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MODELING AND ANALYZING THE EFFECT OF GROUND REFUELING CAPACITY ON AIRFIELD THROUGHPUT

THESIS

W. Heath Rushing, Lt., USAF

AFIT/GOR/ENS/97M - 19

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MODELING AND ANALYZING THE EFFECT OF GROUND REFUELING CAPACITY ON AIRFIELD THROUGHPUT

THESIS

Presented to the Faculty of the Graduate School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Operations Research

W. Heath Rushing, B.S

Lt., USAF

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THESIS APPROVAL

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Abstract

This study develops five analytical models to understand the current ground refueling process, to optimize the airfield configuration and to determine the refueling policy which maximizes throughput, the primary measure of airfield efficiency. The airfield refueling system is a complex network of aircraft arrivals and departures from two refueling systems, a hydrant system and a truck system. While there is no significant difference in each system's service rates, there are many more trucks than hydrants. However, trucks have a limited capacity and must refill. Although simulations have been developed to understand this process, they do not provide an optimal airfield configuration which minimizes the average time in the system or a refueling policy which maximizes throughput. In order to fulfill this need, a linear program was developed to maximize airfield throughput, but because it fails to adequately represent variable ground times, airfield capabilities are overestimated. This study models the airfield refueling process as a continuous time Markov process to adequately represent the inherent stochastic nature of the transitory ground refueling system and provide an analytical evaluation of various airfield configurations. Also, the study provides an optimal refueling policy to minimize the number of aircraft on the ground which in turn minimizes the average amount of time aircraft spend on the ground. By accomplishing this, higher throughput rates can be achieved by allowing a higher aircraft arrival rate into the airfield. The first four models are demonstrated using data from a transient airbase,

Hickam AFB, HI while the fifth model is demonstrated using a small, capacitated airfield.

MODELING AND ANALYZING THE EFFECT OF GROUND REFUELING CAPACITY ON AIRFIELD THROUGHPUT

Chapter 1

Introduction

1.1 Background.

"As our military increasingly becomes a US-based[,] power projection force, our transportation assets become an even more crucial element of the national defense. Without possessing the ability to rapidly and efficiently move our service personnel and their equipment into an overseas theater of operations, all of the money we have spent and all of the effort we have put into building the strongest armed forces in the world would be for naught."

-Senator John Warner, addressing the Senate, May 19, 1996.

In the post-Cold War era, the evolution of our national defense strategy required the US military to become a more responsive force, ready for any contingency that might arise, worldwide. This strategy of global involvement caused our policy of *forward basing* of troops to change to a policy of *forward presence* of troops, meaning, if a conflict should arise, troops and equipment would require deployment (12:18). Because of this change in strategy, global mobility has become the foundation of our national security strategy (7:6). This concept was never more apparent than during Operation Desert Storm, which required the largest airlift in the history of the world.

The lessons learned from Desert Storm showed that "Air Mobility Command (AMC) needed to make changes to improve airlift efficiency" (5:4). Using the insights gained from Desert Storm, AMC laid out a strategy to meet the nation's new national security strategy. AMC's strategy was based on achieving four goals, the first of which was "to improve mission effectiveness by optimizing its force structure despite limited resources to produce more efficient and effective air mobility operations that support a wide range of contingencies" (6:5). Emphasizing this goal in their strategy provided AMC with a vision for future analysis of operations: to produce more efficient operations through the optimal use of current strategic airlift resources.

The primary measure of efficiency for mobility analysis is throughput, the maximum amount of cargo that can flow through an airfield in a day. Throughput depends on the amount of cargo each plane carries and the working maximum-on-ground (MOG) of an airfield (20:22). MOG is defined as "the maximum number of aircraft on the ground that can land, taxi-in, park, be unloaded, refueled, maintained, inspected, loaded, taxi-out, be cleared for departure and takeoff within a planned time interval" (24:1). In an unconstrained world, the key to maximizing this throughput is to allow more aircraft in the system, especially those aircraft that deliver "the most on every arrival" (25:3).

The two factors believed to have the greatest potential to increase the throughput of an airfield are ramp parking and refueling capacities (4:1). Refueling capacity has historically constrained the efficiency of the airlift system, even when ramp space is unconstrained. For example, in Desert Storm, ramp parking space was virtually infinite at Dhahran, Saudi Arabia. However, as more aircraft arrived, ground times increased due to the limited capacity of the refueling system (26:4). As more aircraft waited, takeoffs were delayed, which ultimately caused a bottleneck on the ground. This bottleneck caused the overall airlift schedule to be delayed, resulting in an inefficient airlift operation.

Although strategic airlift is a composition of many systems working together, this study concentrates on the analysis of ground refueling operations at an airfield. Focusing this study on only one system is a valid approach because each particular system in the strategic airlift system needs "to be scrutinized if the whole is to be utilized to its maximum capacity" (28:2). The organization responsible for strategic airlift operations, AMC, requires analysis of the efficiency of current refueling operations to increase an airfield's ability to rapidly refuel aircraft during a contingency.

Currently, mobility analysis is accomplished primarily through the use of simulation models under various strategic airlift scenarios (the principal model used is MASS (Military Airlift Support System)). These simulations require the building of a database of input files with the necessary routes (origin to destination) and the current airfield capabilities. Although these simulations are useful, the processing of input data takes an average of two weeks to accomplish and the results of these models do not provide immediate insight into any one area of the airlift system. Therefore, AMC has begun modeling crucial areas of the airlift system to gain insight into the individual areas of the system composition in order to provide larger models such as MASS with more accurate representations of the true system.

In particular, AMC is developing a model called BRACE (Base Resource and Capability Estimator) to estimate the relationships between the various airlift characteristics of an airfield. BRACE is an interactive model that estimates airfield throughput capacity based on current "in-place" resources. This model is also used to evaluate any planned changes in airfield resources (increase or decrease) and to plan for contingencies which require pre-positioned resources in order to meet an airlift need. Additionally, BRACE is used to estimate the working MOG of an airfield, ground time distributions and airfield queuing.

Simulation models are generally used to understand a complex, real world system. The large mobility simulations are generally used to determine if required due dates (for logistics) can be met, given a number of inputs including an airfield's working MOG. This MOG constraint is found using smaller simulation models (BRACE for example) and encompasses all aspects of an airfield including ramp space, onloading/offloading resources, maintenance capabilities, and refueling. These models answer questions concerning the adequacy of airfield configurations but do not answer questions concerning the optimality of ground operations (23:5). For example, BRACE evaluates the "in-place" refueling system efficiency using the current refueling policy and represents the amount of fuel needed per aircraft as a deterministic value.

Two techniques are used in this study to model and analyze the ground refueling policy/operations: stochastic modeling and mathematical programming. Stochastic modeling is used in order to capture the true non-deterministic nature of arrivals and service times in an airfield refueling system. Mathematical programming uses sound mathematical analysis techniques to provide optimal solutions to problems (23:7). This mathematical modeling technique calculates optimal values for variables based on estimates of coefficients and constraints. This technique is used in order to find the optimal airfield refueling policy using constraints determined through stochastic analysis and cost coefficients provide by AMC.

Aircraft are ground refueled in one of two ways: by a hydrant system or by a truck system. Hydrant outlets are available on most, but not all, runway parking spaces. If a parking space with a hydrant outlet is available, an aircraft parks in the space and becomes part of the hydrant system or the truck system, whichever becomes available first. A typical airfield has hydrant outlets on all tanker parking spaces and 75% of all strategic airlift parking spaces. If no parking space with a hydrant outlet is available, the aircraft joins the truck refueling system. Each aircraft utilizes one truck at a time for refueling. The trucks have a capacity of 5600-5700 gallons and require time to refill once they are empty. Truck refilling at a fillstand takes approximately 15-20 minutes to accomplish. A typical airfield has 9-30 refueling trucks depending on the current hydrant system, the number of aircraft at the airfield and the expected flow rate of aircraft through the airfield.

The per-aircraft rate at which each hydrant system refuels an aircraft is faster than that of an individual truck. Trucks refuel at a rate of 550 gallons per minute (gpm) while hydrants refuel at a rate of 600 gpm. Hydrants can refuel at a much faster rate, but are constrained by the aircraft intake rate. Although the refuel rates are not much different, trucks have to refill upon emptying their tanks. This may cause delays in the refueling process. AMC's current refueling policy is to send an arriving aircraft to the fastest available refueling resource. AMC has requested research support to provide insight on airfield refueling operations and to determine if this refueling policy maximizes the throughput of the airfield. If the current policy is not optimal, they request an airfield refueling policy which maximizes throughput.

1.2 Initial Research.

AMC provided airfield data for one simulated contingency that requires operations at Hickam AFB, Hawaii, a transient airbase. A transient airbase only performs refueling and emergency maintenance on arriving aircraft. For this reason, evaluating operations at this type of airfield isolates the refueling system and its effect on throughput. The data lists aircraft, arrival times, and fuel required to accomplish the next leg of the mission. The next leg is defined as the distance until the next refueling, either at another base or at an air-to-air refueling point. The data reflects operations over a period of 798 hours. The represented aircraft are the C-141B, C-17, C-5A and Wide Body Craft (WBC).

Aircraft arrival times and the amount of fuel demanded are the two crucial data elements required to model the airfield refueling process. From the listed data, the interarrival times are calculated and tested to ensure the validity of the assumption that the underlying distribution is exponential. If the assumption is valid, the aircraft arrivals can be assumed to be generated by a Poisson process. The service rate depends on the individual refueling rates (for each system) and the amount of fuel needed per aircraft. Although the specific fuel system service rate is constant, the amount of fuel the aircraft needs is a random variable. As the state of the system changes as a result of an event (such as an aircraft arrival or departure), the refueling system "restarts" the refueling process. Therefore, the remaining service time for an aircraft being refueled does not depend on the amount of time the aircraft has already been in service. For this reason, the departure or service rates are assumed to be exponentially distributed and the time between system state transitions are assumed to be "memoryless".

This study models the airfield refueling system using five models which sequentially focus on creating a more detailed representation of the aircraft refueling process. Two of the five models determine any impacts due to trucks being delayed while refilling at a fillstand. For these models, the rate at which the truck refills at a fillstand is represented by ε_1. The rate the trucks refill is 1-3 per hour per fillstand at the airfield. For example, Hickam AFB has 6 fillstands, so the trucks refill at a rate of 6-18 per hour. The rate at which trucks refuel an aircraft is represented by γ. The fuel trucks carry approximately 5500 gallons of gasoline and refuel aircraft at a rate of 550 gallons per minute (gpm). Since the amount of fuel an aircraft needs is assumed to be exponentially distributed, a truck either refuels an aircraft or runs out of fuel before the aircraft is refueled. Likewise, there are associated probabilities p_1 and p_2 (which is equal to $1 - p_1$) associated with each event. These probabilities depend on the average amount of fuel demanded by the aircraft and the amount each truck carries, 5500 gallons. This probability along with the refuel rate γ determines the system state upon transition.

The data shows that a proportion of the WBCs do not require fuel. WBCs are civilian aircraft that are refurbished for use in the transportation of military loads during contingencies. Air Force resources are not always used to refuel these aircraft, but they still occupy Air Force resources during unloading/loading of cargo. These aircraft refuel at civilian airports and are not considered as arrivals to the airfield refueling system.

1.3 Definition of Terms.

Truck system - The refueling system consisting of those aircraft being refueled by refueling trucks. Trucks carry approximately 5500-6500 gallons of fuel and pump fuel at a rate of approximately 550 gpm.

Hydrant system - The system consisting of those aircraft being refueled by airfield hydrants. Because only a certain number of hydrants are used at one time (designated as "active" hydrants), some of the "hydrant" aircraft are located on "inactive" hydrant system spaces, waiting for a hydrant to become available. The hydrant system pumps fuel at an overall rate of 2400 gpm, which remains constant regardless of the number of aircraft being serviced by the hydrant system. For example, if the hydrant system is refueling six aircraft, each receives fuel at a rate of 400 gpm. All aircraft are limited by an individiual receiving rate of 600 gpm. The hydrant system therefore discharges fuel at a rate less than its full capacity when it is servicing fewer than four aircraft.

Fillstand - The system the trucks return to in order to refill once they have either emptied their tank or refueled an aircraft. The fillstands' service rate is approximately 600 gpm.

Demand for fuel - This is the amount of fuel an aircraft needs until its next scheduled refueling (either air-to-air or ground).

Aircraft Arrival - An event which occurs when an aircraft arrives at the refueling system.

Aircraft Departure - An event which occurs when an aircraft leaves the refueling system for taxi.

Single Server Process Sharing System with Capacity - This characterizes a queuing system with one server that can simultaneously serve *n* customers. The overall service rate is constant, with service evenly distributed equally over all customers in service.

Markov decision process - A Markov process where a decision is made at each state of the system. Each decision will lead to a different future distribution of system states.

1.4 Problem Statement.

The efficiency of strategic airlift capability can be measured by aggregating the cargo throughput at an individual airfield. For our purposes, throughput depends on the aircraft load sizes and the rate at which aircraft can be refueled. AMC's current airfield refueling policy assigns each arriving aircraft to the fastest available server. This policy has never been shown to maximize throughput. Therefore, the airfield refueling system needs to be modeled and analyzed to evaluate the current policy and find a policy which maximizes airfield throughput (if appropriate).

1.5 Objectives.

The primary objective of this study is to develop a tool which determines the refueling policy that maximizes throughput or the amount of cargo that can flow through an airfield per day. Secondary objectives include a better understanding of airfield refueling operations and how their efficient use can increase airfield throughput.

1.6 Scope.

Four models are developed to analyze the current refueling policy, and a fifth model is developed to optimize the refueling policy for a particular airfield. The first four models are used to understand the refueling process and to determine the refueling system state space that best represents the refueling operations of an airfield. With this knowledge, the fifth model is built to determine an optimal refueling policy, with the objective of maximizing throughput. This model produces the constraints for a linear program with objective function coefficients provided by AMC. Although the objective of this demonstrated linear program is to maximize throughput by minimizing the number of aircraft on the ground, it can be changed within the program to account for any required objective. A comparison of the current refueling policy and the refueling policy given by the fifth model is made in order to determine if the current refueling policy is optimal.

1.7 Approach.

All models are continuous time Markov processes, with the state of the system depending only on the present state, and independent of past states. The refueling system can be modeled as Markov processes if all aircraft arrivals are assumed to be generated by a Poisson process and all service times are assumed to be exponentially distributed. This assumption about the service times means the remaining time in service is independent of the amount of time the aircraft/truck has been in service. Therefore the amount of time the system is in that state is assumed to be memoryless. The hydrant system is modeled as a single server process sharing system with capacity. The capacity of the hydrant system is the maximum number of aircraft that can be simultaneously serviced by the system. The truck system is modeled as a multi-server queuing system, with the service rate depending on the number of available trucks and the number of aircraft in the truck system.

The first model is a birth and death process. Each transition from a state can only move to an adjacent state. This model has one state variable, the number of aircraft in the system. Each arriving aircraft is served by the first available server. Since this model is based on the current refueling policy, the first four arriving aircraft are sent to a hydrant, while succeeding aircraft are sent to a truck system. If an aircraft is sent to the truck system, it is served by the first available truck server. This model represents a system in which, whenever an aircraft leaves the hydrant system, an aircraft from the truck system is immediately moved to the hydrant system for servicing. This characteristic of the modeled refueling process does not represent the actual operation. Although model two represents the system in the same way, models three, four and five avoid this simplification.

The second model is a continuous time Markov process with an additional state variable, the number of fuel trucks in the system (not at a fillstand). Once again, aircraft are assigned to a refueling system per current policy. When a truck finishes servicing an aircraft (either the aircraft has obtained the required fuel or the truck is empty), the truck returns to a fillstand to refill. Once it is refilled, the truck returns to the airfield refilling system. This model assesses the delay due to truck refueling in order to determine how significantly this delay impacts the throughput. This model also represents a system where, if an aircraft leaves the hydrant system, an aircraft from the truck system is immediately moved to the hydrant system for servicing.

The third model is a continuous time Markov process with state vectors (i, j) representing the number of aircraft on the hydrant system and the total number of aircraft in the system. This model represents a system in which each aircraft, upon arrival, is placed in a refueling system and remains in that refueling system until servicing is complete. Because this model uses a more realistic assignment policy, the results of this model should better represent the airfield refueling system.

The fourth model is a continuous time Markov process with an additional state variable - the number of trucks in the system (not at a fillstand). This model is used to account for the delay due to trucks refilling at fillstands. Because this model uses a realistic assignment policy and models the delay due to truck refilling, the results of this model should provide an even better representation of the airfield refueling system.

The fifth model is a Markov decision process in which a refueling policy is determined that minimizes the airlift capability on the ground, and therefore maximizes throughput. This optimal sequence of actions assigns each arriving aircraft to a refueling system, or waits for the state of the system to change. The model has four state variables: the number of aircraft in the refueling system, the number of aircraft in the hydrant system, the number of aircraft in the truck system and the number of refueling trucks available to refuel aircraft. At each state of the system with an aircraft not assigned a refueling system (an aircraft arrival), a decision is made as to which refueling system the arriving aircraft should be sent to (if any), in order to minimize the total "cost" to the system. The cost coefficients are measurements of the importance of aircraft on the ground. The resulting Markov decision process is formulated as the constraints for a linear program. A comparison of this refueling policy with the current refueling policy is made in order to determine if the current refueling policy is optimal.

1.8 Thesis Overview.

This thesis is organized into six chapters: Introduction, Literature Review, Markovian Modeling, Methodology, Findings and Analysis, and Conclusions. Chapter 1 presents the need and importance of this research, all background information on the AMC airfield refueling problem, an outline of the input data and an overview of the models used in the research. Chapter 2 outlines USAF documents relevant to the refueling process, previous research on airfield throughput and refueling, and literature on the queuing and mathematical programming theory used in this study. Chapter 3 describes Markovian modeling theory to justify its use in the airfield refueling study. This chapter also presents analysis of the input data used for the model. Chapter 4 presents the application of the Markovian modeling theory used and outlines the important characteristics of the five models. The fifth chapter presents results and analysis of the models, as well as any further understanding gained from the study. Chapter 6 gives insights and conclusions found from the modeling and analysis of the refueling system and suggests future research opportunities.

Chapter 2

Literature Review

This chapter outlines the literature which pertains to this thesis. The first section outlines the US Air Force documents used to understand the refueling system. Each subsequent section outlines the research that has been done on throughput and refueling, as well as literature on the theory used to model the airfield refueling system.

Even before Desert Storm, the United States Air Force knew the importance of throughput and refueling in the strategic airlift problem. Because of this, simulation models were developed to better understand these two areas of strategic airlift and answer "what-if" questions. These simulation models either assume the refueling process cannot be modeled using queuing theory (because of the steady state assumption) or represent the refueling process without varying the current refueling policy. Although these simulations are useful, in recent years, many decision makers have requested analytical models to answer "what's optimal" questions. While these analytical throughput models do recognize the importance of variable (non-deterministic) ground times, they all assume that the refueling capability of an airfield can be modeled with a measure that is a combination of all ground operations. These models, which take the form of mathematical programs, fail to recognize that an airfield's throughput capacity can be increased using the existing refueling resources by providing an improved refueling policy for arriving aircraft. These simulation models and mathematical programs are outlined in section 2.2.

This study develops five models to understand the refueling process and to maximize the throughput of an airfield through the use of an optimal refueling policy. Four models are continuous time Markov processes. Details on Markovian theory are presented in Chapter 3. The fifth model, which calculates this refueling policy, is developed as a Markov decision process. A Markov decision process is a Markov process where a decision is made at each state of the system. Each system state has an associated probability governing the decisions for that state and the subsequent action chosen. The object of the process is to find the sequence of decisions to maximize some expected gain or minimize some loss. The optimal sequence is found using computational iteration algorithms or linear programming. Section 2.3 outlines literature on Markov processes and the mathematical programming solution of Markov decision processes.

2.1 Airfield Ground Operations.

All Air Force ground operations are outlined in Air Force Materiel Command Technical Order 00-25-172, Ground Servicing of Aircraft and Static Grounding/Bonding. This manual gives all procedures for refueling aircraft with the truck system and hydrant system as well as concurrent activities that may take place while refueling.

Air Force Pamphlet 144-4 outlines airfield servicing operations at other than US Air Force airfields. The pamphlet provides an overview of all airfield refueling operations, as well as pump rates and capacities for various hydrant and truck systems. The manual lists all Air Force aircraft and the average amount of fuel required by each modeled aircraft.

2.2 Refueling capabilities and strategic airlift.

Johnson (1984) uses a SLAM Network to model fueling operations at an airfield. He chooses not to use a Jackson Network queuing system because this approach assumes a system is in steady state. The author argues that an airfield may never reach steady state. The author fails to recognize that if operations exhibit a constant arrival stream and the maximum service rate is greater than the arrival rate, the system may approximate steady-state operation very quickly.

Loden (1986) develops a SLAM simulation model of a network process to find the optimal configuration of refueling equipment to meet sortie generation requirements for Tactical Air Command's (TAC) peak operation hours. TAC's aircraft refuel with the use of a hydrant or a fuel truck which requires refueling at a fillstand. A hydrant system can be converted to a fillstand. This conversion requires the use of scarce manpower. Given a contingency or "surge operation", as well as the number of hydrants and fuel trucks, the developed model optimizes the configuration of refueling equipment to maximize resource utilization. The model also allows fuel managers to assess each airfield's refueling capabilities during peak operations. Through this assessment, managers can determine bottlenecks and under-utilized resources in order to propose alternatives to the current configurations. The model assumes that each arriving aircraft chooses the refueling system with the shortest queue, with the hydrant system (because of the higher refueling rate) being chosen in case of a tie.

Donnelly and Hill (1986) develop a SLAM simulation model to analyze interactions between deploying C-130s and other strategic airlift aircraft during a conflict.

Their model is used to assess support requirement tradeoffs during a strategic airlift. The authors determine the limiting factors for an airfield and the number of fuel trucks required at each airfield "to support the transient aircraft", given a number of fuel pits (10:5). All refueling times are based on empirical data from prior Military Airlift Command (MAC) simulation runs and are not considered random variables (10:31). The authors' simulation utilizes a FORTRAN model that allocates refueling resources to aircraft. The current refueling policy of assigning each arriving aircraft to hydrant systems until they are full, then to the truck system, is represented in this FORTRAN model. The authors' simulation shows "refueling to be a major source of interaction between the strategic airlifters and C-130s" (10:75).

Needham (1987) employs a methodology used by the US Army to evaluate their transportation subsystems and applies it to the US Air Force transportation system. Harriot (1988) extends the work of Needham (1987) by developing a computer assessment tool for each subsystem of an air transportation infrastructure. Her model evaluates the present and future capability of a transportation system given a requirement, "identifying any equipment or facility shortfalls" (27:6). The author's model evaluates many areas of an airfield including all onloading/offloading equipment, pallets and airfield capability, but does not optimize the use of resources to increase throughput. The author addresses only aircraft parking when evaluating the airfield, and does not present any information about refueling of aircraft.

Yost (1994) develops a linear program called the THRUPUT Strategic Airlift Flow Optimization Model that is primarily used to assess constrained resources in a strategic airlift scenario. The model uses a constraint for the working MOG, which represents the maximum number of aircraft an airfield can simultaneously service. The measurement is an input to the model (predetermined for the entire run) and accounts for all factors which may effect MOG including ramp space, fuel availability, maintenance and the amount of time it takes to service the aircraft. The author's use of MOG and a deterministic ground time limits how well the model represents an airfield. The model also represents an airfield's MOG with a deterministic linear constraint, which makes it "difficult to capture stochastic parameters" (34: 4). One recommendation proposed by the author for this study is a model upgrade which considers better MOG methodology since the linear program is "sensitive" to the MOG input (34:27).

Lim (1994) enhances this network-based linear program to maximize the effectiveness of airlift assets subject to physical and policy constraints. The effectiveness measure he uses is the minimization of penalties incurred by late loads (20:7). His model determines the maximum amount of cargo that can be delivered on time, and his analysis uses an Operation Desert Storm scenario (of 30 days) to gain insights into the model outputs. Lim models the airfield operations capacity with the same working MOG measure as the original THRUPUT model. The working MOG approximation is a measurement which encompasses all parking, maintenance, loading and refueling capabilities. The airfield capacity constraints ensure that the number of aircraft handled at each airfield is within the airfield's limits. Because of the recommendations from Yost, Lim uses a MOG efficiency factor to account for the variability caused by ground operations. Lim also notes that random aircraft down times can "significantly affect the performance of an airlift system" (20:49). The results of Lim's model show that these MOG limits constrain the airlift operation during the middle phase of the operation because more and shorter flights consume MOG at a faster rate. Lim shows the system performs better (a decrease in penalties) by adding more efficient aircraft ("high ratio of cargo-delivered-per-plane to MOG-hours-consumed-per-plane") (26:64).

Goggins (1995) advances the deterministic model developed by Lim (1994) by modeling the assumption that aircraft reliability is known prior to making a decision. He shows how larger-than-expected ground times are the major contributing factor to congestion in contingency airlift operations. He chooses to evaluate aircraft reliability because he believes aircraft reliability is the one area that most constrains an airfield's MOG. The author argues that unreliable aircraft seriously degrade an airlift system and models which do not account for this are "too optimistic with respect to throughput capability" (13:6). He uses aircraft reliability data to stochastically extend Lim's model to encompass this data, arguing that not modeling aircraft reliability may lead to an overestimation of airfield capacities (13:1). The model presents an optimal solution which maximizes system throughput performance that is not seriously degraded by aircraft reliability events. This model also assumes a deterministic ground time (by using the same MOG constraint), which includes loading/unloading, maintenance and refueling in one measurement.

The model proposed by Goggins, called THRUPUT 2, is being combined with a RAND throughput model, CONOP, to adjust for the shortcomings of both models. This new model, called the NPS (Naval Postgraduate School)/RAND Mobility Optimizer is an on-going research project between NPS, RAND and the University of Texas at Austin (31:1). Although this model will address some disadvantages of THRUPUT 2, it does not account for varying ground times any differently, and makes no recommendation on how throughput can be increased through better utilization of current resources.

2.3. Markov decision processes.

Howard (1960) recognizes the need for a model technique to solve problems containing "probabilistic and decision-making features" (16:1). He formulates the problem as a Markov process and then uses an iteration scheme taken from dyamic programming to solve for optimality. Howard introduces a set of Markov processes that have rewards (where a negative reward should be considered a cost) associated with each state the system may occupy. He then constructs a Markov decision process by introducing alternative decisions at each state of the system. With this process, a decision needs to be made as to which alternative should be selected at each state of the system in order to optimize the objective. Howard's *value iteration technique* divides the process into stages and seeks to find what decision should be made at stage n in order to maximize the expected return (or minimize expected loss) at stage $n+1$. Through solving for a decision at each stage of the process, an optimal policy is reached which maximizes the expected return (or minimizes expected loss).

Manne (1960) examines how to represent a sequential decision model with a Markov decision process and optimize it by use of a linear program. He uses an inventory example to represent an infinite process with finite states. The decision is how many items to produce at the end of each month. This decision needs to be made at each state of the system (inventory) which leads to a transition from state *i* to *j*, designated x_{ij}. Each transition has an associated cost, with only one transition being made for each state of the system. Through the series of decisions, a sequence is built. Solving the linear program finds the sequence which minimizes the total cost, subject to constraints. One constraint ensures the probability of transitioning from any state sums to one. The other constraints, equilibrium constraints, ensure that the inventory at the end of the month is equal to the inventory at the beginning of the month (there is such a demand that if an item is produced, it will be sold) (22:262). The solution gives the probability of being in a state and a decision rule if the system is in that state.

Chapter 3

Markovian Modeling

Markovian modeling is used to model stochastic processes which exhibit transitory behavior and the Markovian property. Many transportation systems continuously change from one state of the system to another, as events such as an aircraft arrival or departure occur. The transition probabilities associated with this change or "transition" from one possible state i to another state j (under appropriate conditions), determine the steady-state probability distribution of system states. Let the probability of the system, starting in state i, being in state j at some time period t be $P_{ij}(t)$. Also, let any past state of the system at time u, designated by $X(u)$, be state $x(u)$. If the probability of being in state j some time in the future t, depends only on the most recent state i, and not on any other past state $x(u)$, the process is known as a continuous time Markov process. That is, if the stochastic process exhibits the ***Markovian property,***

$$P_{ij}(t) = P\{X(t+s) = j \mid X(s) = i, X(u) = x(u), 0 < u < s\} = P\{X(t+s) = j \mid X(s) = i\}$$

then the stochastic process is a continuous time Markov process. The Markovian property states that the conditional distribution of some future state of the system given the present and past states only depends on the present state and is independent of the past states (29: 256).

This property is the underlying assumption needed for modeling the airfield refueling system as a continuous time Markov process. In order to use Markovian modeling, this study assumes that the aircraft interarrival times and service times are exponentially distributed. If this is true, the times between system state transitions are exponentially distributed and hence, memoryless. System transitions are caused by four events: an aircraft arrival, an aircraft departure, a truck arrival into the system, and a truck departure from the system. The analysis accomplished on the aircraft arrival data supports the assumption that aircraft arrivals are generated by a Poisson process. Therefore, the interarrival times are exponentially distributed and the time between aircraft arrivals is memoryless. The times between aircraft departures from the system depend on the refueling rate and the amount of fuel each aircraft demands. Because of the nature of the refueling system (arrivals and departures), the amount of fuel an aircraft needs following a state transition is assumed to be expontially distributed. Hence, the aircraft service times are exponentially distributed and the time between aircraft departures from the system is memoryless. Similarly, the time until a truck departure from the system depends on the amount of fuel the aircraft needs and the rate at which the truck refuels. Since the amount of fuel the aircraft needs is assumed to be exponentially distributed, the time between truck departures from the system is also memoryless. Furthermore, the time between a truck departure from the system until the truck arrives back in the system depends on the fillstand refill rate and the amount of fuel remaining in the truck when it left the system. Since this amount of fuel is also assumed to be exponentially distributed, this refilling time is assumed to be exponentially distributed. Therefore, the time between arrivals to the fillstand and departures from the fillstand is memoryless.

The airfield refueling system exhibits transitory behavior and, because the times between these transitions are assumed to be exponentially distributed, the system can be modeled as a continuous time Markov process. These unique features of Markovian modeling allow for the use of transition probabilities to completely describe the system state distribution. Steady-state probabilities, P_j, describe the long term probability of being in state *j*. These states can describe such system characteristics as the number of aircraft in the system or the number of refueling trucks in the system. Knowledge of the steady-state distribution allows analysis of current airfield configurations and of the effect of changes in these configurations. For the models used to evaluate the current system, the primary performance measure is the average time an aircraft spends in the system. For the Markov decision model, the performance results are evaluated based on the average number of aircraft at the airfield. This performance measure should be minimized in order to maximize throughput. By modeling the process as a continuous time Markov process, performance measures can be established to allow users to assess any current refueling system, and how changes to the system affect performance.

3.1 Data Analysis

Currently, AMC uses results from the MASS simulation to plan all routing, refueling and loading of aircraft for each contingency. For this reason, this study uses the MASS output data as an input to the models in order to evaluate the airfield refueling process and determine an optimal refueling policy for a given contingency. The MASS output data required for this model is the actual time of arrival (to calculate an interarrival distribution) and the amount of fuel demanded by an arriving aircraft (to calculate a service rate for each refueling system). The simulation determines the actual time of arrival based on the prior actual departure time and the duration of the previous flight leg.

MASS determines the required ramp fuel based on prior refuelings (ground and air), the duration of the previous flight leg and the duration of the next flight leg.

AMC provided airfield data for one simulated contingency. The simulation results are for the operations at Hickam Air Force Base (AFB) (a transient airfield). Because this study seeks to isolate this one aspect of an airfield, the simulation data used is from Hickam AFB, a transient airfield. The data lists aircraft, aircraft arrival times, and required fuel. The data captures operations over a period of 798 hours, and has 2876 data points. The aircraft that are modeled are the C-141B, C-17, C-5A and Wide Body Craft (WBC).

The data shows that a proportion of the WBCs do not require fuel. WBCs are civilian aircraft that are refurbished for transportation of military loads during contingencies. Air Force resources are not always used to refuel these aircraft, but they still occupy Air Force resources during unloading/loading of cargo. These aircraft refuel at civilian airports, and do not arrive to the airfield refueling system. Therefore, the interarrivals for these aircraft are not considered in the arrival process or in the service process (demand for fuel).

Three primary ground operations occur at an airfield: maintenance (routine and emergency), loading/unloading and refueling. A transient airfield only performs emergency maintenance and refueling during contingencies. Because the purpose of this study is to isolate the refueling system in order to evaluate the current and future system configurations' effect on the throughput of the airfield and to determine an optimal refueling policy, data from a transient base is used.

This study uses the actual arrival time and the required ramp fuel from the MASS simulation. Using the actual arrival times for each aircraft, interarrival times are computed. Using BestFit software, a data fit of the exponential distribution is compared to a data fit of all other continuous distributions. This is accomplished to evaluate the validity of the assumption that the interarrival times are exponentially distributed. The rate of refueling aircraft for each system depends on the number of aircraft in/on the system. When either an aircraft departs the system upon service completion or a truck departs the system to refill (or a truck re-enters the system from the fillstand), the system configuration and service rate changes. The departure or service rate of the system depends on the specific refueling system service rate and the amount of fuel needed by the aircraft. At each state transition, the aircraft's required fuel is the amount not filled by the previous system configuration. Because the amount of required fuel each aircraft demands is assumed to be exponetially distributed, the time between transitions due to service completion is memoryless. The required ramp fuel is used to determine each refueling system's mean service rate.

3.1.1 Arrival Process.

From the MASS simulation data, the aircraft interarrivals are evaluated for use as input to the model. The aircraft interarrivals are plotted to ensure that the exponential distribution is a valid option. As can be seen from the initial plot of interarrivals versus time (Fig. 3-1), the data appears to be exponentially distributed.

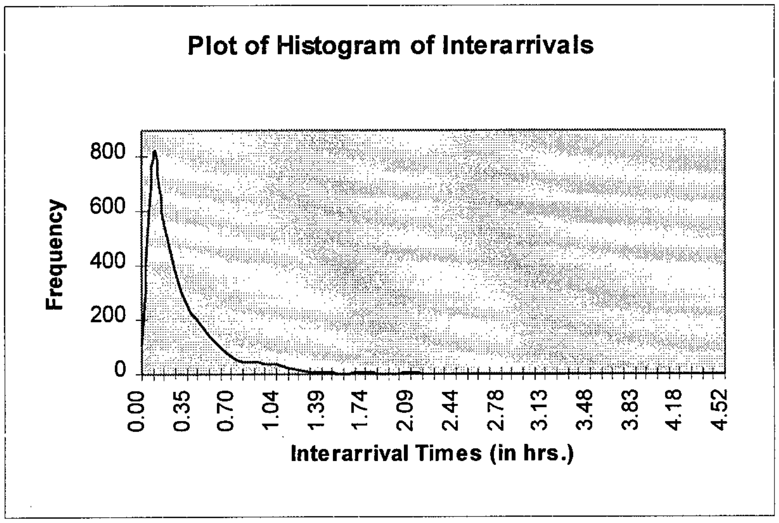

Figure 3-1. Histogram of Interarrival Data.

The data is fit to the exponential distribution and compared with other continuous distributions to ensure the validity of the assumption that aircraft arrivals are generated by a Poisson process. If this assumption is valid, the aircraft interarrivals can be modeled using the exponential distribution.

Using BestFit software, the interarrival data is fit to the exponential as well as other continuous distributions. The graph of the interarrivals support the assumption that the aircraft arrival process is a Poisson process (Fig. 3-2). The aircraft interarrivals are exponentially distributed with a rate of 3.57 aircraft per hour (mean 0.28 hrs per aircraft arrival).

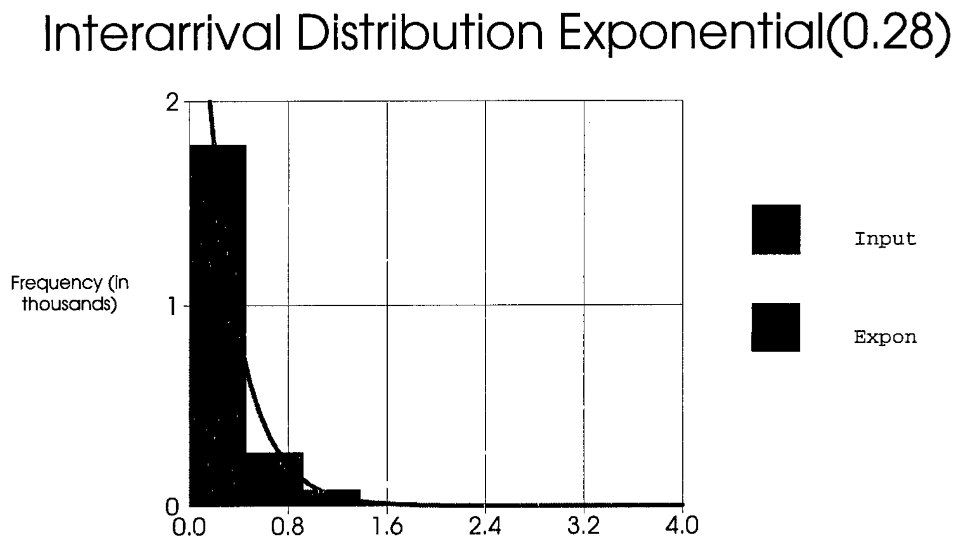

Figure 3-2. BestFit histogram.

Chi-square, Kolmogorov-Smirnoff (KS) and Andrson-Darling goodness-of-fit parameters are used to compare and rank the continuous distributions (Fig. 3-3). Although each test concluded that aircraft interarrivals should be modeled using an empirical distribution, the comparison of the distributions are used to validate the assumption that the interarrivals are exponentially distibuted. Both the Kolmogorov-Smirnoff (recognized as the most powerful test for fitting continuous distributions) and the Anderson-Darling test rank the exponential as the best fit for the data. It can now be assumed that the interarrivals are exponentially distributed and aircraft arrivals are generated by a Poisson process.

Table 3-1. BestFit Goodness-of-fit comparisons.

Best Fit Results			
Function	Chi-Square*	K-S Test	A-D Test
Gamma (.73, .37)	1	8	6
Weibull (.91, .31)	2	2	2
Erlang (1.0, .36)	3	3	4
Lognormal2 (-1.73, 2.14)	4	4	5
Expon (.28)	5	1	1
Chisq (1.0)	8	13	3
Lognormal (1.17E+2, 4.0E+4)	11	7	9
Triang (0, 0, 4.61)	12	10	12
Logistic (.28, .19)	13	5	7
Beta (.39, 8.06)* 4.61	14	11	10
Pareto (1, 0, 0)	15	12	13
Normal (.28, .35)	16	6	8
Erf(4.0, 4.0, 66.0)	17	9	11

*** The Chi-square test also fits the data to discrete distributions. Therefore, these distributions are not listed. Consequently, lower ranked continuous distributions are listed.**

3.1.2 Service Process.

This study used the required ramp fuel from the MASS simulation. The aircraft departure rate depends on the number of aircraft and trucks in the system, the pump rate of each fuel system, and the intake rate of the aircraft. The truck arrival/departure rate depends on the number of aircraft in/on the system, the pump rate of the truck system, the fillstand pump rate and the number of trucks at the fillstand. When either an aircraft departure occurs or a truck arrival or departure occurs, the system configuration changes and subsequently, the system departure rate changes.

The times between these system transitions depend on the specific refueling system service rate, any aircraft receiving limitations, and the amount of fuel remaining in either the aircraft or truck (for the fillstand). Each system has an associated service rate. For the truck system, this service rate is 550 gpm. The hydrant system has an overall service rate of 2400 gpm and is equally distributed as aircraft are placed on the system. This hydrant service rate is much faster than the truck system, but is limited by the maximum aircraft receive rate, which is 600 gpm. Therefore, the service rate for each hydrant system is 600 gpm. The fillstand takes 15-20 minutes to fill the trucks and there is approximately 15-20 minutes travel time (each way) to and from the fillstand. Therefore the service rate of a single fillstand is approximately 1 truck per hour. Because of the random nature of the travel times, truck maintenance times at the aircraft, and truck/fillstand maintences times, this service rate assumes the fillstands do not operate while trucks are traveling. Although this causes system performance measures to be conservative, this service rate is a variable in the model and can be changed if fillstand configurations warrant a significant change in truck arrivals to the refilling system.

At each state transition, an aircraft's required fuel is the amount it enters the state from the previous system state. An aircraft can reach this state by arriving in the system or by having an event occur outside of its control (as in a truck arriving to the system). In both cases, the amount of fuel the aircraft demands is assumed to be exponentially distributed. Therefore, the time between system state transitions is memoryless.

Similarly, the truck's remaining fuel upon arrival at a fillstand is the amount not released into an aircraft before the last system transition. A truck can reach this state by having an aircraft complete refueling (truck has fuel remaining in its tank) or by emptying all 5500 gallons before aircraft refueling is complete (truck is empty). Because this study assumes the aircraft demands an exponential amount of fuel, the truck enters and departs the state demanding an amount of fuel that is assumed to be exponentially distributed; therefore, the time between transitions is memoryless.

Since the time between system transitions depends on each system's service rate, the amount of fuel needed by an arriving aircraft is used to determine this rate. For this reason, the demonstation of the models for this study use the average amount of fuel needed by each aircraft. This value is calculated from the MASS simulation data to determine each system's refuel service rate. The mean amount of fuel needed by the aircraft is 24,727 gallons. This allows for the following aircraft departure rates, μ_i (where i is the number of aircraft in the system) for the hydrant system and γ for the truck system, for the scenario used in this study:

Table 3-2, Hickam AFB System Service Rates.

	System service rate (in aircraft per hour)
μ_1	1.456
μ_2	2.912
μ_3	4.368
μ_4	5.824
γ	1.335 (per truck)

3.2 Steady-State Probabilities.

As shown earlier, the Markovian property states that the conditional distribution of some future state of the system given the present and past states only depends on the present state and is independent of the past states (29:256). Therefore, the amount of time the system has been in a certain state is irrelevant to the remaining time the system is in that state.

Now let q_{ij} be the rate the process transitions from state *i* to *j* and v_i be the rate the process transitions from *i*. We can state that the transition rate from *i* to *j* is equal to the transition rate from *i* times the probability of transitioning from *i* to *j*, or:

$q_{ij} = v_i * P_{ij}$

These are known as the instantaneous transition rates from *i* to *j*. State transitions are known to be governed by the Chapman-Kolmogorov equations:

$P_{ij}'(t) = \sum_{k \neq i} q_{ik}(t)P_{kj}(t) - v_i P_{ij}(t)$ backward equation
$P_{ij}'(t) = \sum_{k \neq j} q_{kj}(t)P_{ik}(t) - v_j P_{ij}(t)$ forward equation

where $P_{ij}'(t)$ is the instantaneous rate of change of the probability of transitioning from *i* to *j* by some time period *t*.
q_{ik} is the instantaneous transition rate from *i* to *k* by some time period *t*.
$P_{kj}(t)$ is the probability of transitioning from *k* to *j* by some time period *t*.
v_i is the transition rate from *i*.
$P_{ij}(t)$ is the probability of transitioning from *i* to *j* by some time period *t*.

q_{kj} is the instantaneous transition rate from *k* to *j* by some time period *t*.
$P_{ik}(t)$ is the probability of transitioning from *i* to *k* by some time period *t*.
v_j is the transition rate from *j*.
$P_{ij}(t)$ is the probability of transitioning from *i* to *j* by some time period *t*.

As time approaches infinity, each $P_{ij}'(t)$ converges to 0. Therefore, the Chapman-Kolmogorov forward and backward equations reduce to:

$v_j P_j = \sum_{k \neq j} q_{kj} P_k$

where P_j is the long run probability of the system being in state *j*.
P_k is the long run probability of the system being in state *k*.

Simply stated, the rate out of state *j* equals the rate into state *j*.

This system of simultaneous linear equations, along with the conservation of probability equation,

$\sum P_i = 1.0 \qquad \text{for all } i$

are formulated in a transition matrix and solved to obtain the steady-state probabilities that completely describe the system. This "transition matrix" is used in the first four models to solve for the steady-state probabilities. These steady-state probabilities are used to calculate two queuing performance measures for the system, the average time in system and the average number in system.

3.3 Summary.

All models in this study represent the airfield refueling system as a continuous time Markov process. This approach is valid because the refueling process is a stochastic process that exhibits transitory behavior and has the Markovian property. The data analysis accomplished on the aircraft interarrival data supports the assumption that the aircraft arrivals are generated by a Poisson process. The time between departures depends upon the remaining fuel of the aircraft. In order to have a mathematically tractable model, an assumption is made that the remaining amount of fuel in an aircraft is exponentially distributed. Therefore, the time between system state transitions is memoryless. The unique features of Markovian modeling allow for the use of transition probabilities to completely describe the distribution of the state of the system. This distribution allows analysis of current system configurations using queuing performance measures. These measurements allow users to assess current refueling systems, and how resource changes may affect system performance.

Chapter 4

Continuous Time Markov Models

It has been shown that modeling the airfield refueling system as a continuous time Markov process is a valid approach. Therefore, four models are sequentially developed to gain a more complete understanding of the system and current refueling policy. The knowledge gained from this building process is used to construct a Markov decision process model which presents an improved representation of the system and optimizes the refueling policy.

The first and fifth models are more specific cases of continuous time Markov processes. The first model is a birth and death process while the fifth model is a Markov decision process. In a birth and death process, each transition from a state can only move to an adjacent state. The steady-state probabilities are calculated by solving the system of linear differential equations in a transition rate matrix. These steady-state probabilities, P_j, are used to completely describe the system (using the state distribution, such as *j* being the number of aircraft in the system) and solve for queuing performance measures. In a Markov decision process, an action is taken at each state of the system. This action determines which state the system proceeds to when the action is taken. Using reward/cost coefficients and constraints generated by the transition rate matrix, a linear program is formulated and solved for the optimal sequence of decisions which maximize/minimize an objective function. This sequence is the optimal policy which minimizes the number of aircraft on the airfield. This essentially minimizes the time aircraft spend in the refueling system, allowing the airfield to sustain a higher aircraft arrival rate. This in turn, increases the throughput of the airfield.

4.1 Model Assumptions

The assumptions reflected in the models are:

1. Unlimited number of Type III hydrant spaces. Currently, hydrant systems are of three types: I, II, or III. Type I and II hydrant systems have very restrictive parking requirements, such as two aircraft cannot be parked next to each other and refuel using the hydrant system. Because of these limitations, each airfield's hydrant system will be retrofitted to Type III hydrant systems (no parking restrictions) in the future.

2. Refueling service is not interrupted by other airfield operations.

3. As soon as an aircraft arrives, it enters the refueling system and departs when refueling service is complete.

4. Steady state conditions exist at the airfield during a contingency.

5. The airfield has a limited capacity (an unlimited number of aircraft cannot occupy the airfield). Making this assumption leads to conclusions being stated only for the aircraft that are allowed to land at the airfield.

6. Different aircraft types are aggregated in order to make the problem mathematically tractable.

7. The fuel requirements for the aircraft are exponentially distributed.

8. Aircraft arrivals are generated by a Poisson process (aircraft interarrivals are exponentially distributed).

These assumptions allow modeling of the system as a continuous time Markov process. The system can then be analyzed using queuing performance measures. Also, in order for complete analysis of the refueling system's effect on the airfield's throughput, assumptions are made in order to isolate the refueling system.

4.2 Birth and Death Process Model.

A continuous time Markov process where each transition from a state can only move to an adjacent (or nearest neighbor) state is known as a discrete space birth and death process (30:233). If a process is in state *n*, an event occurs that either increases the state of the process to $n+1$ (a birth) or decreases the state of the process to $n-1$ (a death). Births occur at a rate of λ_n (the birth rate depends on the current population) and deaths occur at a rate of μ_n (the death rate depends on the current population). Obviously when there is no one in the population, deaths cannot occur so $\mu_0 = 0$ and if the population has a capacity of *C*, births cannot occur while the population size is *C*, so $\lambda_C = 0$. Births and deaths are independent of one another and the amount of time between births is exponentially distributed with mean $1/\lambda_n$ while the amount of time between deaths is exponentially distributed with mean $1/\mu_n$. Because of this, the process has stationary transition probabilities.

The first model is a birth and death process because each transition from a state moves to an adjacent state. This model has one state variable, the number of aircraft in the system. Each arriving aircraft is served by the first available server. Arriving aircraft are sent to "active" hydrant spaces until all spaces are filled, while subsequent arriving aircraft are sent to a truck system. If an aircraft is sent to the truck system, it is served by the first available truck server. If a hydrant becomes available, the aircraft ends service by the truck and begins service by a hydrant.

The instantaneous transition rates for the simple birth and death process model are shown below:

i = number of aircraft in the system.

N = number of refueling trucks at the airfield.
H = number of active hydrants at the airfield
C = airfield capacity.

λ = aircraft arrival rate.

μ_i = service rate for the number of aircraft, i, in the system up to i = H because the hydrant service rate does not change after H aircraft are on the system.

γ represents the service rate of 1 truck.

q_{ij} is the instantaneous transition rate from i to j.

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$$q_{i,\,i+1} = \begin{cases} \lambda & \forall\ i < C \\ 0 & \forall\ i \geq C \end{cases}$$

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$$q_{i,\,i-1} = \begin{cases} \mu_i & \forall\ 0 < i \leq H \\ \mu_H + \gamma_*(i-H) & \forall\ H < i \leq N + H \\ \mu_H + \gamma_*(N) & \forall\ N + H < i \leq C \\ 0 & \forall\ i \geq C \end{cases}$$

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4.3 Continuous Time Markov Process Models.

Because the state space for the first model is the number of aircraft in the system, it could be represented by a simple birth and death process. Subsequent models add additional state variables and refueling specifics to provide a better representation of the airfield refueling system. This sequential model building process is done to gain a more complete understanding of the refueling process.

The second model adds the number of fuel trucks in the system (not at a fillstand). Once again, the aircraft are assigned to a refueling system as per current policy. If an aircraft departure leads to an available hydrant, the aircraft ends service by the truck and begins service on the hydrant system. Because the aircraft intake rate constrains the hydrant refueling rate, this rate is approximately equal to the truck refueling rate. The major difference between the two is that the trucks run out of fuel and have to refill at a fillstand. This model assesses the delay due to truck refilling in order to determine how this delay impacts the average number of aircraft in the system and the average time an aircraft is in the system.

The state space representation for this model is i, the number of aircraft in the system and j, the number of trucks in the system. Four events determine the state the system transitions to: an aircraft arrival, an aircraft departure due to refueling being accomplished, a truck's fuel being depleted prior to an aircraft being filled, and a truck rejoining the system after being refilled at a fillstand. The probability associated with a truck completing service (fuel depleted) before an aircraft is completely refueled depends on the average amount of fuel an aircraft needs (user-defined input to the model) and the service rate of the truck system.

The instantaneous transition rates are shown below:

i = number of aircraft in the system.
j = number of trucks in the system.

N = number of refuel trucks at the airfield.
H = number of active hydrants at the airfield.
C = airfield capacity.

λ = aircraft arrival rate.

μ_i = service rate for the number of aircraft, i, in the system up to i = H because the hydrant service rate does not change after H aircraft are on the system.

γ represents the service rate of 1 truck.

ε_1 = the rate trucks refill at the fillstand.

p_1 = probability truck refuels the aircraft before it empties.
= probalility amount of fuel aircraft needs is less than the amount the truck has.
= P(X ≤ amount of fuel carried in truck)
= P(X ≤ 5500 gallons)
Since the amount the aircraft demands is assumed to be exponentially distributed:
= 1 - exp(-5500/average amount needed by an aircraft)

p_2 = probability truck empties before it completes refueling of the aircraft.
= $(1 - p_1)$

$q_{i,j;\,k,l}$ is the instantaneous transition rate from state i,j to state k,l.

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$$q_{i,j;\,i+1,j} = \begin{cases} \lambda & \forall\, i < C, j \\ 0 & \forall\, i \geq C, j \end{cases}$$

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$$q_{i,j;\,i,j+1} = \begin{cases} \varepsilon_1 & \forall\; i, j < N \\ 0 & \forall\; i, j \geq N \end{cases}$$

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$$q_{i,j;\,i-1,j} = \begin{cases} 0 & \forall\ i = 0, j \\ \mu_i & \forall\ 0 < i \le C, j \end{cases}$$

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$$q_{i,j;\,i-1,j-1} = \begin{cases} 0 & \forall\ 0 \le i \le H, j \\ p_1 * \gamma * (j) & \forall\ H < i \le N + H, j \\ p_1 * \gamma * (N) & \forall\ N + H < i \le C, j \end{cases}$$

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$$q_{i,j;\,i,j-1} = \begin{cases} 0 & \forall\ 0 \le i \le H, j \\ 0 & \forall\ i, j = 0 \\ p_2 * \gamma * (j) & \forall\ H < i \le N + H, j > 0 \\ p_2 * \gamma * (N) & \forall\ N + H < i \le C, j > 0 \end{cases}$$

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The third model advances the first model by allowing aircraft placed on a refueling system to remain on that system until service is complete. Once again, the aircraft are assigned to a refueling system as per current policy. Although this model does not include the delay due to trucks refilling at a fillstand, it provides a better representation of ground refueling operations which does not allow aircraft to "switch" refueling systems. The state space used for this model is *i*, the number of aircraft on the hydrant system, and *j*, the number of aircraft in the system.

The instantaneous transition rates are shown below:

i = the number of aircraft on hydrants.
j = the number of aircraft in the system.

N = number of refuel trucks at the airfield.
H = number of active hydrants at the airfield.
C = airfield capacity.

λ = aircraft arrival rate.

μ_i = service rate for the number of aircraft, *i*, in the system up to *i* = H because the hydrant service rate does not change after H aircraft are on the system.

γ represents the service rate of 1 truck.

$q_{i,j;k,l}$ is the instantaneous transition rate from state *i,j* to state *k,l*.

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$$q_{i,j;\, i+1,j+1} = \begin{cases} \lambda & \forall\ i, j < C \\ 0 & \forall\ i, j \geq C \end{cases}$$

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$$q_{i,j;\, i-1,j-1} = \begin{cases} 0 & \forall\ i = 0, j \\ \mu_i & \forall\ i > 0, j \end{cases}$$

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$$q_{i,j;\, i,j-1} = \begin{cases} 0 & \forall\ i = j \\ \gamma_*(j) & \forall\ i = 0, j \leq N \\ \gamma_*(N) & \forall\ i = 0, j > N \\ \mu_i + \gamma_*(j-i) & \forall\ i \neq 0, j - i \leq N \\ \mu_i + \gamma_*(N) & \forall\ i \neq 0, j - i > N \end{cases}$$

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By adding the number of trucks in the system to the state space representation, the fourth model provides a more complete description of the aircraft refueling system. The refueling process allows each arriving aircraft to complete service on its original system. This model is used to gain an understanding of the actual system and to provide a more complete state space representation for the final model. The transition rates are shown below:

i = number of aircraft on hydrants.
j = number of aircraft in the system.
k = number of trucks in the system.

N = number of refueling trucks at the airfield.
T = number of trucks in the system, not at a fillstand.
H = the number of active hydrants at the airfield.
C = airfield capacity.

λ = aircraft arrival rate

μ_i = service rate for the number of aircraft, i, in the system up to i = H because the hydrant service rate does not change after H aircraft are on the system.

γ represents the service rate of 1 truck.

ε_1 = the rate trucks refill at the fillstand.

p_1 = probability truck refuels the aircraft before it empties.
= probalility amount of fuel aircraft needs is less than the amount the truck has.
= $P(X \leq$ amount of fuel carried in truck$)$
= $P(X \leq 5500$ gallons$)$
Since the amount the aircraft demands is assumed to be exponentially distributed:
= 1 - exp(-5500/average amount needed by an aircraft)

p_2 = probability truck empties before it completes refueling of the aircraft.
= $(1- p_1)$

$q_{i,j,k;\, l,\, m,\, n}$ is the instantaneous transition rate from state *i,j,k* to state *l,m,n.*

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$$q_{i,j,k;\, i+1,j+1,k} = \begin{cases} \lambda & \forall\, i < H, j < C, k \\ 0 & \forall\, i < H, j \geq C, k \end{cases}$$

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$$q_{i,j,k;\, i,j+1,k} = \begin{cases} \lambda & \forall\, i \geq H, j < C, k \\ 0 & \forall\, i \geq H, j \geq C, k \end{cases}$$

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$$q_{i,j,k;\, i,j,\, k+1} = \begin{cases} \varepsilon_1 & \forall\, i, j, k < N \\ 0 & \forall\, i, j, k \geq N \end{cases}$$

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$$q_{i,j,k;\, i-1,j-1,k} = \begin{cases} 0 & \forall\, i = 0, j, k \\ \mu_i & \forall\, i > 0, j, k \end{cases}$$

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$$q_{i,j,k;\, i,j-1,k-1} = \begin{cases} 0 & \forall\, i = j \text{ and } i, j, k = 0 \text{ and } i, j = 0, k \\ p_1 * \gamma & \forall\, i, j > 0, k > 0 \end{cases}$$

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$$q_{i,j,k;\, i,j,k-1} = \begin{cases} 0 & \forall\, i = j \text{ and } i, j, k = 0 \text{ and } i, j = 0, k \\ p_2 * \gamma & \forall\, i, j > 0, k > 0 \end{cases}$$

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Because this model has the most comprehensive state space description, it is used to build a complete system description for the fifth model, the Markov decision process.

4.4 Markov Decision Process.

A continuous time Markov decision process can be in any of a finite number of states 1,2.....N. After observing the state of the process, an action is chosen from a finite set of actions which causes the process to change states. The rate the process transitions to that state depends on the action chosen. Subsequently, the conditional probability of being in a state j depends on the present state X_n and the action a chosen and not on previous states and/or actions chosen. That is,

$P_{ij}(a) = P\{X_{n+1} = j \mid X_0, a_0, X_1, a_1, \ldots X_n, a_n\} = P\{X_{n+1} = j \mid X_n, a_n\}$ (29:182).

The transition probabilities form the constraints for the Markov decision process, ensuring that the rate into a state equals the rate out of a state. By formulating these constraints along with the cost coefficients associated with each state of the system (the objective function), a linear program is formulated. The solution of this linear program provides a sequence of refueling system decisions which form a policy or "a rule for choosing actions" to maximize airfield throughput (29:182).

AMC's current refueling policy is to send an arriving aircraft to a hydrant system space if one is available. If all hydrant spaces are occupied, the truck system is utilized. The fifth model presents three different refueling options in a Markov decision process, formulates the problem as a linear program and solves for the optimal refueling policy. The three different refueling options are: send the arriving aircraft to the truck system, send the arriving aircraft to the hydrant system, or wait for the state of the system to change. The linear program seeks to minimize the number of aircraft on the airfield subject to the "rate in equals rate out" and the conservation of probability constraints.

This model uses four variables to completely describe the system state: the number of aircraft in the system, the number of aircraft on the hydrant system, the number of aircraft on the truck system and the number of trucks available for servicing. At each state of the system, a decision is made as to which refueling system the arriving aircraft should be sent. For the airfield refueling model, three options are possible when aircraft arrive: wait for the system to transition to another state, send the aircraft to a hydrant system, or send the aircraft to a truck system.

The constraints for the linear program are the Chapman-Kolmogorov equations (represented by the transition rate diagram), the conservation of probability equations and the probability bound constraints. The Chapman-Kolmogorov equations ensure that the rate out of each state equals the rate into each state. The conservation of probability equations and probability bound constraints ensure the solution adheres to the fundamental assumptions of probability theory.

The objective function's cost coefficients are measurements of the importance of aircraft on the ground and the variables for the program are the steady-state probabilities for each state of the system and action chosen. For this model, a cost of one unit per aircraft is assigned. The linear program selects a sequence of decisions, or a refueling policy based on minimizing the "cost" to the system. By minimizing this cost, the total number of aircraft on the ground is minimized, so the throughput of the airfield is maximized.

4.4.1 Markov Decision Process Linear Programming Formulation.

The Markov decision process linear programming formulation is shown below:

i = the number of aircraft on the airfield
j = the number of aircraft on the hydrant system
k = the number of aircraft on the truck system
l = the number of trucks in the system, not at a fillstand and not refueling an aircraft.
a = action (decision) chosen

N = number of refueling trucks at the airfield.
T = number of trucks in the system, not at a fillstand.
H = the number of active hydrants at the airfield.
C = airfield capacity.

$P^{a}_{i,j,k,l}$ = steady-state probability associated with state i, j, k, l and decision d.

λ = aircraft arrival rate

μ_i = service rate for the number of aircraft, i, in the system up to i = H because the hydrant service rate does not change after H aircraft are on the system.

γ represents the service rate of 1 truck.

ε_1 = the rate trucks refill at the fillstand.

p_1 = probability truck refuels the aircraft before it empties.
= probalility amount of fuel aircraft needs is less than the amount the truck has.
= P(X $\leq$ amount of fuel carried in truck)
= P(X $\leq$ 5500 gallons)
Since the amount the aircraft demands is assumed to be exponentially distributed:
= 1 - exp(-5500/average amount needed by an aircraft)

p_2 = probability truck empties before it completes refueling of the aircraft.
= $(1- p_1)$

Minimize $C_{i,j,k,l} * P^a_{i,j,k,l}$

subject to:

$$\Sigma_a [v^a_{(i,j,k,l)} * P^a_{(i,j,k,l)}] = \Sigma_a [\Sigma_{(i,j,k,l)' \neq (i,j,k,l)} q^a_{((i,j,k,l)';(i,j,k,l))} * P^a_{(i,j,k,l)'}]$$

for all states *(i,j,k,l)* and possible actions *a.*

where: $v^a_{i,j,k,l}$ is the rate out of state *i,j,k,l* when action *a* is chosen.
$P^a_{(i,j,k,l)}$ is the steady-state probability of the system being in state *(i,j,k,l)* when action *a* is chosen.
$q^a_{((i,j,k,l)';(i,j,k,l))}$ is the instantaneous transition rate out of state *(i,j,k,l)'* to state *(i,j,k,l).*
$P^a_{(i,j,k,l)'}$ is the steady-state probability of the system being in state *(i,j,k,l)'* when action *a* is chosen.

where:

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$$q^a_{i,j,k,l;i+1,j,k,l} = \begin{cases} \lambda & \forall\ a,\ i < \text{capacity},\ j,\ k,\ l \\ 0 & \forall\ a,\ i \geq \text{capacity},\ j,\ k,\ l \end{cases}$$

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$$q^a_{i,j,k,l;i-1,j-1,k,l} = \begin{cases} 0 & \forall\ a,\ i,\ j = 0,\ k,\ l \\ \mu_j & \forall\ a,\ i,\ 0 < j < H,\ k,\ l \\ \mu_H & \forall\ a,\ i,\ j \geq H,\ k,\ l \end{cases}$$

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$$q^a_{i,j,k,l;i-1,j,k,l} = \begin{cases} 0 & \forall\ a \neq 2,\ i,\ j,\ k,\ l \text{ and } a = 2,\ i = j + k,\ l \\ \mu_j & \forall\ a = 2,\ i,\ j < H - 1,\ k,\ l \\ \mu_H & \forall\ a = 2,\ i,\ j \geq H - 1,\ k,\ l \end{cases}$$

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$$q^{a}_{i,j,k,l;i-1,j,k-1,l} = \begin{cases} 0 & \forall\, a \neq 3,\, i, j, k = 0, l \\ p_1 * \gamma * (k) & \forall\, a \neq 3,\, i, j, k > 0, l \end{cases}$$

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$$q^{a}_{i,j,k,l;i,j,k-1,l} = \begin{cases} 0 & \forall\, a \neq 3,\, i, j, k = 0, l \text{ and } a \neq 3,\, i, j, k, l \neq 0 \\ p_2 * \gamma * (k) & \forall\, a \neq 3,\, i, j, k \neq 0, l = 0 \end{cases}$$

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$$q^{a}_{i,j,k,l;i,j,k,l-1} = \begin{cases} 0 & \forall\, a \neq 3,\, i, j, k = 0, l \text{ and } a \neq 3,\, i, j, k, l = 0 \\ p_2 * \gamma * (k) & \forall\, a \neq 3,\, i, j, k \neq 0, l \neq 0 \end{cases}$$

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$$q^{a}_{i,j,k,l;i-1,j,k-1,l} = \begin{cases} 0 & \forall\, a = 3,\, i, j, k = 0, l \text{ and } a = 3,\, i \neq j + k, l \neq 0 \\ p_1 * \gamma * (k) & \forall\, a = 3,\, i, j, k > 0, l = 0 \text{ and } a = 3,\, i = j + k, l \neq 0 \end{cases}$$

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$$q^{a}_{i,j,k,l;i-1,j,k,l-1} = \begin{cases} 0 & \forall\, a = 3,\, i, j, k, l = 0 \text{ and } a = 3,\, i = j + k, l \neq 0 \\ p_1 * \gamma * (k+1) & \forall\, a = 3,\, i \neq j + k, l \neq 0 \end{cases}$$

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$$q^{a}_{i,j,k,l;i,j,k-1,l} = \begin{cases} 0 & \forall\, a = 3,\, i, j, k = 0, l \text{ and } a = 3,\, i, j, k, l \neq 0 \\ p_2 * \gamma * (k) & \forall\, a = 3,\, i, j, k \neq 0, l = 0 \end{cases}$$

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$$q^{a}_{i,j,k,l;i,j,k,l-1} = \begin{cases} 0 & \forall\, a = 3,\, i, j, k, l = 0 \\ p_2 * \gamma * (k+1) & \forall\, a = 3,\, i, j, k, l \neq 0 \end{cases}$$

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$$q^{a}_{i,j,k,l;i,j,k,l+1} = \begin{cases} 0 & \forall \quad a, i, j, k+1 \geq \text{number of trucks} \\ \varepsilon_1 & \forall \quad a, i, j, k+1 < \text{number of trucks} \end{cases}$$

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***Conservation of probability*:**
$\sum P^{d}_{i,j,k,l} = 1.0$

***Bound constraints*:**
$0 \leq P^{d}_{i,j,k,l} \leq 1$ for $i = 0,1\ldots\ldots$capacity
$j = 0, 1\ldots\ldots$the number of hydrants
$k = 0,1\ldots\ldots$the number of refueling trucks such that
$l\text{-}k \leq$ the number of refueling trucks
$l = 0,1\ldots\ldots\ldots$the number of refueling trucks such that
$l\text{-}k \leq$ the number of refueling trucks

Since solving the constraint matrix provides us with transition probabilities, if a state has an associated transition probability, the linear program provides the decision to be made if the system is in that state. This decision is the one which minimizes the number of aircraft in the system. The sequence of decisions built constitute the refueling policy which maximizes the throughput by minimizing the number of aircraft in the system.

4.5 Queue Performance Measures.

The advantage of modeling the airfield refueling process as a continuous time Markov process is that queuing performance measures can be derived from the system state distribution. These measures can be used to evaluate various configurations of the system.

AMC plans individual missions to implement large contingency operations. The success of the contingency relies heavily on aircraft scheduling. This schedule is dependent on how accurately the aircraft flies the mission leg and stays within the scheduled ground time. AMC's primary measure of efficiency is the throughput for an airfield. Naturally, in order to provide a more efficient airfield operation, steps need to be taken in order to increase the throughput of an airfield. The queuing performance measure that evaluates this is the mean time each aircraft is in the system. In order to increase the airfield throughput, the aircraft arrival rate the airfield can sustain has to be increased. By changing airfield configurations or utilizing current resources with an optimal refueling policy, this study provides two tools (models four and five) which can be used to determine the maximum arrival rate the airfield can sustain, based on a pre-determined mean time in system. In order to calculate the mean time in system, the number of aircraft in the system (on the ground) is first determined using:

$N = \sum k * p(k)$

where k is the number of aircraft in the system.
$p(k)$ is the long run probability of having k aircraft in the system.

Then the average time in system is calculated using Little's Law and the previous queuing measure, N:

$T = N / \lambda(1 - P_C)$

where N is the mean number of aircraft in the system (calculation shown above).
λ is the aircraft arrival rate to the system.
P_C represents the proportion of aircraft which find the system at capacity, C.
$(1 - P_C)$ represents the proportion of aircraft that do arrive to the airfield. Aircraft which arrive and find the system at capacity, C, do not arrive (land) at the airfield.
$\lambda(1 - P_C)$ is the essential aircraft arrival rate. This measure is the arrival rate for the proportion of aircraft which actually arrive to the airfield.

A misconception may lead one to believe that in order to stay within the scheduled ground time, this measurement is minimized. By minimizing this measurement, the aircraft depart the system earlier than expected. This causes aircraft to arrive at bases earlier and may cause a bottleneck at an airfield. Since this ultimately leads to a more inefficient contingency operation, the average time in the refueling system should be close to the expected (pre-determined) average ground time (due to refueling) proposed by MASS. The models can be used to determine the maximum aircraft arrival rate the airfield can sustain using the pre-determined average ground time due to refueling.

4.6 Computer Implementation.

The models discussed previously are implemented using the FORTRAN computer language. The formulations are solved using IMSL for the first four models and using CPLEX for the Markov decision process model. These imbedded subroutines solve either the matrix for the steady-state probabilities or the linear program, and output the results. The FORTRAN, IMSL and CPLEX code, as well as instructions for use, are included in Appendix A.

Chapter 5

Model Results

5.1 Introduction.

Five models are developed to understand current ground refueling operations at AMC airfields. The user inputs five characteristics that define the airfield refueling configuration and planned mission: arrival rate, average amount of fuel demanded, the number of "active" hydrants, the number of fuel trucks and the number of fillstands at the airfield. Using these inputs, the models output the steady-state distribution of the number of aircraft in the system. Since the steady-state distribution completely describes the system, any queuing proficiency measure can be calculated within the model. The output of the results for the first four models concentrates on the average time spent in the system. This output can be used to evaluate the sensitivity to changes in the airfield configuration (by varying the number of trucks, fillstands or active hydrants) or to changes in the contingency scenario (by varying the aircraft arrival rate).

The fifth model, the Markov decision process, is used to optimize the refueling policy. Therefore, the output of the model indicates which decision should be made at each state of the system to minimize the number of aircraft on the ground for each system. For a given arrival rate, this objective minimizes the average time each aircraft spends in the refueling system. By minimizing this time, the refueling policy decreases resource utilization and allows the airfield to possibly sustain a higher aircraft arrival rate, which increases airfield throughput. An aircraft arrival rate can be determined by comparing the mean time in system (provided from various arrival rate inputs) with a pre-determined maximum mean time in system.

The MASS output for a contingency operation through Hickam AFB is used to demonstrate the first four models using a typical mission scenario and airbase. Hickam has between 20-30 fuel trucks, 6 fillstands, and 4 "active" hydrants. Aircraft arrive at a rate of 3.57 (aircraft) per hour and demand approximately 24,767 gallons of fuel. The presented results vary the aircraft arrival rate between 3.0 and 4.0 aircraft per hour and the number of trucks between 5 and 30. The first four models present comparisons of the airfield configuration using the average time in system. The fifth model's use is demonstrated using a small, capacitated airfield and presents an optimal refueling policy. This policy is then compared to the current AMC refueling policy. Also, results from modifications to the objective function are shown and comparisons are made for various airfield configurations and aircraft arrival rates.

5.2 Birth and Death Process Results.

The first model uses a birth and death process to represent the airfield refueling process. The number of aircraft in the system is the only variable used in the system state description and the refueling process forces an aircraft to switch refueling systems when a hydrant becomes available. Using the current airfield configuration of Hickam AFB, the average time in the system is 0.6295 hours.

As expected, increasing the arrival rate in the model shows a slight increase in the average time the aircraft are in the system (Figure 5-1). As the number of trucks is increased in the model, no difference in the mean time in system is noticed when the number of fuel trucks at the airfield is varied between 10 to 20 and little difference between 5 and 10.

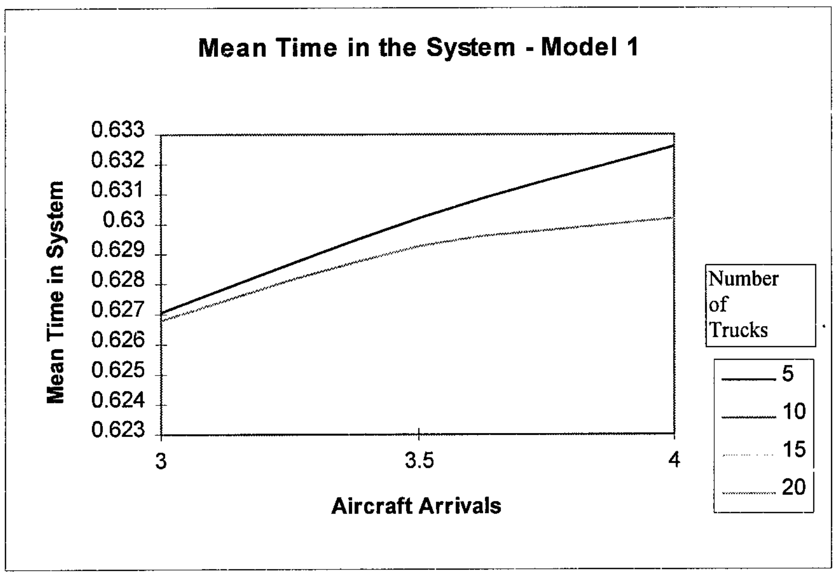

Figure 5-1, Mean Time in System, Model 1.

Because of the low fidelity of the first model, the results do not give any appreciable insight into the airfield refueling process. For this reason, two additional characteristics are modeled in the succeeding models: the delay due to trucks having to refill at a fillstand and the refueling policy allowing aircraft to stay on the same refueling system until completion.

5.3 Continuous Time Markov Processes Results.

The results from all three models which represent the system as a continuous time Markov process are presented. The second model adds the number of trucks in the system and not at a fillstand. The third model does not include trucks in the system but does represent the true refueling policy, allowing each aircraft to stay on the original refueling system until completion. The fourth model combines these two characteristics to gain a more complete description of the process. Once again, the aircraft arrival rate and the number of trucks are varied in order to compare airfield configurations. The configurations are evaluated using the same queuing performance measure, average time in the system. By presenting results from the first three models, a comparison can be made between the fidelity of each so the important modeling characteristics are realized for use in the fourth model (used to evaluate current system) and in the fifth model (used to evaluate a new refueling policy).

The first continuous time Markov process enhances the birth and death process by modeling the number of trucks in the system. Because the delay due to trucks having to refill at the fillstand is represented in this model, higher delays are expected. The mean time each aircraft is in the system is 0.7611 hours. Once again, as the aircraft arrival rate increases, the mean time in service increases (Fig. 5-2). The mean time in the system then decreases as the number of trucks increase because the departure rate from the system is a function of the number of trucks on the airfield. The results from this model show that the delay due to trucks should be represented in the fourth model.

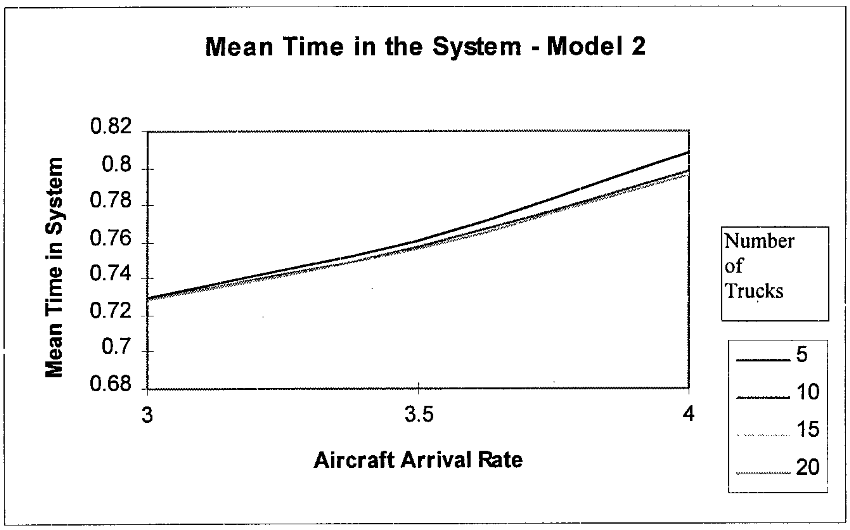

Figure 5-2, Mean Time in System, Model 2.

Although this model's results present a more appreciable difference in the queuing performance measure, it does not allow aircraft to remain on the truck system if a hydrant becomes available. Because the current policy forces each aircraft to remain on the refueling system, this is modeled to provide a more complete representation of the airfield refueling process.

The next continuous time Markov process does not model the delay due to trucks refilling at the fillstand but does allow each aircraft to stay with its original refueling system. This model is used to assess the significance of a specific aspect of the refueling policy. In comparison to the second model, the mean time in the system is approximately the same - 0.7512 hours. As shown below (Fig. 5-3), the refueling policy addition without modeling the delay due to trucks refilling does not significantly affect the model when the number of trucks is increased.

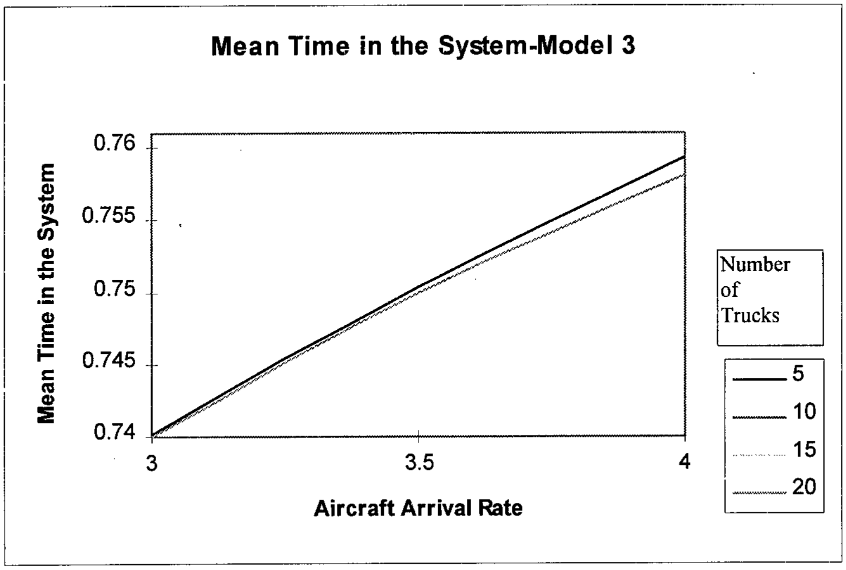

Figure 5-3, Mean Time in System, Model 3.

The second and third models show a slight difference in mean time in the system from the simple birth and death process. Because the results show that both the delay due to trucks refilling at the fillstand and the true the refueling policy provide results that differ from the first model, the next model assesses the combined effects of representing these two airfield characteristics.

The fourth model uses the previous two continuous time Markov process models to construct a better representation of the airfield refueling system. It is the primary model to be used to assess varying airfield configurations using the current refueling policy. Due to the added fidelity of this model, the results show a significant difference in the mean time in the system from all previous models (Fig. 5-4). Using the Hickam AFB baseline, the average time each aircraft is in the refueling system for this model is 1.3354 hours versus 0.6295 hours for the simple birth and death process, or a difference of over 42 minutes per aircraft.

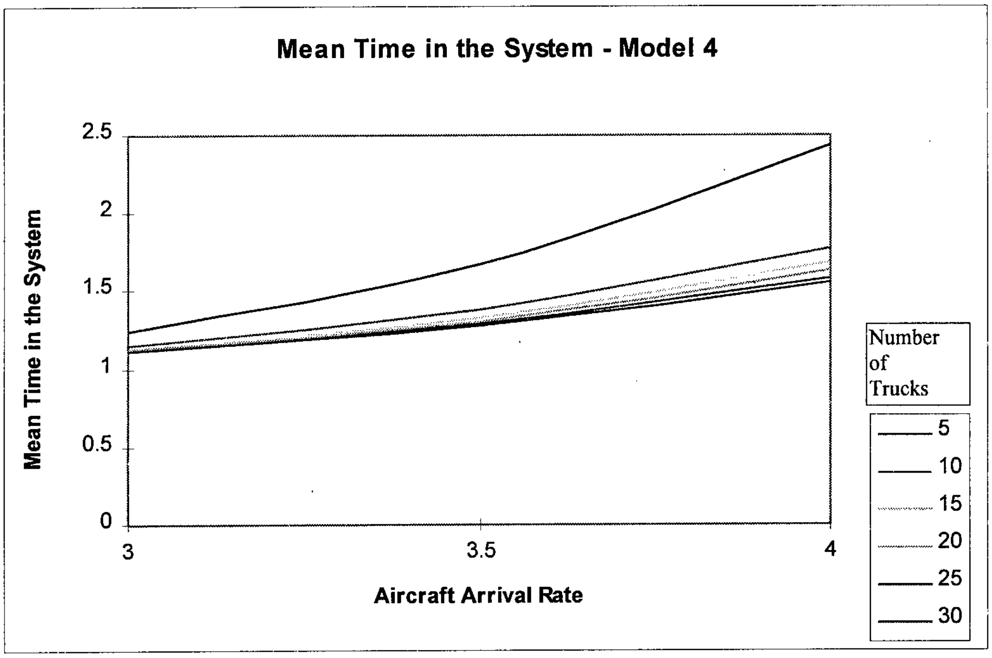

Figure 5-4, Mean Time in System, Model 4.

The difference between the queuing performance measures of the first three models and the fourth warrants use of the final continuous time Markov process to evaluate current airfield configurations over all previous models. By using this model, a more complete representation of ground refueling operations leads to more accurate results and a better understanding of the system.

5.3.1 Additional Results of Model 4.

Because the fourth model provides the best representation of the ground refueling operations, additional runs are made using this model to demonstate its contribution to AMC. These additional runs are used to gain insight into the refueling system and to demonstrate how the model can be used to evaluate an airfield by varying the configuration. This demonstration varies two aspects of the airfield, the number of fillstands and "active" hydrants, in order to find the optimal configuration of the airfield for the desired throughput or aircraft time in the system. Results of the model are then presented that use the current Hickam AFB configuration and show various possible arrival rates that the airfield can sustain using the average time in the system as the criteria for evaluation.

The result show that when varying the number of active hydrants in the model, there

is

no appreciable difference in terms of the average time in the system after 4 hydrants (Fig. 5-5). Under the current configuration, the present number of active hydrants seems appropriate.

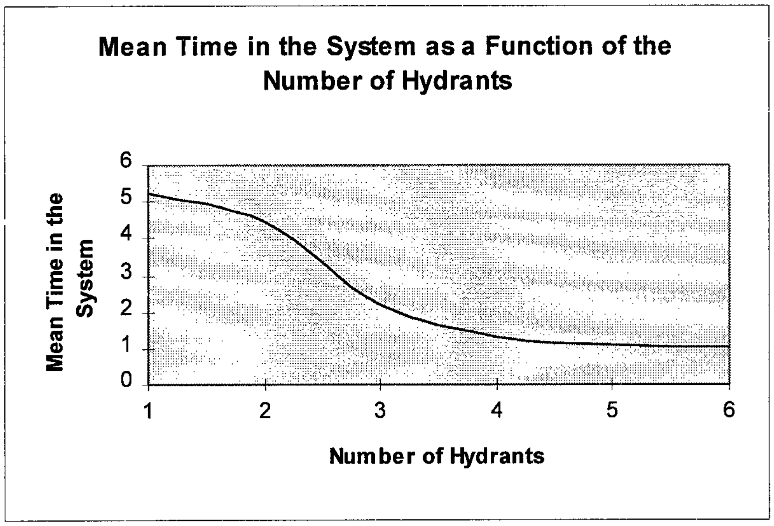

Figure 5-5, Mean Time in System, Hydrants.

Also, the results show that there is no appreciable gain in terms of the average time in the system if the airfield has over 5-6 truck fillstands (Fig. 5-6). Under the current configuration, the present number of fillstands seems appropriate.

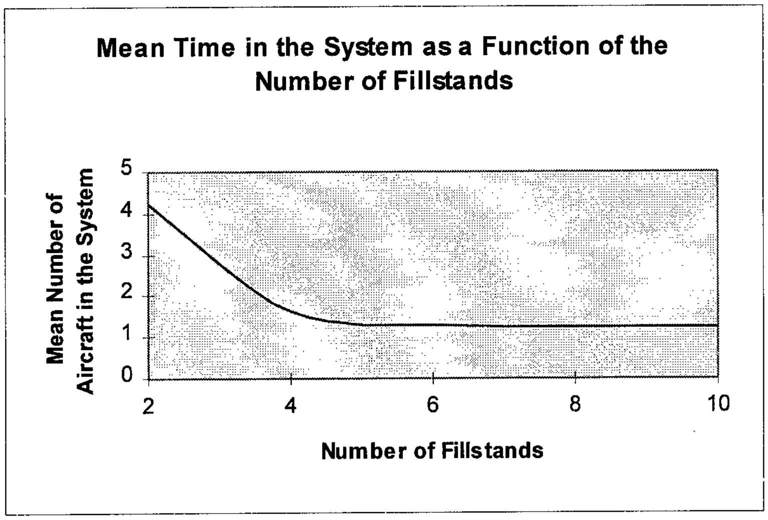

Figure 5-6, Mean Time in System, Fillstands.

Although our data shows that the current configuration is appropriate for an aircraft arrival rate of 3.57 aircraft, results are presented to determine the arrival rate the current airfield can sustain without excessive delay. This is done to demonstate how this model can be used to increase throughput capacity. The results below (Fig. 5-7) show how an increasing arrival rate effects the average time in system.

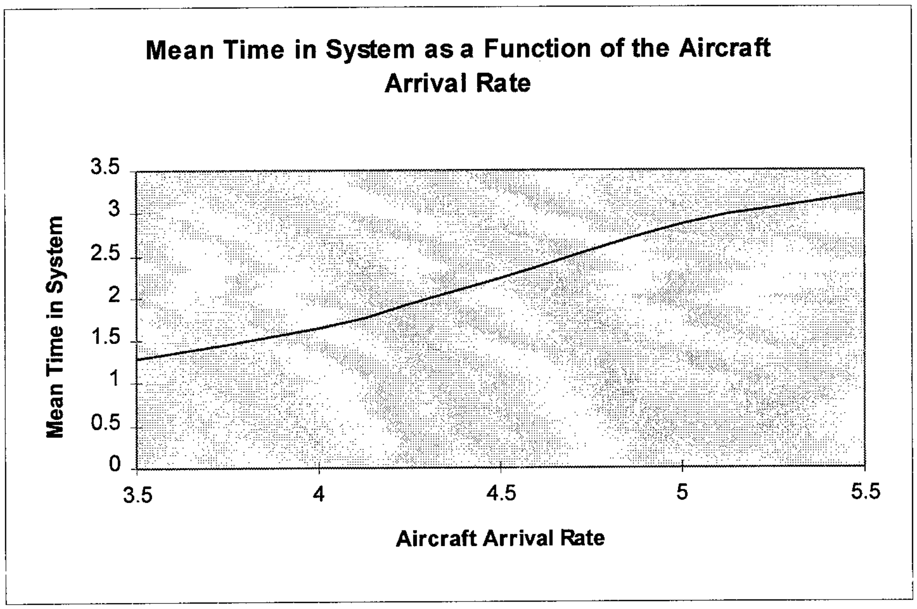

Figure 5-7, Mean Time in System, Arrival Rate.

Because many airfields are not limited by capacity, the primary evaluation means for AMC is the average time in the system. Currently, AMC has a maximum ground time (including unloading/loading, maintenance and refueling) limitation of 2.5 hours. Analysts can determine the maximum amount of time an aircraft can spend in the refueling system to meet this requirement and choose the appropriate arrival rate to try to increase throughput.

The fourth model can also be used to find the sensitivity of specific characteristics of the airfield, such as the sensitivity of the refill rate to the mean time in the system. By varying the per-fillstand rate that trucks refill from 1 per hour to 2 per hour then to 3 per hour, we can show the effect of this airfield characteristic on the mean time in the system. Because Hickam has 6 fillstands, the rate at which trucks are refilled is 6, 12 and 18 per hour. As shown below, using the baseline airfield, the time to refill trucks at a fillstand has a small effect on the mean time in system (Fig. 5-8):

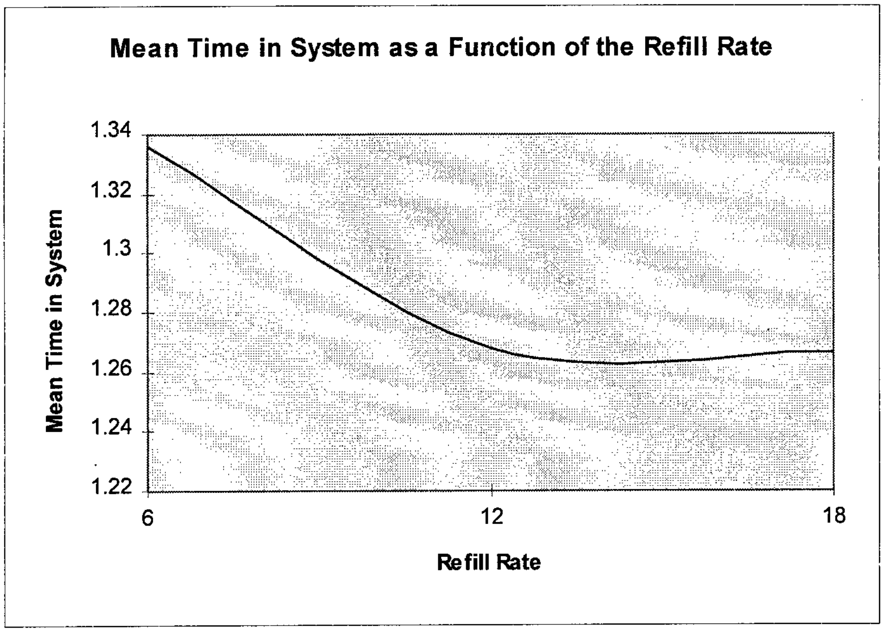

Figure 5-8, Mean Time in System, Refill Rate.

As shown, there is not a substantial difference in the mean time in system when varying the amount of time it takes to refill trucks. This finding is a result of the large number of fillstands and the large number of trucks occupying the airfield. With such a large number, the truck system essentially represents a continuous flowing refueling system for the airfield.

It has been shown that this model is an accurate representation of the airfield and that it can be used to evaluate configurations of various airfields and mission scenarios. It has also been shown how this model can be used to increase throughput for an airfield and provide insight into the sensitivity of airfield characteristics. This model remains a flexible tool that can be used to vary any configuration or mission scenario to try to gain insight into and optimize ground refueling operations at AMC airfields.

5.4 Markov Decision Process Results.

The system state notation used in the fifth model completely describes the airfield refueling system by representing the number of aircraft in the system, on which system each aircraft is being serviced, and the number of trucks in the system (not at a fillstand and/or on an aircraft). The model only uses capacity to restrict the number of aircraft placed on any system. Due to its complexity, this model is demonstrated using a small airfield. The capacity of the baseline airfield is set at 8 aircraft with 2-4 active hydrants, 4-8 refueling trucks and 4 fillstands. The aircraft arrival rate varies between 1.0-3.5 aircraft per hour and the average amount of fuel demanded varies from 6,000 to 24,767 gallons. General trends of the optimal airfield refueling policy are presented and compared to the current AMC refueling policy. Also, results are shown for any changes in the refueling policy that may occur with the other presented airfield configurations.

In general, the optimal airfield refueling policy follows a "greedy" algorithm in that it chooses the refueling system that provides the highest immediate *system* departure rate. For this reason, the optimal sequence of decisions follows AMC's current refueling policy of placing each arriving aircraft on the hydrant system (if available) before the truck system because of the hydrant system's higher refuel rate. Also, once all active hydrant systems are full, the optimal refueling policy places the aircraft on the truck system even if the truck system is full, not unlike the current AMC refueling policy. As shown below, as the number of hydrants is increased, the average time in system decreases (Fig. 5-9). This model represents an airfield with an arrival rate of 2.0 aircraft per hour, 4 refueling trucks, 4 fillstands and an average of 24,767 gallons demanded:

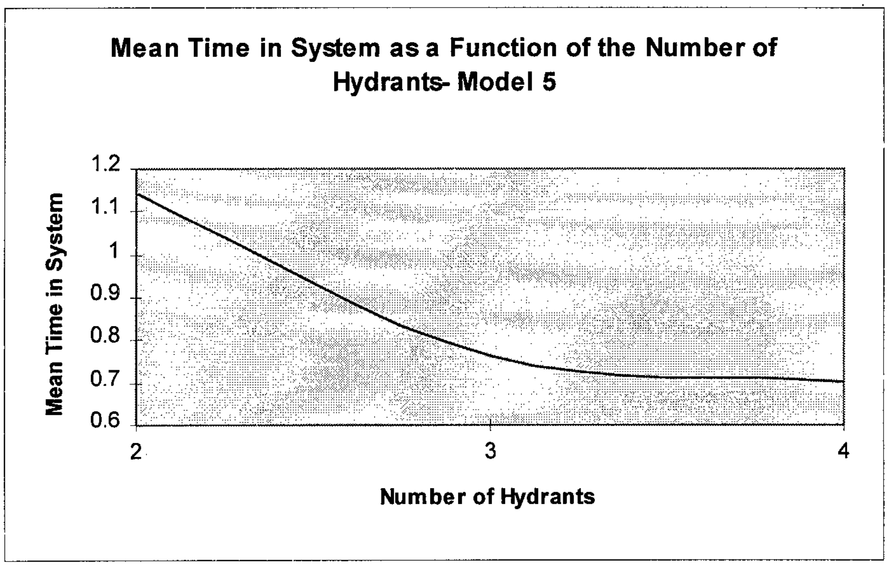

Figure 5-9, Mean Time in System, Model 5 - Hydrants.

As shown, there is no substantial gain after placing 3 hydrants on the airfield. The optimal refueling policy of this model is to send every arriving aircraft to the hydrant system (if available).

The original model places no restrictions on the number of aircraft that can be placed on the hydrant system. For this reason, the model always selects the decision to place the aircraft on the hydrant system because it has a higher aircraft departure rate than the truck system, even when all active hydrants are taken. This forces each aircraft not on an "active" hydrant system to be placed in a hydrant queue, awaiting an active hydrant. Although the model system state description does not represent a policy that would limit the number of aircraft that can be placed in the hydrant system, the objective function can be modified to account for this. By placing a virtual "infinite" cost on a steady-state probability, the linear program, seeking to minimize the objective function, does not select that state. This essentially eliminates a specified number of aircraft from entering the hydrant system and forces the aircraft to be sent to the truck system. To demonstrate this concept, the objective function is modified so no more than 4 aircraft can be placed in the hydrant queue. As shown below (Fig. 5-10), when hydrants are capacitated, the number of hydrants needed to obtain a reasonable mean time in system depends highly on the amount of fuel demanded and the number of hydrants.

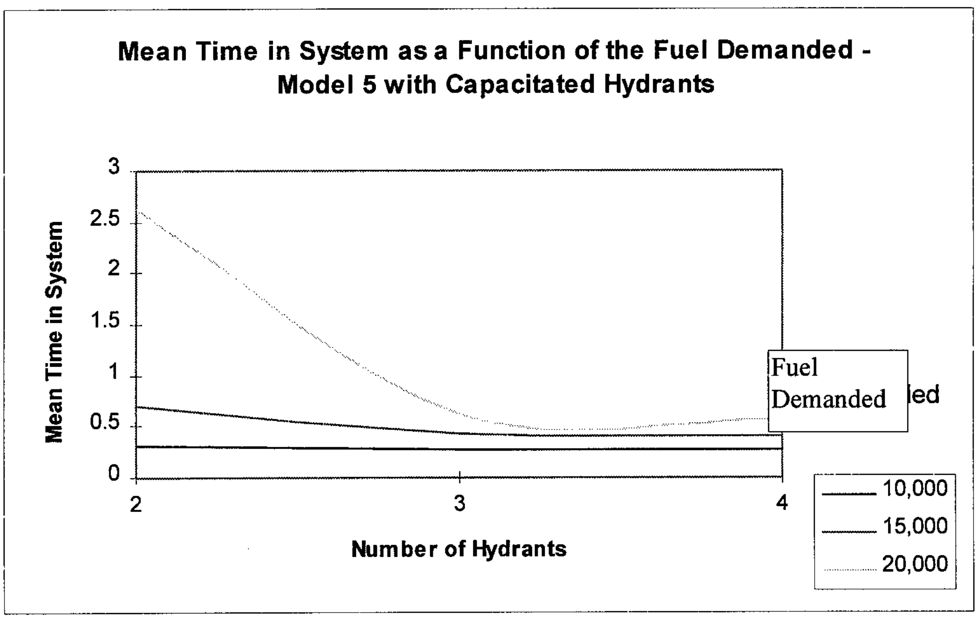

Figure 5-10, Mean Time in System, Model 5 with Capacitated Hydrants.

This is a result of forcing aircraft to go to the truck system once the hydrant queue is full. With two active hydrants, since there can be no more than six aircraft in the hydrant system, the optimal refueling policy sends the first six aircraft to the hydrant system and each subsequent arriving aircraft (maximum of two aircraft because of airfield capacity of eight) to the truck system. With three active hydrants, only one aircraft is sent to the truck system. Because there are only 4 trucks on the airfield, each holding 5500 gallons, the probability a truck refuels an aircraft before it empties is low, if the aircraft demands a comparatively high amount of fuel. Therefore, after each truck empties its tank (approximately 10 minutes), it must refill at a fillstand, taking approximately 1 hour to accomplish. This leads to a bottleneck in the system.

In the previous scenario, with an aircraft arrival rate of 2.0 and 24,767 gallons demanded, three hydrants appeared to be an appropriate configuration. With this configuration, any increase in throughput can be evaluated by comparing the mean time in system (using increasing arrival rates) with a pre-determined maximum mean time in the refueling system (Fig. 5-11). By determining a value for comparison, the maximum arrival rate (and throughput) can be determined for a given airfield configuration.

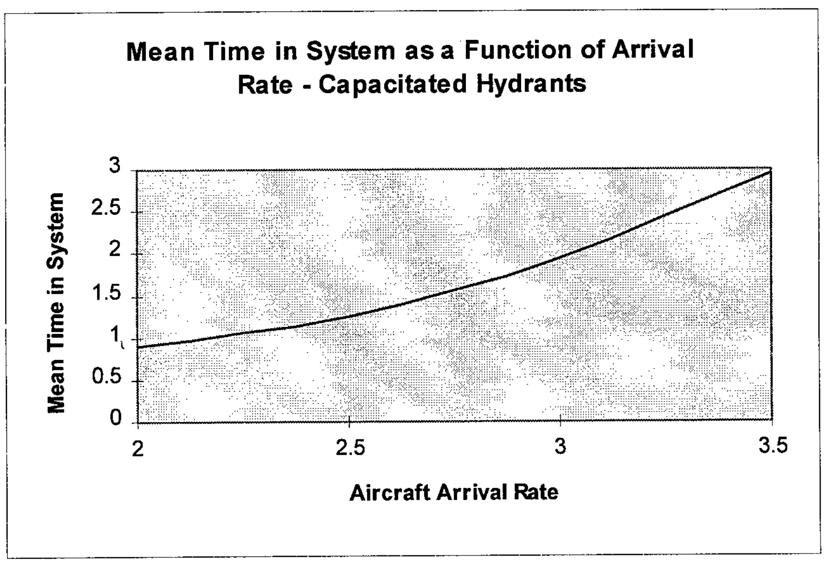

Figure 5-11, Mean Time in System - Arrival Rate, Model 5 - Capacitated Hydrants.

It should be noted that the probability an arriving aircraft cannot enter a capacitated system increases with the aircraft arrival rate. Because "balking" or not entering a capacitated system, may not be a policy for an airfield, the distribution for the steady-state probability needs to be considered when using these models to increase the arrival rate (and throughput).

It has been shown that the fifth model is an accurate representation of the airfield. This model can be used to evaluate configurations of various airfields and mission scenarios and to determine the optimal refueling policy of the airfield. It has also been shown how this model can be used to increase throughput for an airfield by increasing the arrival rate the airfield can sustain. It should be noted that conclusions from all five models are drawn from a specific data set and should not be used to typify any airfield. Because each airfield's configuration and policies (such as truck refilling) are different, this study provides tools to be used to make better decisions on how to increase airfield efficiency.

Chapter 6

Conclusions.

6.1 Overview.

This study develops five analytical models to analyze the current ground refueling process and to determine the refueling policy which minimizes the number of aircraft on the ground. The airfield refueling process is modeled as a continuous time Markov process to adequately represent the inherent stochastic nature of arrivals and departures from the system and to provide an analytical evaluation of various airfield configurations.

In order to use Markovian modeling, the process has to exhibit transitory behavior and the Markovian property. In order to make the problem mathematically tractable, it is assumed that aircraft arrivals are generated by a Poisson process. It is also assumed that the amount of fuel an aircraft or truck needs at any time in the system is exponentially distributed and therefore memoryless. The primary advantage of using Markovian modeling is the ability to determine the complete state. Using this distribution, queuing performance measures are developed and used to compare various airfield configurations and mission scenarios. The model is demonstrated using data from a transient airbase, Hickam AFB, HI.

Four models are sequentially developed in order to represent the current ground refueling process. As each model is built, the system state description becomes more complex to provide a better representation of the ground refueling process. The fourth model provides a comprehensive yet flexible tool that can be used to evaluate the current airfield refueling policy with various resource configurations and mission scenarios.

A fifth model is built that represents the system as a continuous time Markov decision process, where one of three decisions are made at each state of the system: send the arriving aircraft to the hydrant system, send the arriving aircraft to the truck system or wait for the system to change. A refueling policy is chosen by a linear program in order to minimize the number of aircraft on the ground. The results of the model provide the refueling policy, the mean number of aircraft in the system and the mean time in the system. By increasing the aircraft arrival rate, the mean time in the system can be compared to a pre-determined maximum value to maximize airfield throughput using the provided refueling decisions. This sequence of decisions is the optimal refueling policy for the airfield.

6.2 Applications.

Although three other models are developed, the fourth model should be used to evaluate various airfield configurations to determine the minimum number of resources needed to meet a required contingency need. For each mission scenario, the average amount of time an aircraft should spend in the refueling system should be determined and unchanging resources should be fixed (such as the number of active hydrants). Then, by varying the airfield characteristics that may change (such as the number of trucks), airfield interactions can be shown and compared to determine the number of each resource that is needed to meet the required objective. Also, using a current airfield configuration and a pre-determined average time in the refueling system, a maximum aircraft arrival rate can be calculated to increase the throughput of the airfield. The model can also be modified to accommodate changes in refueling rates (of the hydrants and trucks) and refilling rates (of the fillstands).

The fifth model provides a more complete but less flexible representation of the airfield refueling process to be used to determine the optimal refueling policy that minimizes the number of aircraft on the ground (on the average). The results of this model provide the refueling system decision that should be made at each state of the system. Because this approach is not flexible enough for contingency use, the model should not be used to provide the refueling policy decision maker an action to be taken at each aircraft arrival. Rather, the model should be used as a tool to determine and report general trends seen on a particular airfield and mission scenario prior to contingency operations. This approach provides refueling personnel with a tool that *aids* in decision making and does not *make* a decision void of intuition and experience.

6.3 Recommendations for Further Research.

Global mobility became the foundation of our national security strategy when it changed from a strategy of *forward basing* of troops to one of *forward presence* of troops. AMC realized that, with this change, airlift would be required to produce more efficient operations through the optimal use of current strategic airlift resources. For this reason, this study develops models to try to understand how one aspect of the strategic airlift process, ground refueling, effects the airlift efficiency. All models assumed that the aircraft arrivals are generated by a Poisson process (hence, the interarrival times are exponentially distributed) and the times between departures are exponentially distributed, and therefore, memoryless. Although the models provide AMC with a valuable tool to not only evaluate airfields but to optimize their ground refueling operations, future modeling and analysis should concentrate on four areas: modeling a large airfield, the aircraft arrival process, the departure process, and modeling of all airfield operations.

Because no contingency aircraft arrival data was available, the input data used to justify the assumption that aircraft arrivals are generated from a Poisson process is from MASS data output. The analysis showed that the assumption is reasonable although no standard distribution provides a "good" fit for the data. Because aircraft arrivals are strategically scheduled, relaxation of the assumption that aircraft interarrivals are exponentially distributed might yield additional insight into the ground refueling process.

The system service rate depends on the specific refueling system service rate and the average amount of fuel demanded by aircraft. Although the refueling system service rate is constant, the amount of fuel each aircraft demands is assumed to be exponentially distributed. Oher distributions could be examined to model the amount of fuel each aircraft demands. This would change the distribution of the aircraft departure process.

This study sought to isolate one aspect of airfield operations, the ground refueling operations. Further research should concentrate on modeling all three operations of the airfield (refueling, maintenance and unloading/onloading) as a Markov process. Using the assumption that aircraft arrivals are generated by a Poisson process, a further assumption could be made that the time between departures from each separate operation or "phase" is exponentially distributed. This would allow for the service times to be modeled with the Erlang distribution. After justifying the validity of this assumption, the airfield operations could be modeled as a network of queues. The results of this model can be used for comparison with and evaluate of BRACE output.

Appendix A

FORTRAN Programs and Instructions

Five FORTRAN programs are written to represent the ground refueling operations at AMC airfields. The first four models use imbedded IMSL subroutines for the solution. The fifth model, the Markov decision process uses imbedded CPLEX subroutines for the solution due to the complexity of the system state description.

Each program requests input from the user to define the airfield and mission scenario:

```
PRINT*,'What is the aircraft arrival rate?'
READ*, lambda

PRINT*,'How many active hydrants are available?'
READ*, numhydrants

PRINT*,'How many fueling trucks?'
READ*, numtrucks

PRINT*,'What is the aircraft receiving rate?'
READ*, acrec

PRINT*,'What is the average amount of fuel needed per
aircraft?'
READ*, amtfuel

PRINT*,'What is the refueling system capacity?'
READ*,capacity

PRINT*,'How many fillstands are in use at the
airfield?'
READ*, numfill
```

This input is used to form the Markov process transition matrix. The solution for this matrix defines the transition probabilities. These probabilities are used to solve for the queuing performance measures. Within the programs, the queuing performance measures and objective function coeffiecients can be modified. Directions are imbedded in the programs as to where and how these two characteristics can be modified.

The programs output a system state distribution and the queuing performance measures. For the first four models, the number of aircraft in the system are the only description that is used. For the fifth model, the entire state description is output in order to decide what characteristics effect the refueling decision. A sample output (the fifth model) is shown below:

```
System State Description:
6  aircraft in the system.
2  aircraft on hydrants.
3  aircraft in the truck system.
5  trucks waiting to refuel an aircraft.
Probability of being in this state .230678

Assign the aircraft to a hydrant.

...


Queuing Performance Measures.

The average number in the system is   2.4645 aircraft.

The average time in the system is   .857638 hours.')
```

Following is the FORTRAN code with imbedded IMSL and CPLEX subroutines for the five models (FORTRAN programs):

```
PROGRAM THESIS1

C  This program is written by LT W Heath Rushing.  This program computes the
C  the queuing measurements for AMC's current refueling policy using only the
C  number of aircraft in the system to describe the state space.  The aircraft
C  refueling system is modeled as a birth and death process with the
C  population being the number of aircraft.  A birth is an arrival and a death
C  is a departure from the system after service is completed.

C  Describes the matrix used to solve the system of equations.

      PARAMETER (IPATH = 1, LDA = 1000, N = 1000)
      REAL A(LDA, LDA), B(N), X(N)

C  Parameter definition.

      REAL W, L
      REAL lambda, numtrucks, amtfuel
      REAL mu1, mu2, mu3, TEMP
      REAL mu4, mu5, rout
      INTEGER m, capacity, numhydrants, I

      COMMON /WORKSP/  RWKSP
       REAL RWKSP(1002022)
       CALL IWKIN(1002022)

C  Gain all airfield specific characteristics needed for analysis:  aircraft
      arrival
C  rate, the number of active hydrants, the number of trucks and the amount of
      fuel
C  needed per aircraft.

      PRINT*,'What is the aircraft arrival rate?'
      READ*, lambda

c     PRINT*,'How many active hydrants are available?'
c     READ*, numhydrants
      numhydrants = 4

      PRINT*,'How many fueling trucks?'
      READ*, numtrucks
      TEMP = numtrucks

c     PRINT*,'What is the average amount of fuel needed per aircraft?'
c     READ*, amtfuel
      amtfuel = 24727

C  Set the capacity of the airfield, this limits the Markov process.
C  It is assumed that no more than capacity aircraft can be at the airfield
C  at one time.

c     PRINT*,'What is the airfield capacity?'
c     READ*,capacity
      capacity = 20
```

```
C       Represents service rate of hydrant system as aircraft arrive on the
        system.

        mu1 = 600.0*60.0/amtfuel
        mu2 = 1200.0*60.0/amtfuel
        mu3 = 1800.0*60.0/amtfuel
        mu4 = 2400.0*60.0/amtfuel

C       Represents service rate of truck system.

        mu5 = 550.0*60.0/amtfuel

C       Outputs inputs to user.

c       PRINT 10, lambda
c       10      FORMAT (1X, 'Aircraft arrive at a rate of ',F3.1,' per hr.')

c       PRINT 15, numhydrants, numtrucks
c       15      FORMAT (1X, 'There are ',I2,' active hydrants and ',F5.1,'
                trucks.')

        PRINT 20, mu1
20      FORMAT (1x, 'Mu 1 is ', F6.3)

        PRINT 21, mu2
21      FORMAT (1x, 'Mu 2 is ', F6.3)
        PRINT 22, mu3
22      FORMAT (1x, 'Mu 3 is ', F6.3)
        PRINT 23, mu4
23      FORMAT (1x, 'Mu 4 is ', F6.3)

        PRINT 24, mu5
24      FORMAT (1x, 'Mu 5 is ', F6.3)

C       Calculates specific probabilities of birth and death process.

        DO 32 I = 1, N
          DO 31 J = 1, N
             A(I,J) = 0
31        CONTINUE
          A(I,I) = 1.0
32      CONTINUE

C       Second row of matrix.

        rout = lambda + mu1

        A(2,1) = -(lambda/(rout))
        A(2,3) = -mu3/(rout)


        DO 36 I = 3, capacity + 1
          A(2,I) = 0.0
36      CONTINUE

C       Remaining probabilities.

        numtrucks = 0
```

```
      DO 50 m = 2, capacity
         IF (m.EQ.2) THEN
           muo = mu2
         ELSEIF (m.EQ.3) THEN
           muo = mu3
         ELSEIF (m.EQ.4) THEN
           muo = mu4
        ELSE
           muo = mu4 + mu5*numtrucks
        ENDIF

        IF (m.EQ.capacity+1) THEN
         rout = mu0
        ELSE
         rout = lambda + muo
        ENDIF

        IF (m.EQ.2) THEN
          mui = mu3
        ELSEIF (m.EQ.3) THEN
          mui = mu4
        ELSE
          mui = mu4 + mu5*numtrucks+1
        ENDIF

         A(m+1,m+1) = 1.0
         A(m+1,m) = -lambda/rout
         IF (m.LT.capacity+1) A(m+1,m+2) = - mui/rout

c       rin = lambda/rout + mui/rout

        IF (m.GE.numhydrants) numtrucks = numtrucks + 1
        IF (numtrucks.GT.TEMP) numtrucks = TEMP

50    CONTINUE


C     Using forward and backward equations and the conservation of
      probabilities equation (in 1st row).


      DO 68 I = 1, capacity + 1
        A(1,I) = 1.0
68    CONTINUE

C     Set RHS matrix.

      DO 70 I = 1, N
        IF (I.EQ.1) THEN
          B(I) = 1.0
        ELSE
          B(I) = 0.0
        ENDIF
70    CONTINUE


C     This is the matrix and the printout of the probabilities.

      CALL LSARG (N, A, LDA, B, IPATH, X)
```

```
c       CALL WRRRN ('Probabilities', N, 1, X, N, 0)

C       Calculation of the average number in the system.

        L = 0
        DO 80 I = 1, capacity+1
          PRINT 75, I-1, X(I)
75        FORMAT('The probability of ',I2,' aircraft on the ground is ',
               F8.3,'.')
          L = L + (I-1)*X(I)
80      CONTINUE

C       Using Little's law, calculating the average time in system.
        W = L/(lambda*(1-X(I)))

        PRINT*

        PRINT 85, L
85      FORMAT('The average number of aircraft in the system is ',F8.4, '.')

        PRINT 90, W
90      FORMAT('The average time in the refueling system is ',F8.4, ' hours.')

        STOP

        END
```

```
      PROGRAM THESIS2

C  This program is written by LT W Heath Rushing.  This program computes the
C  the queuing measurements for AMC's current refueling policy accounting for
C  truck refueling.

c     Setup for IMSL subroutines.

      PARAMETER (IPATH = 1, LDA = 500, N = 500)
      REAL A(LDA, LDA), B(N), X(N)

c     Declarations.

      REAL  mu5i, mu5o, calci, calco, L, W, calcii, mu3, mu4, mu5, account
      REAL lambda, amtfuel, gamma1, rout, gamma1i, gamma1o, mu5, mu5i
      REAL mu5o
      REAL mu1, mu2, mu5, numfill, mu1i, mu1o, lambdai, lambdao
      INTEGER m, capacity, numhydrants, temp2, numtrucks,I, J, K

        COMMON /WORKSP/ RWKSP
        REAL RWKSP(1002022)
        CALL IWKIN(1002022)

c     Inputs

      PRINT*,'What is the aircraft arrival rate?'
      READ*, lambda

      PRINT*,'How many active hydrants are available?'
      READ*, numhydrants

      PRINT*,'How many fueling trucks?'
      READ*, numtrucks
      temp2 = numtrucks

      PRINT*,'What is the average amount of fuel needed per aircraft?'
      READ*, amtfuel
      amtfuel = 24727

      PRINT*,'What is the system capacity?'
      READ*,capacity

      PRINT*,'How many fillstands are there?'
      READ*,numfill

c     the refuel rate for the trucks at the fillstand
      gamma1 = 1.0*numfill

c     the rate at which trucks empty while refueling aircraft
      mu5 = 6.0

c     The hydrant system service rate according to how many aircraft are on
      the system.
      mu1 = 600.0*60.0/amtfuel
      mu2 = 1200.0*60.0/amtfuel
      mu3 = 1800*60.0/amtfuel
      mu4 = 2400*60.0/amtfuel

C     The rate at which trucks refuel aircraft
      mu5 = 550.0*60.0/amtfuel
```

```
      PRINT 10, lambda
10    FORMAT (1X, 'Aircraft arrive at a rate of ',F3.1,' per hr.')

      PRINT 15, numhydrants, numtrucks
15    FORMAT (1X, 'There are ',I2,' active hydrants and ',I2,' trucks.')

      PRINT 20, mu1, mu2, mu5
20    FORMAT (1X, 'Mu 0 is ',F6.4, ' and Mu1 is ',F6.4,' Mu2 is ', F6.4)

      PRINT 25, gamma1, mu5
25    FORMAT (1X, 'Gamma 1 is ',F8.4,' and Gamma 2 is ', F8.4, '.')

c     the probability an aircraft runs out of gasoline first.
      probac = 1 - EXP(-5500.0/amtfuel)

c     the probability a truck runs out of gasoline first.
      probtr = 1 - probac

c     set up the matrix.
      DO 27 I = 1, N
        DO 26     J = 1, N
          A(I,J) = 0.0
          IF (I.EQ.J) A(I,J) = 1.0
26      CONTINUE
27    CONTINUE

c     formulate the transition matrix
      m=1
      DO 65 J = 0, temp2
        DO 60 I = 0, capacity

          IF (I.EQ.capacity) THEN
             lambdao = 0.0
           ELSE
             lambdao = lambda
           ENDIF

          IF (I.EQ.0) THEN
             mu1o = 0.0
          ELSEIF (I.EQ.1) THEN
             mu1o = mu1
          ELSEIF (I.EQ.2) THEN
             mu1o = mu2
          ELSEIF (I.EQ.3) THEN
             mu1o = mu3
          ELSE
             mu1o = mu4
          ENDIF

          IF (J.EQ.temp2) THEN
             gamma1o = 0.0
          ELSE
             gamma1o = gamma1
          ENDIF

          IF (I.LE.numhydrants.OR.J.EQ.0) THEN
             mu5o = 0.0
```

```
          ELSE
             mu5o = mu5
          ENDIF

          numtrucks = I - numhydrants
           IF (numtrucks.GT.temp2) numtrucks = temp2
           IF (numtrucks.GT.J) numtrucks = J

           calco=probtr*(numtrucks*mu5o) + probac*(numtrucks*mu5o)
            rout = mu1o + lambdao + gamma1o + calco
c          PRINT*,'Rout is ', rout

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

          IF (I.EQ.0) THEN
             lambdai = 0.0
           ELSE
             lambdai = lambda
           ENDIF

          IF (I.EQ.capacity) THEN
             mu1i = 0.0
           ELSEIF (I.EQ.0) THEN
             mu1i = mu1
           ELSEIF (I.EQ.1) THEN
             mu1i = mu2
           ELSEIF (I.EQ.2) THEN
             mu1i = mu3
          ELSE
            mu1i = mu4
           ENDIF

          IF (J.EQ.0) THEN
             gamma1i = 0.0
           ELSE
             gamma1i = gamma1
           ENDIF


          IF (I.LT.numhydrants.OR.J.EQ.temp2) THEN
             mu5i = 0.0
          ELSE
             mu5i = mu5
           ENDIF


          IF (I.LE.numhydrants.OR.J.EQ.temp2) THEN
             mu5i = 0.0
           ELSE
             mu5i = mu5
           ENDIF


            numtrucks = I - numhydrants + 1
           IF (numtrucks.GT.temp2) numtrucks = temp2
           IF (numtrucks.GT.J+1) numtrucks = J+1
            calci = probac*((numtrucks)*mu5i)

          numtrucks = I - numhydrants
           IF (numtrucks.GT.temp2) numtrucks = temp2
```

```
          IF (numtrucks.GT.J+1) numtrucks = J+1
          calcii = probtr*(numtrucks*mu5i)

          IF ((I.LT.numhydrants).OR.(J.EQ.temp2).OR.(I.EQ.capacity)) THEN
            calci = 0.0
          ENDIF

           IF ((I.LE.numhydrants).OR.(J.EQ.temp2)) THEN
             calcii = 0.0
          ENDIF

C          rin = lambdai + calci + +calcii + gammali



          IF (I.LT.numhydrants) THEN

             A(m,m) = 1.0
             A(m, m-1) = -lambdai/rout
             A(m,m+1) = -muli/rout
             A(m,m-(capacity+1)) = -gammali/rout



          ELSE

             A(m,m) = 1.0
             A(m, m-1) = -lambdai/rout
             A(m,m-(capacity+1)) = -gammali/rout
             A(m,m+(capacity+2)) = -calci/rout
             A(m,m+(capacity+1)) = -calcii/rout
             A(m,m+1) = -muli/rout

           ENDIF

          m = m+1

60       CONTINUE
65     CONTINUE


c      set the conservation of probability equation for the 1st row of the
       transition matrix
       DO 75 I = 1, N
         A(1,I) = 1.0
75     CONTINUE

c      set the RHS
       DO 80 I = 1, N
         IF (I.EQ.1) THEN
           B(I) = 1.0
          ELSE
            B(I) = 0.0
          ENDIF

80     CONTINUE


         DO 82 I = 1,(capacity+1)*(temp2+1)
```

```
          DO 81 J = 1,(capacity+1)*(temp2+1)
c            PRINT*, I,' ', J,' ', A(I,J)

81        CONTINUE
c          PRINT*,'B ',I,' is ', B(I)
82      CONTINUE

c       use IMSL subroutines to solve for the transiton probabilities

        CALL LSARG (N, A, LDA, B, IPATH, X)

c       CALL WRRRN ('Probabilities', LDA, 1, X, LDA, 0)

c       solve for the average number in system
        L = 0.0

        DO 94 K = 0, capacity
        m = K+1
        account = 0.0

          DO 90 J = 0, temp2
            account = account + X(m)
              m = m+capacity+1
90          CONTINUE

        L = L + K*(account)
        PRINT 91, K, account
91      FORMAT('The probability of ',I2,' aircraft is ', F6.3,'.')

94      CONTINUE

c       use Little's Law to calculate average time in refueling system

        W = L/(lambda*(1-account))

        PRINT 95, L
95      FORMAT('The average number of aircraft in the system is ',F8.4, '.')

        PRINT 100, W
100     FORMAT('The average time in the refueling system is ',F8.4, '.')

        STOP
        END
```

```
      PROGRAM THESIS3

C  This program is written by LT W Heath Rushing.  This program computes the
C  the queuing measurements for AMC's current refueling policy using only the
C  number of aircraft in the system to describe the state space.  The aircraft
C  refueling system is such that once an aircraft is on a refueling system,
C  the aircraft stays on that system.

C  Describes the matrix used to solve the system of equations.

      PARAMETER (IPATH = 1, LDA = 441, N = 441)
      REAL A(LDA, LDA), B(N), X(N)

C  Parameter definition.

      REAL W, L, AC, calc
      REAL lambda, numtrucks, amtfuel, TTEMP
      REAL mu1, mu2, mu3, mu4, mu5, TEMP, cap
      REAL mu5i, mu5o, rout, muo, mui, lambdai, lambdao
      INTEGER capacity, numhydrants, I, J, syslimit, m, K

      COMMON /WORKSP/ RWKSP
      REAL RWKSP(195385)
      CALL IWKIN(195385)

C  Gain all airfield specific characteristics needed for analysis:  aircraft
C arrival rate, the number of active hydrants, the number of trucks and
C the amount of fuel needed per aircraft.

      PRINT*,'What is the aircraft arrival rate?'
      READ*, lambda

      PRINT*,'How many active hydrants are available?'
      READ*, numhydrants

      PRINT*,'How many fueling trucks?'
      READ*, numtrucks
      TEMP = numtrucks

      PRINT*,'What is the average amount of fuel needed per aircraft?'
      READ*, amtfuel

C  Set the capacity of the airfield, this limits the Markov process.
C  It is assumed that no more than capacity aircraft can be at the airfield
C  at one time.

      PRINT*,'What is the refueling system capacity?'
      READ*,capacity

C     Represents service rate of hydrant system as aircraft arrive on the c
      system.
      mu1 = 600.0*60.0/amtfuel
      mu2 = 1200.0*60.0/amtfuel
      mu3 = 1800.0*60.0/amtfuel
      mu4 = 1800.0*60.0/amtfuel

C     Gives service rate of one truck.  This will be used for service rate of c
      multiple servers.
      mu5 = 550.0*60.0/amtfuel
```

```
C       Outputs inputs to user.

        PRINT 10, lambda
10      FORMAT (1X, 'Aircraft arrive at a rate of ',F3.1,' per hr.')

        PRINT 15, numhydrants, numtrucks
15      FORMAT (1X, 'There are ',I2,' active hydrants and ',F5.1,' trucks.')

        PRINT 20, mu1, mu2, mu5
20      FORMAT (1X, 'Mu 1 is ',F6.3, ' and Mu2 is ',F6.3,' Mu3 is ', F6.3)

C       Sets probability matrix equal to the identity.  The program fills in c
        the matrix as needed.
        DO 22 I = 1, N
          DO 21       J = 1, N
            A(I,J) = 0.0
            IF (I.EQ.J) A(I,J) = 1.0
21        CONTINUE
22      CONTINUE


C       Syslimit is the number of aircraft that can arrive to the system once
        hydrants are filled.
        syslimit = capacity - numhydrants
        TEMP2 = syslimit + 1

c       formulates the transition matrix.
        m = 1
        DO 70 I = 0, numhydrants
        numtrucks = 0

          DO 65 J = I, syslimit

            IF (I.EQ.0) THEN
               muo = 0.0
            ELSEIF (I.EQ.1) THEN
              muo = mu1
            ELSEIF (I.EQ.2) THEN
               muo = mu2
             ELSEIF (I.EQ.3) THEN
               muo = mu3
             ELSE
               muo = mu4
             ENDIF

            IF (J.GT.I) THEN
               mu5o = mu5
            ELSE
               mu5o = 0.0
             ENDIF

            IF (J.EQ.capacity) THEN
               lambdao = 0.0
             ELSE
               lambdao = lambda
             ENDIF

            rout = lambdao + (numtrucks*mu5o) + muo

              IF (I.EQ.0) THEN
```

```
          lambdai = 0.0
        ELSE
         lambdai = lambda
        ENDIF

          IF (I.EQ.numhydrants) THEN
           mui = 0.0
          ELSEIF (I.EQ.0) THEN
           mui = mu1
          ELSEIF (I.EQ.1) THEN
           mui = mu2
          ELSEIF (I.EQ.2) THEN
           mui = mu3
          ELSE
             mui = mu4
          ENDIF

      IF (J.EQ.syslimit) THEN
          mu5i= 0.0
          IF (J.EQ.capacity) THEN
            mui = 0.0
          ELSE
            mui = mui
          ENDIF
     ELSE
          mu5i = mu5
     ENDIF


      TTEMP = numtrucks+1
      IF (TTEMP.GT.TEMP) TTEMP = TEMP


      IF (I.EQ.numhydrants) THEN
        A(m, m) = 1.0
        A(m, (m-TEMP2)) = -lambdai/rout
         A(m,  (m+1)) = - ((TTEMP)*mu5i)/rout
         IF (I.EQ.J) THEN
         A(m, (m-1)) = 0.0
      ELSE
          A(m, (m-1)) = - lambdai/rout
      ENDIF
    ELSE
        A(m, m) = 1.0
        A(m, (m-TEMP2)) = -lambdai/rout
         A(m,  (m+1)) = - ((TTEMP)*mu5i)/rout
         A(m, (m + TEMP2)) = - mui/rout
    ENDIF
    m = m+1

    numtrucks = numtrucks + 1

    IF (numtrucks.GT.TEMP) numtrucks = TEMP

65  CONTINUE
  syslimit = syslimit + 1

70  CONTINUE
```

```
C       The conservation of probability equation is set on the first row of the
        matrix.
        DO 75 I = 1, N
          A(1,I) = 1.0
75      CONTINUE

C       sets RHS.
        DO 80 I = 1, N
          IF (I.EQ.1) THEN
            B(I) = 1.0
           ELSE
             B(I) = 0.0
           ENDIF

80      CONTINUE

C       Calls IMSL subroutines to solve the matrix of linear differential
        equations and
C       output them to the screen.

        CALL LSARG (N, A, LDA, B, IPATH, X)

c       CALL WRRRN ('Probabilities', LDA, 1, X, LDA, 0)

C       Calculation of the average number in the system.
        TEMP3 = TEMP2*(numhydrants+1)

        L = 0.0
        cap = 0.0
        DO 90 I = 1, (capacity-numhydrants + 1)
          AC = I-1
          J=I
          DO 89 K = 1, (numhydrants+1)
            calc = X(J) * AC
               L = L + calc
               J = J + (capacity - numhydrants+1)
               IF (AC.EQ.capacity) cap = cap + X(J)
            AC = AC + 1
89        CONTINUE

90      CONTINUE

C       Using Little's law, calculating the average time in system.
        W = L/(lambda*(1-cap))

        PRINT 95, L
95      FORMAT('The average number of aircraft in the system is ',F8.4, '.')

        PRINT 100, W
100     FORMAT('The average time in the refueling system is ',F8.4, ' hrs.')

        STOP

        END
```

```
PROGRAM THESIS4

C  This program is written by LT W Heath Rushing.  This program computes the
C  the queuing measurements for AMC's current refueling policy using only the
C  number of aircraft and trucks in the system to describe the state space.
C  The aircraft refueling policy is such that once an aircraft is on a
c  refueling system it stays there.

C  Describes the matrix used to solve the system of equations using IMSL.

      PARAMETER (IPATH = 1, LDA = 3000, N = 3000)
      REAL A(LDA, LDA), B(N), X(N)

C  Parameter definition.

      REAL W, L, AC, calc, TTEMP, TEMP3, TEMP4, probac, probtr
      REAL lambda, numtrucks, amtfuel, gamma1o, mu5oo, mu5ii
      REAL mu1, mu2, mu5, TEMP, gamma1, TTEMP2, T, cap
      REAL mu5i, mu5o, rout, muo, mui, lambdai, lambdao, gamma1i
      INTEGER capacity, numhydrants, I, J, syslimit, m, K, numfill, M

      COMMON /WORKSP/ RWKSP
      REAL RWKSP(25010022)
      CALL IWKIN(25010022)

C  Gain all airfield specific characteristics needed for analysis:  aircraft
C  arrival rate, the number of active hydrants, the number of trucks and
C  the amount of fuel needed per aircraft.

      PRINT*,'What is the aircraft arrival rate?'
      READ*, lambda


      PRINT*,'How many active hydrants are available?'
      READ*, numhydrants


      PRINT*,'How many fueling trucks?'
      READ*, numtrucks

      PRINT*,'What is the average amount of fuel needed per aircraft?'
      READ*, amtfuel


C  Set the capacity of the airfield, this limits the Markov process.
C  It is assumed that no more than capacity aircraft can be at the airfield
C  at one time.

      PRINT*,'What is the refueling system capacity?'
      READ*,capacity

      PRINT*,'How many fillstands are in use at the airfield?'
      READ*, numfill
      PRINT*,'Numfill = ',numfill

C     refuel rate of the fillstands
      gamma1 = numfill*1.0


c     the probability an aircraft runs out of gasoline first.
```

```
      probac = 1 - EXP(-5500.0/amtfuel)

c     the probability a truck runs out of gasoline first.
       probtr = 1 - probac

C     Represents service rate of hydrant system as aircraft arrive on the
      system.
      mu1 = 600.0*60.0/amtfuel
      mu2 = 1200.0*60.0/amtfuel
      mu3 = 1800.0*60.0/amtfuel
      mu4 = 2400.0*60.0/amtfuel

C     Gives truck service rate.
      mu5 = 550.0*60.0/amtfuel

C     Outputs inputs to user.

      PRINT 10, lambda
10    FORMAT (1X, 'Aircraft arrive at a rate of ',F3.1,' per hr.')

c     PRINT 15, numhydrants, numtrucks
c15   FORMAT (1X, 'There are ',I2,' active hydrants and ',F5.1,' trucks.')

      PRINT 16, gamma1
16    FORMAT (1X, 'Gamma 1 equals ',F9.3,'.')

      PRINT 17, mu1, mu2, mu5
17    FORMAT (1X, 'Mu 1 is ',F6.3, ' and Mu2 is ',F6.3,' Mu3 is ', F6.3)

      numtrucks = 0
      DO 200 T = 1, 6
         IF (T.EQ.1) numtrucks = 5
         IF (T.EQ.2) numtrucks = 10
         IF (T.EQ.3) numtrucks = 15
         IF (T.EQ.4) numtrucks = 20
         IF (T.EQ.5) numtrucks = 25
         IF (T.EQ.6) numtrucks = 30

        TEMP = numtrucks
        PRINT*, 'numtrucks = ', numtrucks

C     Sets probability matrix equal to the identity.  The program will fill in
      the matrix as needed.
      DO 22 I = 1, N
        DO 21      J = 1, N
          A(I,J) = 0.0
          IF (I.EQ.J) A(I,J) = 1.0
21      CONTINUE
22    CONTINUE


c     Formulates the transition matrix.
      m = 1
      DO 75 K = 0, TEMP

      syslimit = capacity - numhydrants
      TEMP2 = syslimit + 1

        DO 70 I = 0, numhydrants
        numtrucks = 0
```

```
          DO 65 J = I, syslimit

             IF (I.EQ.0) THEN
                muo = 0.0
             ELSEIF (I.EQ.1) THEN
               muo = mu1
             ELSEIF (I.EQ.2) THEN
                muo = mu2
              ELSEIF (I.EQ.3) THEN
                muo = mu3
              ELSE
                muo = mu4
              ENDIF

              IF (J.LE.I.OR.K.EQ.0) THEN
                mu5o = 0.0
                mu5oo = 0.0
              ELSE
                mu5o = mu5
                mu5oo = mu5
              ENDIF

               IF (J.EQ.capacity) THEN
                lambdao = 0.0
               ELSE
                lambdao = lambda
               ENDIF

               IF (K.EQ.TEMP) THEN
                gammalo = 0.0
               ELSE
                gammalo = gamma1
               ENDIF

            IF (numtrucks.GT.K) numtrucks = K
             rout=lambdao+probac*(numtrucks*mu5o)+muo+
                   gammalo+probtr*(numtrucks*mu5oo)
C           PRINT*,'Rout is ',rout

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

              IF (I.EQ.0) THEN
                lambdai = 0.0
               ELSE
                lambdai = lambda
              ENDIF

             IF (I.EQ.numhydrants) THEN
                mui = 0.0
             ELSEIF (I.EQ.0) THEN
                mui = mu1
             ELSEIF (I.EQ.1) THEN
               mui = mu2
             ELSEIF (I.EQ.2) THEN
                mui = mu3
             ELSE
                mui = mu4
              ENDIF
```

```
         IF (J.EQ.syslimit.OR.K.EQ.TEMP) THEN
           mu5i= 0.0
           IF (J.EQ.capacity) THEN
            mui = 0.0
           ELSE
             mui = mui
           ENDIF
         ELSE
           mu5i = mu5
         ENDIF

          IF (J.LE.I.OR.K.EQ.TEMP) THEN
           mu5ii = 0.0
         ELSE
           mu5ii = mu5
         ENDIF

         IF (K.EQ.O) THEN
           gamma1i = 0.0
         ELSE
           gamma1i = gamma1
         ENDIF

        TTEMP = numtrucks + 1
        IF (TTEMP.GT.TEMP) TTEMP = TEMP
        IF (TTEMP.GT.K+1) TTEMP = K+1
        TTEMP2 = J-I
         IF (TTEMP2.GT.TEMP) TTEMP2 = TEMP
         IF (TTEMP2.GT.K+1) TTEMP2 = K+1

        TEMP3 = probac*(((TTEMP)*mu5i))
        TEMP4 = probtr*(((TTEMP2)*mu5ii))
        IF (I.EQ.numhydrants) THEN
            A(m, m) = 1.0
            A(m, (m-TEMP2)) = -lambdai/rout
           A(m,m+((numhydrants+1)*(TEMP2))+1) = -TEMP3/rout
            A(m, (m-((numhydrants+1)*(TEMP2)))) = -gamma1i/rout
           A(m, (m+((numhydrants+1)*(TEMP2)))) = -TEMP4/rout
          IF (I.EQ.J) THEN
            A(m, (m-1)) = 0.0
          ELSE
            A(m, (m-1)) = - lambdai/rout
          ENDIF
        ELSE
           A(m, m) = 1.0
           A(m, (m-TEMP2)) = -lambdai/rout
           A(m,(m+((numhydrants+1)*(TEMP2))+1)) = -TEMP3/rout
           A(m, (m + TEMP2)) = - mui/rout
           A(m, (m-((numhydrants+1)*(TEMP2)))) = -gamma1i/rout
           A(m, (m+((numhydrants+1)*(TEMP2)))) = -TEMP4/rout
        ENDIF

        m = m+1

      numtrucks = numtrucks + 1

      IF (numtrucks.GT.TEMP) numtrucks = TEMP

65    CONTINUE
```

```
          syslimit = syslimit + 1

70        CONTINUE

75      CONTINUE

C       Sets the conservation of probality equation.

        DO 80 I = 1, N
          A(1,I) = 1.0
80      CONTINUE

        DO 85 I = 1, N
          IF (I.EQ.1) THEN
            B(I) = 1.0
           ELSE
             B(I) = 0.0
           ENDIF

85      CONTINUE

c       Calls the IMSL subroutines to solve the transition matrix, and write c
        them to the screen.

        CALL LSLRG (N, A, LDA, B, IPATH, X)

c       CALL WRRRN ('Probabilities', LDA, 1, X, LDA, 0)


C       Calculation of the average number in the system.

        TEMP3 = TEMP2*(numhydrants+1)

        L = 0.0
        cap = 0.0

        DO 91 K = 1, (TEMP+1)

              IF (K.EQ.1) THEN
                J = 1
              ELSE
                J = ((K-1)*(numhydrants+1)*(TEMP2))+1
             ENDIF

          DO 90 I = 1, (capacity-numhydrants + 1)

                  AC = I-1

            DO 89 M = 1, (numhydrants+1)
              calc = X(J) * AC
              L = L + calc

              IF (M.NE.numhydrants+1) J = J + (capacity-numhydrants+1)
              IF (AC.EQ.capacity) cap = cap + X(J)

              AC = AC + 1
89          CONTINUE

             J = J - (numhydrants*(capacity-numhydrants+1)) + 1
```

```
90        CONTINUE
91      CONTINUE

C       Using Little's law, calculating the average time in system.
        W = L/(lambda*(1-cap))

        PRINT 95, L
95      FORMAT('The average number of aircraft in the system is ',F8.4, '.')

        PRINT 100, W
100     FORMAT('The average time in the refueling system is ',F8.4, '.')


200     CONTINUE
        STOP

        END
```

```
      PROGRAM THESIS5

C  CPLEX
C  This program is written by LT W Heath Rushing.  This program computes the
C  the queuing measurements for AMC's current refueling policy using only the
C  number of aircraft in the system to describe the state space.  The aircraft
C  refueling system is modeled as a birth and death process with the
C  population being the number of aircraft.  A birth is an arrival and a death
C is a departure from the system after service is completed.

C  Describes the matrix used to solve the system of equations.

       external slogfo !$pragma C (slogfo)
       external sscrin !$pragma C (sscrin)
       external sitfoi !$pragma C (sitfoi)
       external sitlim !$pragma C (sitlim)
       external iloadp !$pragma C (iloadp)
       external iloadl !$pragma C (iloadl)
       external ibarop !$pragma C (ibarop)
       external iopt   !$pragma C (iopt)
       external gx     !$pragma C (gx)
c      external gmar   !$pragma C (gmar)
c      external gmac   !$pragma C (gmac)
c      external ilpwr  !$pragma C (ilpwr)
       external isolut !$pragma C (isolut)
c      external iaddr  !$pragma C (iaddr)
c      external icbds  !$pragma C (icbds)
       external slogfc !$pragma C (slogfc)

c  Part I constants
      integer            mac
      parameter          (mac=1600)
      integer            mar
      parameter          (mar=1590)
      integer            macsz
      parameter          (macsz=1600)
      integer            marsz
      parameter          (marsz=1590)
      integer            matsz
      parameter          (matsz=mac*mar)
c     integer            cstsz
c     parameter          (cstsz=macsz*3+1)
c     integer            rstsz
c     parameter          (rstsz=marsz*3+1)
c     integer            cex
c     parameter          (cex=macsz-mac)
c     integer            rex
c     parameter          (rex=marsz-mar)
      integer            namlen
      parameter          (namlen = 0)

c  Part I declarations
      integer            objsen        / 1 /
      double precision   objx(macsz)  /macsz*0.0/
      double precision   rhsx(marsz)  /marsz*0.0/
      character*1        senx(marsz)
      integer            matbeg(macsz) /macsz*0/
      integer            matcnt(macsz) /macsz*0/
      integer            matind(0:matsz-1) /matsz*0/
      double precision     matval(0:matsz-1) /matsz*0/
```

```
      double precision  bdl(macsz)
      double precision  bdu(macsz)
      character*3       datanm   /'  '/
      character*3       objnm    /'  '/
      character*3       rhsnm    /'  '/
      character*3       rngnm    /'  '/
      character*3       bndnm    /'  '/
      character*3       cstore   /'  '/
      character*3       rstore /'  '/
      character*3       estore /'  '/
      integer           idummy(1)
      double precision  ddummy(1)
      integer           lpstat
      double precision  obj
      double precision  x(macsz)
      double precision  pi(macsz)
      double precision  slack(macsz)
      double precision  dj(macsz)

      REAL TEMP3, TEMP4, TEMP5, mu3,mu4, W, L, mu5oo
      REAL lambda, amtfuel, gamma1o, mu5o, mu5i, mu5ii
      REAL probac, probtr, mu5iii, mu5iv, mu5ooo, mu5oiv
c     INTEGER new
      REAL mu1, mu2, mu5, gamma1, mu5ii, mu5oo, mu2o, add, add2, cap
      REAL mu5i, mu5o, rout, muo, mui, lambdai, lambdao, gamma1i, mu2i
      INTEGER capacity, numhydrants, I, J, syslimit, m, K, numfill
      INTEGER Am, limit, T, Z, limit2, limitcalc, count, TEMP6
      INTEGER numtrucks, TEMP, TTEMP2, TTEMP3, TTEMP4, TTEMP, prvcnt
      INTEGER toosmall, toobig, matcount, D, B, C
      REAL A(mar,mac)

      integer status

c  Functions
      integer           sscrin
      integer           slogfo
      integer           sitfoi
      integer           sitlim
      integer           iloadp
      integer           iloadl
      integer           ibarop
      integer           iopt
      integer           isolut
      integer           gx
c     integer           iaddr
c     integer           icbds
c     integer           ilpwr
      integer           slogfc

c     Request inputs used to describe the system.


      PRINT*,'What is the aircraft arrival rate?'
      READ*, lambda

      PRINT*,'How many active hydrants are available?'
      READ*, numhydrants
```

```
      PRINT*,'How many fueling trucks?'
      READ*, numtrucks

      TEMP = numtrucks

      PRINT*,'What is the aircraft receiving rate?'
      READ*, acrec

      PRINT*,'What is the average amount of fuel needed per aircraft?'
      READ*, amtfuel

C  Set the capacity of the airfield, this limits the Markov process.
C  It is assumed that no more than capacity aircraft can be at the airfield
C  at one time.

      PRINT*,'What is the refueling system capacity?'
      READ*,capacity


      PRINT*,'How many fillstands are in use at the airfield?'
      READ*, numfill


      gamma1 = numfill*1.0

c     the probability an aircraft runs out of gasoline first.
      probac = 1 - EXP(-5500.0/amtfuel)

c     the probability a truck runs out of gasoline first.
      probtr = 1 - probac

      IF (numhydrants.EQ.1) THEN
        mu1 = acrec*60.0/amtfuel
        mu2 = acrec*60.0/amtfuel
        mu3 = acrec*60.0/amtfuel
        mu4 = acrec*60.0/amtfuel
     ELSEIF (numhydrants.EQ.2) THEN
        mu1 = acrec*60.0/amtfuel
        mu2 = 2*acrec*60.0/amtfuel
        mu3 = 2*acrec*60.0/amtfuel
        mu4 = 2*acrec*60.0/amtfuel
    ELSEIF (numhydrants.EQ.3) THEN
        mu1 = acrec*60.0/amtfuel
        mu2 = 2*acrec*60.0/amtfuel
        mu3 = 3*acrec*60.0/amtfuel
        mu4 = 3*acrec*60.0/amtfuel
    ELSE (numhydrants.EQ.2) THEN
        mu1 = acrec*60.0/amtfuel
        mu2 = 2*acrec*60.0/amtfuel
        mu3 = 3*acrec*60.0/amtfuel
        mu4 = 4*acrec*60.0/amtfuel
    ENDIF

C     Represents service rate of hydrant system when more than one aircraft
      is in the system.

      mu5 = 550.0*60.0/amtfuel
C     Gives service rate of one truck.  This will be used for service rate of
      multiple 550
```

```
C       Outputs inputs to user.

c       PRINT 10, lambda
c10     FORMAT (1X, 'Aircraft arrive at a rate of ',F3.1,' per hr.')

c       PRINT 11, numhydrants, numtrucks
c11     FORMAT (1X, 'There are ',I2,' active hydrants and ',I3,' trucks.')

c       PRINT 12, gamma1, mu5
c12     FORMAT (1X, 'Gamma 1 equals ',F9.3,' and gamma 2 equals ',F9.3,'.')

c       PRINT 13, mu1, mu2, mu5
c13     FORMAT (1X, 'Mu 1 is ',F6.3, ' and Mu2 is ',F6.3,' Mu3 is ', F6.3)

C       Sets probability matrix equal to the identity.  The program will fill
        in the matrix as needed.
        DO 15 I = 1, mar
          DO 14       J = 1, mac
            A(I,J) = 0.0
14        CONTINUE
15      CONTINUE

        DO 16 I = 1, mac
          objx(I) = 0.0
16      CONTINUE

        DO 17 I = 1, mar
          senx(I) = 'E'
17      CONTINUE

        DO 18 I = 1, mac
          bdl(I) = 0.0
18      CONTINUE

        DO 19 I = 1, mac
          bdu(I) = 1.0
19      CONTINUE

         m = 1
         count = 0

         IF (capacity.LT.numtrucks) THEN
           TTEMP2 = capacity+1
         ELSE
           TTEMP2 = TEMP + 1
         ENDIF

        TTEMP3 = TEMP+1
         DO 76 T = 0, TEMP
            prvcnt = count
           count = 0

           TTEMP2 = TTEMP2 - 1
           DO 30 D = 0, TTEMP3
             TTEMP = D-1
             DO 29 B = 0, numhydrants
             TTEMP = TTEMP+1
               DO 28 C = TTEMP, capacity

                 count = count + 1
```

```
28          CONTINUE
29        CONTINUE
30      CONTINUE


         DO 75 K = 0, TTEMP2

           TTEMP = K-1
           DO 70 J = 0, numhydrants
             limit  =  limit - 1
             limit2 = limit - 1

              TTEMP = TTEMP+1

                  DO 65 I = TTEMP, capacity

              IF (I.EQ.K) THEN
                 limit = 0.0
                 limitcalc = capacity - K + 1
                 DO 31 Z = 0, numhydrants
                   limit = limit + limitcalc
                   limitcalc = limitcalc - 1
31              CONTINUE
                 limit2 = limit-1
               ENDIF

              IF (J.EQ.0) THEN
                 muo = 0.0
              ELSEIF (J.EQ.1) THEN
                muo = mu1
              ELSEIF (J.EQ.2) THEN
                 muo = mu2
              ELSEIF (J.EQ.3) THEN
                muo = mu3
              ELSE
                muo = mu4
              ENDIF

              IF (K.EQ.0) THEN
                mu5o = 0.0
              ELSE
                mu5o = mu5
              ENDIF

               IF (I.EQ.capacity) THEN
                 lambdao = 0.0
               ELSE
                 lambdao = lambda
               ENDIF

               IF (K+T.EQ.TEMP) THEN
                 gamma1o = 0.0
               ELSE
                 gamma1o = gamma1
               ENDIF

              numtrucks = K
              rout = lambdao + probac*(numtrucks*mu5o) + muo
               rout = rout + gamma1o + probtr*(numtrucks*mu5o)
```

```
c               PRINT*,'Rout is ',rout

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

                IF (J+K.EQ.I) THEN
                  lambdai = 0.0
                ELSE
                 lambdai = lambda
                ENDIF

               IF (J.EQ.numhydrants.OR.I.EQ.capacity)   THEN
                  mui = 0
               ELSEIF (J.EQ.0) THEN
                  mui = mu1
               ELSEIF (J.EQ.1) THEN
                  mui = mu2
                ELSEIF (J.EQ.2) THEN
                  mui = mu3
                ELSE
                  mui = mu4
                ENDIF

                IF (I.EQ.capacity.OR.K+T.EQ.TEMP) THEN
                  mu5i= 0.0
                ELSE
                  mu5i = mu5
                ENDIF

                IF (I.EQ.J+K.OR.T.NE.0.OR.K+T.EQ.TEMP) THEN
                  mu5ii = 0.0
                ELSE
                  mu5ii = mu5
                ENDIF

                 IF (K.EQ.0.OR.K+T.EQ.TEMP.OR.I.EQ.K+T) THEN
                  mu5iii = 0.0
                ELSE
                  mu5iii = mu5
                ENDIF

                 IF (T.EQ.0) THEN
                  gamma1i = 0.0
                ELSE
                  gamma1i = gamma1
                ENDIF


c       calculate the number of trucks for truck refueling aircraft, mu5i.
                numtrucks = K + 1
                IF (numtrucks.GT.TEMP) numtrucks = TEMP



c               PRINT*

                TEMP3 = probac*(((numtrucks)*mu5i))
                TEMP4 = probtr*(((numtrucks)*mu5ii))
                TEMP5 = probtr*(((K)*mu5iii))
                TEMP6 = -1
```

```
               IF (m+TEMP6.GT.0) A(m, m+TEMP6) = -lambdai
                A(m, m + limit) = -TEMP3
                A(m, (m+(capacity-TTEMP+1))) = - mui
                IF ((m-prvcnt).GT.0) A(m,m-prvcnt) = -gammali
                A(m,m+limit2) = -TEMP4
                A(m,m+count) = -TEMP5
                A(m, m) = rout


              objx(m) = REAL(I)

              m = m+1

65          CONTINUE

70         CONTINUE

75        CONTINUE

76       CONTINUE

C---------------------------------------------------------------------------
C     This is the hyrant decision matrix.

       Am = m
      m = 1

        IF (capacity.LT.numtrucks) THEN
          TTEMP2 = capacity+1
        ELSE
          TTEMP2 = TEMP + 1
        ENDIF

        DO 110 T = 0, TEMP

        prvcnt = count
          count = 0

          TTEMP2 = TTEMP2 -1
          DO 88 D = 0, TTEMP2
           TTEMP = D-1
            DO 87 B = 0, numhydrants
              TTEMP = TTEMP+1
             DO 86 C = TTEMP, capacity

               count = count + 1

86          CONTINUE
87         CONTINUE
88        CONTINUE

          DO 105 K = 0, TTEMP2


          TTEMP = K-1
           DO 100 J = 0, numhydrants

             limit  = limit - 1
              limit2 = limit - 1
```

```
      TTEMP = TTEMP+1

      DO 95 I = TTEMP, capacity

       IF (I.EQ.K) THEN
          limit = 0.0
          limitcalc = capacity - K + 1
          DO 89 Z = 0, numhydrants
            limit = limit + limitcalc
            limitcalc = limitcalc - 1
89      CONTINUE
        limit2 = limit-1
       ENDIF

       IF (J.EQ.0) THEN
         muo = 0.0
       ELSEIF (J.EQ.1) THEN
        muo = mu1
       ELSEIF (J.EQ.2) THEN
         muo = mu2
       ELSEIF (J.EQ.3) THEN
         muo = mu3
       ELSE
         muo = mu4
       ENDIF

       IF (I.EQ.J+K) THEN
          mu2o = 0.0
       ELSEIF (J.EQ.numhydrants) THEN
         IF (J.LT.4) THEN
           mu2o = numhydrants*mu1
         ELSE
          mu2o = mu4
         ENDIF
       ELSE
         IF ((I-K).LT.numhydrants) THEN
          mu2o = (I-K)*mu1
         ELSE
           mu2o = numhydrants*mu1
         ENDIF
       ENDIF


         IF (K.EQ.0) THEN
          mu5o = 0.0
         ELSE
          mu5o = mu5
         ENDIF

        IF (I.EQ.capacity) THEN
          lambdao = 0.0
        ELSE
          lambdao = lambda
        ENDIF

        IF (K+T.EQ.TEMP) THEN
          gamma1o = 0.0
        ELSE
          gamma1o = gamma1
        ENDIF
```

```
            numtrucks = K
            rout = lambdao + probac*(numtrucks*mu5o) + muo + mu2o
             rout = rout + gammalo + probtr*(numtrucks*mu5o)
c           PRINT*,'Rout is ',rout

ccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

             IF (J+K.EQ.I) THEN
               lambdai = 0.0
            ELSE
             lambdai = lambda
             ENDIF

            IF (J.EQ.numhydrants.OR.I.EQ.capacity)   THEN
               mui = 0
            ELSEIF (J.EQ.0) THEN
               mui = mu1
            ELSEIF (J.EQ.1) THEN
             mui = mu2
            ELSEIF (J.EQ.2) THEN
               mui = mu3
            ELSE
               mui = mu4
            ENDIF

            IF (I.EQ.capacity) THEN
               mu2i = 0.0
            ELSEIF (J.EQ.numhydrants) THEN
              IF (J.LT.4) THEN
                mu2i = numhydrants*mu1
              ELSE
               mu2i = mu4
              ENDIF
            ELSE
              IF ((I-K+1).LT.numhydrants) THEN
               mu2i = (I-K+1)*mu1
              ELSE
                mu2i = numhydrants*mu1
              ENDIF
            ENDIF

             IF (I.EQ.capacity.OR.K+T.EQ.TEMP) THEN
               mu5i= 0.0
             ELSE
               mu5i = mu5
             ENDIF

             IF (I.EQ.J+K.OR.T.NE.0.OR.K+T.EQ.TEMP) THEN
               mu5ii = 0.0
             ELSE
               mu5ii = mu5
             ENDIF


             IF (I.EQ.J+K.OR.K.EQ.0.OR.K+T.EQ.TEMP) THEN
               mu5iii = 0.0
             ELSE
               mu5iii = mu5
             ENDIF
```

```
              IF (T.EQ.0) THEN
                gammali = 0.0
              ELSE
                gammali = gammal
              ENDIF

C        calculate the number of trucks for truck refueling aircraft, mu5i.
              numtrucks = K + 1
              IF (numtrucks.GT.TEMP) numtrucks = TEMP

c             PRINT*

              TEMP3 = probac*(((numtrucks)*mu5i))
              TEMP4 = probtr*(((numtrucks)*mu5ii))
              TEMP5 = probtr*(((K)*mu5iii))
                A(m, Am) = rout
                A(m, (Am-1)) = -lambdai
                A(m, Am+limit) = -TEMP3
                A(m, (Am+(capacity-TTEMP+1))) = - mui
                A(m, Am - prvcnt) = -gammali
                A(m,Am+limit2) = -TEMP4
                A(m,Am+count) = -TEMP5
                A(m,Am+1) = - mu2i

               objx(Am) = REAL(I)

                m = m + 1
               Am = Am +1
95          CONTINUE

100       CONTINUE

105      CONTINUE

110      CONTINUE

C ------------------------------------------------------------------
C       This is the truck decision matrix.

        Am = Am
        m = 1

        IF (capacity.LT.numtrucks) THEN
          TTEMP2 = capacity+1
        ELSE
          TTEMP2 = TEMP + 1
        ENDIF

        DO 210 T = 0, TEMP

          prvcnt = count
          count = 0

          TTEMP2 = TTEMP2 -1
          DO 130 D = 0, TTEMP2
            TTEMP = D-1
            DO 129 B = 0, numhydrants
```

```
              TTEMP = TTEMP+1
               DO 128 C = TTEMP, capacity

                count = count + 1

128             CONTINUE
129           CONTINUE
130         CONTINUE

          DO 205 K = 0, TTEMP2


          TTEMP = K-1
           DO 200 J = 0, numhydrants

             limit = limit - 1
              limit2 = limit - 1
              TTEMP = TTEMP+1

              DO 195 I = TTEMP, capacity

                IF (I.EQ.K) THEN
                    limit = 0.0
                    limitcalc = capacity - K + 1
                    DO 131 Z = 0, numhydrants
                      limit = limit + limitcalc
                      limitcalc = limitcalc - 1
131               CONTINUE
                  limit2 = limit - 1
                 ENDIF

                IF (J.EQ.0) THEN
                    muo = 0.0
                ELSEIF (J.EQ.1) THEN
                   muo = mu1
                ELSEIF (J.EQ.2) THEN
                    muo = mu2
                ELSEIF (J.EQ.3) THEN
                  muo = mu3
                ELSE
                   muo = mu4
                ENDIF

                IF (K.EQ.0.OR.(T.NE.0.AND.I.NE.J+K)) THEN
                    mu5o = 0.0
                ELSE
                    mu5o = mu5
                ENDIF

                 IF (K.EQ.0.OR.T.NE.0) THEN
                    mu5oo = 0.0
                  ELSE
                    mu5oo = mu5
                  ENDIF

                 IF (T.EQ.0.OR.I.EQ.J+K) THEN
                    mu5ooo = 0.0
                  ELSE
                    mu5ooo = mu5
                  ENDIF
```

```
         IF (T.EQ.0) THEN
           mu5oiv = 0.0
          ELSE
           mu5oiv = mu5
          ENDIF


          IF (I.EQ.capacity) THEN
           lambdao = 0.0
          ELSE
           lambdao = lambda
          ENDIF

          IF (K+T.EQ.TEMP) THEN
           gamma1o = 0.0
         ELSE
           gamma1o = gamma1
          ENDIF

          numtrucks = K

              rout = lambdao + probac*(K*mu5o) + muo
              rout = rout + probtr*(K)*mu5oo

               IF (K.EQ.TEMP) THEN
                 rout = rout+gamma1o+(probac*(K)*mu5ooo))
                 rout = rout +(probtr*(K)*mu5oiv)
              ELSE
                 rout = rout+gamma1o+(probac*(K+1)*mu5ooo))
                 rout = rout +(probtr*(K+1)*mu5oiv)
              ENDIF

c        PRINT*,'Rout is ',rout

cccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccccc

         IF (J+K.EQ.I) THEN
           lambdai = 0.0
        ELSE
          lambdai = lambda
         ENDIF

        IF (J.EQ.numhydrants.OR.I.EQ.capacity)   THEN
           mui = 0
        ELSEIF (J.EQ.0) THEN
           mui = mu1
        ELSEIF (J.EQ.1) THEN
          mui = mu2
        ELSEIF (J.EQ.2) THEN
           mui = mu3
         ELSE
           mui = mu4
          ENDIF

          IF (I.EQ.capacity.OR.K+T.EQ.TEMP.OR.(T.NE.0.AND.I.NE.J+K))THEN
            mu5i= 0.0
          ELSE
            mu5i = mu5
          ENDIF
```

```
              IF (I.EQ.J+K.OR.T.NE.0.OR.K+T.EQ.TEMP) THEN
                mu5ii = 0.0
               ELSE
                mu5ii = mu5
               ENDIF

               IF (K+T.EQ.TEMP.OR.I.EQ.capacity) THEN
                mu5iii = 0.0
               ELSE
                mu5iii = mu5
               ENDIF

               IF (K+T.EQ.TEMP) THEN
                mu5iv = 0.0
               ELSE
                mu5iv = mu5
               ENDIF

               IF (T.EQ.0) THEN
                gamma1i = 0.0
               ELSE
                gamma1i = gamma1
               ENDIF

C         calculate the number of trucks for truck refueling aircraft, mu5i.
              numtrucks = K + 1
              IF (numtrucks.GT.TEMP) numtrucks = TEMP


c             PRINT*

              TEMP3 = probac*(((numtrucks)*mu5i))
              TEMP4 = probtr*(((numtrucks)*mu5ii))

                A(m,Am) = rout
                A(m, (Am-1)) = -lambdai
                A(m,Am+limit) = -TEMP3
                A(m, (Am+(capacity-TTEMP+1))) = - mui
                A(m,Am-prvcnt) = -gamma1i
                A(m,Am+limit2) = -TEMP4
                A(m,Am+count+1) = - (probac*(numtrucks)*mu5iii)
                A(m,Am+count) = -(probtr*(numtrucks)*mu5iv)

               objx(Am) = REAL(I)

              Am = Am + 1
              m = m+1

195         CONTINUE
            syslimit = syslimit + 1

200       CONTINUE

205     CONTINUE

210     CONTINUE

      J = m+1
```

```
      DO 215 I = Am, mac
        A(J,I) = 1.0
        J = J + 1
215   CONTINUE

      DO 216 I = 1,mac
         A(m,I) = 1.0
216   CONTINUE


c     change back to mar then mac
      DO 220 I = 1, marsz
        rhsx(I) = 0.0
220   CONTINUE
      rhsx(m) = 1.0


c     DO 230 I = 1, mac
c       PRINT*, bdl(I)
c       PRINT*, bdu(I)
c       PRINT*, senx(I)
c       PRINT*, I, rhsx(I)
c         PRINT*, objx(I)
c230  CONTINUE

      PRINT*

C

c     mac is number of variables and mar is the number of constraints

      z = 0
      matcount = 0
      add2 = 0

      DO 255 I = 1, mac
        matbeg(I) = z
        matcnt(I) = 0


        DO 255 J = 1, mar

          IF (A(J, I).NE.0.0) THEN
             matval(z) = A(J, I)
c           PRINT*, J, I, matval(z)
            add2 = add2 + 1
            matcnt(I) = matcnt(I) + 1
            matind(z) = J-1
            z = z + 1
          ENDIF

250     CONTINUE

255    CONTINUE

      add = 0
c     DO 257 I = 1, mac

c       DO 256 J = 1, mar
c         IF (A(J,I).NE.0.0) THEN
```

```
c              PRINT*,J,I, A(J,I)
c              PRINT*, matval(add)
c                PRINT*, matind(add)
c                add = add+1
c              ENDIF
c256       CONTINUE
c        PRINT*, 'matbeg is ',I, matbeg(I)
c          PRINT*, 'matcnt is ',I, matcnt(I)
c257    CONTINUE


c        iloadp = loadprob

c    "Using the Callable Library."
c
c    NUL character appended to strings as required by C
c


c
c    Set up CPLEX message output. slogfo opens a logfile of the given name.
c    sscrin sets the screen indicator to on, sending messages to
c    stdout.
c

      if (sscrin (1) .ne. 0  .or.
     .     slogfo ('refuel.log'//char(0)) .ne. 0) then
          write (*, 263)
263     format ('Error on setting up log.')
         goto 99000
      endif
       PRINT*,'status 1 = ', status

      status = iloadp ('refuel'//char(0),
     .  mac, mar, 0, objsen, objx, rhsx,
     .  senx, matbeg, matcnt, matind, matval,
     .  bdl, bdu, idummy, idummy, idummy, idummy, idummy,
     .  idummy, ddummy, datanm//char(0), objnm//char(0),
     .  rhsnm//char(0), rngnm//char(0), bndnm//char(0),
     .  cstore//char(0), rstore//char(0), estore//char(0),
     .  macsz, marsz, matsz,
     .  0, 0, 0, 0, 0, namlen)

c         status = iloadl ('refuel'//char(0),
c        .  mac, mar, objsen, objx, rhsx,
c        .  senx, matbeg, matcnt, matind, matval,
c        .  bdl, bdu, idummy, macsz, marsz, matsz)
       PRINT*, 'status 2 = ',status

c
c    Set iteration logging to log every factorization
c
      status = sitfoi (1, toosmall, toobig)
      if (status .ne. 0) goto 99000
c
c    Set iteration limit to 10000 or largest value possible
c
      status = sitlim (10000, toosmall, toobig)
      if (status .ne. 0) status = sitlim (toobig, toosmall, toobig)
c
c    Optimize the problem and obtain the solution
```

```
c

        if (status .ne. 0) goto 99000
       PRINT*,'status 3  = ', status

        status = ibarop ()

c         status = iopt ()
       PRINT*,'status 4 = ', status
        if (status .ne. 0) goto 99000

      status = isolut (lpstat, obj, x, pi, slack, dj,
     .                1, 1, 0, 0, 0 )

c       status = gx (x, 1, macsz-1)
       if (status .ne. 0) goto 99000

c
c  Close the CPLEX log file.
c
      status = slogfc ()
      PRINT*, 'Status 5 = ', status

      PRINT 291, lpstat
291   FORMAT (' Solution status = ', i2)
      PRINT 292, obj
292   FORMAT (' Solution value  = ', f15.6)

      PRINT*

c      DO 310 j = 1, mac
c         PRINT 294, x(j)
c294      FORMAT (' Value = ', f15.6)
c310   CONTINUE

       PRINT*



cccccccccccccccccccccccccccccccccccccccccccccccccccc

       m = 1
       L = 0.0
       W = 0.0
        cap = 0.0

       DO 600 Y = 1, 3


            IF (capacity.LT.numtrucks) THEN
             TTEMP2 = capacity+1
            ELSE
             TTEMP2 = TEMP + 1
            ENDIF

           DO 576 T = 0, TEMP

             TTEMP2 = TTEMP2 -1
             DO 575 K = 0, TTEMP2
```

```
                TTEMP = K-1
                DO 570 J = 0, numhydrants
                limit  =  limit - 1
                limit2 = limit - 1

                 TTEMP = TTEMP+1

                  DO 565 I = TTEMP, capacity

                   IF (I.EQ.K) THEN
                      limit = 0.0
                      limitcalc = capacity - K + 1
                      DO 531 Z = 0, numhydrants
                        limit = limit + limitcalc
                        limitcalc = limitcalc - 1
531                 CONTINUE
                      limit2 = limit-1
                    ENDIF

                   IF (x(m).GT.0.0000001) THEN
                     PRINT*
                     PRINT*,'System State Description:'
                     PRINT*,I,' aircraft in the system.'
                     PRINT*,J,' aircraft on hydrants. '
                      PRINT*,K,' aircraft in the truck system.'
                      PRINT*,T,' trucks waiting to refuel an aircraft.'

                      IF (I.EQ.capacity) cap = cap + x(m)

                       PRINT*

                     WRITE(NOUT, 540), x(m)
540                  FORMAT(2X,'Probability of being in this state ',f10.9)

                    PRINT*
                    IF (I.GT.J+K) THEN
                         IF (Y.EQ.1) THEN
                           PRINT*,'Wait for system change.'
                         ELSEIF (Y.EQ.2) THEN
                           PRINT*,'Assign the aircraft to a hydrant.'
                         ELSE
                           PRINT*,'Assign a truck to the aircraft.'
                         ENDIF
                      ENDIF


c Calculate the average number in system.

                    L = L + (x(m)*I)

                   ENDIF

                   m = m+1

565                CONTINUE

570              CONTINUE

575            CONTINUE
```

```
576       CONTINUE

600     CONTINUE

        PRINT*
        PRINT*,'Queuing Performance Measures.'
        PRINT*

c  Calculate the average time in system using Little's Law.

        W = L/(lambda*(1-cap))

        PRINT 605, L
605     FORMAT(1x,'The average number in the system is ',f9.4,' aircraft.')

        PRINT*

        PRINT 610, W
610     FORMAT(1x,'The average time in the system is   'f9.4,' hours.')

        PRINT*

99000 continue
       write (*, *) 'Error, status = ', status


        STOP

        END
```

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13. ABSTRACT *(Maximum 200 words)*

This thesis develops five analytical models to understand the current ground refueling process, to optimize the airfield configuration and to determine the refueling policy which maximizes throughput, the primary measure of airfield efficiency. This study models the airfield refueling process as a continuous time Markov process to adequately represent the inherent stochastic nature of the transitory ground refueling system and provide an analytical evaluation of various airfield configurations. Also, the study provides an optimal refueling policy to minimize the number of aircraft on the ground which in turn minimizes the average amount of time aircraft spend on the ground in a fifth model, a Markov decision process solved by a linear program. By accomplishing this, higher throughput rates can be achieved by allowing a higher aircraft arrival rate into the airfield.

14. SUBJECT TERMS

Markovian modeling; continuous time Markov process; Markov decision process; Throughput; Airfield refueling; Ground refueling; FORTRAN; CPLEX; IMSL; stochastic modeling.

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