	NORTHCOM	5	1	12
8	Maneuver - 3	Reserve	NORTHCOM	5	1	12
9	Maneuver - 3	Active	NORTHCOM	6	1	12
10	Maneuver - 3	Reserve	NORTHCOM	6	1	12
11	Maneuver - 3	Active	NORTHCOM	6	1	12
12	Maneuver - 3	Reserve	NORTHCOM	6	1	12
13	Maneuver - 3	Active	NORTHCOM	6	1	12
14	Maneuver - 3	Reserve	NORTHCOM	6	1	12
15	Maneuver - 3	Active	NORTHCOM	6	1	12
16	Maneuver - 3	Reserve	NORTHCOM	6	1	12
17	Maneuver - 3	Active	NORTHCOM	6	1	12
18	Maneuver - 3	Reserve	NORTHCOM	6	1	12
19	Maneuver - 3	Active	NORTHCOM	6	1	12
20	Maneuver - 3	Reserve	NORTHCOM	6	1	12
21	Maneuver - 3	Active	NORTHCOM	6	1	12
22	Maneuver - 3	Reserve	NORTHCOM	6	1	12
23	Maneuver - 3	Active	NORTHCOM	6	1	12
24	Maneuver - 3	Reserve	NORTHCOM	6	1	12
25	Maneuver - 3	Active	NORTHCOM	6	1	12
26	Maneuver - 3	Reserve	NORTHCOM	6	1	12
27	Maneuver - 3	Active	NORTHCOM	6	1	12
28	Maneuver - 3	Reserve	NORTHCOM	6	1	12
29	Maneuver - 3	Active	NORTHCOM	7	1	12
30	Maneuver - 3	Reserve	NORTHCOM	7	1	12
31	Maneuver - 3	Active	NORTHCOM	7	1	12
32	Maneuver - 3	Reserve	NORTHCOM	7	1	12
33	Maneuver - 3	Active	NORTHCOM	7	1	12
34	Maneuver - 3	Reserve	NORTHCOM	7	1	12
35	Maneuver - 3	Active	NORTHCOM	7	1	12
36	Maneuver - 3	Reserve	NORTHCOM	7	1	12
37	Maneuver - 3	Active	NORTHCOM	7	1	12
38	Maneuver - 3	Reserve	NORTHCOM	7	1	12
39	Maneuver - 3	Active	NORTHCOM	8	1	12
40	Maneuver - 3	Reserve	NORTHCOM	8	1	12
41	Maneuver - 3	Active	NORTHCOM	8	1	12
42	Maneuver - 3	Reserve	NORTHCOM	8	1	12
43	Maneuver - 3	Active	NORTHCOM	8	1	12
44	Maneuver - 3	Reserve	NORTHCOM	8	1	12
45	Maneuver - 3	Active	NORTHCOM	5	1	12
46	Maneuver - 3	Reserve	NORTHCOM	5	1	12
47	Maneuver - 3	Active	NORTHCOM	5	1	12
48	Maneuver - 3	Reserve	NORTHCOM	5	1	12
49	Maneuver - 3	Active	NORTHCOM	5	1	12
50	Maneuver - 3	Reserve	NORTHCOM	5	1	12

Figure 20: Maneuver WfF Mission Demand (1 of 3)

Mission Demand (j)	Preferred Unit Type (u)	Preferred Component (c)	Location (l)	Initial Month (t)	Duration (Months)	Interval Between Mission Demands (Months)
51	Maneuver - 3	Active	NORTHCOM	5	1	12
52	Maneuver - 3	Reserve	NORTHCOM	5	1	12
53	Maneuver - 3	Active	NORTHCOM	6	1	12
54	Maneuver - 3	Reserve	NORTHCOM	6	1	12
55	Maneuver - 3	Active	NORTHCOM	6	1	12
56	Maneuver - 3	Reserve	NORTHCOM	6	1	12
57	Maneuver - 3	Active	NORTHCOM	6	1	12
58	Maneuver - 3	Reserve	NORTHCOM	6	1	12
59	Maneuver - 3	Active	NORTHCOM	6	1	12
60	Maneuver - 3	Reserve	NORTHCOM	6	1	12
61	Maneuver - 3	Active	NORTHCOM	6	1	12
62	Maneuver - 3	Reserve	NORTHCOM	6	1	12
63	Maneuver - 3	Active	NORTHCOM	6	1	12
64	Maneuver - 3	Reserve	NORTHCOM	6	1	12
65	Maneuver - 3	Active	NORTHCOM	6	1	12
66	Maneuver - 3	Reserve	NORTHCOM	6	1	12
67	Maneuver - 3	Active	NORTHCOM	6	1	12
68	Maneuver - 3	Reserve	NORTHCOM	6	1	12
69	Maneuver - 3	Active	NORTHCOM	6	1	12
70	Maneuver - 3	Reserve	NORTHCOM	6	1	12
71	Maneuver - 3	Active	NORTHCOM	6	1	12
72	Maneuver - 3	Reserve	NORTHCOM	6	1	12
73	Maneuver - 3	Active	NORTHCOM	7	1	12
74	Maneuver - 3	Reserve	NORTHCOM	7	1	12
75	Maneuver - 3	Active	NORTHCOM	7	1	12
76	Maneuver - 3	Reserve	NORTHCOM	7	1	12
77	Maneuver - 3	Active	NORTHCOM	7	1	12
78	Maneuver - 3	Reserve	NORTHCOM	7	1	12
79	Maneuver - 3	Active	NORTHCOM	7	1	12
80	Maneuver - 3	Reserve	NORTHCOM	7	1	12
81	Maneuver - 3	Active	NORTHCOM	7	1	12
82	Maneuver - 3	Reserve	NORTHCOM	7	1	12
83	Maneuver - 3	Active	NORTHCOM	8	1	12
84	Maneuver - 3	Reserve	NORTHCOM	8	1	12
85	Maneuver - 3	Active	NORTHCOM	8	1	12
86	Maneuver - 3	Reserve	NORTHCOM	8	1	12
87	Maneuver - 3	Active	NORTHCOM	8	1	12
88	Maneuver - 3	Reserve	NORTHCOM	8	1	12
89	Maneuver - 4	Active	NORTHCOM	10	2	10
90	Maneuver - 4	Active	NORTHCOM	10	2	10
91	Maneuver - 4	Active	NORTHCOM	10	2	10
92	Maneuver - 4	Active	NORTHCOM	10	2	10
93	Maneuver - 4	Active	NORTHCOM	10	2	10
94	Maneuver - 4	Active	NORTHCOM	10	2	10
95	Maneuver - 4	Active	NORTHCOM	10	2	10
96	Maneuver - 4	Active	NORTHCOM	10	2	10
97	Maneuver - 5	Reserve Only	EUCOM	6	2	10
98	Maneuver - 5	Reserve Only	EUCOM	6	2	10
99	Maneuver - 5	Reserve	EUCOM	6	6	6
100	Maneuver - 5	Reserve	EUCOM	6	6	6

Figure 21: Maneuver WfF Mission Demand (2 of 3)

Mission Demand (*j*)	Preferred Unit Type (*u*)	Preferred Component (*c*)	Location (*l*)	Initial Month (*t*)	Duration (Months)	Interval Between Mission Demands (Months)
101	Maneuver - 5	Active	EUCOM	8	4	8
102	Maneuver - 5	Active	EUCOM	8	4	8
103	Maneuver - 5	Active	EUCOM	8	4	8
104	Maneuver - 5	Active	EUCOM	8	4	8
105	Maneuver - 5	Reserve Only	EUCOM	6	2	10
106	Maneuver - 5	Reserve Only	EUCOM	6	2	10
107	Maneuver - 5	Reserve	EUCOM	6	6	6
108	Maneuver - 5	Reserve	EUCOM	6	6	6
109	Maneuver - 5	Active	EUCOM	8	4	8
110	Maneuver - 5	Active	EUCOM	8	4	8
111	Maneuver - 5	Active	EUCOM	8	4	8
112	Maneuver - 5	Active	EUCOM	8	4	8
113	Maneuver - 2	Active	SOUTHCOM	8	1	11
114	Maneuver - 2	Active	SOUTHCOM	8	1	11
115	Maneuver - 2	Active	SOUTHCOM	8	1	11
116	Maneuver - 2	Active	SOUTHCOM	8	1	11
117	Maneuver - 2	Active	SOUTHCOM	8	1	11
118	Maneuver - 2	Active	SOUTHCOM	8	1	11
119	Maneuver - 2	Active	SOUTHCOM	10	2	10
120	Maneuver - 2	Active	SOUTHCOM	10	2	10
121	Maneuver - 2	Active	SOUTHCOM	10	2	10
122	Maneuver - 2	Active	SOUTHCOM	10	2	10
123	Maneuver - 2	Active	SOUTHCOM	8	1	11
124	Maneuver - 2	Active	SOUTHCOM	8	1	11
125	Maneuver - 2	Active	SOUTHCOM	8	1	11
126	Maneuver - 2	Active	SOUTHCOM	8	1	11
127	Maneuver - 2	Active	SOUTHCOM	8	1	11
128	Maneuver - 2	Active	SOUTHCOM	8	1	11
129	Maneuver - 2	Active	SOUTHCOM	10	2	10
130	Maneuver - 2	Active	SOUTHCOM	10	2	10
131	Maneuver - 2	Active	SOUTHCOM	10	2	10
132	Maneuver - 2	Active	SOUTHCOM	10	2	10
133	Maneuver - 1	Reserve	CENTCOM	1	1	1
134	Maneuver - 1	Active	CENTCOM	1	1	1
135	Maneuver - 1	Reserve	CENTCOM	1	1	1
136	Maneuver - 1	Active	CENTCOM	1	1	1
137	Maneuver - 1	Reserve	INDOPACOM	1	1	1
138	Maneuver - 1	Active	INDOPACOM	1	1	1
139	Maneuver - 1	Reserve	INDOPACOM	1	1	1
140	Maneuver - 1	Active	INDOPACOM	1	1	1
141	Maneuver - 1	Reserve	CENTCOM	1	1	1
142	Maneuver - 1	Active	CENTCOM	1	1	1
143	Maneuver - 1	Reserve	CENTCOM	1	1	1
144	Maneuver - 1	Active	CENTCOM	1	1	1
145	Maneuver - 1	Reserve	INDOPACOM	1	1	1
146	Maneuver - 1	Active	INDOPACOM	1	1	1
147	Maneuver - 1	Reserve	INDOPACOM	1	1	1
148	Maneuver - 1	Active	INDOPACOM	1	1	1

Figure 22: Maneuver WfF Mission Demand (3 of 3)

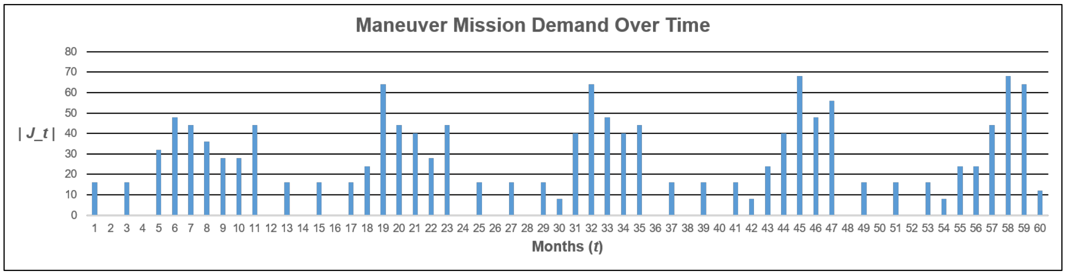


Figure 23: Maneuver WfF Consolidated Mission Demand Over Time

4.4.7 Maneuver WfF Results and Analysis

Table 11 provides the initial summary results from the Maneuver WfF instance, illustrating that the instance solved to optimality in 24.04 seconds.

Table 11: Maneuver WfF Instance Computational Results

OBJ Value (z)	**# Cols (D.V.)**	**# Rows (Constraints)**	**Processing Time (s)**	**OPT Gap**
78,940.31	2,667,660	29,832	24.04	0.00%

The solution to the Maneuver WfF instance produces the unit mix outlined in Table 12. This instance's mission demand data contains 10 different sets of mission demands that prefer a specific unit for a set of mission demands. Interestingly this instance results in employing 13 different sets of units by type, Component status, and Geographic COCOM alignment. This instance does not employ two sets of units that have corresponding mission demand preferences: $(Maneuver-2, Active, SOUTHCOM)$ and $(Maneuver-3, Reserve, NORTHCOM)$. Instead, it employs $(Maneuver-1, Reserve, NORTHCOM)$, $(Maneuver-4, Active, INDOPACOM)$, $(Maneuver-4, Active, SOUTHCOM)$, $(Maneuver-4, Reserve, EUCOM)$, and $(Maneuver-4, Reserve, NORTHCOM)$ units; these units do not have any corresponding missions demands that they are fully suitable for. This significantly contributes to the total number of mission demands that are met by non-preferred units, given in Table 13. The key takeaway from this result is that the units employed in this instance have a larger utility in meeting many different mission demands than units that are perfectly suitable (or preferred) to meet the mission demands. This further elaborates on the application of α_j^{ucl} and the degree of unit suitability provided in Figure 5.

The Maneuver WfF instance's AMPU is approximately 10.58 mission demands per unit employed. This AMPU is considered mid-range among the different WfF

Table 12: Maneuver WfF Instance Unit Employment Mix

Unit Type Employed	**# Units Employed** (v^{ucl})
$(Maneuver-1, Active, CENTCOM)$	15
$(Maneuver-1, Active, INDOPACOM)$	15
$(Maneuver-1, Reserve, CENTCOM)$	10
$(Maneuver-1, Reserve, INDOPACOM)$	8
$(Maneuver-1, Reserve, NORTHCOM)$	1
$(Maneuver-3, Active, NORTHCOM)$	4
$(Maneuver-4, Active, INDOPACOM)$	1
$(Maneuver-4, Active, NORTHCOM)$	24
$(Maneuver-4, Active, SOUTHCOM)$	16
$(Maneuver-4, Reserve, EUCOM)$	4
$(Maneuver-4, Reserve, NORTHCOM)$	16
$(Maneuver-5, Active, EUCOM)$	16
$(Maneuver-5, Reserve, EUCOM)$	8

Table 13: Maneuver WfF Use of Non-Preferred Units

Total Mission Demands	1,460
Routine Deployments with Non-Preferred Unit	740
Surges with Non-Preferred Unit	4
Extensions with Non-Preferred Unit	0
% of Mission Demands with Non-Preferred Units	50.96%

instances. This instance involves both semi-continuous and singular mission demands that initially begin between months 1 through 10, with intervals of recurrence between 1 to 12 months. Although clustering of mission demands over time still occurs, these clusters are not as severe or defined when compared to other WfF instances. This allows units to meet more mission demands during their deployments, as illustrated in Figures 23 and 24.

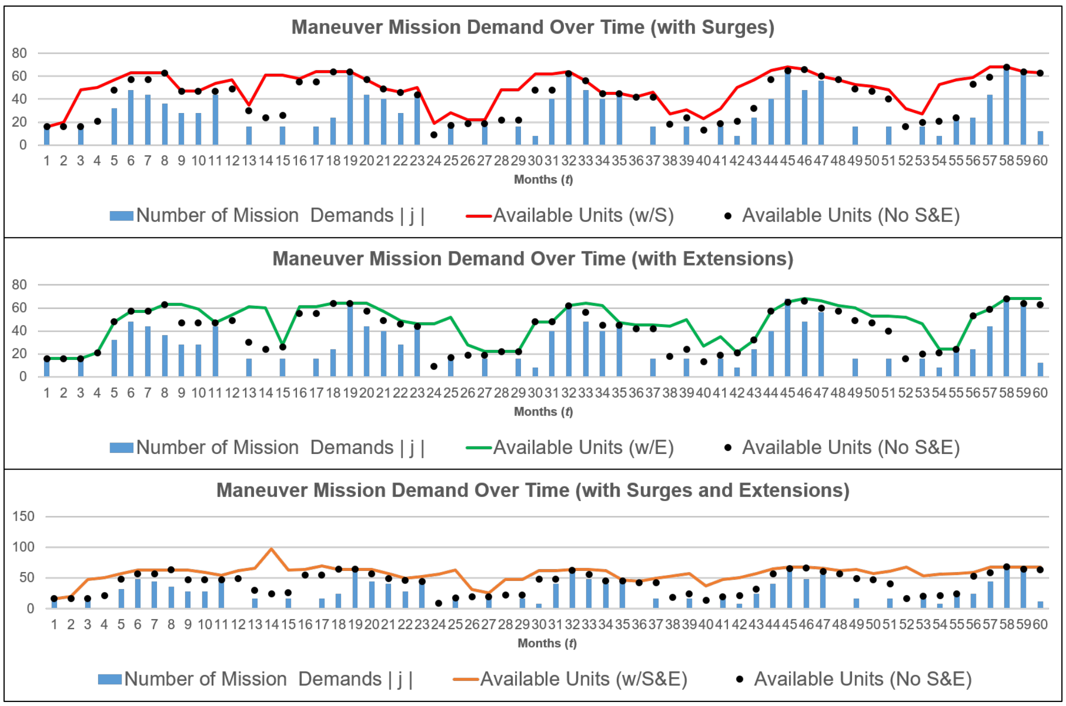


Figure 24: Maneuver WfF Consolidated Mission Demand Over Time - Sufficiency Results

Although the size of this instance must be considered, Table 14 shows that the solution results in a high degree of under-utilization. Examining only the utilization of units during routine deployments, this instance only results in 9 months where all units deployed are meeting mission demand. With an average of 16.50 units not utilized per month, these results show that Maneuver units regularly have the ability to meet more mission demand.

This instance requires two units to surge during time period 32, and three units

Table 14: Maneuver WfF - Unit Utilization

Metric	R	RSE
Months with 100% Utilization	9	5
Months with < 100% Utilization	51	55
Average Units Not Utilized per Month	16.50	32.87
Total Unit-Months Not Utilized	990	1972
Unit-Months Not Utilized per Unit Employed	7.17	14.29

to surge during period 45. This is tabulated in Table 15 and graphically illustrated in Figure 24.

Table 15: Maneuver WfF Instance Surges and Extensions

Total Mission Demands	1,460
Routine Deployments	1,455
Surges	5
Extensions	0

Figures 25-37 show the unit employment plateau level for each set of units employed. These plateaus represent the same values as those presented in Table 12.

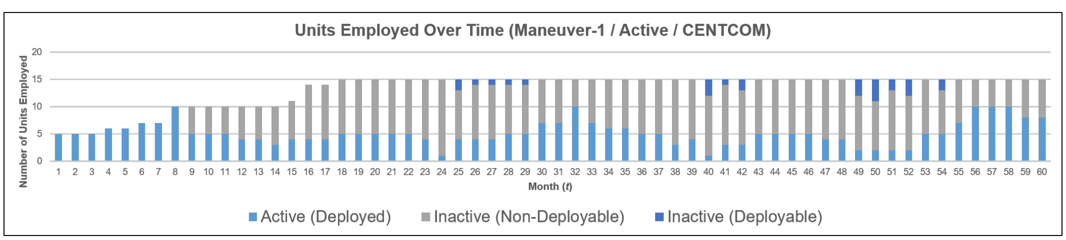


Figure 25: Maneuver WfF Unit Employment Over Time - Part 1 of 13

As discussed previously the following sets of units are employed in the solution to this instance, but do not have corresponding mission demands that prefer these units:

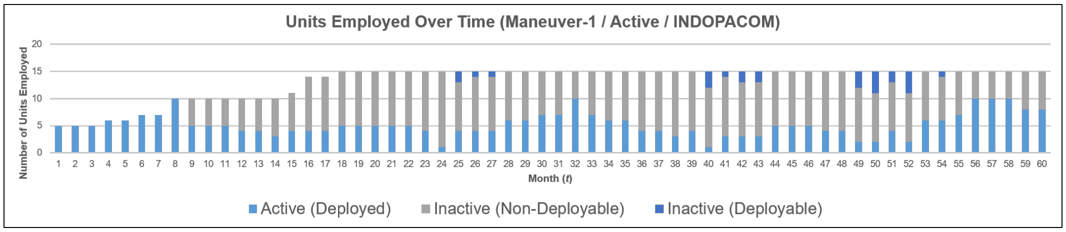


Figure 26: Maneuver WfF Unit Employment Over Time - Part 2 of 13

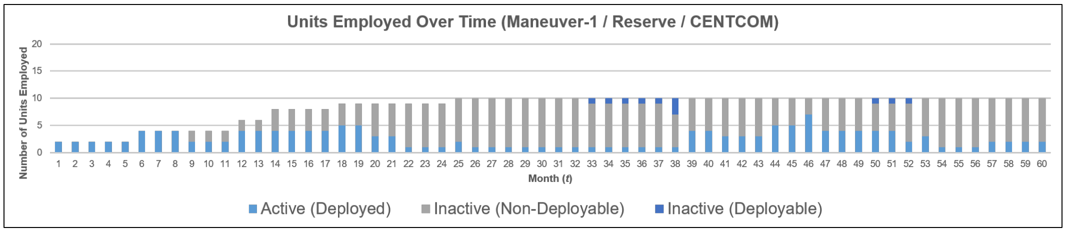


Figure 27: Maneuver WfF Unit Employment Over Time - Part 3 of 13

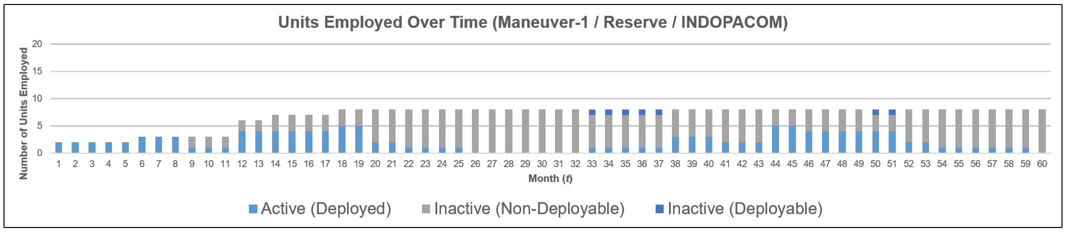


Figure 28: Maneuver WfF Unit Employment Over Time - Part 4 of 13

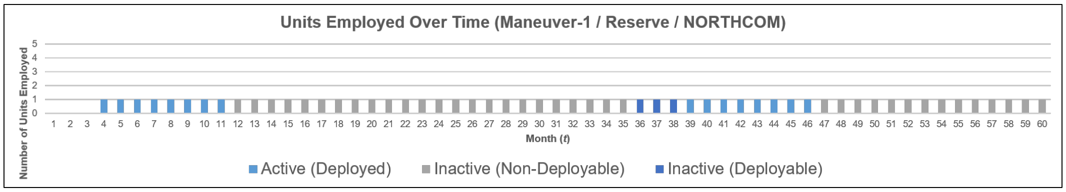


Figure 29: Maneuver WfF Unit Employment Over Time - Part 5 of 13

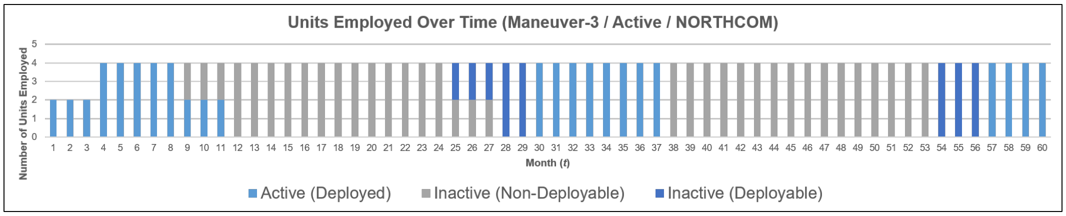


Figure 30: Maneuver WfF Unit Employment Over Time - Part 6 of 13

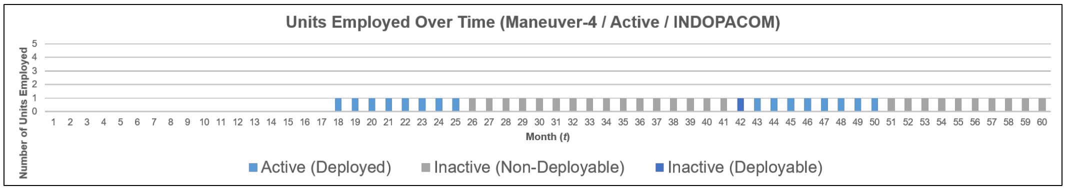


Figure 31: Maneuver WfF Unit Employment Over Time - Part 7 of 13

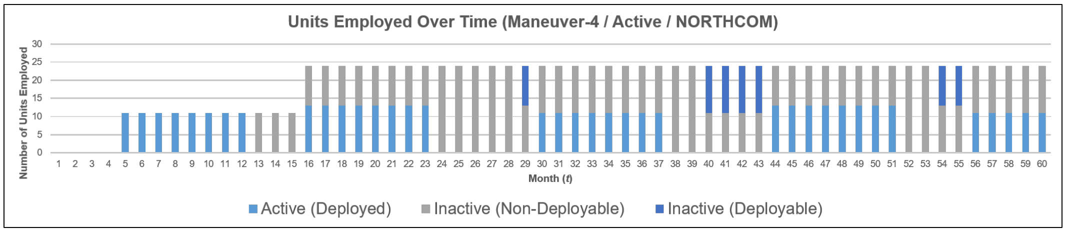


Figure 32: Maneuver WfF Unit Employment Over Time - Part 8 of 13

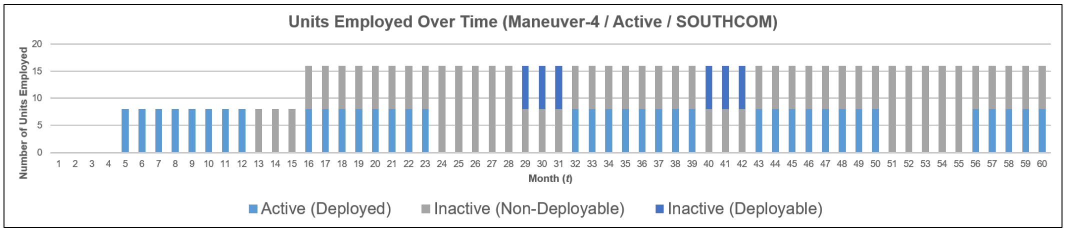


Figure 33: Maneuver WfF Unit Employment Over Time - Part 9 of 13

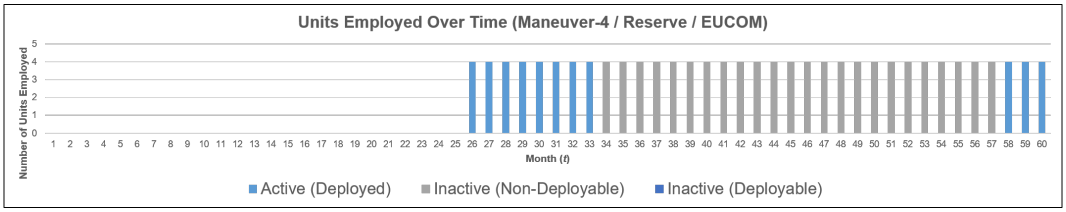


Figure 34: Maneuver WfF Unit Employment Over Time - Part 10 of 13

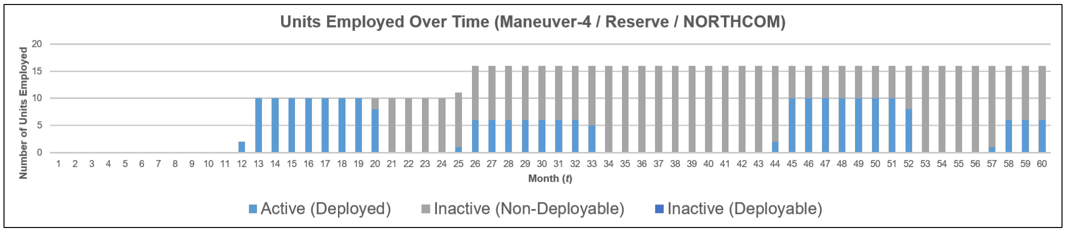


Figure 35: Maneuver WfF Unit Employment Over Time - Part 11 of 13

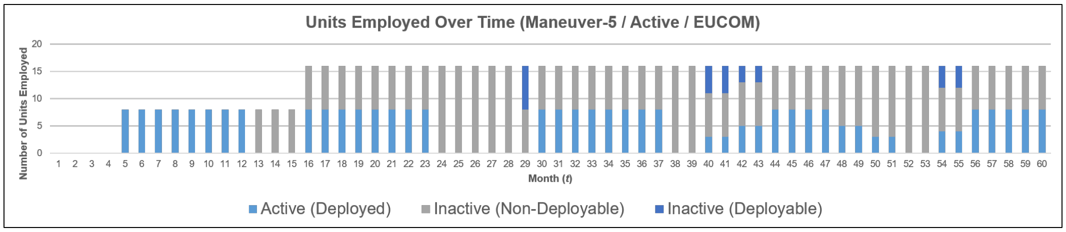


Figure 36: Maneuver WfF Unit Employment Over Time - Part 12 of 13

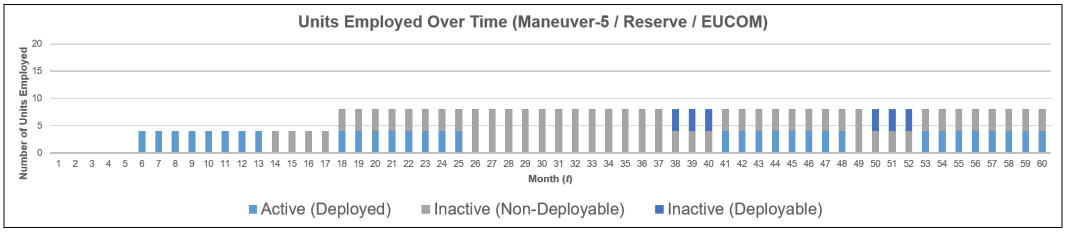


Figure 37: Maneuver WfF Unit Employment Over Time - Part 13 of 13

$(Maneuver-4, Active, INDOPACOM)$, $(Maneuver-4, Active, SOUTHCOM)$, $(Maneuver-4, Reserve, EUCOM)$, and $(Maneuver-4, Reserve, NORTHCOM)$. Therefore, every time these units are utilized to meet a mission demand, a penalty cost is incurred. These units account for approximately 38% of all mission demands met by non-preferred units. In fact, all $Maneuver-4$ type units employed in this solution account for 66% of all mission demands met by non-preferred units. Given the various duration and frequency of the indexed mission demands throughout the time horizon, these results demonstrate that the U.S. Army should use these $Maneuver-4$ type units as "catch all" units when necessary. This is further re-enforced when evaluating the employment cost of $Maneuver-4$ type units as well as these units suitability scores, specifically when meeting mission demands preferring $Maneuver-2$ or $Maneuver-3$ type units. This also explains why no $Maneuver-2$ type units are employed in this instance's solution.

4.4.8 Mission Command WfF Input Data

This Mission Command WfF instance can be described as completely unrestricted in terms of which units can be employed and which units are capable of meeting each of the mission demands. This WfF instance has two unique unit MTOE types; $\mathcal{U} = \{Mission\ Command - 1, Mission\ Command - 2\}$. In this instance, every unit regardless of MTOE unit type, Component status, or Geographic COCOM location alignment can be employed and each of these units can meet any of the mission demands. The location for mission demands in this instance are $SOUTHCOM$ and $NORTHCOM$. Figures 38-40 provides the mission demand input data; this data allows for the construction of the sets $\mathcal{A}$, $\mathcal{A}_j$, $\mathcal{J}$, $\mathcal{J}_t$, and $\mathcal{J}_{ucl}$ as described in Section 3.2.1.

Initially occurring during different months, each of the 132 indexed mission demands for the Mission Command WfF instance are non-continuous mission demands. Figure 41 presents the total number of mission demands that must be met each month. Each unique mission demand occurs during a single month. Additionally, the gaps between indexed mission demands uniformly occur at 12 month intervals. As seen in Figure 41, the Mission Command WfF instance's mission demand occurs completely cyclically throughout the time horizon. This instance requires 560 unique mission demands be met; that is, $\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} |\mathcal{J}_t| = 560$.

4.4.9 Mission Command WfF Results and Analysis

The Mission Command WfF instance solves to optimality in 2.56 seconds, with additional preliminary results provided in Table 16.

The solution to the Mission Command WfF instance produces the unit mix outlined in Table 17. This instance primarily employs ($Mission\ Command - 2, Active$, $SOUTHCOM$) units, which corresponds to the majority of unique mission demands.

Mission Demand (j)	Preferred Unit Type (u)	Preferred Component (c)	Location (l)	Initial Month (t)	Duration (Months)	Interval Between Mission Demands (Months)
1	Mission Command - 2	Reserve	SOUTHCOM	7	1	12
2	Mission Command - 2	Reserve	SOUTHCOM	7	1	12
3	Mission Command - 2	Reserve	SOUTHCOM	8	1	12
4	Mission Command - 2	Reserve	SOUTHCOM	8	1	12
5	Mission Command - 2	Active	SOUTHCOM	8	1	12
6	Mission Command - 2	Active	SOUTHCOM	8	1	12
7	Mission Command - 2	Active	SOUTHCOM	8	1	12
8	Mission Command - 2	Active	SOUTHCOM	8	1	12
9	Mission Command - 2	Active	SOUTHCOM	9	1	12
10	Mission Command - 2	Active	SOUTHCOM	9	1	12
11	Mission Command - 2	Active	SOUTHCOM	9	1	12
12	Mission Command - 2	Active	SOUTHCOM	9	1	12
13	Mission Command - 2	Active	SOUTHCOM	9	1	12
14	Mission Command - 2	Active	SOUTHCOM	9	1	12
15	Mission Command - 2	Active	SOUTHCOM	9	1	12
16	Mission Command - 2	Active	SOUTHCOM	9	1	12
17	Mission Command - 2	Active	SOUTHCOM	9	1	12
18	Mission Command - 2	Active	SOUTHCOM	9	1	12
19	Mission Command - 2	Active	SOUTHCOM	9	1	12
20	Mission Command - 2	Active	SOUTHCOM	9	1	12
21	Mission Command - 2	Active	SOUTHCOM	9	1	12
22	Mission Command - 2	Active	SOUTHCOM	9	1	12
23	Mission Command - 2	Active	SOUTHCOM	9	1	12
24	Mission Command - 2	Active	SOUTHCOM	9	1	12
25	Mission Command - 2	Active	SOUTHCOM	9	1	12
26	Mission Command - 2	Active	SOUTHCOM	9	1	12
27	Mission Command - 2	Active	SOUTHCOM	9	1	12
28	Mission Command - 2	Active	SOUTHCOM	9	1	12
29	Mission Command - 2	Active	SOUTHCOM	10	1	12
30	Mission Command - 2	Active	SOUTHCOM	10	1	12
31	Mission Command - 2	Active	SOUTHCOM	10	1	12
32	Mission Command - 2	Active	SOUTHCOM	10	1	12
33	Mission Command - 2	Active	SOUTHCOM	10	1	12
34	Mission Command - 2	Active	SOUTHCOM	10	1	12
35	Mission Command - 2	Active	SOUTHCOM	10	1	12
36	Mission Command - 2	Active	SOUTHCOM	10	1	12
37	Mission Command - 2	Active	SOUTHCOM	11	1	12
38	Mission Command - 2	Active	SOUTHCOM	11	1	12
39	Mission Command - 2	Active	SOUTHCOM	11	1	12
40	Mission Command - 2	Active	SOUTHCOM	11	1	12
41	Mission Command - 2	Active	SOUTHCOM	12	1	12
42	Mission Command - 2	Active	SOUTHCOM	12	1	12
43	Mission Command - 2	Reserve	SOUTHCOM	7	1	12
44	Mission Command - 2	Reserve	SOUTHCOM	7	1	12
45	Mission Command - 2	Reserve	SOUTHCOM	8	1	12
46	Mission Command - 2	Reserve	SOUTHCOM	8	1	12
47	Mission Command - 2	Active	SOUTHCOM	8	1	12
48	Mission Command - 2	Active	SOUTHCOM	8	1	12
49	Mission Command - 2	Active	SOUTHCOM	8	1	12
50	Mission Command - 2	Active	SOUTHCOM	8	1	12

Figure 38: Mission Command WfF Mission Demand (1 of 3)

Table 16: Mission Command WfF Instance Computational Results

OBJ Value (z)	**# Cols (D.V.)**	**# Rows (Constraints)**	**Processing Time (s)**	**OPT Gap**
37,492.29	951,864	16,308	2.56	0.00%

Mission Demand (j)	Preferred Unit Type (u)	Preferred Component (c)	Location (l)	Initial Month (t)	Duration (Months)	Interval Between Mission Demands (Months)
51	Mission Command - 2	Active	SOUTHCOM	9	1	12
52	Mission Command - 2	Active	SOUTHCOM	9	1	12
53	Mission Command - 2	Active	SOUTHCOM	9	1	12
54	Mission Command - 2	Active	SOUTHCOM	9	1	12
55	Mission Command - 2	Active	SOUTHCOM	9	1	12
56	Mission Command - 2	Active	SOUTHCOM	9	1	12
57	Mission Command - 2	Active	SOUTHCOM	9	1	12
58	Mission Command - 2	Active	SOUTHCOM	9	1	12
59	Mission Command - 2	Active	SOUTHCOM	9	1	12
60	Mission Command - 2	Active	SOUTHCOM	9	1	12
61	Mission Command - 2	Active	SOUTHCOM	9	1	12
62	Mission Command - 2	Active	SOUTHCOM	9	1	12
63	Mission Command - 2	Active	SOUTHCOM	9	1	12
64	Mission Command - 2	Active	SOUTHCOM	9	1	12
65	Mission Command - 2	Active	SOUTHCOM	9	1	12
66	Mission Command - 2	Active	SOUTHCOM	9	1	12
67	Mission Command - 2	Active	SOUTHCOM	9	1	12
68	Mission Command - 2	Active	SOUTHCOM	9	1	12
69	Mission Command - 2	Active	SOUTHCOM	9	1	12
70	Mission Command - 2	Active	SOUTHCOM	9	1	12
71	Mission Command - 2	Active	SOUTHCOM	10	1	12
72	Mission Command - 2	Active	SOUTHCOM	10	1	12
73	Mission Command - 2	Active	SOUTHCOM	10	1	12
74	Mission Command - 2	Active	SOUTHCOM	10	1	12
75	Mission Command - 2	Active	SOUTHCOM	10	1	12
76	Mission Command - 2	Active	SOUTHCOM	10	1	12
77	Mission Command - 2	Active	SOUTHCOM	10	1	12
78	Mission Command - 2	Active	SOUTHCOM	10	1	12
79	Mission Command - 2	Active	SOUTHCOM	11	1	12
80	Mission Command - 2	Active	SOUTHCOM	11	1	12
81	Mission Command - 2	Active	SOUTHCOM	11	1	12
82	Mission Command - 2	Active	SOUTHCOM	11	1	12
83	Mission Command - 2	Active	SOUTHCOM	12	1	12
84	Mission Command - 2	Active	SOUTHCOM	12	1	12
85	Mission Command - 1	Active	NORTHCOM	9	1	12
86	Mission Command - 1	Reserve	NORTHCOM	9	1	12
87	Mission Command - 1	Active	NORTHCOM	9	1	12
88	Mission Command - 1	Reserve	NORTHCOM	9	1	12
89	Mission Command - 1	Active	NORTHCOM	9	1	12
90	Mission Command - 1	Reserve	NORTHCOM	9	1	12
91	Mission Command - 1	Active	NORTHCOM	10	1	12
92	Mission Command - 1	Reserve	NORTHCOM	10	1	12
93	Mission Command - 1	Active	NORTHCOM	10	1	12
94	Mission Command - 1	Reserve	NORTHCOM	10	1	12
95	Mission Command - 1	Active	NORTHCOM	11	1	12
96	Mission Command - 1	Reserve	NORTHCOM	11	1	12
97	Mission Command - 1	Active	NORTHCOM	11	1	12
98	Mission Command - 1	Reserve	NORTHCOM	11	1	12
99	Mission Command - 1	Active	NORTHCOM	12	1	12
100	Mission Command - 1	Reserve	NORTHCOM	12	1	12

Figure 39: Mission Command WfF Mission Demand (2 of 3)

Mission Demand (*j*)	Preferred Unit Type (*u*)	Preferred Component (*c*)	Location (*l*)	Initial Month (*t*)	Duration (Months)	Interval Between Mission Demands (Months)
101	Mission Command - 1	Active	NORTHCOM	1	1	12
102	Mission Command - 1	Reserve	NORTHCOM	1	1	12
103	Mission Command - 1	Active	NORTHCOM	1	1	12
104	Mission Command - 1	Reserve	NORTHCOM	1	1	12
105	Mission Command - 1	Active	NORTHCOM	1	1	12
106	Mission Command - 1	Reserve	NORTHCOM	1	1	12
107	Mission Command - 1	Active	NORTHCOM	2	1	12
108	Mission Command - 1	Reserve	NORTHCOM	2	1	12
109	Mission Command - 1	Active	NORTHCOM	9	1	12
110	Mission Command - 1	Reserve	NORTHCOM	9	1	12
111	Mission Command - 1	Active	NORTHCOM	9	1	12
112	Mission Command - 1	Reserve	NORTHCOM	9	1	12
113	Mission Command - 1	Active	NORTHCOM	9	1	12
114	Mission Command - 1	Reserve	NORTHCOM	9	1	12
115	Mission Command - 1	Active	NORTHCOM	10	1	12
116	Mission Command - 1	Reserve	NORTHCOM	10	1	12
117	Mission Command - 1	Active	NORTHCOM	10	1	12
118	Mission Command - 1	Reserve	NORTHCOM	10	1	12
119	Mission Command - 1	Active	NORTHCOM	11	1	12
120	Mission Command - 1	Reserve	NORTHCOM	11	1	12
121	Mission Command - 1	Active	NORTHCOM	11	1	12
122	Mission Command - 1	Reserve	NORTHCOM	11	1	12
123	Mission Command - 1	Active	NORTHCOM	12	1	12
124	Mission Command - 1	Reserve	NORTHCOM	12	1	12
125	Mission Command - 1	Active	NORTHCOM	1	1	12
126	Mission Command - 1	Reserve	NORTHCOM	1	1	12
127	Mission Command - 1	Active	NORTHCOM	1	1	12
128	Mission Command - 1	Reserve	NORTHCOM	1	1	12
129	Mission Command - 1	Active	NORTHCOM	1	1	12
130	Mission Command - 1	Reserve	NORTHCOM	1	1	12
131	Mission Command - 1	Active	NORTHCOM	2	1	12
132	Mission Command - 1	Reserve	NORTHCOM	2	1	12

Figure 40: Mission Command WfF Mission Demand (3 of 3)

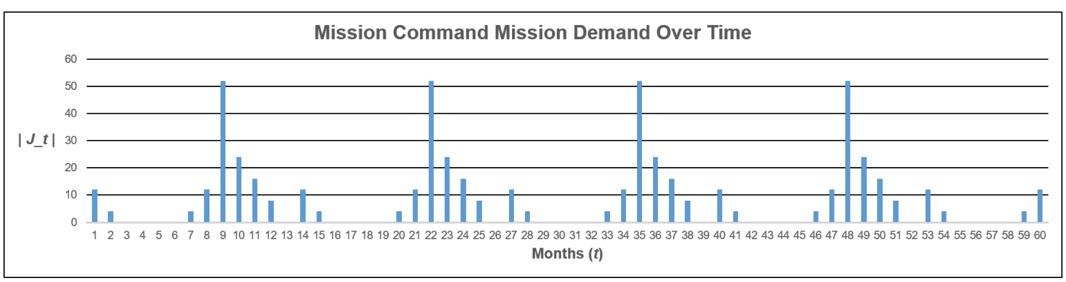


Figure 41: Mission Command WfF Consolidated Mission Demand Over Time

In addition to this set of units, the remaining units employed are used to compensate for the lack of units with unit type $Mission\ Command{-}1$ and location alignment with $NORTHCOM$. This is why the percent of mission demands met by non-preferred units is approximately 46.43%, as outlined in Table 18.

Table 17: Mission Command WfF Instance Unit Employment Mix

Unit Type Employed	**# Units Employed (v^{ucl})**
$(Mission\ Command-2, Active, NORTHCOM)$	24
$(Mission\ Command-2, Active, SOUTHCOM)$	60
$(Mission\ Command-2, Reserve, NORTHCOM)$	12
$(Mission\ Command-2, Reserve, SOUTHCOM)$	8

Table 18: Mission Command WfF Use of Non-Preferred Units

Total Mission Demands	560
Routine Deployments with Non-Preferred Unit	245
Surges with Non-Preferred Unit	5
Extensions with Non-Preferred Unit	10
% of Mission Demands with Non-Preferred Units	46.43%

In addition to having a high number of mission demands met by a non-preferred unit, shown in Table 18, the average number of mission demands met per unit employed is only approximately 5.38 (AMPU). This is the lowest AMPU among all of the WfF instances. The severe clustering of singular mission demands as well as the significant difference between the maximum number of mission demands that occur in a single month and every other month forces units to only complete 2 to 3 mission demands during each deployment. This can be seen in Figures 41 and 42.

The solution to this instances utilizes both unit surges and extensions, as shown in Table 19. All of the unit surges occur during month 1, while the extensions occur during months 9, 10, and 22.

Similarly to the low AMPU, this instance results in significant unit under-utilization

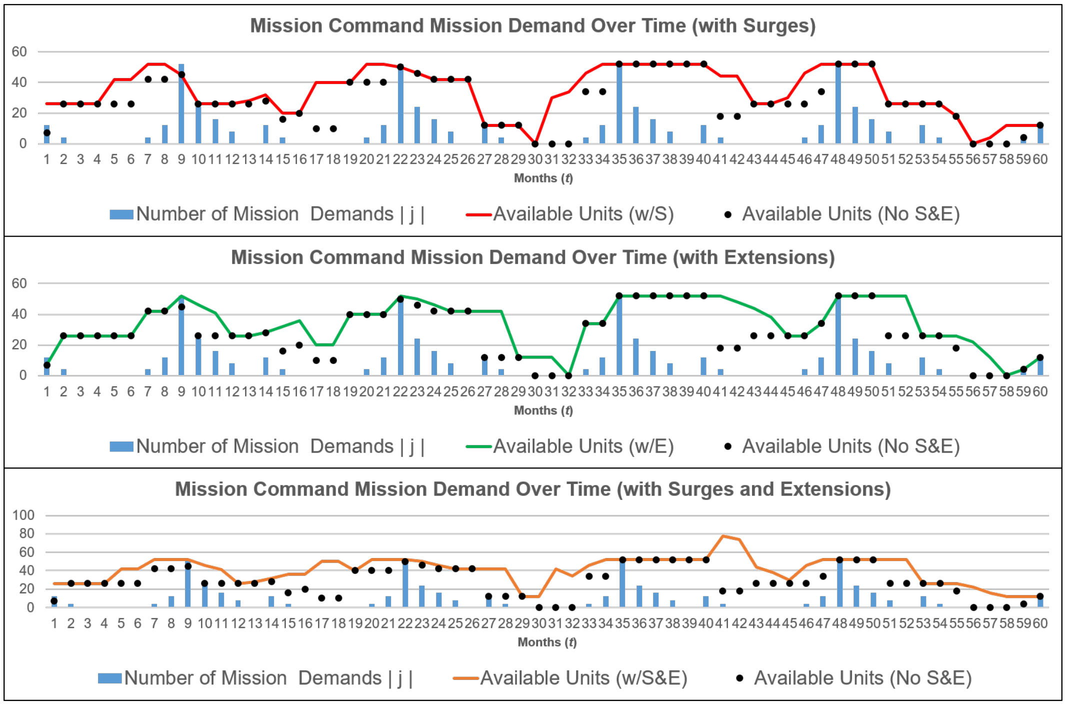


Figure 42: Mission Command WfF Consolidated Mission Demand Over Time - Sufficiency Results

Table 19: Mission Command WfF Instance Surges and Extensions

Total Mission Demands	560
Routine Deployments	545
Surges	5
Extensions	10

as shown in Table 20. Despite 14 months involving the full utilization of units on routine deployments, the average number of units on routine deployments not utilized per month is still 18.67 units.

Table 20: Mission Command WfF - Unit Utilization

Metric	R	RSE
Months with 100% Utilization	14	5
Months with $<$ 100% Utilization	46	55
Average Units Not Utilized per Month	18.67	31.22
Total Unit-Months Not Utilized	1120	1873
Unit-Months Not Utilized per Unit Employed	10.77	18.01

Figures 43-46 show the unit employment plateau level for each set of units employed. These plateaus are the same value as those presented in Table 17.

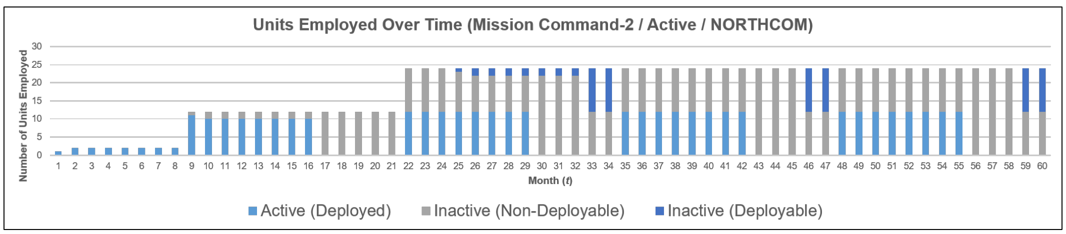


Figure 43: Mission Command WfF Unit Employment Over Time - Part 1 of 4

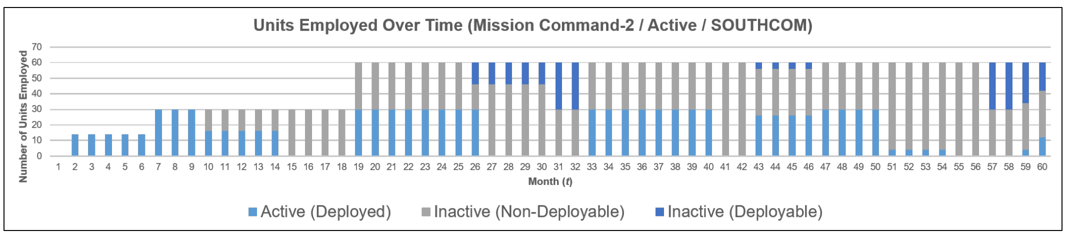


Figure 44: Mission Command WfF Unit Employment Over Time - Part 2 of 4

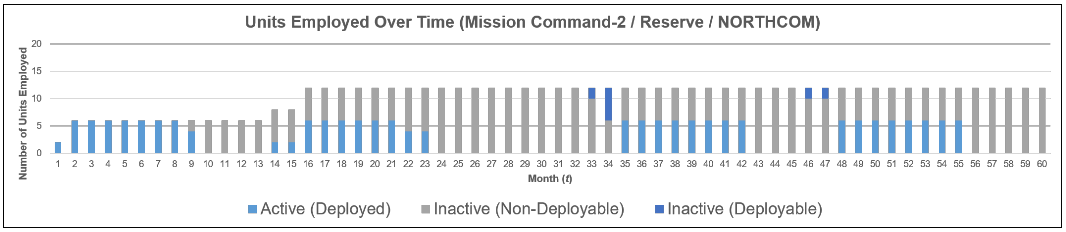


Figure 45: Mission Command WfF Unit Employment Over Time - Part 3 of 4

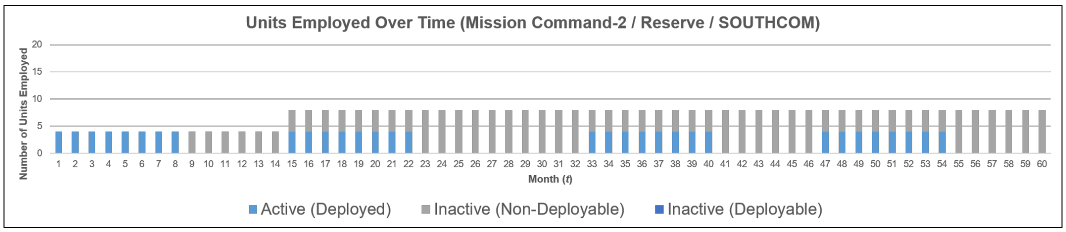


Figure 46: Mission Command WfF Unit Employment Over Time - Part 4 of 4

Similarly to the Fires WfF, these results illustrate the cost ineffectiveness of utilizing $Mission\ Command-1$ units. The employment cost of a $Mission\ Command-1$ unit is a 133% increase over the employment cost of a $Mission\ Command-2$ unit for both $Active$ and $Reserve$ status units. Despite the fact that a $Mission\ Command-2$ unit is only given a non-normalized suitability score of 4 when meeting a mission demand preferring $Mission\ Command-1$ units, the penalty cost incurred for using these non-preferred units, even for an entire deployment length of mission demands, does not approach the employment cost of employing a $Mission\ Command-1$ unit. Further, it would not be cost effective to employ $Mission\ Command-1$ units even if the suitability score is 10 for a Mission Command - 2 unit accomplishing a mission preferring a $Mission\ Command-1$ unit. Given the construction of the α_j^{ucl} parameter, the U.S. Army should not employ $Mission\ Command-1$ units, and instead only employ $Mission\ Command-2$ units.

4.4.10 Protection WfF Input Data

This Protection WfF instance can also be described as completely unrestricted, as every unit regardless of type, Component status, or Geographic COCOM alignment can be employed and each of these units can meet any of the mission demands given in this instance. This instance has two unique unit MTOE types; $\mathcal{U} = \{Protection-1, Protection-2\}$. Every mission demand for this instance occurs within $AFRICOM$ or $SOUTHCOM$. This instance's set of mission demands is shown in Figure 47. This information allows for the construction of the sets $\mathcal{A}$, $\mathcal{A}_j$, $\mathcal{J}$, $\mathcal{J}_t$, and $\mathcal{J}_{ucl}$ as described in Section 3.2.1.

This instance is unique when compared with many of the other WfF instances because of the duration and frequency of each of the mission demands. Every mission demand for this instance can be described as semi-continuous, with unique mission demands occurring every month, over a four to six month period of time. However, this instance also involves "one-off" or non-reoccurring mission demands. Specifically, mission demands $j \in \{15, ..., 54\}$ begin at month $t = 12$ and last until month $t = 15$. These mission demands do not cyclically repeat, only occurring over the course of a four month time period. Each of the remaining indexed mission demands last for six months with an additional six month interval between occurrences. This is illustrated in Figure 48. This instance contains 54 indexed mission demands, while requiring 580 unique mission demands be met; that is, $\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} |\mathcal{J}_t| = 580$.

Mission Demand (j)	Preferred Unit Type (u)	Preferred Component (c)	Location (l)	Initial Month (t)	Duration (Months)	Interval Between Mission Demands (Months)
1	Protection - 2	Reserve	AFRICOM	1	6	6
2	Protection - 2	Reserve	AFRICOM	1	6	6
3	Protection - 2	Active	AFRICOM	1	6	6
4	Protection - 2	Active	AFRICOM	1	6	6
5	Protection - 2	Active	AFRICOM	1	6	6
6	Protection - 2	Active	AFRICOM	1	6	6
7	Protection - 2	Active	AFRICOM	1	6	6
8	Protection - 2	Reserve	AFRICOM	1	6	6
9	Protection - 2	Reserve	AFRICOM	1	6	6
10	Protection - 2	Active	AFRICOM	1	6	6
11	Protection - 2	Active	AFRICOM	1	6	6
12	Protection - 2	Active	AFRICOM	1	6	6
13	Protection - 2	Active	AFRICOM	1	6	6
14	Protection - 2	Active	AFRICOM	1	6	6
15	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
16	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
17	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
18	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
19	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
20	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
21	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
22	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
23	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
24	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
25	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
26	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
27	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
28	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
29	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
30	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
31	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
32	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
33	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
34	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
35	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
36	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
37	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
38	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
39	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
40	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
41	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
42	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
43	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
44	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
45	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
46	Protection - 1	Reserve	SOUTHCOM	12	4	Not Reoccuring
47	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
48	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
49	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
50	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
51	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
52	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
53	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring
54	Protection - 1	Active	SOUTHCOM	12	4	Not Reoccuring

Figure 47: Protection WfF Mission Demand

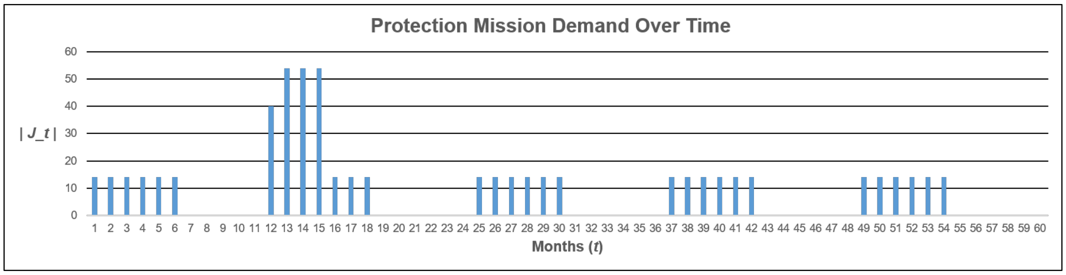


Figure 48: Protection WfF Consolidated Mission Demand Over Time

4.4.11 Protection WfF Results and Analysis

The Protection WfF instance solves to optimality in 1.43 seconds, as shown in Table 21.

Table 21: Protection WfF Instance Computational Results

OBJ Value (z)	**# Cols (D.V.)**	**# Rows (Constraints)**	**Processing Time (s)**	**OPT Gap**
55,282.99	390,264	11,628	1.43	0.00%

The solution to the Protection WfF instance produces the unit mix outlined in Table 22. Interestingly this is the first instance encountered where the instance results in employing the same sets of units by type, Component status, and Geographic COCOM alignment as the preferred units according to each set of mission demand unit preferences. Further, mission demand unit preferences are all represented among the units the instance employs. However, this does not indicate that non-preferred units are unused. If the cost of employing another preferred unit, or the cost of using an existing preferred unit through surges or extensions to meet a single mission demand is less cost effective then the incurred penalty cost of using an existing and available non-preferred unit, then a non-preferred unit is used to meet the mission demand. This situation does occur for this instance, albeit rarely. This is evident by the comparatively low percent of mission demands met with non-preferred units shown in Table 23, approximately 17.24% of mission demands.

Table 22: Protection WfF Instance Unit Employment

Unit Type Employed	**# Units Employed (v^{ucl})**
$(Protection-1, Active, SOUTHCOM)$	16
$(Protection-1, Reserve, SOUTHCOM)$	20
$(Protection-2, Active, AFRICOM)$	28
$(Protection-2, Reserve, AFRICOM)$	4

Table 23: Protection WfF Use of Non-Preferred Units

Total Mission Demands	580
Routine Deployments with Non-Preferred Unit	100
Surges with Non-Preferred Unit	0
Extensions with Non-Preferred Unit	0
% of Mission Demands with Non-Preferred Units	17.24%

The average number of mission demands met per unit employed is 8.53 (AMPU). This AMPU once again is attributed to the semi-continuous nature of each of the instance's mission demand, seen in Figures 48 and 49. The average number of mission demands met per unit is lowered by the "one-off" mission demands that occur between months 12-15. This is further illustrated in Figure 49, where the number of units available without surging or extending closely corresponds with the number of mission demands each month for the majority of months within the time horizon. This is not true when examining the months outside of the "one-off" mission demand's associated months, and the months prior to and after the occurrence of these "one-off" demands.

This instance results in a high degree of unit utilization, as shown in Table 24. When examining units during routine deployments, 44 of 60 months completely utilize every deployed unit.

Table 24: Protection WfF - Unit Utilization

Metric	**R**	**RSE**
Months with 100% Utilization	44	29
Months with $<$ 100% Utilization	16	31
Average Units Not Utilized per Month	5	10.93
Total Unit-Months Not Utilized	300	656
Unit-Months Not Utilized per Unit Employed	4.41	9.65

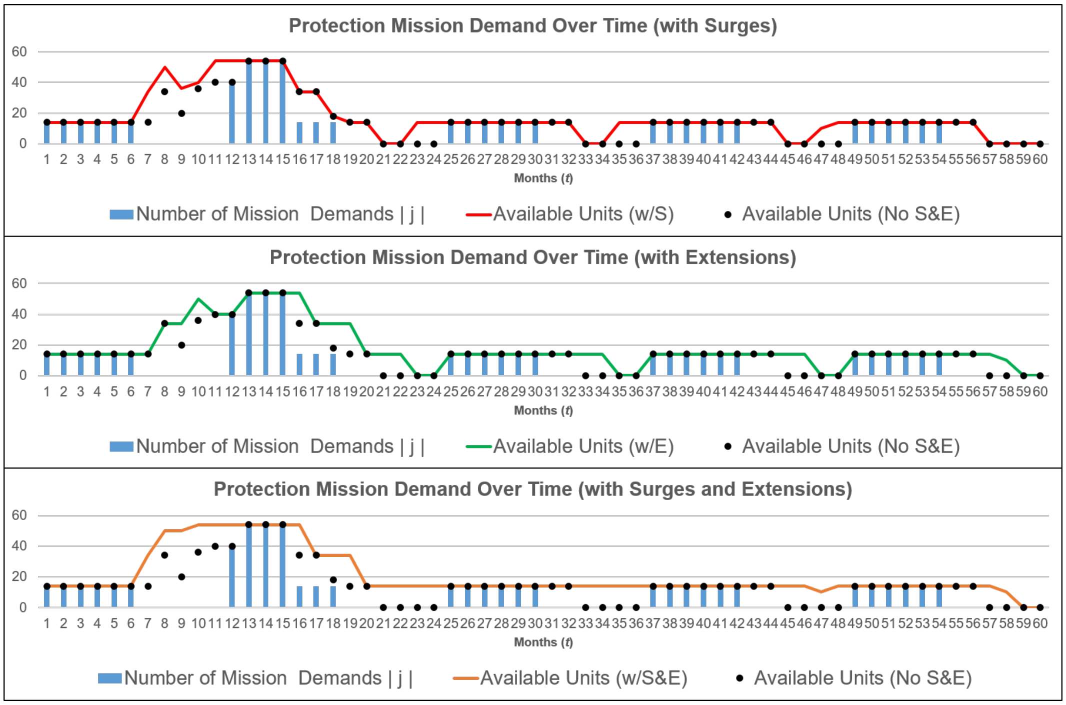


Figure 49: Protection WfF Consolidated Mission Demand Over Time - Sufficiency Results

This instance does not require a single unit surge or extension, as shown in Table 23. Figures 50-53 show the unit employment plateau level for each set of units employed. These plateaus are the same value as those presented in Table 22.

Table 25: Protection WfF Instance Surges and Extensions

Total Mission Demands	580
Routine Deployments	580
Surges	0
Extensions	0

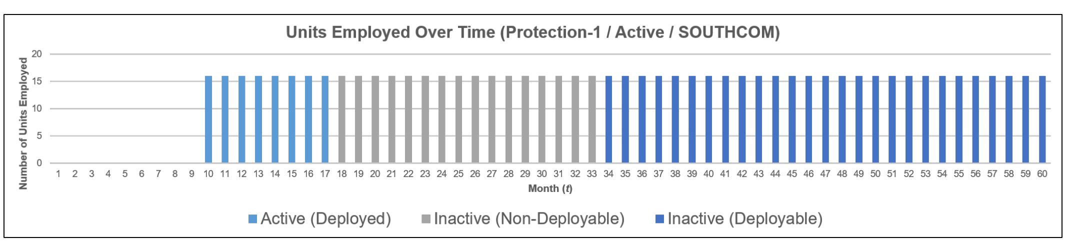


Figure 50: Protection WfF Unit Employment Over Time - Part 1 of 4

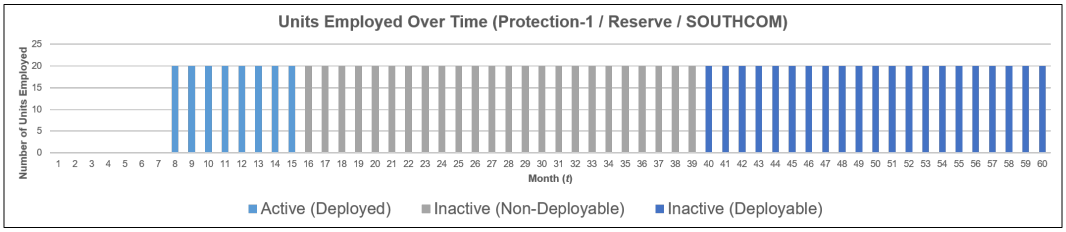


Figure 51: Protection WfF Unit Employment Over Time - Part 2 of 4

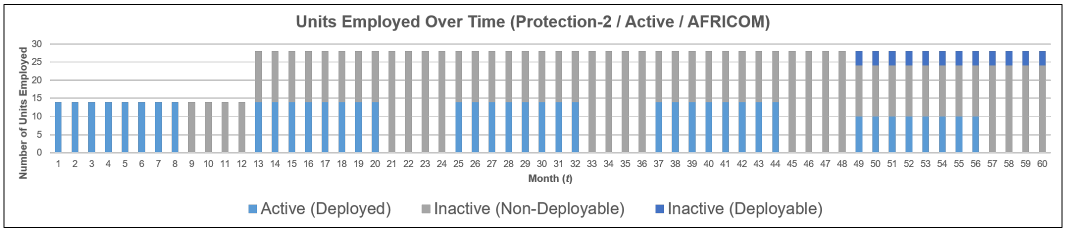


Figure 52: Protection WfF Unit Employment Over Time - Part 3 of 4

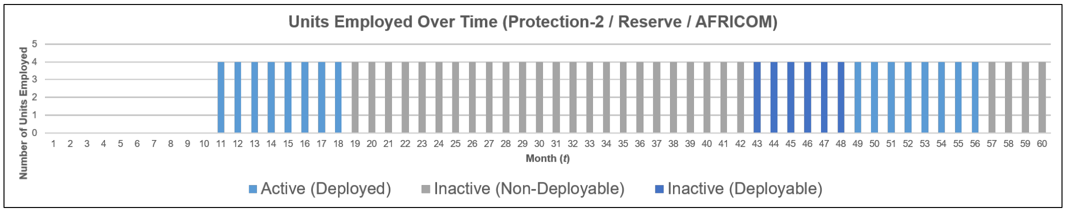


Figure 53: Protection WfF Unit Employment Over Time - Part 4 of 4

Only 160 of the 580 unique mission demands prefer $Protection-1$ type units, these mission demands are the "one-off" mission demands that occur during $t = (12, ..., 15)$. Despite this, $Protection - 1$ units are still employed in the instance's solution. The employment cost associated with $Protection - 1$ and $Protection - 2$ units are equal, and the suitability scores associated with each unit type meeting a mission demand intended for the other unit type are comparable. However, because the mission demand's preferring $Protection - 1$ units are the "one-off" mission demands, the relative utility and cost effectiveness of employing and utilizing a $Protection - 1$ unit over a $Protection - 2$ unit will be diminished over time if the cyclically reoccurring $Protection - 2$ preference mission demands continue over a longer time horizon, beyond the 60 month horizon currently modeled. The U.S. Army would need to consider this when evaluating the frequency of reoccurring mission demands, and is explored further in the the Sensitivity Analysis section.

4.4.12 Special Operations WfF Input Data

Once again, the input data used in the construction of this instance is completely unrestricted, as the mission demand input data shows in Figure 54. The Special Operations WfF instance also incorporates two unique unit MTOE types; $\mathcal{U} = \{Special\ Operations - 1, Special\ Operations - 2\}$. Here the Special Operations WfF instance is also similar to the Intelligence WfF instance; mission demands only occur within one Geographic COCOM, with all mission demands taking place in $EUCOM$. The information provide in Figure 54 allows for the construction of the sets $\mathcal{A}$, $\mathcal{A}_j$, $\mathcal{J}$, $\mathcal{J}_t$, and $\mathcal{J}_{ucl}$ as described in Section 3.2.1.

The cardinality of $\mathcal{J}$ for the Special Operations WfF instance is the smallest of all baseline instances tested, with only 26 indexed mission demands. Figure 55 presents the total number of unique mission demands that must be met each month. Indexed

Mission Demand (j)	Preferred Unit Type (u)	Preferred Component (c)	Location (l)	Initial Month (t)	Duration (Months)	Interval Between Mission Demands (Months)
1	Special Operations - 2	Active	EUCOM	1	Continuous	N/A
2	Special Operations - 2	Active	EUCOM	1	Continuous	N/A
3	Special Operations - 2	Active	EUCOM	1	Continuous	N/A
4	Special Operations - 2	Reserve	EUCOM	1	Continuous	N/A
5	Special Operations - 2	Reserve	EUCOM	1	Continuous	N/A
6	Special Operations - 2	Reserve	EUCOM	1	Continuous	N/A
7	Special Operations - 2	Active	EUCOM	9	1	11
8	Special Operations - 2	Active	EUCOM	9	1	11
9	Special Operations - 2	Reserve	EUCOM	9	1	11
10	Special Operations - 2	Active	EUCOM	1	Continuous	N/A
11	Special Operations - 2	Active	EUCOM	1	Continuous	N/A
12	Special Operations - 2	Active	EUCOM	1	Continuous	N/A
13	Special Operations - 2	Reserve	EUCOM	1	Continuous	N/A
14	Special Operations - 2	Reserve	EUCOM	1	Continuous	N/A
15	Special Operations - 2	Reserve	EUCOM	1	Continuous	N/A
16	Special Operations - 2	Active	EUCOM	9	1	11
17	Special Operations - 2	Active	EUCOM	9	1	11
18	Special Operations - 2	Reserve	EUCOM	9	1	11
19	Special Operations - 1	Active	EUCOM	11	1	11
20	Special Operations - 1	Reserve	EUCOM	11	1	11
21	Special Operations - 1	Active	EUCOM	11	1	11
22	Special Operations - 1	Reserve	EUCOM	11	1	11
23	Special Operations - 1	Active	EUCOM	11	1	11
24	Special Operations - 1	Reserve	EUCOM	11	1	11
25	Special Operations - 1	Active	EUCOM	11	1	11
26	Special Operations - 1	Reserve	EUCOM	11	1	11

Figure 54: Special Operations WfF Mission Demand

mission demands are a mix between those that occur continuously throughout the time horizon and those that occur singularly with an 11 month gap as intervals between occurrences. This instance requires 790 unique mission demands be met; that is, $\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}|\mathcal{J}_t| = 790$.

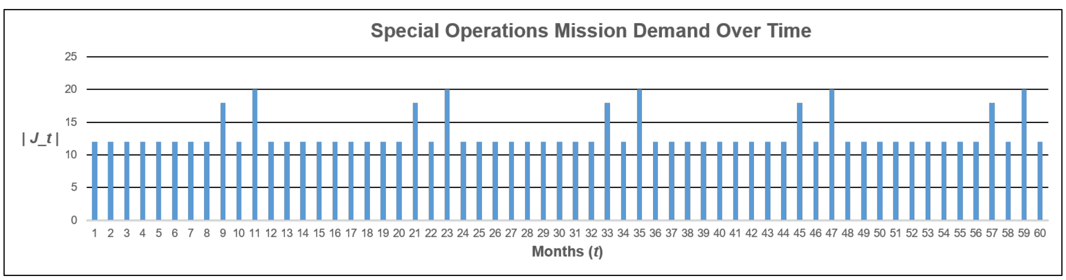


Figure 55: Special Operations WfF Consolidated Mission Demand Over Time

4.4.13 Special Operations WfF Results and Analysis

Table 26 presents the initial results from the Special Operations WfF instance, which solves to optimality in 1.95 seconds.

Table 26: Special Operations WfF Instance Computational Results

OBJ Value (z)	**# Cols (D.V.)**	**# Rows (Constraints)**	**Processing Time (s)**	**OPT Gap**
30,952.40	188,664	9,948	1.95	0.00%

This instance is also the smallest instance in terms of the IP's constraint matrix. The solution to the Special Operations WfF instance produces the unit mix outlined in Table 27. Despite having a series of mission demands that prefer $Special\ Operations-1$ type units, this instance does not employ any units matching that unit type. However this instance produces a relatively low number of mission demands met with non-preferred units, only 9.87% of units, as outlined in Table 28. This is primarily due to the fact that the vast majority of unique mission demands prefer $Special\ Operations - 2$ type units.

Table 27: Special Operations WfF Instance Unit Employment

Unit Type Employed	**# Units Employed (v^{ucl})**
$(Special\ Operations - 2, Active, EUCOM)$	24
$(Special\ Operations - 2, Reserve, EUCOM)$	26

Table 28: Special Operations WfF Use of Non-Preferred Units

Total Mission Demands	790
Routine Deployments with Non-Preferred Unit	70
Surges with Non-Preferred Unit	6
Extensions with Non-Preferred Unit	2
% of Mission Demands with Non-Preferred Units	9.87%

The average number of mission demands met per unit employed is 15.80 (AMPU).

This relatively high AMPU is once again directly attributed to the continuous nature of a majority of the indexed mission demands. This is clearly on display in Figures 55 and 56. This high AMPU is also reinforced by unit utilization as shown in Table 29.

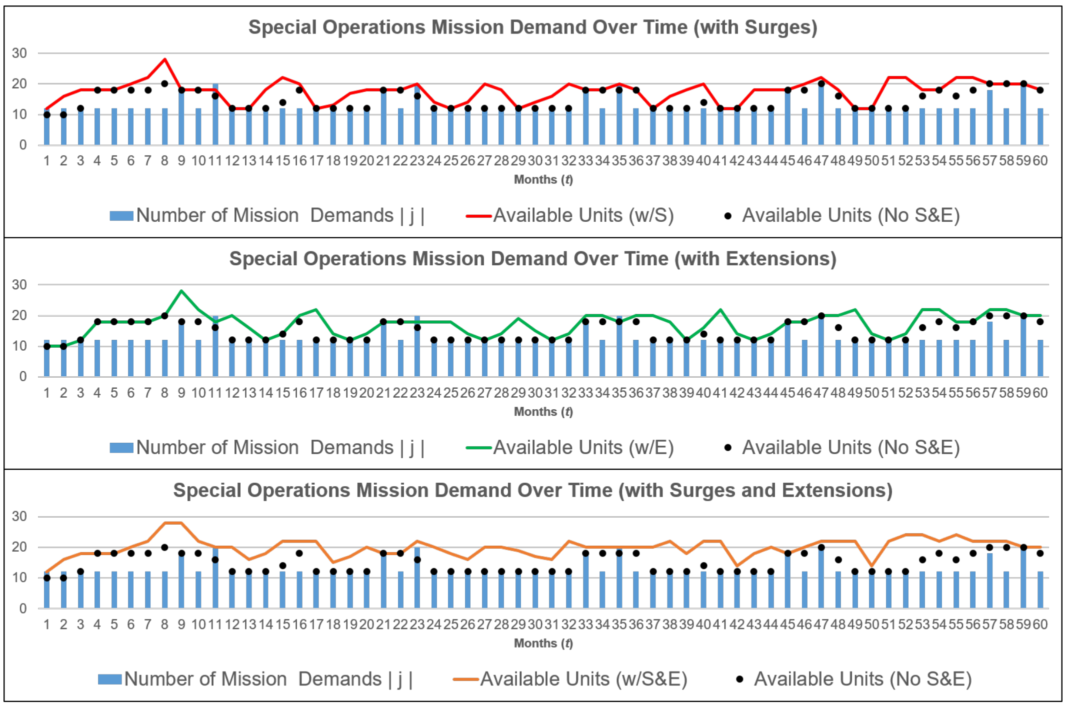


Figure 56: Special Operations WfF Consolidated Mission Demand Over Time - Sufficiency Results

This instance does require a significant number of surges and extension, as shown in Table 30. These surges and extensions take place throughout the time horizon as shown in Figure 56. The number of surges and extensions can also be attributed to the continuous nature of the mission demands; allowing some units to meet 9 or even 10 consecutive mission demands prior to redeployment.

Figures 57 and 58 show the unit employment plateau level for each set of units employed. These plateaus are the same value as those presented in Table 27.

Given this mission demand input data, the U.S. Army should not employ *Special Operations* − 1 type units. Although the employment cost associated with

Table 29: Special Operations WfF - Unit Utilization

Metric	R	RSE
Months with 100% Utilization	39	6
Months with < 100% Utilization	21	54
Average Units Not Utilized per Month	1.63	6.77
Total Unit-Months Not Utilized	98	406
Unit-Months Not Utilized per Unit Employed	1.96	8.12

Table 30: Special Operations WfF Instance Surges and Extensions

Total Mission Demands	790
Routine Deployments	774
Surges	10
Extensions	6

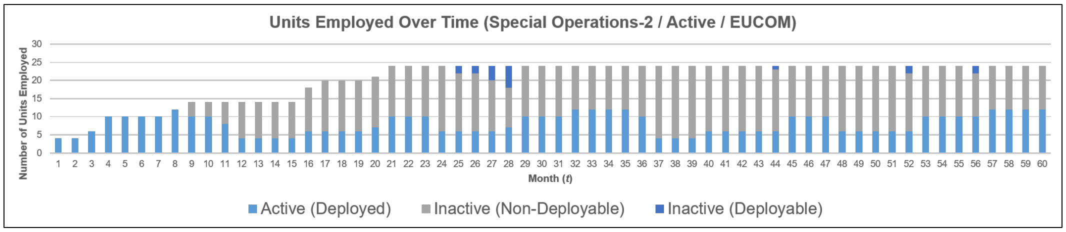


Figure 57: Special Operations WfF Unit Employment Over Time - Part 1 of 2

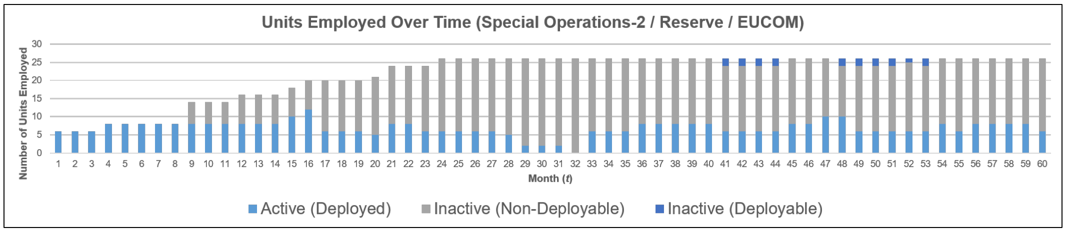


Figure 58: Special Operations WfF Unit Employment Over Time - Part 2 of 2

employing $Special\ Operations - 1$ units is more than the cost of employing a $Special\ Operations - 2$ type unit, regardless of unit Component status, the difference between these costs is not as drastic as the differences seen in the Fires and Mission Command WfF instances. The reason why the U.S. Army should not employ $Special\ Operations - 1$ type units is because of the small percentage of mission demands that prefer these units. This is why over 90% of mission demands are still met by the preferred unit in this instance, despite not employing $Special\ Operations - 1$ type units. These results also provide insight concerning the use of surges and extensions, especially when compared to the Intelligence WfF. Both of these instances involve primarily continuously reoccurring mission demands, however the Intelligence WfF instance solution only involves a single extension, while this instance's solution involves a combined 16 surges and extensions. The reason for this is related to when the remaining singular or semi-continuous mission demand initially occur. In the Intelligence WfF instance, these singular mission demands begin to occur immediately at $t = 1$, while the Special Operations WfF instance has singular mission demands that do not begin to occur until $t = 9$. In this situation the Intelligence WfF instance must meet mission demand through routine deployments, while the Special Operations WfF instance has the flexibility to utilize more surges and extensions.

4.4.14 Sustainment WfF Input Data

This instance places no constraints on which units by type, Component status, and Geographic COCOM alignment can be employed or utilized to meet each of the given mission demands. Similarly to the Mission Command, Protection, and Special Operations WfF instances, this instance's input data is completely unrestricted. This instance incorporates the use of two unique unit MTOE types; $\mathcal{U} = \{Sustainment - 1, Sustainment - 2\}$. The location of mission demands for this instance occur within

$CENTCOM$, $SOUTHCOM$, $INDOPACOM$, and $EUCOM$. The mission demand input data shown in Figure 59 allows for the construction of the sets $\mathcal{A}$, $\mathcal{A}_j$, $\mathcal{J}$, $\mathcal{J}_t$, and $\mathcal{J}_{ucl}$ as described in Section 3.2.1.

Mission Demand (*j*)	Preferred Unit Type (*u*)	Preferred Component (*c*)	Location (*l*)	Initial Month (*t*)	Duration (Months)	Interval Between Mission Demands (Months)
1	Sustainment - 1	Active	CENTCOM	2	3	9
2	Sustainment - 1	Active	CENTCOM	2	3	9
3	Sustainment - 1	Active	CENTCOM	2	3	9
4	Sustainment - 1	Active	SOUTHCOM	2	3	9
5	Sustainment - 1	Active	SOUTHCOM	2	3	9
6	Sustainment - 1	Active	SOUTHCOM	2	3	9
7	Sustainment - 1	Active	SOUTHCOM	2	3	9
8	Sustainment - 1	Active	INDOPACOM	2	3	9
9	Sustainment - 1	Active	INDOPACOM	2	3	9
10	Sustainment - 1	Active	INDOPACOM	2	3	9
11	Sustainment - 1	Active	CENTCOM	2	3	9
12	Sustainment - 1	Active	CENTCOM	2	3	9
13	Sustainment - 1	Active	CENTCOM	2	3	9
14	Sustainment - 1	Active	SOUTHCOM	2	3	9
15	Sustainment - 1	Active	SOUTHCOM	2	3	9
16	Sustainment - 1	Active	SOUTHCOM	2	3	9
17	Sustainment - 1	Active	SOUTHCOM	2	3	9
18	Sustainment - 1	Active	INDOPACOM	2	3	9
19	Sustainment - 1	Active	INDOPACOM	2	3	9
20	Sustainment - 1	Active	INDOPACOM	2	3	9
21	Sustainment - 2	Reserve	EUCOM	12	2	Not Reoccurring
22	Sustainment - 2	Reserve	EUCOM	12	2	Not Reoccurring
23	Sustainment - 2	Active	EUCOM	12	2	Not Reoccurring
24	Sustainment - 2	Active	EUCOM	12	2	Not Reoccurring
25	Sustainment - 2	Active	EUCOM	12	2	Not Reoccurring
26	Sustainment - 2	Reserve	EUCOM	12	2	Not Reoccurring
27	Sustainment - 2	Reserve	EUCOM	12	2	Not Reoccurring
28	Sustainment - 2	Active	EUCOM	12	2	Not Reoccurring
29	Sustainment - 2	Active	EUCOM	12	2	Not Reoccurring
30	Sustainment - 2	Active	EUCOM	12	2	Not Reoccurring

Figure 59: Sustainment WfF Mission Demand

This instance consist of 30 indexed mission demands. Figure 60 presents the total number of mission demands that must be met each month. The duration and frequency of the mission demands share the same qualities as those associated with the Protection WfF instance, with each of the mission demands being semi-continuous, and taking place over 2-3 month consecutive time periods. There are also mission demands that are non-reoccurring, similar to the Protection WfF instance. These mission demands begin at month $t = 12$ and last for two months, this is also graphically depicted in Figure 60. This instance requires 320 unique mission demands be met; that is, $\sum_{j \in \mathcal{J}} \sum_{t \in \mathcal{T}} |\mathcal{J}_t| = 320$. This is also the fewest amount of individual mission demands that must be met of all the WfF instances.

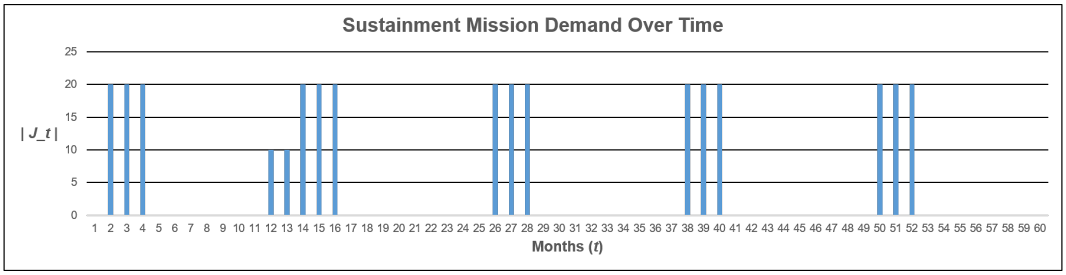


Figure 60: Sustainment WfF Consolidated Mission Demand Over Time

4.4.15 Sustainment WfF Results and Analysis

The Sustainment WfF instance solves to optimality in 0.68 seconds, demonstrated in Table 31. This instance requires the least amount of processing time when compared to the other WfF instances.

Table 31: Sustainment WfF Instance Computational Results

OBJ Value (z)	**# Cols (D.V.)**	**# Rows (Constraints)**	**Processing Time (s)**	**OPT Gap**
26,336.53	217,464	10,188	0.68	0.00%

The solution to the Sustainment WfF test instance produces the unit mix outlined in Table 32. The instance does not require the employment of any $Sustainment-2$ type units or any units aligned with $EUCOM$. This is due to the fact that the indexed mission demands that prefer $Sustainment-2$ units and occur within $EUCOM$ are non-reoccurring. This implies that these mission demands do not impact force planning as much, particularly in comparison to the remaining mission demands that do not prefer $Sustainment-2$ units nor occur within $EUCOM$, and do occur semi-continuously and repetitively. This is why the instance produces the lowest percentage of mission demands met with non-preferred units, at 6.25%, as shown in Table 33.

Table 32: Sustainment WfF Instance Unit Employment Mix

Unit Type Employed	**# Units Employed (v^{ucl})**
$(Sustainment-1, Active, CENTCOM)$	12
$(Sustainment-1, Active, INDOPACOM)$	12
$(Sustainment-1, Active, SOUTHCOM)$	16

Table 33: Sustainment WfF Use of Non-Preferred Units

Total Mission Demands	320
Routine Deployments with Non-Preferred Unit	20
Surges with Non-Preferred Unit	0
Extensions with Non-Preferred Unit	0
% of Mission Demands with Non-Preferred Units	6.25%

The average number of mission demands met per unit employed is exactly 8 (AMPU). This AMPU can be directly attributed to the intervals in which the semi-continuous mission re-occur. As can be seen in Figures 60 and 61. Similarly, this mid-range AMPU value is reinforced by the average degree of unit utilization among each WfF instance, as shown in Table 34.

Table 34: Sustainment WfF - Unit Utilization

Metric	**R**	**RSE**
Months with 100% Utilization	31	17
Months with $<$ 100% Utilization	29	43
Average Units Not Utilized per Month	8.00	14.00
Total Unit-Months Not Utilized	480	840
Unit-Months Not Utilized per Unit Employed	12.00	21.00

This instance does not require any surges or extension, as shown in Table 35. Figures 62-64 show the unit employment plateau level for each set of units employed.

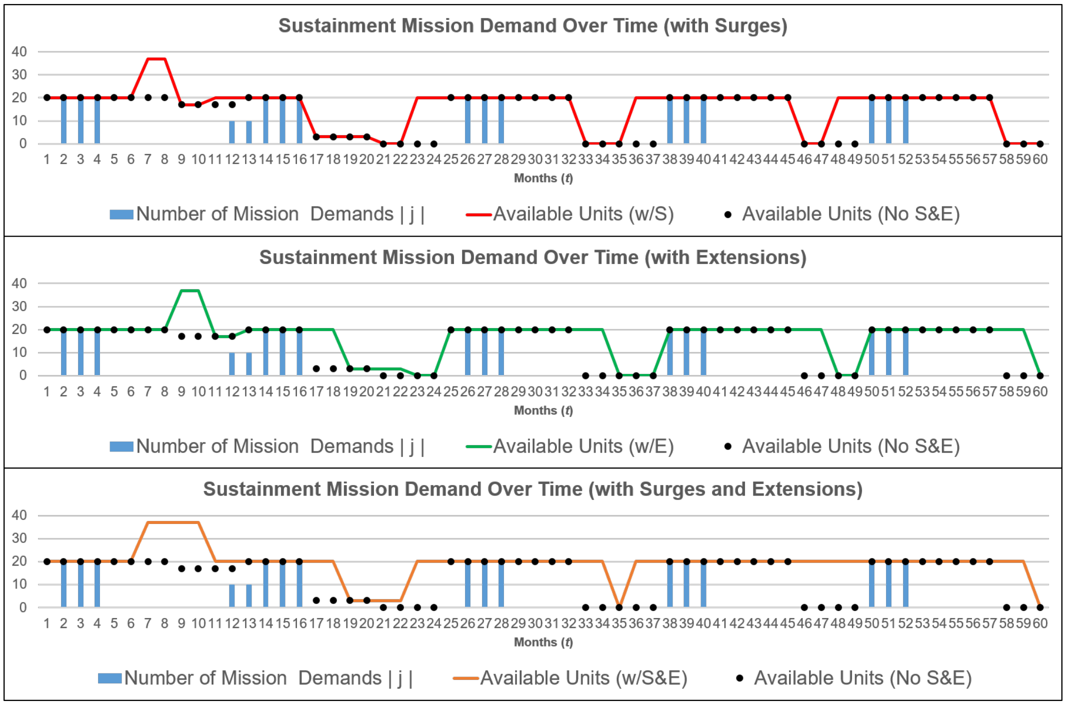


Figure 61: Sustainment WfF Consolidated Mission Demand Over Time - Sufficiency Results

These plateaus are the same value as those presented in Table 32.

Table 35: Sustainment WfF Instance Surges and Extensions

Total Mission Demands	320
Routine Deployments	320
Surges	0
Extensions	0

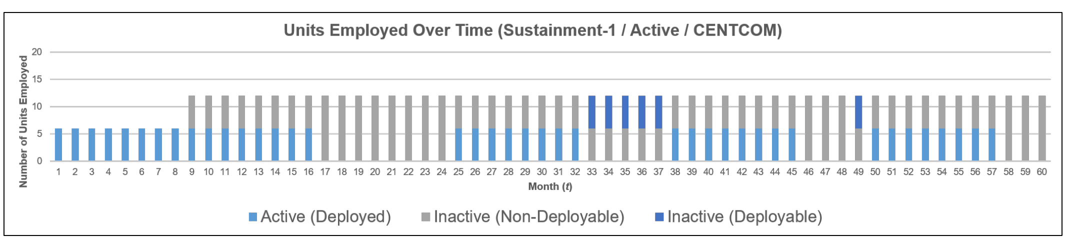


Figure 62: Sustainment WfF Unit Employment Over Time - Part 1 of 3

From this WfF instance, the U.S. Army should not employ $Sustainment-2$ type units for a number of reasons. The first reason is that only 20 of the 320 unique

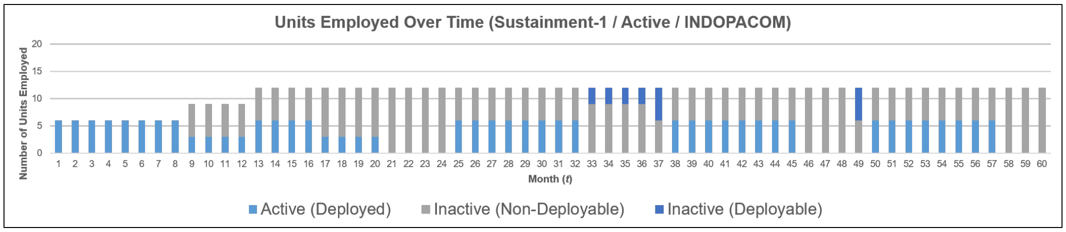


Figure 63: Sustainment WfF Unit Employment Over Time - Part 2 of 3

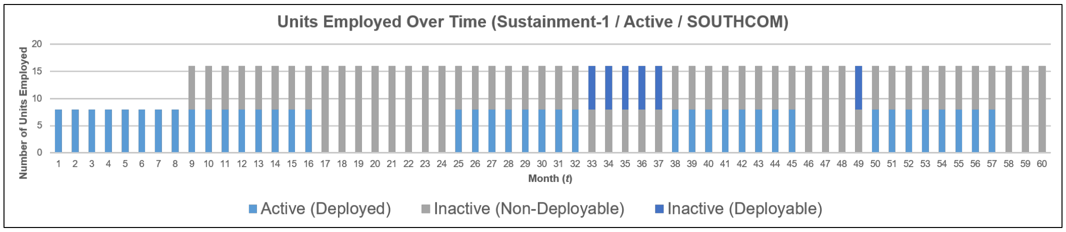
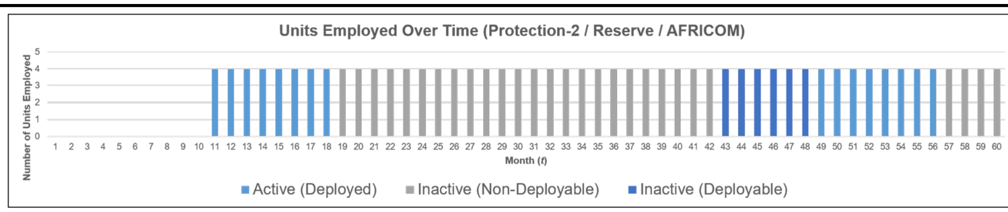


Figure 64: Sustainment WfF Unit Employment Over Time - Part 3 of 3

mission demands prefer $Sustainment-2$ type units. These mission demands are also "one-off" mission demands that do not cyclically repeat throughout the time horizon. This is similar to the Protection WfF instance, where the importance associated with these "one-off" mission demands will decrease if the time horizon is increased. Additionally, a $Sustainment-2$ type unit has a relatively poor suitability score when meeting mission demands preferring $Sustainment-1$ type units. A $Sustainment-2$ type unit also has a higher employment costs than a $Sustainment-1$ type unit, regardless of unit Component status.

4.4.16 Example Unit Deployment Schedule

The solution to a WfF instance can be obtained and subsequently translated into a deployment schedule for a unique unit or a series of unique units. This is not completed explicitly, but rather implicitly, given an instance's solution. Figure 65 depicts this deployment schedule construction for the four $(Protection-2, Reserve, AFRICOM)$ units that the model employs for the Protection WfF baseline instance. However, this implicit construction of a unit deployment schedule can be completed for any unit employed in any instance.



Month

Unique Unit	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	D	D	D	D	D	D	D	D	D	D	R	R - 19	R - 1	R - 1	R - 1	R - 1	R - 1	R - 1	I	I	I	I	I	I	I	I	I	I	I	I
2	D	D	D	D	D	D	D	D	D	D	R	R - 26	R - 2	R - 2	R - 2	R - 2	R - 2	R - 2	I	I	I	I	I	I	I	I	I	I	I	I
3	D	D	D	D	D	D	D	D	D	D	R	R - 39	R - 8	R - 8	R - 8	R - 8	R - 8	R - 8	I	I	I	I	I	I	I	I	I	I	I	I
4	D	D	D	D	D	D	D	D	D	D	R	R - 44	R - 9	R - 9	R - 9	R - 9	R - 9	R - 9	I	I	I	I	I	I	I	I	I	I	I	I

Surge Possible (month 10) Extension Possible (month 19)

Month

Unique Unit	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
1	I	I	I	I	I	I	I	I	I	I	I	I	D	D	D	D	D	D	R - 1	R - 1	R - 1	R - 1	R - 1	R - 1	R	R	I	I	I	I
2	I	I	I	I	I	I	I	I	I	I	I	I	D	D	D	D	D	D	R - 2	R - 2	R - 2	R - 2	R - 2	R - 2	R	R	I	I	I	I
3	I	I	I	I	I	I	I	I	I	I	I	I	D	D	D	D	D	D	R - 8	R - 8	R - 8	R - 8	R - 8	R - 8	R	R	I	I	I	I
4	I	I	I	I	I	I	I	I	I	I	I	I	D	D	D	D	D	D	R - 9	R - 9	R - 9	R - 9	R - 9	R - 9	R	R	I	I	I	I

Surge Possible (month 48) Extension Possible (month 57)

D: Deployable; R: Routine Deployment (Unit not utilized); R - *j*: Routine Deployment Meeting demand *j*; I: Inactive (Non-Deployable)

Figure 65: Example Deployment Schedule

As shown above, the four units first deploy at $t = 11$, initiating the deployment cycle for each unit. These units redeploy to begin their inactive period at $t = 18$. After the completion of the units' inactive period, which is also the conclusion of their cycles at $t = 42$, the units are then available to deploy again and are classified as "deployable". However, there are not mission demands occurring at this time and the units are not needed to deploy or meet a mission demand until time $t = 49$. At this point the units begin their second deployment, and therefore cycle, subsequently redeploying for modernization and training at time $t = 57$. This information is explicitly derived from the partial instance solution given below:

$$y_{11}^{(Protection-2,Reserve,AFRICOM)} = 4$$
$$y_{49}^{(Protection-2,Reserve,AFRICOM)} = 4$$

The decision variables concerning how unique mission demands are met provide the information required for an analyst or planner to then build a deployment schedule. These solution variables are presented below:

$$x_{(12,j)}^{(Protection-2,Reserve,AFRICOM)} = 1, \; j = (19, 26, 39, 44)$$
$$x_{tj}^{(Protection-2,Reserve,AFRICOM)} = 1, \; t = (13, .., 18, 49, ..., 54), \; j = (1, 2, 8, 9)$$

This solution means that all four units begin their first deployment at $t = 11$, but do not begin to meet unique mission demands until $t = 12$. At $t = 12$ the units individually meet one of the following mission demands, $j = (19, 26, 39, 44)$. The solution does not directly assign these four units to each mission demand, but they can be arbitrarily assigned to the first, second, third, and fourth unit, respectively. The preferred unit for each of these four mission demands are $(Protection - 1, Reserve, SOUTCOM)$ units, meaning that this instance incurs a non-preferred penalty cost for each of these mission demands being met, specifically for unit type and Geographic COCOM location alignment.

At $t = (13, ...18)$, mission demands $j = (1, 2, 8, 9)$ must be met, these mission demands can then also be systematically assigned to the the first unit, second unit, third unit, and forth unit respectively. Each of the indexed mission demands must be met during each of the months discussed, although an analyst or planner would not necessarily need to assign the same indexed mission demand to the same unit in consecutive months, they most likely would; this potentially avoids issues associated with units unnecessarily changing indexed mission demands month to month.

This mission demand assignment process continues when each of the units deploy a second time at $t = (49, ..., 56)$. Once again these units are assigned the same mission demands to meet that they were tasked with on their first deployments. This contributes to satisfying the ReARMM philosophy of preparing for and executing the same missions over time to better obtain and retain the institutional knowledge of operating in the same environment against the same threat. It is also important to note that $(Protection - 2, Reserve, AFRICOM)$ units are the preferred units for mission demands $j = (1, 2, 8, 9)$.

This example deployment schedule also allows analyst or planners to observe the operational flexibility that the employed units maintain given the mission demand input data. As shown in Figure 65, these units could be deployed earlier than either of their deployments to meet other mission demands if required. Additionally, none of these units exercise a surge or extension to meet mission demands, however, additional mission demands could be added to the deployment schedule during these potential surge or extension periods if required.

4.5 Consolidated Baseline WfF Instance Results

This section briefly provides consolidated results for all of the baseline WfF instances. These results examine the combined output of each instance, and ultimately allow the questions posed in this research's problem statement to be answered. Figure 66 illustrates the total number of missions each month that the must be met over the time horizon.

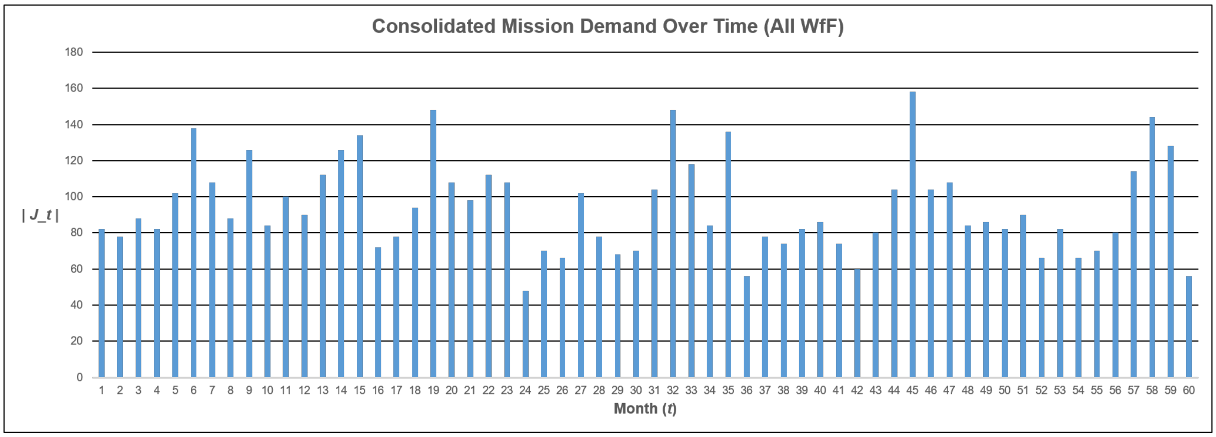


Figure 66: Consolidated Mission Demand Over Time (All WfF)

Table 36 provides the complete unit mix of all WfF instances. This output directly correlates to the question of "How many U.S. Army units, by MTOE type, Component status, and Geographic COCOM location alignment are required to meet all missions over a planning horizon".

Table 37 further addresses the research question by providing the degree to which the U.S. Army is able to adhere to its desired ReARMM goals. The mission demand input data effects these numbers, but more importantly the α_j^{ucl} parameter dictating the non-preferred penalty cost, significantly impacts how often non-preferred units meet mission demand.

Similarly, Table 38 provides the consolidated output showing the degree to which the U.S. Army is adhering to its policies mandating deployment lengths. The values

Table 36: Consolidated WfF Instances Unit Employment Mix

Unit Type Employed	**# Units Employed** (v^{ucl})
$(Fires-1, Active, AFRICOM)$	72
$(Fires-1, Active, EUCOM)$	16
$(Fires-1, Active, SOUTHCOM)$	16
$(Intelligence-1, Reserve, NORTHCOM)$	34
$(Intelligence-2, Active, NORTHCOM)$	45
$(Maneuver-1, Active, CENTCOM)$	15
$(Maneuver-1, Active, INDOPACOM)$	15
$(Maneuver-1, Reserve, CENTCOM)$	10
$(Maneuver-1, Reserve, INDOPACOM)$	8
$(Maneuver-1, Reserve, NORTHCOM)$	1
$(Maneuver-3, Active, NORTHCOM)$	4
$(Maneuver-4, Active, INDOPACOM)$	1
$(Maneuver-4, Active, NORTHCOM)$	24
$(Maneuver-4, Active, SOUTHCOM)$	16
$(Maneuver-4, Reserve, EUCOM)$	4
$(Maneuver-4, Reserve, NORTHCOM)$	16
$(Maneuver-5, Active, EUCOM)$	16
$(Maneuver-5, Reserve, EUCOM)$	8
$(Mission\ Command-2, Active, NORTHCOM)$	24
$(Mission\ Command-2, Active, SOUTHCOM)$	60
$(Mission\ Command-2, Reserve, NORTHCOM)$	12
$(Mission\ Command-2, Reserve, SOUTHCOM)$	8
$(Protection-1, Active, SOUTHCOM)$	16
$(Protection-1, Reserve, SOUTHCOM)$	20
$(Protection-2, Active, AFRICOM)$	28
$(Protection-2, Reserve, AFRICOM)$	4
$(Special\ Operations-2, Active, EUCOM)$	24
$(Special\ Operations-2, Reserve, EUCOM)$	26
$(Sustainment-1, Active, CENTCOM)$	12
$(Sustainment-1, Active, INDOPACOM)$	12
$(Sustainment-1, Active, SOUTHCOM)$	16
TOTAL	583

Table 37: Consolidated WfF Instances Use of Non-Preferred Units

WfF	All	Fires	Intel	Man	MC	Prot	SO	Sust
Total Mission Demands	5,660	640	1,310	1,460	560	580	790	320
Routine Deployments with Non-Preferred Unit	1,666	160	331	740	245	100	70	20
Surges with Non-Preferred Unit	15	0	0	4	5	0	6	0
Extensions with Non-Preferred Unit	12	0	0	0	10	0	2	0
% of Mission Demands with Non-Preferred Units	29.91%	25.00%	25.27%	50.96%	46.43%	17.24%	9.87%	6.25%

presented are still heavily influenced by the mission demand input data as well as the γ^{ucl} and β^{ucl} penalties incurred when surging or extending a unit to meet mission demand.

Table 38: Consolidated WfF Instances Surges and Extensions

WfF	All	Fires	Intel	Man	MC	Prot	SO	Sust
Total Mission Demands	5,660	640	1,310	1,460	560	580	790	320
Routine Deployments	5,623	640	1,309	1,455	545	580	774	320
Surges	20	0	0	5	5	0	10	0
Extensions	17	0	1	0	10	0	6	0

In addition to providing each of the WfF instance's AMPU, Table 39 provides the consolidated AMPU for all WfF instance, this value provides an indication of the utilization of the units the U.S. Army employs. An AMPU of 9.71 is a mid-range value and suggest an average level of efficiency of utilizing the all of the U.S. Army units employed. As discussed previously, this value is still heavily influenced by each of the WfF's input mission demand, specifically the duration and frequencies of indexed mission demands.

Table 39: Consolidated WfF Instances - Average Mission Demands Met per Unit Employed

WfF Instance	**AMPU**
Fires	6.15
Intelligence	16.58
Maneuver	10.58
Mission Command	5.83
Protection	8.53
Special Operations	15.80
Sustainment	8.00
Consolidated Results	9.71

4.6 Sensitivity Analysis

In order to test the effects of penalty cost in this model, a sensitivity analysis for the parameter α_j^{ucl}, the cost for a unit to meet a mission demand, is completed. A sensitivity analysis is also conducted for the parameters β^{ucl} and γ^{ucl}, the cost for a unit to operate under extension and surge conditions, respectively. Finally, a sensitivity analysis examining the effects of extending the time horizon T is completed.

Due to the nature of the baseline WfF instances, specifically their differences in mission demand input data, it would be exhaustive and not assuredly informative to complete a sensitivity analysis for each of the WfF instances. With that in mind, a single WfF instance is used in the sensitivity analysis investigation.

To determine which WfF instance on which to conduct sensitivity analysis, three distinct metrics are examined. These metrics are the number of combined surges and extensions, the percent of mission demand met by non-preferred units, and the average number of mission demands met per unit over the entire time horizon (AMPU). In a perfect world one of the baseline WfF instances would fit between both extremes of the evaluated metrics. This, however, does not occur, with each of the baseline WfF instances containing a metric result on at least one extreme. Of the three metrics, the combined number of surges and extension was deemed to be the least consequential metric of the three.

The Special Operations WfF baseline instance's only extreme result among the three metrics was the number of combined surges and extensions, specifically this WfF results in the most surges and extensions used among the WfF instances, both in pure quantity (16) and percent of total mission demands. The Fires and Protection WfF instances are conversely both tied with the fewest number of combined surges and extensions used (0). Of these three WfFs, it is determined that sensitivity analysis on the Protection WfF would result in the most robust findings. Compared to the Special Operations and Fires WfF, the mission demands for the Protection WfF still require each of the three unit factors to be included, preferring units of different types, and Component statuses, while also occurring within different Geographic COCOMs. Conversely, the Special Operations WfF instance's mission demands only occur within *EUCOM*, and the Fires WfF instance does not allow for *Reserve* Component status units.

4.6.1 Non-Preferred Penalty Cost - Sensitivity Analysis 1

In order to conduct a sensitivity analysis of the α_j^{ucl} penalty cost parameter, two additional instances are created. A '50% Cost' instance reflects a 50% reduction uniformally to the α_j^{ucl} parameter. The intent associated with this instance is to determine how much of an effect a given reduction in non-preferred penalty cost has when compared with the baseline instance. An 'M Cost' instance was also created, and is equivalent to not allowing units to meet a mission demand that they are not fully suitable for.

Table 40 provides the objective function value for each new instance. It is once again important to note that each of these new instances solve to optimality.

Table 40: Protection WfF Instance Computational Results - Sensitivity Analysis 1

Instance	**OBJ Value (z)**	**OPT Gap**
Baseline	55,282.99	0.000%
50% Cost	52,815.00	0.000%
M Cost	56,700.00	0.000%

When evaluating the new unit mixes (Table 41) produced by the both the '50% Cost' and 'M Cost' instances the effect of the α_j^{ucl} penalty cost is clearly seen. Specifically when evaluating the '50% Cost' instance, the total number of units employed remains the same, however the unit mix is changed. Here, the '50% Cost' instance employs only *Active* Component status units. This does not result from the increased ability to surge or extend units, as shown in Table 42. Instead this change to unit mix is derived from the unit employment costs Ω^{ucl} of the instance. In the Protection WfF input data, the employment cost of an *Active* unit is less than that of a *Reserve* unit. Reducing the non-preferred penalty cost allows the model to employ a less costly unit while still affording the penalty cost. The 'M Cost' instance provides us the exact number of units needed to meet all mission demands while utilizing only preferred units, fully embracing the ReARMM policy. Comparatively the 'M Cost' solution requires 72 total units employed versus the 68 total units required by the baseline solution.

Table 43 is also reflective of the changing α_j^{ucl} values. As previously mentioned, the '50% cost' instance employs only *Active* Component status units, with these units also required to meet all mission demands preferring *Reserve* status units. While still more cost effective, this results in the increase to percent of missions met with non-preferred units.

Both of these new instances involving changes to α_j^{ucl} do not result in any changes to unit surges or extensions, as seen in Table 42.

When evaluating both Tables 41 and 44, the '50% Cost' instance still employs the

Table 41: Protection WfF Instances Unit Employment Mix - Sensitivity Analysis 1

Instance	**Unit Type Employed**	**# Units Employed (v^{ucl})**
Baseline	$(Protection-1, Active, SOUTHCOM)$	16
Baseline	$(Protection-1, Reserve, SOUTHCOM)$	20
Baseline	$(Protection-2, Active, AFRICOM)$	28
Baseline	$(Protection-2, Reserve, AFRICOM)$	4
50% Cost	$(Protection-1, Active, SOUTHCOM)$	40
50% Cost	$(Protection-2, Active, AFRICOM)$	28
M Cost	$(Protection-1, Active, SOUTHCOM)$	16
M Cost	$(Protection-1, Reserve, SOUTHCOM)$	24
M Cost	$(Protection-2, Active, AFRICOM)$	20
M Cost	$(Protection-2, Reserve, AFRICOM)$	12

Table 42: Protection WfF Instance Surge and Extensions - Sensitivity Analysis 1

Instance	**Surges**	**Extensions**	**Combined**
Baseline	0	0	0
50% Cost	0	0	0
M Cost	0	0	0

Table 43: Protection WfF Instances Non-Preferred Unit Usage - Sensitivity Analysis 1

Instance	**% Mission Demand met by Non-Preferred Units**
Baseline	17.24%
50% Cost	37.24%
$\mathcal{M}$ Cost	0.00%

same number of units despite the difference in unit mix. This results in the same average number of mission demands met per unit employed as the baseline instance (AMPU). The 'M Cost', due to its additional constraints requires the employment of additional units. This unit employment increase is then reflected in a decrease to AMPU.

Table 44: Protection WfF Instances Average Mission Demands Met per Unit - Sensitivity Analysis 1

Instance	**AMPU**
Baseline	8.53
50% Cost	8.53
M Cost	8.06

4.6.2 Surges or Extensions Penalty Costs - Sensitivity Analysis 2

For the sensitivity analysis on the penalty cost associated with extending or surging a unit, three new instances are created to compare with the baseline results. The first new instance 'No Cost' applies no penalty for surging or extending a unit to meet a mission demand. The '50% Cost' instance decreases the cost of each surge or extension penalty cost, β^{ucl} and γ^{ucl} by 50%. Finally, the 'M Cost' instance completely prevents the use of surges or extensions. Table 45 provides the computational results for each of these new instances.

Table 45: Protection WfF Instance Computational Results - Sensitivity Analysis 2

Instance	**OBJ Value (z)**	**OPT Gap**
Baseline	55,282.99	0.000%
No Cost	54,991.32	0.000%
50% Cost	55,282.99	0.000%
M Cost	55,282.99	0.000%

The insignificance of changing the surge and extension penalty costs given this Protection WfF input data is even more clear when evaluating the remaining tables in this sensitivity analysis. Table 46 does show that the 'No Cost' instance employs a different unit mix when compared with each of the other instances. However the total number of units employed for each instance is 68. For this reason, the average number of mission demands met by units employed remains 8.53, for each instance, as shown in Table 47.

Table 46: Protection WfF Instances Unit Employment Mix - Sensitivity Analysis 2

Instance	**Unit Type Employed**	**# Units Employed (v^{ucl})**
Baseline	$(Protection - 1, Active, SOUTHCOM)$	16
Baseline	$(Protection - 1, Reserve, SOUTHCOM)$	20
Baseline	$(Protection - 2, Active, AFRICOM)$	28
Baseline	$(Protection - 2, Reserve, AFRICOM)$	4
No Cost	$(Protection - 1, Active, SOUTHCOM)$	25
No Cost	$(Protection - 1, Reserve, SOUTHCOM)$	11
No Cost	$(Protection - 2, Active, AFRICOM)$	24
No Cost	$(Protection - 2, Reserve, AFRICOM)$	8
50% Cost	$(Protection - 1, Active, SOUTHCOM)$	16
50% Cost	$(Protection - 1, Reserve, SOUTHCOM)$	20
50% Cost	$(Protection - 2, Active, AFRICOM)$	28
50% Cost	$(Protection - 2, Reserve, AFRICOM)$	4
M Cost	$Protection - 1, Active, SOUTHCOM)$	16
M Cost	$(Protection - 1, Reserve, SOUTHCOM)$	20
M Cost	$(Protection - 2, Active, AFRICOM)$	28
M Cost	$(Protection - 2, Reserve, AFRICOM)$	4

Table 47: Protection WfF Instances Average Mission Demands Met per Unit - Sensitivity Analysis 2

Instance	**AMPU**
Baseline	8.53
No Cost	8.53
50% Cost	8.53
M Cost	8.53

As expected, the 'No Cost' instance heavily utilizes the now free unit surges and extensions, as shown in Table 48.

Table 48: Protection WfF Instance Surge and Extensions - Sensitivity Analysis 2

Instance	**Surges**	**Extensions**	**Combined**
Baseline	0	0	0
No Cost	105	44	149
50% Cost	0	0	0
M Cost	0	0	0

The most interesting result is shown when analyzing both Tables 46 and 49. The unit mix employed by the 'No Cost' instance is drastically different from the baseline instance, however the percent of missions met by a non-preferred unit is only slightly increased. The 'No Cost' instance employs more *Active* units than the baseline instance employs, leading to additional mission demands intended for *Reserve* units being met by *Active* units. The penalty cost associated with using a non-preferred unit is still applied, however this cost is offset by the additional utilization gained by employing more *Active* units. When a penalty is not applied to surges or extensions, an *Active* unit can deploy for 12 months while a *Reserve* unit can only deploy for 10 months.

Table 49: Protection WfF Instances Non-Preferred Unit Usage - Sensitivity Analysis 2

Instance	**% Mission Demand met by Non-Preferred Units**
Baseline	17.24%
No Cost	17.93%
50% Cost	17.24%
M Cost	17.24%

4.6.3 Increase Time Horizon - Sensitivity Analysis 3

Finally, changing the time horizon is evaluated by examining a new instance with $T = 120$. The indexed set of mission demands for the Protection WfF baseline instance is still used, as are the indexed set $\mathcal{J}_t$ for months 1-60. At month 61, the updated $\mathcal{J}_t$ maintains the same unique missions at equivalent duration and frequencies. The key exception to this is the exclusion of mission demands $j = 15, ..., 54$, in time periods $t = 61, ..., 120$. As discussed in the Protection WfF baseline instance, these indexed mission demands are "one-off" mission demands, and not repeated with 60 month intervals between occurrences.

The '$T = 120$' instance involves 1,000 unique mission demands; that is $\sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{J}} |\mathcal{J}_t| = 1,000$. This is a increase of 420 unique mission demands.

Table 50 provides the computational results associated with the '$T = 120$' instance. The constraint matrix for this new instance is also twice the size of the baseline instance. This is reflective in the increase to processing time, however the optimality gap remains 0%.

Table 50: Protection WfF Instance Computational Results - Sensitivity Analysis 3

Instance	**OBJ Value (z)**	**# Cols (D.V.)**	**# Rows (Constraints)**	**Processing Time (s)**	**OPT Gap**
Baseline	55,282.99	390,264	11,628	1.43	0.00%
$T = 120$	56,206.11	780,504	23,508	5.12	0.00%

The resulting unit mix is slightly altered when comparing the baseline instance to the '$T = 120$' instance, shown in Table 51. Both instances employed the same number of units, 68, however the '$T = 120$' instance employs fewer $Protection - 1$ type units and fewer $SOUTHCOM$ aligned units. This can be accredited to the increase in the occurrences of the indexed mission demands $j = 1, ..., 14$ without the increase to mission demands $j = 15, ..., 54$. This change further exacerbates the utility of using $Active$ or $Reserve$, $Protection - 2$ $AFRICOM$ units.

The '$T = 120$' instance does not require any unit surges or extensions. This is expected because the duration and frequency of indexed mission demands does not change, despite the longer time horizon. This is explicitly stated in Table 52.

The percent of mission demands met by non-preferred units, presented in Table

Table 51: Protection WfF Instances Unit Employment Mix - Sensitivity Analysis 3

Instance	**Unit Type Employed**	**# Units Employed (v^{ucl})**
Baseline	$(Protection-1, Active, SOUTHCOM)$	16
Baseline	$(Protection-1, Reserve, SOUTHCOM)$	20
Baseline	$(Protection-2, Active, AFRICOM)$	28
Baseline	$(Protection-2, Reserve, AFRICOM)$	4
$T=120$	$(Protection-1, Active, SOUTHCOM)$	12
$T=120$	$(Protection-1, Reserve, SOUTHCOM)$	16
$T=120$	$(Protection-2, Active, AFRICOM)$	24
$T=120$	$(Protection-2, Active, SOUTHCOM)$	4
$T=120$	$(Protection-2, Reserve, AFRICOM)$	12

Table 52: Protection WfF Instance Surge and Extensions - Sensitivity Analysis 3

Instance	**Surges**	**Extensions**	**Combined**
Baseline	0	0	0
$T=120$	0	0	0

53, also illustrates the increased utility of using *Active* or *Reserve*, $Protection-2$ $AFRICOM$ units. With an increased ratio of unique mission demands preferring these units, combined with more of these units being employed; fewer mission demands will be met by non-preferred units.

Table 53: Protection WfF Instances Non-Preferred Unit Usage - Sensitivity Analysis 3

Instance	**% Mission Demand met by Non-Preferred Units**
Baseline	17.24%
$T=120$	12.00%

Finally, Table 54 provides the AMPU of the '$T = 120$' instance. This value is greater than the baseline AMPU, however this also accounts for a time horizon that is twice as long. If this AMPU value is normalized to read 'per 60 months', the normalized AMPU is 7.35. This can then be viewed as a decrease when compared with the baseline AMPU value of 8.53. However the '$T = 120$' instance does not involve a 100% increase to unique mission demands, only a 72% increase. In considering the “normalized” AMPU, there is not a notable change in unit utilization between instances.

Table 54: Protection WfF Instances Average Mission Demands Met per Unit - Sensitivity Analysis 3

Instance	**AMPU**
Baseline	8.53
$T = 120$	14.71

4.7 Significant Takeaways

The primary purpose of this chapter is to provide validation for this research’s model. Through both baseline WfF instances as well as sensitivity analysis, the efficacy of this model is proven. Each of the imposed constraints functions correctly, and the solution to each instance is not only feasible but optimal. However, these results clearly demonstrate how sensitive the model is to the input it is provided.

As shown throughout this chapter, the structure of the mission demand input, specifically the duration and frequency of indexed mission demands, influences the employment and utilization of units. The greater the clustering of mission demands, the more units must be employed, and utilization of individual units likely decreases. The Fires and Mission Command WfFs’ mission demands input data incorporates unique mission demands that are heavily clustered, non-continuous mission demands with regular and large frequency intervals. Conversely, the Intelligence and Special Operations WfFs’ mission demands are mostly continuous.

Additionally, the parameter values can greatly effect which units are employed. The individual α_j^{ucl} penalty costs can be completely dominated by the unit employment cost, Ω^{ucl}. A very clear example of this is found in the Fires WfF. The solution for the Fires WfF instance only employs $Fires-1$ type units. This means that for every mission preferring a $Fires-2$ or $Fires-3$ type units, a penalty is incurred. This penalty cost is derived from the employment costs of the preferred units, but considering the significant increase of employment cost between $Fires-1$ and either $Fires-2$ or $Fires-3$ units, combined with the suitability scores of a $Fires-1$ unit meeting mission demand preferring $Fires-2$ or $Fires-3$, this non-preferred penalty cost is an acceptable and more cost effective alternative over employing the preferred units for each mission demand. In this situation it is worth using the non-preferred unit. These observations can inform parameter construction for future test instances, both in terms of penalty weights, or even the scale of suitability scores.

4.8 Alternative Model

The model used for this research can be reframed to limit the number of non-preferred units used to meet mission demands and to limit the number of unit surges or extensions. This is easily accomplished through the use of additional parameters that limit the number of mission demands that can be met by non-preferred units, or by units surging/extending their deployment. In doing so, the objective function **(1)** is also more easily interpreted directly in terms of the total number of personnel employed. For example if U.S. Army policy wants to strictly limit the number of mission demands met by non-preferred units, that value can be used as the right hand side of the new constraint. The new constraint can apply to all mission demands, mission demands per time, or a specific mission demand or set of mission demands. Alternatively, this parameter can be further specified to limit use of a specific non-preferred unit by u, c, l, or any combination of the three factors. A similar process can be used to limit the number of surges or extensions.

The alternative model presented below, limits the total number of mission demands met by non-preferred units, as well as the total occurrences of unit surges or extensions. This is a partial model presentation; any set, parameter, decision variable, objective function component, or constraint from the original model not specifically discussed also still applies, unaltered, to this alternative model.

4.8.1 Alternative Model - Parameters Redefined

Alternative Parameters:

$\alpha_j^{ucl} = 1$ if unit (u, c, l) is a non-preferred unit to meet mission demand j,

0 otherwise.

β^{ucl}: Not needed.

γ^{ucl}: Not needed.

π^1: Max number of mission demands that are met by a non-preferred unit.

π^2: Max number of mission demands that are met by a unit executing a surge or extensions.

4.8.2 Alternative Model Formulation

Min

$$\sum_{(u,c,l)\in\mathcal{A}} \Omega^{ucl} v^{ucl} \tag{1}$$

Subject To:

$$\sum_{(u,c,l)\in\mathcal{A}} \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{J}} \alpha_j^{ucl} x_{tj}^{ucl} + \sum_{(u,c,l)\in\mathcal{A}} \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} \alpha_j^{ucl} (s_{tjk}^{ucl} + w_{tjk}^{ucl}) = \pi^1 \tag{20}$$

$$\sum_{(u,c,l)\in\mathcal{A}} \sum_{t\in\mathcal{T}} \sum_{j\in\mathcal{J}} \sum_{k\in\mathcal{K}} s_{tjk}^{ucl} + w_{tjk}^{ucl} = \pi^2 \tag{21}$$

Constraints **(6)** - **(19)** from original model.

4.8.3 Alternative Model Review of Objective Function and Constraints

This alternative model's objective function only relies on a single component from the original objective function, **(1)**. This function minimizes the total unit employment cost of an instance. Components **(2)** to **(5)** from the original model's objective function are unnecessary in this alternative model, as they are now modeled in the constraints.

Constraints **(6)** to **(19)** from the original model still apply to this alternative model. The first new constraint, Constraint **(20)** limits the total number of non-preferred units used to meet mission demands under routine deployment conditions and the number of non-preferred units used to meet mission demands under surge or extension conditions to π^1. Finally, Constraint **(21)** limits the total number of occurrences of unit surges or extensions to π^2.

V. Conclusions

This chapter briefly provides a summary of this research's key findings and contributions, and concludes with potential avenues for future research.

5.1 Summary of Findings and Contributions

The result of this research is an integer programming model that optimally determines how many U.S. Army units by MTOE type, Component status, and Geographic COCOM location alignment are needed to meet all mission demands over a planning horizon, given a set of forecasted mission demands, cost parameters, and operational guidelines. In keeping with the ReARMM framework, this model penalizes the use of non-preferred units. This model also penalizes occurrences of units meeting mission demand outside of their routine deployments, when these units are required to surge or extend their deployment to meet mission demand. When solving the model, all test instances solve to optimality in a timely manner, with solutions obtained in less than 30 seconds.

In addition to obtaining optimal solutions, a key finding of this research is that the number of units required to meet mission demand is heavily dependent on the timing, duration, and recurrence of mission demands. In reality, the U.S. Army would most likely prefer near 100% utilization for units deployed, meaning that these units are actively meeting mission demands while deployed, rather than simply being available. Based on the results in Chapter IV, units are often heavily underutilized if mission demands are clustered together over short time horizons, and units are needed to meet sporadic missions demands rather than those that occur in consecutive time periods. The duration and frequency of mission demands not only dictate when units begin their deployments, but how many unique mission demands are met during each unit's deployment.

This research specifically examines the employment and use of U.S. Army units to meet mission demand given a set of mission demands; each unit and each mission demand account for three different facets of ReARMM, unit type, Component status, and Geographic COCOM location alignment. In addition to penalizing the use of non-preferred units and surges and extensions, this research also accounts for the length of unit deployments and the length of a unit's cycle. This unit cycle consisting of an active period (the deployment), and an inactive period (the modernization and training period). A unit's Component status specifically dictates the capability of a unit in terms of time. In this model a *Reserve* unit has less capacity for surges and extensions, and can also deploy less frequently than it's *Active* counterpart.

Each of these facets to the model can be re-framed to determine a solution for a generic, non-U.S. Army oriented problem instance. This model formulation and diversity to different applications is perhaps the most significant contribution of this research. The model can be used to solve any problem that can be described as a 'Work Force Optimization and Sufficiency Problem'. Unit MTOE type can be translated to 'worker type', Component status can be translated to 'Full-time / Part-time status', location alignment can be translated to 'Department' or 'Area'. Conversely, a similar problem may involve additional or different factors, all of which simply translate to the qualifications (or specialty, utilization, etc.) of a worker, while similarly replacing mission demand unit preference with job/assignment worker preferences. A generic worker, like a U.S. Army unit can only be used for specific amount of time, with the duration and frequency of work hours typically dictated by a company's policies and the specific type of worker. Regardless of the specific work force optimization problem, this model can directly or through small modifications, be used to obtain an optimal solution.

5.2 Future Work

One limitation of this model is that it is not a pure 'assignment' model. The output of this model only conveys which units by type, Component status, and Geographic COCOM location alignment should be tasked with meeting specific mission demand. This model does not assign a specific unit, (i.e. 1-4 Cavalry Squadron) to a specific mission demand, (i.e. $j = 1$ at $t = 1$). Future research should consider a paired model that takes current model's output and actually assigns 'named' units for specific mission demands. This second model would also be able to further correct additional deviations from reality. As discussed in Chapter 3, the original model cannot prevent an individual unit from being assigned consecutive mission demands outside of the same Geographic COCOM, despite the incursion of a penalty cost. The pair model, with named units could rectify this issue.

Another avenue for future research should consider a 'reinterpreted' model that only minimizes cost of unit employment. Here the number of units that are extended or surged or the number of units that meet mission demand that they are not fully suited for would be limited by a series of constraints. These constraints replace their associated components in the current model's objective function. An initial approach to this avenue is presented in Chapter IV, however this only served as brief introduction and can be further altered based on user preferences or restrictions.

A value focused thinking (VFT) approach to weighting the penalties associated with using a non-preferred unit by type, Component status, and Geographic COCOM alignment, could also be used. Although the penalties of the current model were weighted and subsequently applied in an objective manner, a VFT approach with actual conversations between an analyst and a decision maker would remove much of the subjectivity from the current model.

Future research could also consider an analysis of computational complexity of this current integer programming model. Although not proved in this research, the model's constraint matrix may be totally unimodular (TU). This may be why the solver's branch and bound always solves to optimality at the root node. Additionally, exploration on the 'difficulty' of the problem (e.g., NP, NP-Complete, etc.) might inform modeling or solution methodology for larger sized problem instances.

Finally, the current model can be extended. Within the context of the U.S. Army, a WfF can be more accurately described as a classification for a collection of units that can accomplish a similar but not necessarily identical set of tasks. Units classified under a specific WfF can typically accomplish some, but not all of these tasks. For example, a Light Infantry Battalion and an Attack Aviation Squadron are both Maneuver units, they share the ability to execute some of the same tasks, but not all of the same tasks. These units would not normally be tasked to meet identical mission demands. Additionally, not every unit fits neatly into a single WfF. A Brigade Engineer Battalion (BEB) consist of subordinate Companies that can be classified as Maneuver, Mission Command, and Intelligence, WfF elements; therefore the BEB could be given a mission demand corresponding to any of those WfFs. Furthermore, for this research the largest instance tested, the Maneuver WfF, only includes five different unit MTOE types. In reality the Maneuver WfF, at the battalion echelon, may include units classified as Armor, Infantry, Engineers, or Aviation; with the actual number of different Maneuver MTOE unit types closer to 20 then simply 5. Additionally the Reserve Component status can be further separated into the Army Reserve and the Army National Guard. Some MTOE units exist within the Active, Army Reserve, and Army National Guard components, while others only exist in one or two of the components. These are all factors that could be considered and accounted for if this research's model is extended. This avenue of future research can bring the model closer to the real-world input data and parameters.

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14. ABSTRACT

The United States Army perpetually deploys rotational forces across the globe in support of the National Security Strategy. These forces meet a set of discrete mission demands over an extended time period before redeploying, modernizing, and preparing for the next deployment. The U.S. Army now utilizes the Regionally Aligned Readiness and Modernization Model to execute these cyclical stages for unit deployments. Specific emphasis is placed on aligning forces against a Geographic Combatant Command, which allows units to build readiness and lethality oriented towards the same series of threats, physical terrain, and civilian considerations. This research provides an Integer Programming model that offers the U.S. Army an optimal solution outlining how many units by Modification Table of Organizational Equipment, Active or Reserve Component Status, and Geographic Combatant Command location alignment, needed to meet every mission demand, for a prescribed set of time periods, at the battalion level echelon.

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