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Analyzing and Improving Stochastic Network Security: A Multicriteria Prescriptive Risk Analysis Model

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AFIT/GOR/ENS/97M-15

ANALYZING AND IMPROVING STOCHASTIC NETWORK SECURITY: A MULTICRITERIA PRESCRIPTIVE RISK ANALYSIS MODEL

THESIS

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THESIS

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Captain, USAF

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Acknowledgments

As you will probably deduce while reading this thesis, the purpose of this research was to investigate different Risk Assessment methodologies. Although the actual problem which served as the experimental victim upon which different methods were tested, a communication network in need of security improvements, has become increasingly important as the "Information Revolution" has exploded around us, the primary aim was still methodology exploration and development. I believe that the combination of the Risk Assessment/reliability modeling techniques explored and the "guinea pig" upon which they were tested provided an opportunity to learn about two important subject areas not emphasized in the curriculum here, capping what should be recognized as an outstanding Graduate Degree program. I would like to thank Dr. Yupo Chan, my thesis advisor, for encouraging me to explore this topic, letting me get lost without undue pressure, and helping me find my way through the forest after learning from my mistakes. I would also like to thank Dr. James W. Chrissis for being a particularly effective reader. The library staff here at AFIT was outstanding and deserve thanks from every researcher at this institution. To my fellow GORs and GOAs I say good luck in the future. I truly enjoyed the student lounge seminars to improve all the worlds problems, the dinners at the Officers Club, the parties, and all of the other little benefits of being a member of such a great support group. Finally, as all my acquaintances and friends undoubtedly know, I am eternally grateful to my god and my family; I've made my peace with them and nothing else matters.

David L. Lyle

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Abstract

This research optimized two measures of network security by hardening components and improving their reliability. Both measures require quantification of component reliability functions. The descriptive methods used in this research derived component reliability functions by using fault trees. Since the probability basic events will occur are often not known with certainty, Fuzzy Logic and Monte Carlo simulation were used to quantify uncertainty propagation in the fault tree.

Results from the descriptive models were used to develop the prescriptive model, a non-linear multi-criteria optimization model. A common measure of effectiveness (MOE) for networks is statistical reliability, which ignores the effects of hostile actions. A new MOE which includes the effects of hostile actions was developed using a two-person, zero-sum game model of the network. A traditional risk assessment was also conducted, and results compared to the optimization model. All methods and models developed were general and could be easily modified to fit specific applications.

ANALYZING AND IMPROVING STOCHASTIC NETWORK SECURITY: A MULTICRITERIA PRESCRIPTIVE RISK ANALYSIS MODEL

Introduction

The Risk Analysis Model (RAM) development is an effort to create an analytic environment for the assessment and analysis of security risk. One area of interest where RAM can be applied is the analysis of network security. As the information revolution continues, nations that depend upon telecommunications and computer networks, but can not adequately defend them, will become more vulnerable to network compromise (10:26). The problem facing the Department of Defense (DOD) today is that it is becoming more and more reliant on commercial information networks designed and operated by owners with different values (cost and efficiency) than the DOD (10:27). Although the DOD has depended on commercial information systems in the past, the recent infusion of competition in the telecommunication industry has prevented any of the carriers from passing on the cost of protecting the network to customers as they could in the past.

Background

A stochastic communication network can be modeled as a graph, consisting of nodes and arcs. The nodes can be thought of as hardware and the arcs as communication links. The components of the network have attributes which are usually well-defined and easily determined. For example, the capacity of a communication link and the probability that the link is working as a function of time are usually well-known to the designer. However, the risk of a security compromise occurring at the component is unlikely to be quantifiable beyond vague approximations.

Generally, *network security* describes the integrity, availability, authenticatability, non-reputability, and confidentiality of the information being secured on the network. This research focuses on the availability aspect of network security. In times of peace, network availability may seem to be equivalent to statistical reliability. However, during times of increasing tension network availability may be altered by *non-statistical* events such as direct attacks on network components, suggesting that statistical reliability alone may not be a complete measure of network availability.

The design and construction of networks is complicated enough when only statistical reliability is considered (29). Exact calculation of a network's statistical reliability is an NP-hard problem and has received much attention in the literature (3:83, 5:153, 19:496, 20:1105, 22:46, 23:172, 32:516). Despite the attention statistical reliability calculation has received, most network design problems concentrate on minimizing cost within a deterministic network (18:63, 10:26) since the minimum-cost network flow problem subject to perfectly reliable components with limited capacities can be solved very efficiently (using the network simplex algorithm, 200-300 times faster than the standard simplex approach) (8:419). Furthermore, there has been very little research published on quantifying network availability in a hostile environment. It is likely that the existing networks used by the DOD would have different designs had the criteria *availability during hostilities* been given higher priority in the design stages.

At least 90 percent of defense networks depend on commercial systems (10:27). Because the firms responsible for designing commercial systems lack the incentive and the finances to build extra security and robustness needed to defend the networks (10:27), the DOD needs to be able to determine where money can be most effective in reducing the potential for security compromise in existing networks. This study demonstrates a method which can be used to determine the amount and type of effort to expend on network components to improve network availability in times of peace as well as war.

Research Problem

The purpose of this study is to develop a technique to use when optimizing improvements to the network availability component of network security. The technique should include descriptive and prescriptive capabilities. It should apply to prototype networks of sufficient complexity to be of interest to the DOD. The technique should also give critical sensitivity information related to all component assumptions.

There are four overall objectives for this research:

- Quantify component statistical reliability.
- Quantify the network availability component of network security as a function of component statistical reliability and a damage utility which captures information about risk during times of increasing tension.

- Determine the best investment strategies for improving network availability given the option of improving component statistical reliabilities and/or hardening components.
- Determine the sensitivity of the optimum upgrade plan to component assumptions

Scope and Assumptions

This research quantifies the availability component of stochastic network security using two measures of effectiveness (MOE) and finds the optimum investment strategy for improving the network. Analytical approaches are used to calculate the network MOEs derived. The focus is on networks with directional flow. Network statistical reliability is defined as the probability that an operational path exists between a single source and a single sink as a function of time. Extending the methods developed in this study to problems with undirected flow and/or defining network reliability as a function of the connectivity of all (or any size subset of all) network nodes is left as an area recommended for further study.

The following assumptions were made throughout the research, unless otherwise stated:

- Component failure is defined as the inability of the component to adequately perform its specified purpose for a specified period of time, under specified environmental conditions.
- Component failures are independent.
- Both arcs and nodes are subject to failure.

- Components are either failed or functioning.

Chapter II contains selected findings from the open literature which were used in developing models or used as solution techniques. Chapter III describes the method used to quantify statistical reliability of network components, a preliminary task in any attempt to improve aggregate network performance. Chapter IV defines *damage utility,* a measure used to augment the standard *reliability* measure in an attempt to quantify availability security in a hostile environment. Chapter IV also contains the prescriptive models used to identify which types of changes should be made to improve these measures of effectiveness subject to budget constraints. The results of the models and conclusions made as a result of the study are found in Chapter V. Appendices A through F contain the computer input code and the computer output results for all of the prescriptive models when applied to a sample network. Appendices G through I contain this same data for the models when applied to a more realistic (in complexity and size) network. Appendix J contains the computer code and results from the attempt to quantify uncertainty propagation in fault trees via fuzzy logic. Appendices K and L contain linear programs which could be used in the analysis if the only strategy available to a decision-maker was target hardening. Appendix M shows the topology of the realistic network used and the reliability of the components in that network at a given instant in time.

II. Literature Review

Sections included in this chapter include a discussion of network representation and the symbols used in the research, definitions, different techniques for calculating statistical reliability, quantifying potential damage from hostilities, and different methods for analyzing fault trees.

Network Representation

A network is a collection of arcs and nodes. Each arc and node is assumed to be an aggregated collection of parts such that its properties are easily determined. This property allows network representation to be useful in modeling a wide variety of problems.

The structure of the network can be described using a graph or a matrix. An example of a network graph is shown in Figure 1. This graph is described as G = (V,E), where V is a set of nodes and E is a set of edges. Edges with no arrows usually imply flow in both directions (i.e., undirected arcs).

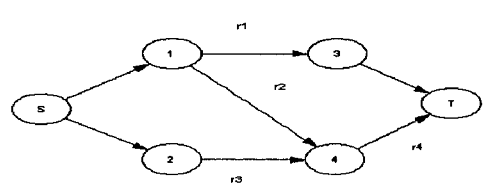

Figure 1. A Sample Network

The flows in the network are usually constrained by component *capacities* corresponding to bandwidth in communication networks. The components are also subject to failure, and the probability that a component is operational at a given instant in time is the component *reliability*. *External flow* is the required quantities of flow entering or leaving the network at each node. The *law of conservation of flow* states that the flow entering the node equals the flow leaving the node.

A *path* is a sequence of arcs in which the initial node of each arc is the same as the terminal node of the preceding arc. Paths are described by listing the order in which nodes are encountered, such as path (s13t) in the example above. A *chain* is a structure similar to a path except that not all arcs are necessarily directed towards the terminal node. A *circuit* is a path from some node r_0 to r_p, plus the arc from r_p back to r_0. Thus a circuit is a closed path. Similarly, a *cycle* is a closed chain. A graph is *connected* if there exists a chain between every pair of nodes in G. A *tree* is a connected graph with no cycles, and a *spanning tree*, defined with respect to some underlying graph G, is a tree that includes every node of the graph.

An *(s,t) path* represents a sequence of arcs which begins at node *s* and ends at node *t*; an *(s,t) cut* represents any minimal set of arcs that intersects every *(s,t)* path. The minimal cuts and paths for the network in Figure 1 are: P1 = (s13t), P2 = (s14t), P3 = (s24t), C1 = (13,14,24), and C2 = (13, 4t).

Notation

The following symbols are used in this thesis:

B = Total budget available for investment

c_i = Cost of increasing the reliability of component i by one unit

y_i = Decision variable (continuous) denoting the percentage of hardening effort to expend at component i

x_i = Decision variable (continuous) denoting how much reliability improvement to add to component i

n = Number of arcs in the network

p_i = Probability component i is functioning or has worked for a given time

q_p = The number of paths in the network

q_c = The number of cuts in the network

r = Any designated node: source, intermediate, or sink node

R_j = Statistical reliability of path or cut j

R0 = The current statistical reliability of the network

R1 = The final statistical reliability of the network after improvements, but before an attack

RI = The final statistical reliability of the network after a successful attack has destroyed component I

s = Source node

t = Sink node

VJ = the amount of damage caused to network statistical reliability by a given attack strategy

V = the network damage utility MOE derived using a game theoretic approach

Definitions

The ultimate goal for this study is to optimize improvements in the availability aspect of network security subject to budgeting constraints. The first step in solving this problem is to define availability security. Throughout this research, network availability security is called *availability*, and is considered to be a function of two criteria: *statistical reliability* and *damage utility*. The definition of *statistical reliability* used here is the probability that a single source is connected to a single sink in a network subject only to statistical failure.

The idea of a damage utility measure for a given network is closely related to what is referred to as "vulnerability" or "importance" measures in some literature. Quantifying the importance of components of the network on network reliability is beginning to attract attention (13, 29), and several measures exist which can be used to distinguish "vulnerable" networks from relatively "invulnerable" ones. One function of importance, proposed by Sengoku, Shinoda, and Yatsuboshi (32: 73), proposes measuring the importance of each component to overall flow. This importance is measured by calculating the amount of flow from i to j which flows through edge k for every possible combination of (i,j,k). The sum of each component's importance measurement is defined as the *system vulnerability*.

Goddard argues that the majority of the importance measures proposed in the past did not incorporate the amount of work required to destroy a component (13). He defines *integrity* as a tradeoff between the amount of work required to remove an edge and the amount of damage removing the edge causes. Perry and Page (25) list several different methods for determining the amount of damage an edge removal causes. Other measures of component importance they propose include considering the number of paths in the graph, the size of the minimum cut sets, the number of minimum cut sets, and how the component affects these numbers.

This research herein uses Game Theory to quantify a new measure of vulnerability or importance. The damage to statistical reliability caused by destroying a component can be measured as the statistical reliability of the network given that component has been destroyed, minus the statistical reliability of the network before the component was destroyed. Since this value ranges from -1 to 0, adding +1 to this quantity generates a utility function ranging from 0 to 1. By setting up a game matrix for a given network where two combatants consider the best choice of attack or defend strategies to achieve their goals, each network has a unique damage utility rating which describes the amount of damage a perfectly rational and operationally effective enemy could inflict on the network in the event of hostilities.

In summary, the following two definitions are used throughout the research:

1. Statistical Reliability - One network performance measure equal to the probability that flow can travel from a single source to a single sink in a directed network where component failures are only the result of the physical properties of the components.

2. Damage utility - A second network performance measure equal to the difference between the statistical reliability of the network before and after an attack by an operationally effective and rational enemy.

Calculating Statistical Reliability

Given a network topology with a single source, a single sink, and the statistical probability each component in the network will be working, calculating the probability that flow from the source can reach the sink (s is connected to t) is one measure of the network's statistical reliability. Assuming only arcs fail, the two simplest networks with two arcs are the *simple series* and the *simple parallel* networks, where r_1 equals the probability component one is up, and r_2 equals the probability component two is up. For each of these networks, the complete event space contains only four events: both components up, both components down, and only one component down.

The probability that flow from the source reaches the sink equals the sum of the probabilities of being in a state where the sink is connected to the source. For the series network the only state where the sink is connected to the source is the state *both components are up*, therefore the reliability is $(r_1)*(r_2)$. Likewise, the probability flow from the source reaches the sink for the parallel network is the sum of the probabilities for the states where at least one of the components is up, and equals $r_1 + r_2 - (r_1)*(r_2)$.

These two fundamental relations can be applied recursively to larger networks to *reduce* the network to a single component network. This method of calculating network reliability is known as *network reduction.* The main difficulty with this method of computation is that as the number of components (n) grows, the number of states (k) grows exponentially ($k = 2^n$). Methods which approximate upper and lower bounds by finding the most likely failure-states (19, 20, 33) are more computationally feasible and have produced tight bounds for a variety of network topologies.

Aggarwal (2, 3, 4) developed a method which requires enumerating all the paths and making them disjoint. Provan and Ball (27) developed a variation of this method which uses minimal cut sets instead of paths. Recent improvements recommended by Heidtmann (16) and Rai (28) make the sum of disjoint products procedures more efficient for complex networks.

Even though path and cut enumeration techniques can reduce the computation required in some network topologies, there is still no guarantee that the number of paths or cuts is not extremely large (8, 11, 24, 27). Page and Perry (24) avoid path or cut enumeration completely with their factoring algorithm, which recursively decomposes the network topology using the probability identity:

$$P(y) = P(y \mid q)*P(q) + P(y \mid q')*P(q') \qquad (1)$$

When P(y) is known without further factoring, that particular branch of the factoring tree can be terminated.

Fault Tree Analysis

Fault trees are used for reliability analysis of complex systems. Applications found in the open literature include rocket engines (15), the space shuttle (35), and nuclear power plants (11). The objective of a fault-tree analysis is to identify and model the various system events that can result in the occurrence of a given top event. A fault-tree analysis may include a quantitative evaluation of the probability of the top event, or a diagnosis which singles out critical components most likely to cause the top event (11).

For information on how to construct a fault tree, the reader is referred to NUREG-0492 "Fault Tree Handbook", published by the US Nuclear Regulatory Commission. This research assumes that the logical relationship of the basic events has been constructed in accordance with the NUREG and only discusses the graphical display of fault trees and the algebra and set theory needed to analyze them.

In standard fault-tree graphics, events are denoted by rectangles. The relationship between events is governed by various logical gates. Although numerous graphical symbols exist for representing various logical relationships (11:48-50), the only two used here are the "AND" and the "OR" gates. An AND gate indicates the preceding (higher level) event occurs only if subsequent events A and B occur, while an OR gate indicates the preceding event occurs if either A or B or both occur.

To analyze a fault tree, the logic must be represented in mathematical form. The output of an OR gate is equivalent to the Boolean + or set theory $\cup$, and is written as $B_0 = B_1 + B_2$ or $P(B_0) = P(B_1 \cup B_2)$ (11:58). Likewise, the output of an AND gate is equivalent to the Boolean * or set theory $\cap$, and is written as $B_0 = B_1B_2$ (11:58).

Given that the probabilities of the basic events are known, calculating the probability of the top event is usually accomplished using one of two methods: *direct reduction* or *cut set enumeration*. Direct reduction of the tree is similar to reducing a network. Each of the gates is replaced by its logical equivalent until only the top event is left. Since a fault tree and a network are different graphical representation of the same logic (an OR gate is equivalent to parallel components, an AND gate equivalent to series components), this method is equivalent to network reduction and suffers from the same potential problems.

According to Dhillon, to quantify the probability of the top event most fault tree analysis practitioners begin by enumerating the *minimal cutsets* of the fault tree, where a *cutset* is a collection of basic events whose presence will cause the top event to occur. A *minimal cutset* is a cutset which cannot be reduced but still insures the occurrence of the top event. The process of enumerating the minimal cutsets can still be very tedious. A cutset in a fault tree is equivalent to a path in a network.

An alternative to the traditional cutset-based solution approach for combinatorial models uses the binary decision diagram (BDD) (12:3). The BDD has primarily been used as a verification technique in circuit theory, but has recently been adapted to solve a fault tree model for both quantitative and qualitative reliability analysis (12:3). The system unreliability is quickly calculated as a sum of branch probabilities from the BDD. This method closely resembles the Page and Perry method of factoring networks. The biggest drawback of BDD is that the size of the graph depends heavily on the input variable ordering used (12:4).

Uncertainty in Fault Trees

Fault-tree reduction techniques require exact values for the probabilities of basic events. This is sometimes an unrealistic requirement, but usually upper and lower bounds on the probability of a basic event occurrence over a given time can be agreed upon.

These bounds describe the uncertainty in the basic events. The propagation of uncertainties in the evaluation of fault trees has been recognized as a very important aspect of any significant risk assessment (26:402). The three different methods most often used to determine uncertainty in the top event probability as a function of basic event uncertainties are Monte Carlo simulation, the Method of Moments, and Discrete Probability Distribution (DPD) methods (21:2). Monte Carlo simulation obtains the shape of the top event distribution from the basic event distributions through a sampling procedure. The Method of Moments consists of expanding the function $f(x_1, x_2, ..., x_n)$ around the mean values of its arguments using a multivariate Taylor series. In the DPD methods each basic event distribution is approximated by a discrete distribution in the form of a histogram. The top-event distribution is then obtained by a combination of these histograms.

A new approach to determining the effect of event uncertainty on the top-event probability is to use fuzzy sets, membership functions, and fuzzy algebra (15:1). A fuzzy set is a set which does not have exact boundaries. The most common example used for illustration in the open literature is the fuzzy set of tall people. The degree to which a person belongs to this fuzzy set is determined using a membership function. A person who is over seven feet tall would have a membership function equal to1. A person only five feet tall may have a membership function equal to 0. An infinite number of functions may be used to describe the degree to which people between five and seven feet tall belong to the set of tall people. The key to using fuzzy sets is the choice of membership function.

In software developed by NASA (FUZZYFTA), uncertainty in basic event probabilities can be described with trapezoidal membership functions, such as the one shown in Figure 2. The membership functions describe the probability of a basic event as being around p, with possibilities ranging as high as p_u or as low as p_l.

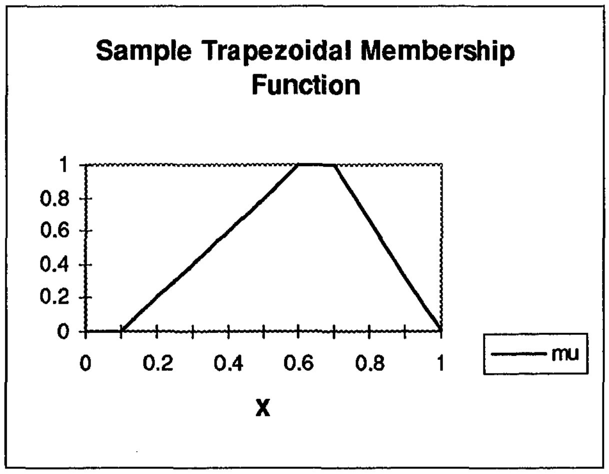

Figure 2. Trapezoidal Membership Function

Summary

Communication networks may consist of different types of equipment organized in a wide variety of topologies. The choice of equipment and topology determines many different MOEs for the network (e.g. cost, reliability, maximum flow, etc.). One MOE of interest to the DOD is availability. This measure should include more than just statistical reliability because of the real threat of an enemy attack on the network in times of hostility. By considering availability a two-criteria quantity consisting of both statistical reliability and damage utility (a measure of robustness in the event of attack), a more complete description of availability is possible. Statistical reliability of a network is a function of the reliability of the network components. Fault trees are often used to determine the statistical reliability of complex equipment. A thorough study of the events which may lead to equipment failure is required to construct the fault tree.

Once constructed, the probability of equipment failure as a function of time can be determined using basic reduction formulas and event occurrence probabilities. Unfortunately, this procedure assumes the event probabilities are fixed numbers. Two possible ways to quantify uncertainty in the component failure probability as a function of the basic event uncertainties are through Monte Carlo simulation or the use of fuzzy sets, membership functions, and fuzzy algebra. Once all of the components of a network have been analyzed, the current availability can be quantified, and further analysis will help make decisions to improve this quantity given a budget constraint, possible improvement strategies, and costs.

III. Descriptive Models and Methodology

This chapter describes how a fault-tree analysis could be applied to each of the stochastic components of a communication network in order to reach the first research objective: quantify component statistical reliability. The results from a fault tree analysis can then be used in the prescriptive models described in Chapter IV.

The Network

Let the following sample network represent a communication network useful for illustrating some methods used in this research. A more complex network (Network B - see Appendix M) was analyzed using the prescriptive methods described in Chapter IV.

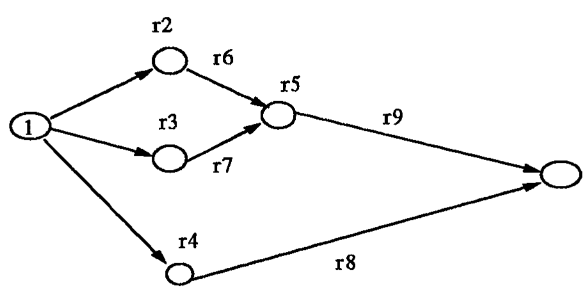

Figure 3. Sample Network

Assume this network consists of two basic types of components, one represented by arcs and the other represented by nodes. The nodes are numbered from 1 to 6, while arcs can be numbered by the nodes they connect (i.e., 2-5, 4-6, etc.). One method to determine the reliability of the components is through a fault-tree analysis.

A Component Fault Tree

Suppose a thorough study of one type of component revealed it would be unavailable or failed only through the series of events shown in Figure 4.

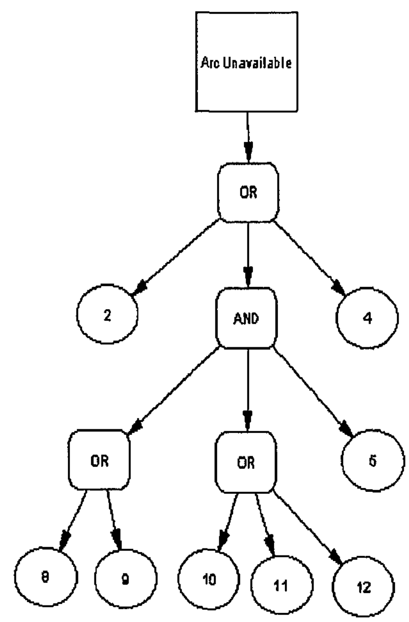

Figure 4. A Component Fault Tree

Further, assume the probability of the basic events can be modeled by probability density functions (pdfs) which can be thought of as the instantaneous rate of event occurrence as a function of time. One possible set of events and their distributions for each arc are shown in Table 1.

Table 1. Arc Event Data

EVENT	TYPE	Arcs:	2-5	3-5	4-6	5-6
2	Exponential	λ	.01	.50	.01	.005
4	Exponential	λ	.08	.08	.08	.04
5	Exponential	λ	.12	.12	.12	.05
8	Binomial	p	.05	.10	.05	.01
9	Weibull	m/α	2/10	1.5/5	2/10	2/5
10	LogNormal	μ/σ	1/1	.5/.5	1/1	1/1
11	Binomial	p	.01	.05	.01	.01
12	Binomial	p	.02	.05	.02	.01

To further understand what this table is communicating, consider event 9 for arc 2-5. The probability this event occurs as a function of time is modeled as a Weibull pdf with parameters m = 2.0, α = 10.0. Loosely, the density at time t is proportional to the probability of event occurrence around time t. To find the probability that event 9 has occurred as a function of time, integrate the pdf from time $t = 0$ to time $t = t_{max}$ yielding the cumulative distribution function (cdf). The cdf at time t thus represents the probability that the event occurs at or before time t.

Exponential pdfs have historically been good models for electronic part failures. The exponential events are described by the pdf and cdf :

$$\text{pdf:} \quad f(t) = 1/\lambda * \exp(-t/\lambda) \tag{2}$$

$$\text{cdf:} \quad P(t) = 1 - \exp(-t/\lambda) \tag{3}$$

A Weibull distribution is usually selected when the event being modeled requires a more general distribution than an exponential. The pdf and cdf for the Weibull used here are given by:

$$f(t) = (m/\alpha)*t^{(m-1)} * \exp(-t^m/\alpha) \tag{4}$$

$$F(t) = 1 - \exp(-t^m/\alpha) \tag{5}$$

Once the cdf is obtained, the probability of event 9 occurring as a function of time can be calculated easily. The graphical display of the pdf and cdf for event 9 for arc 2-5 is given in Figure 5.

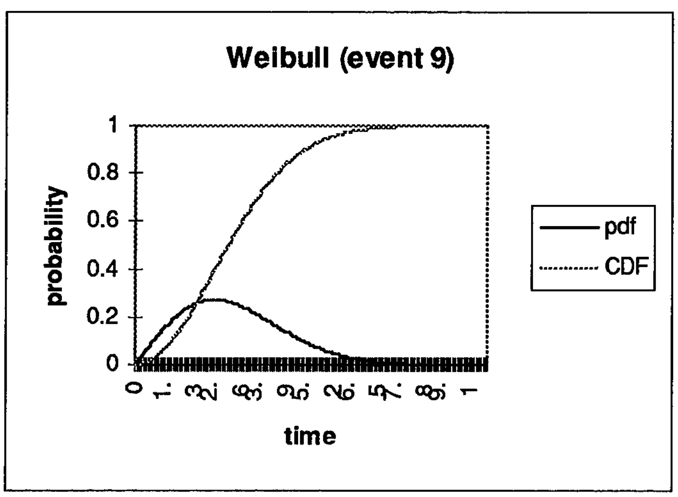

Figure 5. Event 9 for Arc 2-5

The binomial events represent events in the fault tree which require dynamic models. For instance, suppose event 11 corresponds to the event: a part subject to failures and repairs is failed. A Markhov model could be used to find the availability of the part, and this probability used in the fault tree. Event 11 for arc 2-5 occurs with probability .01 regardless of the length of time arc 2-5 is in use.

The lognormal pdf used is shown in equation 6, and has fit a number of datasets containing device times to failure reasonably well. The most useful feature of the lognormal pdf is that it generates a failure rate curve which looks like a "bathtub", a well-known phenomenon among reliability engineers. It describes a part which has a decreasing failure rate at the beginning of its lifetime (burn-in) followed by a long period where the failure rate is relatively constant, and finally an increasing failure rate for extremely long-lived members.

$$f(t) = 1/(2\pi\sigma t)^{.5} * \exp\{-[\ln(t)-\mu]^2/(2\sigma^2)\} \qquad (6)$$

Unfortunately, this pdf has no closed-form cdf. The cdf for the lognormal events in this study was obtained using a numerical integration technique (specifically the Composite Trapezoidal rule). This technique was chosen because it was easy implement the rule to find the value of the cdf from time $t = 0 + 2*h$ ($h = \Delta t$) to t = maximum reasonable operation time. A sample of the spreadsheet implementing the rule and the graphical results for arc 2-5 are shown in Table 2 and Figure 7 .

Table 2. LogNormal Numerical Integration Spreadsheet

mu	1		Numerical Integration for LogNormal			
sigma	1					
t	pdf	CDF	2*pdf	n	h	sum(2pdf)
0	0		0	0		
0.05	0.002723		0.005445	1	0.05	0.005445
0.1	0.017079	0.000563	0.034159	2	0.05	0.039604
0.15	0.040016	0.001991	0.080032	3	0.05	0.119636
0.2	0.066266	0.004648	0.132531	4	0.05	0.252168

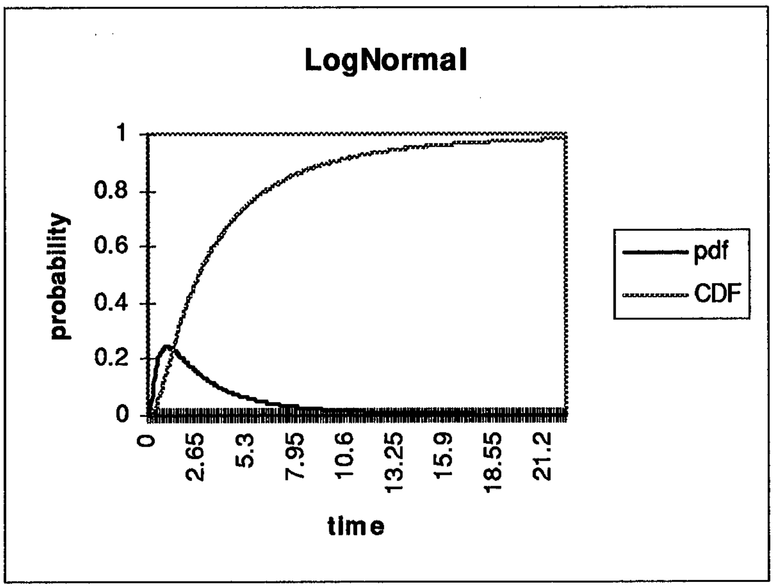

Figure 6. Event 10 for Arc 2-5

Once the cdfs are calculated, all data required to obtain the probability of the top event as a function of time has been collected. Next the functional value for the top-event probability is calculated. For this simple fault tree, converting the fault tree to a reliability block diagram (RBD) and reducing using the network reliability equations from Chapter 2 as shown in the following series of figures was one easy way of determining the top event's probability of occurrence function.

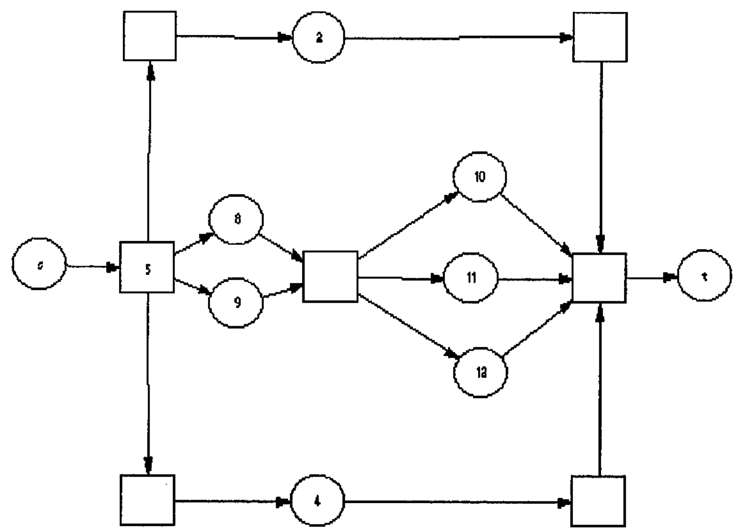

Figure 7. RBD Equivalent for Arc-Component Fault Tree

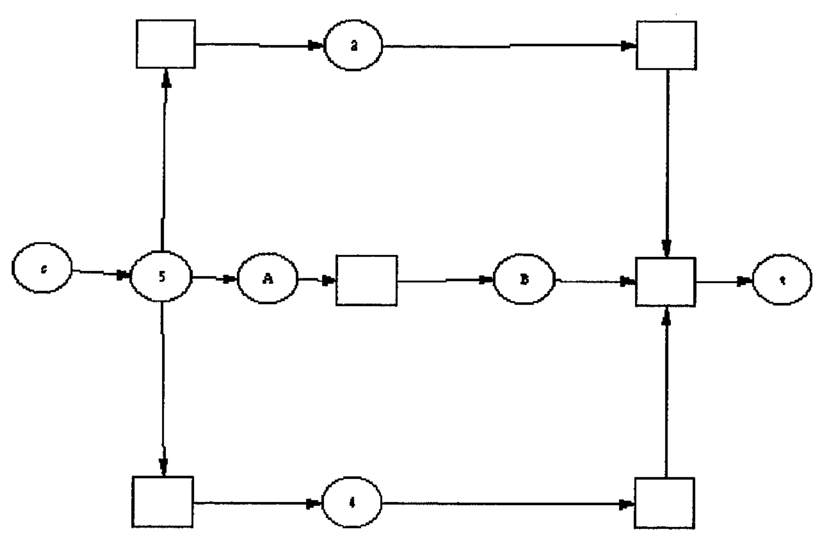

Figure 8. Reduction #1: Parallel Equivalent

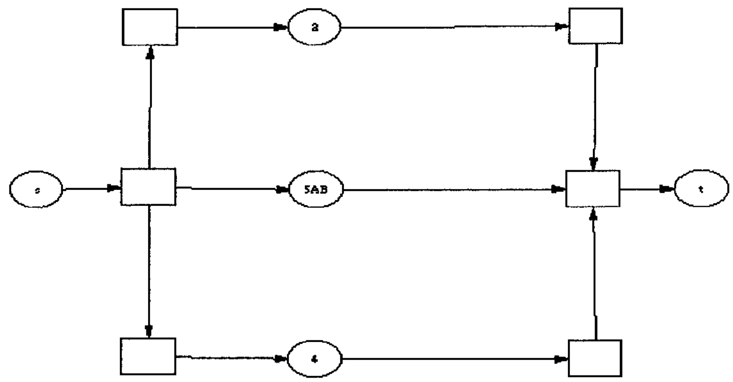

Figure 9. Reduction #2: Series Equivalent

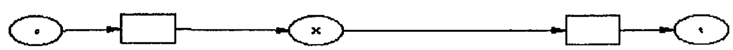

Figure 10. Reduction #3: Final Parallel Reduction

Because the explicit function for the top-event probability even for this simple fault tree becomes a very large expression, a spreadsheet was used to simplify the expression using the basic parallel and series equivalent equations derived in Chapter 2. A sample of the two spreadsheet pages used is shown in Tables 4 and 5. In Table 4, each column indicates the value of the cdf for each event corresponding to the time shown in column 1. In Table 5 the events are combined to determine values for A, B, and finally X as derived using the reduction formulas.

Table 3. Sample Spreadsheet for Arc 2-5 Event Probabilities

					m=2	1		
	0.01	0.08	0.12		alpha=10	1		
t	Exp(2)	Exp(4)	Exp(5)	Bin(8)	Weibull(9)	LogNrml10	Bin(11)	Bin(12)
0	0	0	0	0.05	0	0	0.01	0.02
0.05	0.0005	0.003992	0.005982	0.05	0.00025	0	0.01	0.02
0.1	0.001	0.007968	0.011928	0.05	0.001	0.000563	0.01	0.02
0.15	0.001499	0.011928	0.017839	0.05	0.002247	0.001991	0.01	0.02
0.2	0.001998	0.015873	0.023714	0.05	0.003992	0.004648	0.01	0.02

Table 4. Sample Spreadsheet for Arc2-5 Reduction Formulas

		Event Reduction Page		
t	P(top)	A	B	5AB
0	0	0.05	0.0298	0
0.05	0.004499	0.050237	0.0298	8.96E-06
0.1	0.008978	0.05095	0.030346	1.84E-05
0.15	0.013438	0.052135	0.031731	2.95E-05
0.2	0.017882	0.053792	0.034309	4.38E-05

The probability that Arc 2-5 will be available for use equals 1 minus the probability it is unavailable. This function of time is shown in Figure 11.

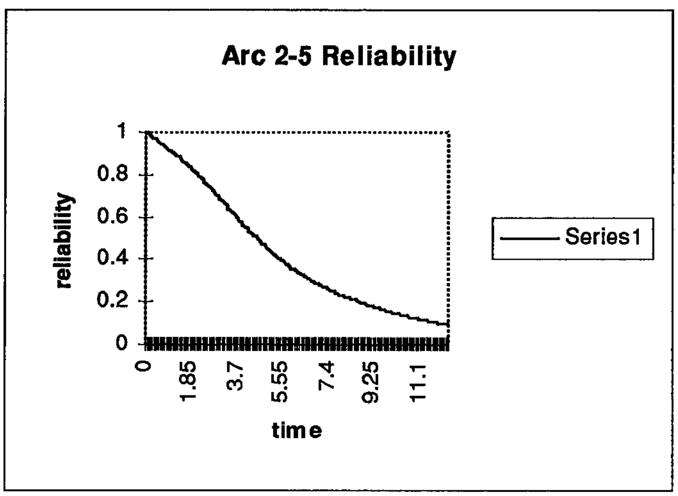

Figure 11. Arc 2-5 Reliability Curve

Modeling Uncertainty in the Fault Tree

Uncertainty in the top-event probability as a function of the basic event uncertainties was quantified using the Monte Carlo simulation approach and the Fuzzy Logic approach. A Monte Carlo simulation was performed on the arc 2-5 component using the spreadsheet designed to calculate the top-event probability as a function of time. A random number for each parameter was drawn and used to evaluate the cdf for each basic event. These randomly derived cdf values were then used to calculate the probability of the top event. The bounds used for each parameter, as well as the initial point estimate, are shown in Table 5.

Table 5. Monte Carlo Simulation Bounds

Event	lb	crisp	ub
2	.00	.01	.02
4	.04	.08	.12
5	.08	.12	.16
8	.025	.05	.075
9	1/5	2/10	3/15
10	N/A	1/1	N/A
11	.00	.01	.02
12	.00	.02	.04

The software package FUZZYFTA was used to quantify uncertainty via the Fuzzy Logic method. Using the same parameter bounds as used in the Monte Carlo simulation, trapezoidal membership functions were derived for each of the basic event probabilities. The software used standard fuzzy algebra to combine the basic event membership functions and derive the top event membership function. The input and output file for this FUZZYFTA run are at Appendix J.

Calculating Event Importance

One advantage to transforming a fault tree to an RBD is the ease in calculating the partial derivatives for each event. Consider the following problem:

Given: P(top event) = 1 - $R(p_1,p_2,\ldots,p_n)$

Required: $\partial(P(\text{top event}))/\partial(p_i)$

When the RBD is used, the P(top event) can always be found exactly using the method of inclusion/exclusion. This method expresses the probability of the source being connected to the sink as the sum of all paths minus the sum of all two-path intersections plus the sum of all three-path intersections, ad infinitum. Because of the Boolean identity of idempotence ($p_i * p_i = p_i$), no terms in this expression will have any of the basic probabilities (p_1, p_2, etc.) raised to any power other than 1 or 0. Therefore, the partial derivative with respect to any of the basic probabilities is always constant. The most important aspect of this property is that the value of the partials can be calculated by knowing the probability of the top event now, and the value of the top event given the specific event cannot occur ($p_i = 0$). Table 6 contains the proof of this feature.

Table 6. Partial Derivatives of the Component Reliability Function

Let m = the number of terms in the reliability function which contain p_i

Let q = the total number of terms in the reliability function

Let n = the total number of events in the fault tree (and reliability function)

Let t_j = the *j*th term in the reliability function

Let p_i = the probability event i occurs

Let $\mathbf{p}$ = the vector $<p_1, p_2, ..., p_n>$

Let $R(\mathbf{p})$ = the reliability function of the RBD equivalent of the fault tree

$P(top) = R(\mathbf{p}) = p_i*[t_1(\mathbf{p})+t_2(\mathbf{p})+...+ t_m(\mathbf{p})] + [t_{m+1}(\mathbf{p}) + ... + t_q(\mathbf{p})]$

Thus $\delta P(top)/ \delta p_i = [t_1(\mathbf{p})+t_2(\mathbf{p})+...+ t_m(\mathbf{p})] = A$

$$P(top \mid p_i = p_i) - P(top \mid p_i = 0) = p_i*[t_1(\mathbf{p})+t_2(\mathbf{p})+...+ t_m(\mathbf{p})] + [t_{m+1}(\mathbf{p}) + ... + t_q(\mathbf{p})] -$$

$$0*[t_1(\mathbf{p})+t_2(\mathbf{p})+...+ t_m(\mathbf{p})] + [t_{m+1}(\mathbf{p}) + ... + t_q(\mathbf{p})]$$

$$= p_i * A$$

Therefore, $\delta P(top)/ \delta p_i = A = [P(top \mid p_i = p_i) - P(top \mid p_i = 0)]/p_i$

Using this result, the partials for each event can be easily computed in the spreadsheet already put together to calculate the probability the component is available. The results for all arcs are given in Chapter V.

IV. Prescriptive Models and Methodology

The sample network improved and the data given for the network is shown in Figure 12, where Node 1 corresponds to the source, Node 6 to the sink, and the reliability for each component as found using fault-tree analysis is shown outside the component. Network B, a much more complex network used to test the robustness of these methods on realistically complex networks, is completely described in Appendix M.

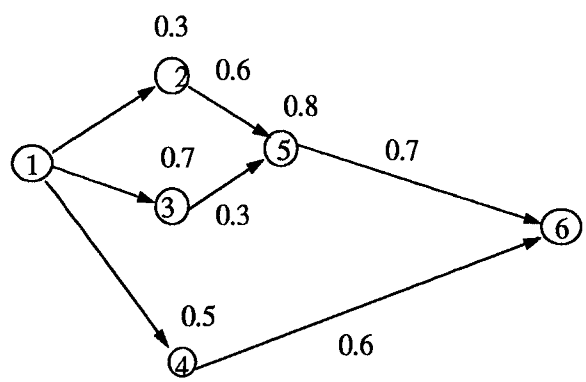

Figure 12. Sample Network.

There are numerous ways to derive the statistical reliability of the network. Since this is a small network, the reduction technique shown in the following illustrations works well.

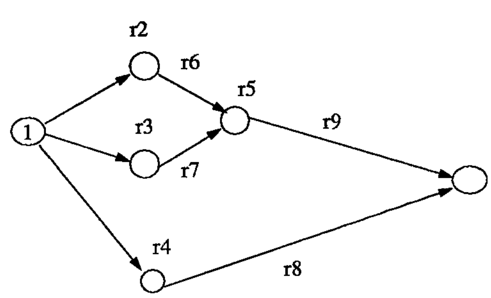

Figure 13. Step one of reduction technique.

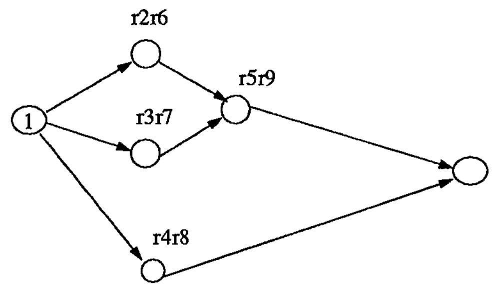

Figure 14. Step two of reduction technique.

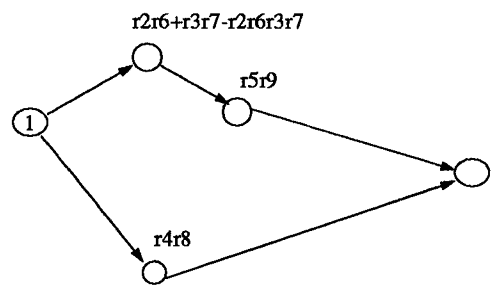

Figure 15. Step three of reduction technique.

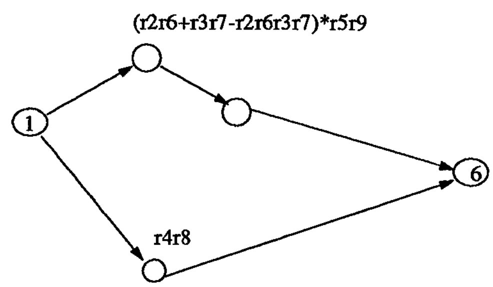

Figure 16. Step four of reduction technique.

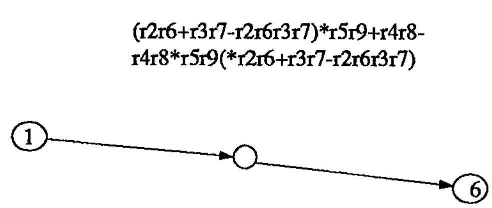

Figure 17. Final step of reduction technique.

Simplifying the expression in the final step of the reduction technique shown in Figure 17 leads to:

$$R0 = r_4r_8 + r_2r_5r_6r_9 + r_3r_5r_7r_9 - r_2r_3r_5r_6r_7r_9 - r_2r_4r_5r_6r_8r_9 - r_3r_4r_5r_7r_8r_9 + r_2r_3r_4r_5r_6r_7r_8r_9 \quad (7)$$

Traditional Risk Analysis Models (RAMs)

Using traditional RAMs to identify network components targeted for improvements requires quantifying the impact of a component failure and the probability of component failure. In peacetime these quantities are relatively easy to determine: the impact of component failure is the partial derivative of the network reliability function with respect to the component times the reliability of the component. The probability of component failure depends on whether non-statistical failures (i.e. enemy attacks) are considered. A simple RAM which ignores the probability of enemy attack for the sample network was accomplished, with results shown in the next chapter.

Model 1: Maximize R1

Since $\mathbf{r'} = \mathbf{r} + \mathbf{x}$, let $R1 = R(\mathbf{r'})$. The single criteria optimization problem which maximizes the statistical reliability can be written as:

Table 7. Model 1 Formulation

$$\max f_1 = R1$$

st	$\mathbf{r'} \leq \mathbf{1}$ (or equivalently $\mathbf{x} \leq \mathbf{1}-\mathbf{r}$)	(upper bound on r_i)
	$\mathbf{c}^t\mathbf{x} \leq B$	(cost constraint)

These two constraints define the **X** space and will henceforth be referred to as $\mathbf{x} \in \mathbf{X}$. Specific results will be discussed later. For the sample network, it was assumed that the cost of improving the reliability of the component was linear and equal to the current reliability of the component.

These assumptions are unrealistic as the following example illustrates: it is probably going to be more expensive to increase the reliability of a component from 0.9 to 1.0 than it is to increase the reliability of the same component from 0.1 to 0.2. However, given a set of real components and real options which will improve the reliability of the components, it would not be difficult to include more realistic cost constraints in the model. Since the purpose of this problem formulation was to develop a method for analyzing security risk, and writing cost constraints is a science thoroughly studied previously, this research kept the simple cost constraint. Furthermore, since the costs were unitless numbers, the budget is also unitless.

This model was evaluated for various budgets B ranging from 0 to 0.5 (see Appendix B). The result when $B = 0$ should be the network's current reliability. The reason 0.5 was the largest B evaluated was because the reliability of the network after improvements given the cost assumptions and a budget of 0.5 was 1.0.

Model 2: Maximize V (without Hardening)

The method for quantifying the second criteria, damage utility, was motivated by Game Theory. Consider a two player game, where the players are "Blue" and "Red".

Suppose the network belongs to Blue, and Blue can improve the network by either hardening some of the network components, improving some of the network components statistical reliability, or a mix of these two types of strategies. Meanwhile, Red has plans to attack the network should hostilities occur. Blue must consider Red's intentions when considering what improvements to make.

This thought process by Blue is the essence of Game Theory. Suppose Blue decides to not to harden any components in the current network, instead spending all available money improving component reliabilities. This is one possible Blue strategy. What is the impact if Red attacks a component? Since the *RI* are calculated as functions of **x**, if Red destroys node 2 the statistical reliability will decrease from *R1* to *R2* regardless of the value of **x** Blue has chosen. The quantity *(R1 - R2)* is the *damage* associated with the intersection of Red's and Blue's strategies from Blue's point of view. If Red is rational, Red will attack the component which results in the maximum damage to Blue. Thus the expected damage of Blue's decision to do nothing will be the maximum *R1-RI* where *I* indicates which component Red decided to attack (*I=1* indicates Red did not attack any components). Note that this value ranges from -1 to 0 since the *RI* after an attack will always be less than or equal to the R1 before the attack.

Let the quantity $V = (1 - damage)_{\min}$ be defined as the *scaled damage utility* resulting from a Blue decision. Now *V* will range from 0 to 1. This leads to a second optimization problem, that of maximizing the damage utility predicted using game theory, where Blue does not have the option of hardening the network components. This model was also evaluated for various budgets B ranging from 0 to 1.25 (see Appendix C).

Table 8. Model 2 Formulation			
	$\max f_2 = V$		
st	$V \leq RI-R1+1$	$I = 1,2,3,\ldots,9$	(Game Constraints)
	$\mathbf{x}\ \varepsilon\ \mathbf{X}$		(X space)

Model 3: Maximize V (with hardening)

The complete model allows Blue the option of hardening network components in addition to improving component reliabilities. Assuming the decision to harden a component makes it impossible for red to destroy the component, Model 2 represents one Blue hardening strategy: harden nothing. In an ordinary game theoretic model, it is assumed that the two combatants are playing a game repeatedly, and the percentage of times Blue should adopt the strategy represented by row *i* is y_i, while the percentage of times Red should adopt the strategy represented by column *j* is z_j. The payoff amounts in the game matrix reflect the outcome of a pure strategy, i.e. the payoff given Red and Blue each pick a specific *i* and *j* strategy. This model constructed a game matrix using these assumptions, but since the game of war on this network is not likely to be played more than once, the **y** (or **z**) results are interpreted as percentage of total hardening (or attacking) effort expended in the single game. The damage utility outcome for each hardening/attack strategy intersection can be identified in the matrix shown below, where each column represents a different Red attack decision, each row represents a different

Blue hardening decision, and Blue's reliability improvement decision is included in the expressions for R1, R2, etc.,:

Table 9. Game Strategy Matrix

	1	2	3	4	5	6	7	8	9
1	1.0	V2	V3	V4	V5	V6	V7	V8	V9
2	1.0	1.0	V3	V4	V5	V6	V7	V8	VJ = 1 - (R1-RJ) = 1 - (R1-R9)
3	1.0	V2	1.0	V4	V5	V6	V7	V8	V9
4	1.0	V2	V3	1.0	V5	V6	V7	V8	V9
5	1.0	V2	V3	V4	1.0	V6	V7	V8	V9
6	1.0	V2	V3	V4	V5	1.0	V7	V8	V9
7	1.0	V2	V3	V4	V5	V6	1.0	V8	V9
8	1.0	V2	V3	V4	V5	V6	V7	1.0	V9
9	1.0	V2	V3	V4	V5	V6	V7	V8	1.0

The model still includes the **x** decision variables, which indicate where reliability improvements should be made. The expected value to Blue of Red's choice of any single strategy z_j (i.e. any column j) equals the sum of the expected payoffs in that column. The expected payoff of any i,j strategy intersection equals the payoff times the probability Blue will choose that row as a single strategy (y_i). For example, the expected value of column six to Blue equals (R6-R1+1)*($y_1 + y_2 + y_3 + y_4 + y_5 + y_7 + y_8 + y_9$) + (1)*$y_6$. A rational Red will choose the column whose sum yields Blue the lowest payoff possible. Using these results leads to Model 3 for Blue (results included at Appendix D for B ranging from 0 to 1.25). The cost of hardening targets is not included in this model.

Table 10. Model 3 Formulation			
max $f_2 = V$			
st	$V \leq (Ri\text{-}R6+1)(\Sigma y_j)+y_i$	$i = 1,2,3,\ldots,9$, $j \neq i$	(Game Constraints)
	$\mathbf{x}\ \varepsilon\ \mathbf{X}$		(X space)

Model 4: Maximize V and R1 Simultaneously (with Hardening)

The single-criteria models were run primarily to show that the damage utility is a different MOE than reliability. Once this was established, a multi-criteria optimization (MCO) model was implemented by using $f_1 = R1$ and $f_2 = V$ as defined above. The clear candidate for parametric evaluation is R1 since it is known to have a lower bound of R0 and an upper bound of 1 for the X-space given when the budget B = 0.5 (see Appendix E). The nonlinear MCO model for the network is then:

Table 11. Model 4 Formulation			
	max $f_2 = V$		
st	$V \leq (Ri\text{-}R6+1)(\Sigma y_j)+y_i$	$i = 1,2,3,\ldots,9$, $j \neq i$	(Game Constraints)
	$\mathbf{x}\ \varepsilon\ \mathbf{X}$		(X space)
	$f_1 = R1 \geq q$		q = R0, .45, ... ,1.0

Model 5: Determining the Monetary Value of Target Hardening

The value of hardening was determined by using the MCO model described above, but the Game Constraints with hardening were replaced with the Game Constraints without hardening. Then the budget was increased from 0.5 (same as with hardening) until the resulting efficient frontier moved beyond the frontier when hardening was allowed and the budget equaled 0.5. The difference between the final budget and 0.5 equals the monetary value of including a hardening strategy (results at Appendix F).

Table 12. Model 5 Formulation

$\max f_2 = V$

st $\quad V \leq R1 - RI + 1 \quad I = 1,2,3,\ldots,9$ (Game Constraint)

$\mathbf{x} \in \mathbf{X}$ (X space)

$f_1 = R1 \geq q \qquad q = R0, .45, \ldots, 1.0$

$B = 0.5, 0.6, \ldots, Bfinal$

V. Results and Analysis

Fault Tree Analysis Results

Three different arc-type components were studied: arc 2-5, arc 3-5, and arc 5-6. The only differences between these components were the parameters of probability functions which model the events which make up the fault tree. The reliability curve for arc 2-5 was given in Chapter III (see Figure 12). The results for arc 3-5 and 5-6 are shown in Figures 18 and 19.

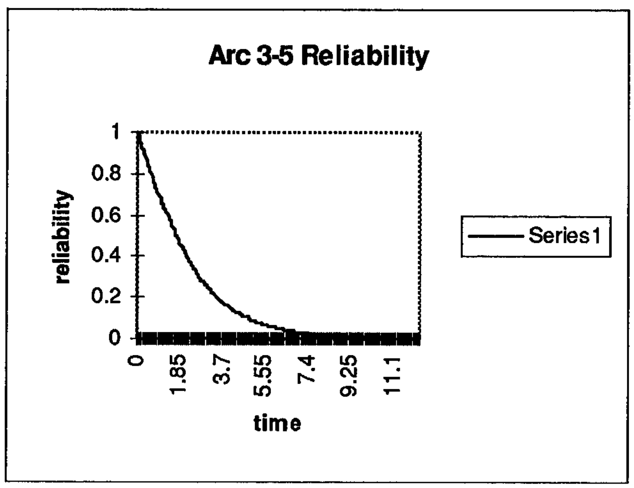

Figure 18. Arc 3-5 Reliability Curve

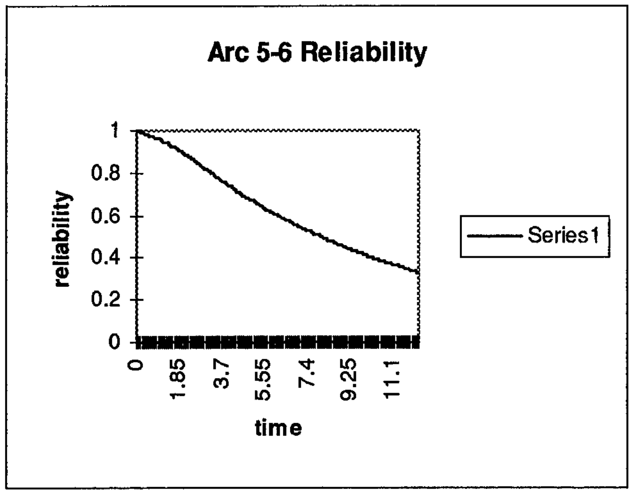

Figure 19. Arc 5-6 Reliability Curve

Uncertainty Modeling Results

Two different methods for quantifying uncertainty propagation in the fault tree were performed. Both methods only work for a given time, $t_{nominal}$. The component chosen for demonstration was arc 2-5, and the nominal time was 3.65 units. The results of the Monte Carlo simulation and the FUZZYFTA analysis are shown in Figures 20 and 21.

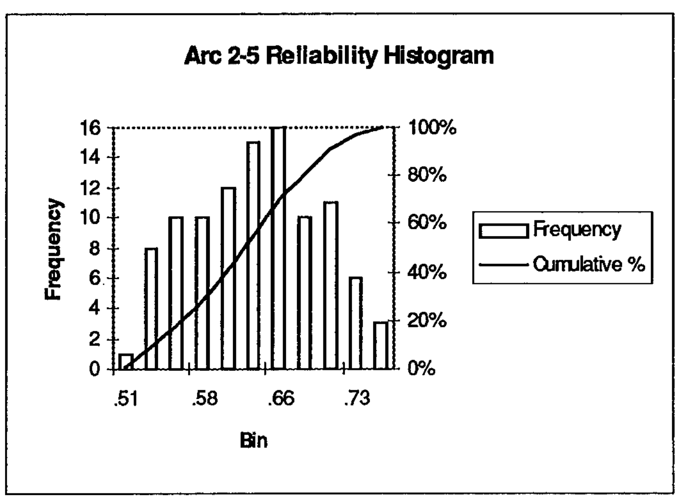

Figure 20. Simulation Results

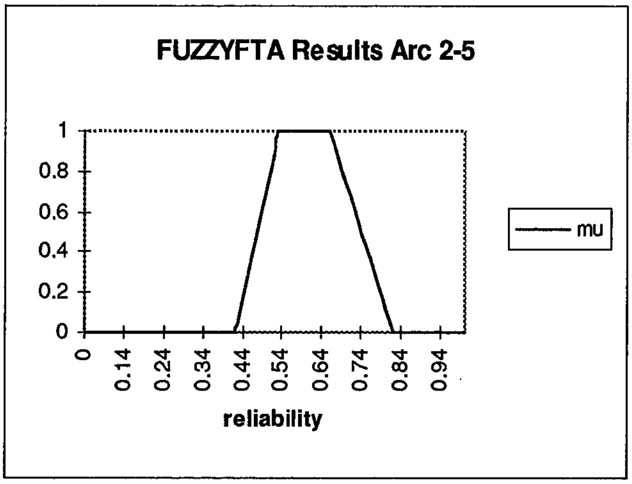

Figure 21. FUZZYFTA Results

Both methods indicate the reliability of arc 2-5 after 3.65 units of time will vary from as low as .44 to as high as .84, with the most likely value being between .5 and .7. The greater amount of deviation or spread in the FUZZYFTA results was expected since this method uses less data to generate results.

Event Importance Results

Using the spreadsheet already built to calculate the current top event probability, calculating the event importance was not time consuming. Results for each arc are shown in Table 13. These calculations for events in a fault-tree are identical to the calculations used to determine the damage caused by destroying components in a network. As such, the event importance results from a fault-tree analysis is closely related to a traditional risk assessment.

Table 13. Arc Event Importance Results

EVENT	ARC 2-5	ARC 3-5	ARC 5-6
2	.621242	.791315	.717888
4	.803267	.341731	.84
5	.33835	.104911	.564
8	.041539	.026	.002646
9	.152141	.041214	.110788
10	.185586	.029519	.160107
11	.068354	.010867	.052891
12	.070926	.010867	.052891

Traditional RAM Results

The expected result from a traditional RAM analysis on the network components is a number, RISK, for each component. Risk equals the probability of a bad event times the impact of the bad event. In this case, the bad event is component failure, while the damage equals the partial derivative times the current reliability. The risks in Figure 23 suggest that node 4 and arc 4-6 should be considered for hardening, but do not indicate where reliability improvements should be made to reduce damage from an attack.

	r	partial	damage	risk
Node 2	.3	.185808	.055742	.04
Node 3	.7	.096432	.067502	.02
Node 4	.5	.48166	.24083	.12
Node 5	.8	.172578	.138062	.03
Arc 2-5	.6	.0929	.055742	.02
Arc 3-5	.3	.22501	.067502	.05
Arc 4-6	.6	.40138	.24083	.10
Arc 5-6	.7	.19723	.138062	.04

Figure 22. Traditional RAM Results

Single Criteria Model Results for the Sample Network

The models described in Chapter IV were coded for the small communication network in GINO (see Appendix A). Only minor changes were required to transform the GINO input file from max f_1 to max f_2 to max f_2 st $f_1 \geq q$. Figure 23 summarizes the resulting reliability of the net designed when single criteria models were run and the budget was increased, where Model 1 results correspond to Max R1, Model 2 results correspond to Max V (without hardening options) and Model 3 results correspond to Max V (with hardening options).

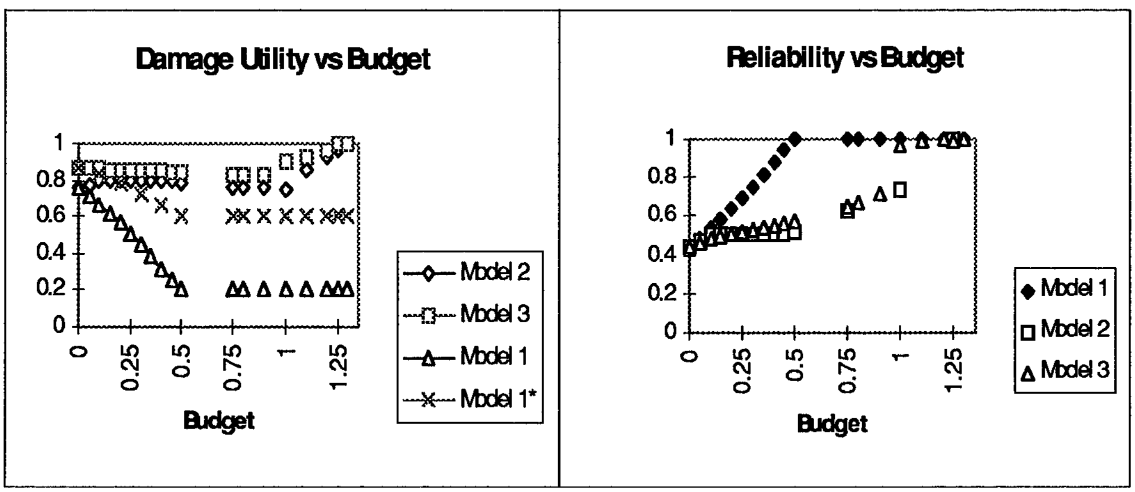

Figure 23. Single Criteria Results

In addition to generating the MOE vs. budget curves shown, at each point on an MOE vs. budget curve the models recommend various decision strategies concerning which components to improve or harden. The following table summarizes the reliability recommendations when the budget was set at 0.5 for each of the models.

Table 14. Recommended Reliability Improvements

Component	Model 1	Model 2	Model 3
None	0	0	0
2	0	0	0
3	0	0.3	0.3
4	0.5	0	0
5	0	0.054767	0
6	0	0	0.240750
7	0	0	0
8	0.4	0.010311	0.242584
9	0	0	0.

Clearly the MOE *damage utility* is a different network performance measure than the standard reliability measure, and a network designed with only one of these MOEs as the objective function may not perform as well when measured using the other MOE.

Multi-Criteria Model Results for the Sample Network

Based on information obtained from Models 1-3, a budget of 0.5 was chosen as the budget used in the MCO models for the sample network. The reason for this choice was because 0.5 was the minimum budget which allowed R1= 1.0 to be feasible. The results of the MCO model 4 are shown in Figure 25. The x-axis begins at the networks current reliability. The first point generated on the graph comes from maximizing V subject only to the budgeting constraints. From this point additional points are generated by successively maximizing V while requiring R1 to increase to 1.0.

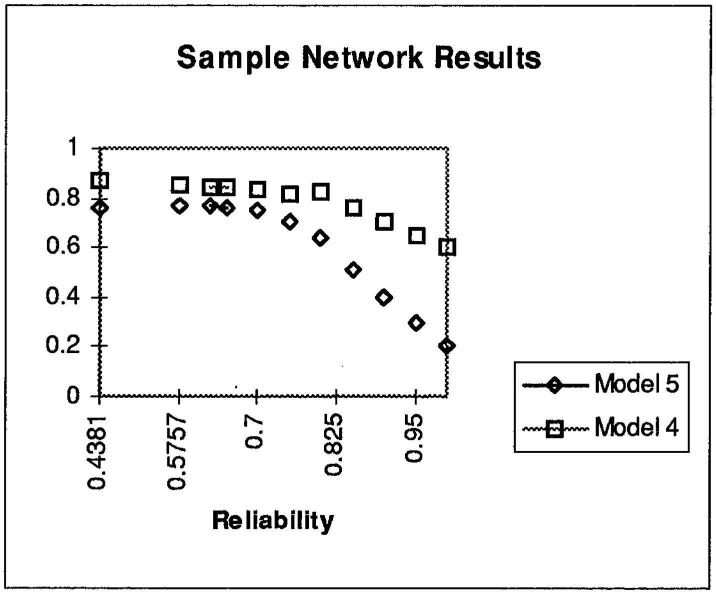

Figure 24. MCO Model Results

In order to help limit the number of efficient points given to a decision-maker, each point was evaluated using the standard *l-norms* (one, two, and infinity) as a means of measuring the distance from an efficient point to the optimal point (network reliability and damage utility both equal to 1). Each norm represents a different decision-maker attitude toward criteria trade-off. The one-norm measures the distance from the optimal point given each criteria affects the distance. The infinity-norm measures the distance from the optimal point given only the criteria furthest from optimal affects the distance. By comparing results from each of the norms, rationally justifiable optimal points on the frontier regardless of attitude toward criteria trade-off are represented somewhere between the optimal one-norm efficient point and the optimal infinity-norm efficient point. The results of this evaluation are summarized in Figure 25.

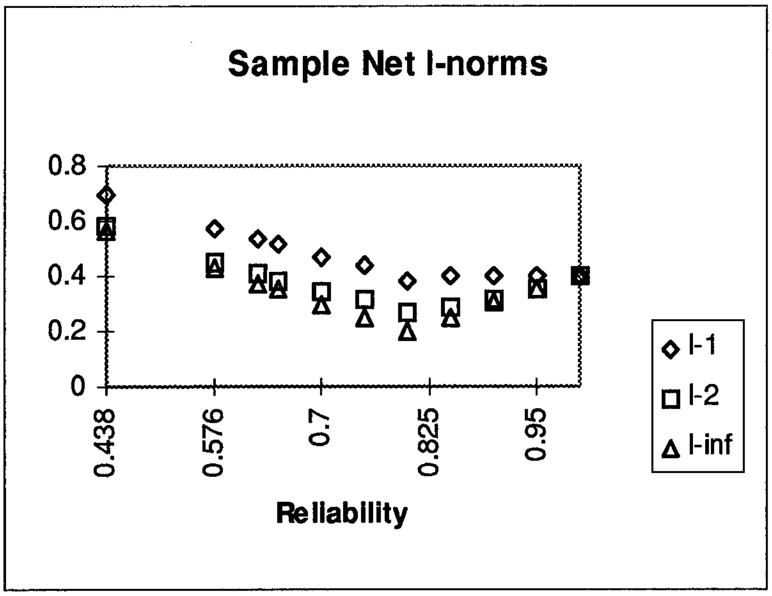

Figure 25. Sample Network Distance Function

The interesting result of this analysis is that the point where R1 = 0.8 is the best point using all three norms. The decision variable values at this point are summarized in Table 15.

Table 15. Decision variables at R = 0.80

i	x_i	y_i	z_i
2			
3			
4	.38	.5	.25
5			
6			
7	.70	.23	.39
8	.13	.27	.36
9	.03		

These number tell the decision-maker to exert 50% of hardening effort on component 4, and the other 50% should be split between component 7 and component 8. Furthermore, the reliability of component 4 should be increased by .38, the reliability of component 7 by .7, the reliability of component 8 by .13, and the reliability of component 9 by .03. If an enemy attacks the network, his best attack strategy would expend 25% of total effort on component 4, and the other 75% between components 7 and 8. Using these decision variables, another traditional RAM was performed, with results compared to the unimproved network as shown:

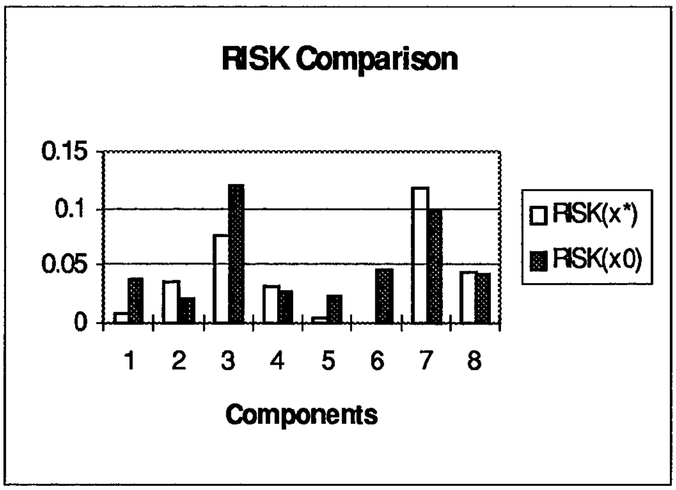

Figure 26. Comparison of RAM Results

Notice that although it is known that the network has better performance at $\mathbf{x}^*$ than at $\mathbf{x}^0$, the traditional RAM results do not show in a conclusive way that this is so. There are two reasons for this lack of information in the traditional RAM. First, there is no way to include the effects of target hardening. Second, there is only risk associated with statistical failures, so enemy attacks and effects are not included.

Value of Hardening

Since costs for target hardening are not explicitly included in the model the value of hardening was determined by iterating model 4 without hardening until a budget was found which resulted in an efficient frontier strategically equivalent to the frontier created with hardening and a budget of 0.5. The value of hardening found using these assumptions was 0.125.

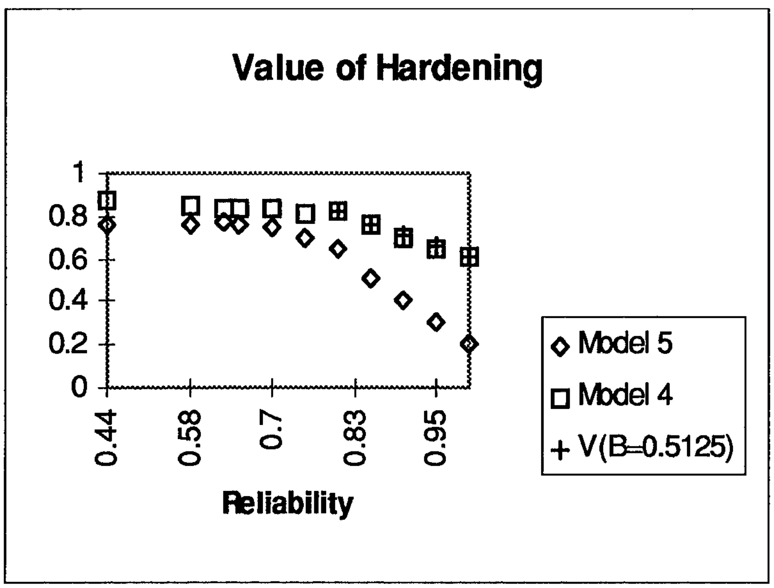

Figure 27. Value of Hardening

Network B Results

Network B (see Appendix M) was analyzed using Model 4. The purpose of the analysis was to determine the optimal target hardening/ component reliability improvement strategy to maximize both network reliability and network damage utility. Although the network is far more complex than the sample network, results are still obtainable.

As a result of a preliminary study on the importance of each component using the partial derivatives as a measure, certain results were expected. Actually implementing the game theory model requires an analytical expression for network reliability after component improvements and after each component is destroyed. In a network of this complexity a model which included these expressions for every component would be extremely large. However, by reducing the network it was shown that the majority of the components would never enter the solution. The exact procedure used to calculate the network reliability is summarized as an algorithm in Appendix M with Network B. The final model included the assumption that these components would not enter the solution.

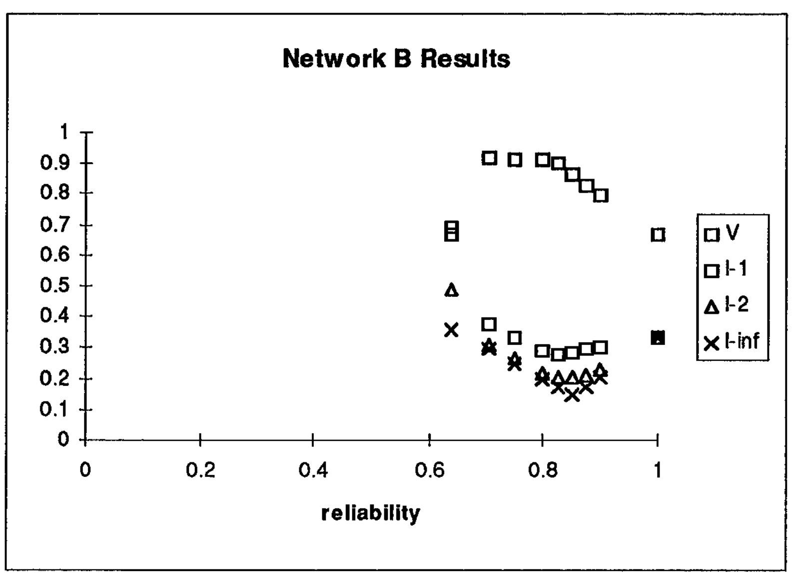

Figure 29. Network B Results

The same distance norms in this instance are not all minimized at the same point, but the total number of possible points required to enumerate all rational optimal points is still very small. The decision variables at the point where both the one and two norms are minimized are shown in Figure 30.

Table 16. Decision Variables for Network B

Component	x_i	y_i	z_i
Node 4		.41	.29
Node 6	.5	.23	.39
Arc 4-24		.36	.32
Node 16			
Arc 4-16			

Optimality

All of the models were solved using GINO. GINO looks for points which satisfy the KKT conditions. Unfortunately, the reliability function is not convex in general, so the KKT conditions are necessary not sufficient to ensure a point is an optimal point. The method used here to explore possible alternate optimal points was to use the GUES command in GINO to see if any other points with different **x** -values satisfied the necessary conditions. None were found. Given the recommended $\mathbf{x}^*$ found using the GINO model, the partial derivatives were calculated and a linear program (LP) was set up to implement a model where only target hardening was allowed. These models found the hardening decision variables **y** which were optimal for a given network topology when no reliability improvements were allowed. Also, the LPs were used to identify possible alternate optimal hardening strategies, and the shadow prices from the game constraints indicate the optimal Red attack strategy (**z**). (36) The LPs used to generate these results for each network are in Appendix K and Appendix L respectively. The results agreed with those obtained using GINO.

Conclusions

Two networks were analyzed for the purpose of performance improvement. Two MOEs were developed: f_1 = statistical reliability of the network and f_2 = damage utility of the network. The model showed that:

- f_1 and f_2 are very different performance measures, and networks designed by only optimizing one may not meet standards for the other
- in the single-criteria models spending strategies are very different depending upon which criteria is optimized
- enemy attack strategies are revealed in the shadow prices for the game constraints when a linear model is used
- including the option to harden components has measurable value even when no information exists concerning the cost of hardening
- minimal decision-maker information is needed to obtain these results
- the model is far superior to traditional RAM for prescriptive purposes

The following areas should be further explored or implemented as the case may be:

- Discretize the possible reliability improvements.
- Cost for reliability improvements should vary with the amount of improvement already implemented since it is likely to be more expensive. to change a reliability from .8 to .9 than it is to change one from .2 to .3.
- Include flow and damage to flow as two more MOEs to consider, using the same game theoretic definition of damage to flow as used here to define damage to reliability.
- Since most existing systems which are important already have highly reliable (i.e., reliability> .95) components, use the partial derivatives as linear coefficients in a pseudo-reliability function. As long as the final reliabilities of network components are not very different from the initial reliabilities, the partials do not change drastically and the solutions will be close to optimal.

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Appendix A. *GINO Input File for Sample Network*

```
MODEL:
  1) R1 = ( .3 + X2 ) * ( .7 + X3 ) * ( .5 + X4 ) * ( .8 + X5 ) * ( .6 +
   X6 ) * ( .3 + X7 ) * ( .6 + X8 ) * ( .7 + X9 ) + ( .3 + X2 ) * ( .8
   + X5 ) * ( .6 + X6 ) * ( .7 + X9 ) + ( .7 + X3 ) * ( .8 + X5 ) * (
   .3 + X7 ) * ( .7 + X9 ) + ( .5 + X4 ) * ( .6 + X8 ) - ( .3 + X2 ) *
   ( .7 + X3 ) * ( .8 + X5 ) * ( .6 + X6 ) * ( .3 + X7 ) * ( .7 + X9 )
   - ( .3 + X2 ) * ( .5 + X4 ) * ( .8 + X5 ) * ( .6 + X6 ) * ( .6 + X8
   ) * ( .7 + X9 ) - ( .7 + X3 ) * ( .5 + X4 ) * ( .8 + X5 ) * ( .3 +
   X7 ) * ( .6 + X8 ) * ( .7 + X9 ) ;
  2) .3 * X2 + .7 * X3 + .5 * X4 + .8 * X5 + .6 * X6 + .3 * X7 + .6 * X8
   + .7 * X9 = 0.00 ;
  3) MAX= V ;
  4) R2 = ( .7 + X3 ) * ( .8 + X5 ) * ( .3 + X7 ) * ( .7 + X9 ) - ( .7 +
   X3 ) * ( .5 + X4 ) * ( .8 + X5 ) * ( .3 + X7 ) * ( .6 + X8 ) * ( .7
   + X9 ) + ( .5 + X4 ) * ( .6 + X8 ) ;
  5) R3 = ( .3 + X2 ) * ( .8 + X5 ) * ( .6 + X6 ) * ( .7 + X9 ) - ( .3 +
   X2 ) * ( .5 + X4 ) * ( .8 + X5 ) * ( .6 + X6 ) * ( .6 + X8 ) * ( .7
   + X9 ) + ( .5 + X4 ) * ( .6 + X8 ) ;
  6) R5 = ( .5 + X4 ) * ( .6 + X8 ) ;
  7) R6 = ( .5 + X4 ) * ( .6 + X8 ) + ( .7 + X3 ) * ( .8 + X5 ) * ( .3 +
   X7 ) * ( .7 + X9 ) - ( .7 + X3 ) * ( .5 + X4 ) * ( .8 + X5 ) * ( .3
   + X7 ) * ( .6 + X8 ) * ( .7 + X9 ) ;
  8) R7 = ( .3 + X2 ) * ( .8 + X5 ) * ( .6 + X6 ) * ( .7 + X9 ) - ( .3 +
   X2 ) * ( .5 + X4 ) * ( .8 + X5 ) * ( .6 + X6 ) * ( .6 + X8 ) * ( .7
   + X9 ) + ( .5 + X4 ) * ( .6 + X8 ) ;
  9) R8 = R4 ;
  10) R4 = ( .3 + X2 ) * ( .8 + X5 ) * ( .6 + X6 ) * ( .7 + X9 ) + ( .7 +
   X3 ) * ( .8 + X5 ) * ( .3 + X7 ) * ( .7 + X9 ) - ( .3 + X2 ) * ( .7
   + X3 ) * ( .8 + X5 ) * ( .6 + X6 ) * ( .3 + X7 ) * ( .7 + X9 ) ;
  11) R9 = R5 ;
  12) R0 = .4380624 ;
  13) V < 1 ;
  14) V < ( R2 - R1 + 1 ) * ( Y1 + Y3 + Y4 + Y5 + Y6 + Y7 + Y8 + Y9 ) + Y2 ;
  15) V < ( R3 - R1 + 1 ) * ( Y1 + Y2 + Y4 + Y5 + Y6 + Y7 + Y8 + Y9 ) + Y3 ;
  16) V < ( R4 - R1 + 1 ) * ( Y1 + Y2 + Y3 + Y5 + Y6 + Y7 + Y8 + Y9 ) + Y4 ;
  17) V < ( R5 - R1 + 1 ) * ( Y1 + Y2 + Y3 + Y4 + Y6 + Y7 + Y8 + Y9 ) + Y5 ;
  18) V < ( R6 - R1 + 1 ) * ( Y1 + Y2 + Y3 + Y4 + Y5 + Y7 + Y8 + Y9 ) + Y6 ;
  19) V < ( R7 - R1 + 1 ) * ( Y1 + Y2 + Y3 + Y4 + Y5 + Y6 + Y8 + Y9 ) + Y7 ;
  20) V < ( R8 - R1 + 1 ) * ( Y1 + Y2 + Y3 + Y4 + Y5 + Y6 + Y7 + Y9 ) + Y8 ;
  21) V < ( R9 - R1 + 1 ) * ( Y1 + Y2 + Y3 + Y4 + Y5 + Y6 + Y7 + Y8 ) + Y9 ;
  22) R1 > R0 ;
```

23) Y1 + Y2 + Y3 + Y4 + Y5 + Y6 + Y7 + Y8 + Y9 = 1.0 ;
END

```
SLB   X2   .000000
SUB   X2   .700000
SLB   X3   .000000
SUB   X3   .300000
SLB   X4   .000000
SUB   X4   .500000
SLB   X5   .000000
SUB   X5   .200000
SLB   X6   .000000
SUB   X6   .400000
SLB   X7   .000000
SUB   X7   .700000
SLB   X8   .000000
SUB   X8   .400000
SLB   X9   .000000
SUB   X9   .300000
SLB   Y1   .000000
SLB   Y3   .000000
SLB   Y4   .000000
SLB   Y5   .000000
SLB   Y6   .000000
SLB   Y7   .000000
SLB   Y8   .000000
SLB   Y9   .000000
SLB   Y2   .000000
```

LEAVE

Appendix B. *Model 1 GINO Output for Sample Network*

Budget = 0.00

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE
3) .438062

VARIABLE	VALUE	REDUCED COST
R1	.438062	.000000
X2	.000000	.103188
X3	.000000	.577893
X4	.000000	.000000
X5	.000000	.598079
X6	.000000	.485089
X7	.000000	.063989
X8	.000000	.176609
X9	.000000	.477093
V	.868331	.000000
R2	.382320	.000000
R3	.370560	.000000
R5	.300000	.000000
R6	.382320	.000000
R7	.370560	.000000
R8	.197232	.000000
R4	.197232	.000000
R9	.300000	.000000
R0	.438062	.000000
Y1	.000000	.000000
Y3	.000000	.000000
Y4	.453355	.000000
Y5	.046645	.000000
Y6	.000000	.000000
Y7	.000000	.000000
Y8	.453355	.000000
Y9	.046645	.000000
Y2	.000000	.000000

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	1.000000
2)	.000000	.963322

4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.131669	.000000
14)	.075926	.000000
15)	.064166	.000000
16)	.000020	.000000
17)	.000046	.000000
18)	.075926	.000000
19)	.064166	.000000
20)	.000020	.000000
21)	.000046	.000000
22)	.000000	.000000
23)	.000000	.000000

Budget = 0.10

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .664495

VARIABLE	VALUE	REDUCED COST
R1	.533997	.000000
X2	.000000	.161542
X3	.000000	1.732736
X4	.197666	.000000
X5	.000000	1.728734
X6	.000000	1.449293
X7	.003889	.000000
X8	.000000	.056600
X9	.000000	1.324019
V	.664495	.000000
R2	.487859	.000000
R3	.477205	.000000
R5	.418600	.000000
R6	.487859	.000000
R7	.477205	.000000
R8	.198482	.000000
R4	.198482	.000000
R9	.418600	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.335515
Y4	.000000	.335515
Y5	.000000	.335515
Y6	.000000	.335515
Y7	.000000	.335515
Y8	.000000	.000000
Y9	.000000	.335515
Y2	.000000	.335515

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	3.161876
2)	.000000	3.041160
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	1.000000
10)	.000000	1.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.335505	.000000
14)	.289367	.000000
15)	.278713	.000000
16)	-.000010	.000000
17)	.220108	.000000
18)	.289367	.000000
19)	.278713	.000000
20)	-.000010	1.000000
21)	.220108	.000000
22)	.095935	.000000
23)	.000000	1.000000
24)	.000000	-.335515
25)	-.000003	-4.161876

Budget = 0.10 (with hardening)

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .534395

VARIABLE	VALUE	REDUCED COST
R1	.534395	.000000
X2	.000000	.135041
X3	.000000	.594424
X4	.200000	.000000
X5	.000000	.627664
X6	.000000	.501015
X7	.000000	.102561
X8	.000000	.016055
X9	.000000	.510904
V	.831388	.000000
R2	.488208	.000000
R3	.478464	.000000
R5	.420000	.000000
R6	.488208	.000000
R7	.478464	.000000
R8	.197232	.000000
R4	.197232	.000000
R9	.420000	.000000
R0	.438062	.000000
Y1	.000000	.000000
Y3	.000000	.000000
Y4	.500000	.000000
Y5	.000000	.000000
Y6	.000000	.000000
Y7	.000000	.000000
Y8	.500000	.000000
Y9	.000000	.000000
Y2	.000000	.000000

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	1.000000
2)	.000000	.963322
4)	.000000	.000000

5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.168612	.000000
14)	.122425	.000000
15)	.112681	.000000
16)	.000030	.000000
17)	.054217	.000000
18)	.122425	.000000
19)	.112681	.000000
20)	.000030	.000000
21)	.054217	.000000
22)	.096332	.000000
23)	.000000	.000000

Budget - 0.20 (with hardening)

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .636145

VARIABLE	VALUE	REDUCED COST
R1	.636145	.000000
X2	.000000	.204810
X3	.000000	.696176
X4	.310000	.000000
X5	.000000	.755245
X6	.000000	.590087
X7	.000000	.179428
X8	.075000	.000000
X9	.000000	.630908
V	.780543	.000000
R2	.600052	.000000
R3	.592438	.000000
R5	.546750	.000000
R6	.600052	.000000
R7	.592438	.000000
R8	.197232	.000000
R4	.197232	.000000
R9	.546750	.000000
R0	.438062	.000000
Y1	.000000	.000000
Y3	.000000	.000000
Y4	.500000	.000000
Y5	.000000	.000000
Y6	.000000	.000000
Y7	.000000	.000000
Y8	.500000	.000000
Y9	.000000	.000000
Y2	.000000	.000000

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	1.000000
2)	.000000	1.083737
4)	.000000	.000000

5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.219457	.000000
14)	.183364	.000000
15)	.175749	.000000
16)	.000000	.000000
17)	.130061	.000000
18)	.183364	.000000
19)	.175749	.000000
20)	.000000	.000000
21)	.130061	.000000
22)	.198083	.000000
23)	.000000	.000000

Budget = 0.2

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS:

OBJECTIVE FUNCTION VALUE

3) .561110

VARIABLE	VALUE	REDUCED COST
R1	.636135	.000000
X2	.000000	.101504
X3	.000000	1.109460
X4	.309987	.000000
X5	.000000	1.106507
X6	.000000	.924490
X7	.000058	.000000
X8	.074981	-.000014
X9	.000000	.848513
V	.561110	.000000
R2	.600042	.000000
R3	.592416	.000000
R5	.546726	.000000
R6	.600042	.000000
R7	.592416	.000000
R8	.197251	.000000
R4	.197251	.000000
R9	.546726	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.438884
Y4	.000000	.438884
Y5	.000000	.438884
Y6	.000000	.438884
Y7	.000000	.438884
Y8	.000000	.000000
Y9	.000000	.438884
Y2	.000000	.438884

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	1.791707
2)	.000000	1.941641
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	1.000000
10)	.000000	1.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.438890	.000000
14)	.402797	.000000
15)	.395171	.000000
16)	.000006	.000000
17)	.349481	.000000
18)	.402797	.000000
19)	.395171	.000000
20)	.000006	1.000000
21)	.349481	.000000
22)	.198073	.000000
23)	.000000	1.000000
24)	.000000	-.438884
25)	-.000010	-2.791707

Budget = 0.30 (with hardening)

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .751209

VARIABLE	VALUE	REDUCED COST
R1	.751209	.000000
X2	.000000	.282995
X3	.000000	.809578
X4	.410000	.000000
X5	.000000	.897618
X6	.000000	.689387
X7	.000000	.265640
X8	.158333	.000000
X9	.000000	.764950
V	.723008	.000000
R2	.726530	.000000
R3	.721323	.000000
R5	.690083	.000000
R6	.726530	.000000
R7	.721323	.000000
R8	.197232	.000000
R4	.197232	.000000
R9	.690083	.000000
R0	.438062	.000000
Y1	.000000	.000000
Y3	.000000	.000000
Y4	.500000	.000000
Y5	.000000	.000000
Y6	.000000	.000000
Y7	.000000	.000000
Y8	.500000	.000000
Y9	.000000	.000000
Y2	.000000	.000000

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	1.000000
2)	.000000	1.217531
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.276992	.000000
14)	.252313	.000000
15)	.247106	.000000
16)	.000004	.000000
17)	.215866	.000000
18)	.252313	.000000
19)	.247106	.000000
20)	.000004	.000000
21)	.215866	.000000
22)	.313146	.000000
23)	.000000	.000000

Budget = 0.3

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS:

OBJECTIVE FUNCTION VALUE

3) .446035

VARIABLE	VALUE	REDUCED COST
R1	.751199	.000000
X2	.000000	.077013
X3	.000000	.841855
X4	.410061	.000116
X5	.000000	.839612
X6	.000000	.701486
X7	.000037	.000000
X8	.158264	.000000
X9	.000000	.643852
V	.446035	.000000
R2	.726519	.000000
R3	.721308	.000000
R5	.690067	.000000
R6	.726519	.000000
R7	.721308	.000000
R8	.197244	.000000
R4	.197244	.000000
R9	.690067	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.553955
Y4	.000000	.553955
Y5	.000000	.553955
Y6	.000000	.553955
Y7	.000000	.553955
Y8	.000000	.000000
Y9	.000000	.553955
Y2	.000000	.553955

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	1.209998
2)	.000000	1.473287
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000

7)	.000000	.000000
8)	.000000	.000000
9)	.000000	1.000000
10)	.000000	1.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.553965	.000000
14)	.529285	.000000
15)	.524074	.000000
16)	.000010	.000000
17)	.492832	.000000
18)	.529285	.000000
19)	.524074	.000000
20)	.000010	1.000000
21)	.492832	.000000
22)	.313137	.000000
23)	.000000	1.000000
24)	.000000	-.553955
25)	-.000010	-2.209998

Budget = 0.4 (with hardening)

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .879585

VARIABLE	VALUE	REDUCED COST
R1	.879585	.000000
X2	.000000	.361568
X3	.000000	.915899
X4	.500000	-.013380
X5	.000000	1.033376
X6	.000000	.782860
X7	.000000	.353168
X8	.250000	.000000
X9	.000000	.894299
V	.658711	.000000
R2	.867640	.000000
R3	.865120	.000000
R5	.850000	.000000
R6	.867640	.000000
R7	.865120	.000000
R8	.197232	.000000
R4	.197232	.000000
R9	.850000	.000000
R0	.438062	.000000
Y1	.000000	.000000
Y3	.000000	.000000
Y4	.500000	.000000
Y5	.000000	.000000
Y6	.000000	.000000
Y7	.000000	.000000
Y8	.500000	.000000
Y9	.000000	.000000
Y2	.000000	.000000

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	1.000000
2)	.000000	1.337947
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.341289	.000000
14)	.329345	.000000
15)	.326825	.000000
16)	.000113	.000000
17)	.311705	.000000
18)	.329345	.000000
19)	.326825	.000000
20)	.000113	.000000
21)	.311705	.000000
22)	.441522	.000000
23)	.000000	.000000

Budget = 0.4

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .317666

VARIABLE	VALUE	REDUCED COST
R1	.879583	.000000
X2	.000000	.063651
X3	.000000	.695848
X4	.500000	-.012175
X5	.000000	.693993
X6	.000000	.579814
X7	.000022	.000000
X8	.249997	.000000
X9	.000000	.532187
V	.317666	.000000
R2	.867638	.000000
R3	.865117	.000000
R5	.849997	.000000
R6	.867638	.000000
R7	.865117	.000000
R8	.197239	.000000
R4	.197239	.000000
R9	.849997	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.682344
Y4	.000000	.682344
Y5	.000000	.682344
Y6	.000000	.682344
Y7	.000000	.682344
Y8	.000000	.000000
Y9	.000000	.682344
Y2	.000000	.682344

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	.910173
2)	-.000004	1.217753
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	1.000000
10)	.000000	1.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.682334	.000000
14)	.670389	.000000
15)	.667868	.000000
16)	-.000010	.000000
17)	.652748	.000000
18)	.670389	.000000
19)	.667868	.000000
20)	-.000010	1.000000
21)	.652748	.000000
22)	.441521	.000000
23)	.000000	1.000000
24)	.000000	-.682344
25)	-.000002	-1.910173

Budget = 0.5 (with hardening)

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) 1.000000

VARIABLE	VALUE	REDUCED COST
R1	1.000000	.000000
X2	.000000	.000000
X3	.000000	.000000
X4	.500000	-.802768
X5	.000000	.000000
X6	.000000	.000000
X7	.000000	.000000
X8	.400000	-.802768
X9	.000000	.000000
V	.598616	.000000
R2	1.000000	.000000
R3	1.000000	.000000
R5	1.000000	.000000
R6	1.000000	.000000
R7	1.000000	.000000
R8	.197232	.000000
R4	.197232	.000000
R9	1.000000	.000000
R0	.438062	.000000
Y1	.000000	.000000
Y3	.000000	.000000
Y4	.500000	.000000
Y5	.000000	.000000
Y6	.000000	.000000
Y7	.000000	.000000
Y8	.500000	.000000
Y9	.000000	.000000
Y2	.000000	.000000

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	1.000000
2)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.401384	.000000
14)	.401384	.000000
15)	.401384	.000000
16)	.000000	.000000
17)	.401384	.000000
18)	.401384	.000000
19)	.401384	.000000
20)	.000000	.000000
21)	.401384	.000000
22)	.561938	.000000
23)	.000000	.000000

Budget = 0.5

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .207947

VARIABLE	VALUE	REDUCED COST
R1	1.000000	.000000
X2	.000000	.063840
X3	.000000	.596960
X4	.500000	-.107147
X5	.000000	.597240
X6	.000000	.514080
X7	.033333	.000000
X8	.400000	.000000
X9	.000000	.452960
V	.207947	.000000
R2	1.000000	.000000
R3	1.000000	.000000
R5	1.000000	.000000
R6	1.000000	.000000
R7	1.000000	.000000
R8	.207947	.000000
R4	.207947	.000000
R9	1.000000	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.792053
Y4	.000000	.792053
Y5	.000000	.792053
Y6	.000000	.792053
Y7	.000000	.792053
Y8	.000000	.000000
Y9	.000000	.792053
Y2	.000000	.792053

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	.811662
2)	.000000	1.071467
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	1.000000
10)	.000000	1.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.792053	.000000
14)	.792053	.000000
15)	.792053	.000000
16)	.000000	.000000
17)	.792053	.000000
18)	.792053	.000000
19)	.792053	.000000
20)	.000000	1.000000
21)	.792053	.000000
22)	.561938	.000000
23)	.000000	1.000000
24)	.000000	-.792053
25)	.000000	-1.811663

Appendix C. *Model 2 GINO Output for Sample Network*

Budget = 0.00

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .759170

VARIABLE	VALUE	REDUCED COST
R1	.438062	.000000
X2	.000000	.016800
X3	.000000	.183680
X4	.000000	.642381
X5	.000000	.183190
X6	.000000	.153048
X7	.000000	.000000
X8	.000000	.594248
X9	.000000	.140480
V	.759170	.000000
R2	.382320	.000000
R3	.370560	.000000
R5	.300000	.000000
R6	.382320	.000000
R7	.370560	.000000
R8	.197232	.000000
R4	.197232	.000000
R9	.300000	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.240830
Y4	.000000	.240830
Y5	.000000	.240830
Y6	.000000	.240830
Y7	.000000	.240830
Y8	.000000	.000000
Y9	.000000	.240830
Y2	.000000	.240830

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-1.000000
2)	.000000	.321440

4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	1.000000
10)	.000000	1.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.240830	.000000
14)	.185088	.000000
15)	.173328	.000000
16)	.000000	.000000
17)	.102768	.000000
18)	.185088	.000000
19)	.173328	.000000
20)	.000000	1.000000
21)	.102768	.000000
22)	.000000	.000000
23)	.000000	1.000000
24)	.000000	-.240830

Budget = 0.10

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .790000

VARIABLE	VALUE	REDUCED COST
R1	.510008	.000000
X2	.000000	.000000
X3	.000000	.000000
X4	.000000	.239993
X5	.008980	.000000
X6	.000000	.000000
X7	.309386	.000000
X8	.000000	.199994
X9	.000000	.000000
V	.790000	.000000
R2	.469093	.000000
R3	.371352	.000000
R5	.300000	.000000
R6	.469093	.000000
R7	.371352	.000000
R8	.300011	.000000
R4	.300011	.000000
R9	.300000	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.146998
Y4	.000000	.146998
Y5	.000000	.146998
Y6	.000000	.146998
Y7	.000000	.146998
Y8	.000000	.000000
Y9	.000000	.083995
Y2	.000000	.146998

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-1.000000
2)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.300000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.700000
10)	.000000	.700000
11)	.000000	.300000
12)	.000000	.000000
13)	.210000	.000000
14)	.169085	.000000
15)	.071344	.000000
16)	.000003	.000000
17)	-.000008	.000000
18)	.169085	.000000
19)	.071344	.000000
20)	.000003	.700000
21)	-.000008	.300000
22)	.071946	.000000
23)	.000000	.936998
24)	.000000	-.146998

Budget = 0.20

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .790000

VARIABLE	VALUE	REDUCED COST
R1	.510006	.000000
X2	.000000	.000000
X3	.000000	.000000
X4	.000000	.239995
X5	.200000	.000000
X6	.000000	.000000
X7	.132824	.000000
X8	.000000	.199996
X9	.000232	.000000
V	.790000	.000000
R2	.448508	.000000
R3	.388229	.000000
R5	.300000	.000000
R6	.448508	.000000
R7	.388229	.000000
R8	.300008	.000000
R4	.300008	.000000
R9	.300000	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.146998
Y4	.000000	.146998
Y5	.000000	.146998
Y6	.000000	.146998
Y7	.000000	.146998
Y8	.000000	.000000
Y9	.000000	.083996
Y2	.000000	.146998

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-1.000000
2)	-.000010	.000000
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.300000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.700000
10)	.000000	.700000
11)	.000000	.300000
12)	.000000	.000000
13)	.210000	.000000
14)	.148502	.000000
15)	.088224	.000000
16)	.000003	.000000
17)	-.000006	.000000
18)	.148502	.000000
19)	.088224	.000000
20)	.000003	.700000
21)	-.000006	.300000
22)	.071943	.000000
23)	.000000	.936998
24)	.000000	-.146998

Budget = 0.3

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .790000

VARIABLE	VALUE	REDUCED COST
R1	.510001	.000000
X2	.000000	.000000
X3	.181128	.000000
X4	.000000	.239999
X5	.200000	.000000
X6	.000000	.000000
X7	.044034	.000000
X8	.000000	.199999
X9	.000000	.000000
V	.790000	.000000
R2	.448538	.000000
R3	.388200	.000000
R5	.300000	.000000
R6	.448538	.000000
R7	.388200	.000000
R8	.300001	.000000
R4	.300001	.000000
R9	.300000	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.147000
Y4	.000000	.147000
Y5	.000000	.147000
Y6	.000000	.147000
Y7	.000000	.147000
Y8	.000000	.000000
Y9	.000000	.084000
Y2	.000000	.147000

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-1.000000
2)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.300000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.700000
10)	.000000	.700000
11)	.000000	.300000
12)	.000000	.000000
13)	.210000	.000000
14)	.148537	.000000
15)	.088199	.000000
16)	.000000	.000000
17)	-.000001	.000000
18)	.148537	.000000
19)	.088199	.000000
20)	.000000	.700000
21)	-.000001	.300000
22)	.071938	.000000
23)	.000000	.937000
24)	.000000	-.147000

Budget = 0.4

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .789995

VARIABLE	VALUE	REDUCED COST
R1	.510012	.000000
X2	.000000	.000000
X3	.300000	.000000
X4	.000000	.239990
X5	.153385	.000000
X6	.112154	.000000
X7	.000000	.000000
X8	.000000	.199991
X9	.000000	.000000
V	.789995	.000000
R2	.440148	.000000
R3	.399807	.000000
R5	.300000	.000000
R6	.440148	.000000
R7	.399807	.000000
R8	.300017	.000000
R4	.300017	.000000
R9	.300000	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.146996
Y4	.000000	.146996
Y5	.000000	.146996
Y6	.000000	.146996
Y7	.000000	.146996
Y8	.000000	.000000
Y9	.000000	.083993
Y2	.000000	.146996

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-1.000000
2)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.300000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.700000
10)	.000000	.700000
11)	.000000	.300000
12)	.000000	.000000
13)	.210005	.000000
14)	.140140	.000000
15)	.099799	.000000
16)	.000010	.000000
17)	-.000008	.000000
18)	.140140	.000000
19)	.099799	.000000
20)	.000010	.700000
21)	-.000008	.300000
22)	.071950	.000000
23)	.000000	.936996
24)	.000000	-.146996

Budget - 0.50

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .787966

VARIABLE	VALUE	REDUCED COST
R1	.517189	.000000
X2	.000000	.072320
X3	.300000	-.047389
X4	.000000	.026317
X5	.054766	.000000
X6	.400000	-.036063
X7	.000000	.072320
X8	.010312	.000000
X9	.000000	.031357
V	.787966	.000000
R2	.429881	.000000
R3	.429881	.000000
R5	.305156	.000000
R6	.429881	.000000
R7	.429881	.000000
R8	.305151	.000000
R4	.305151	.000000
R9	.305156	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.118514
Y4	.000000	.118514
Y5	.000000	.118514
Y6	.000000	.118514
Y7	.000000	.118514
Y8	.000000	.024994
Y9	.000000	.000000
Y2	.000000	.118514

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-1.000000
2)	.000000	-.113253
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.558945
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.441055
10)	.000000	.441055
11)	.000000	.558945
12)	.000000	.000000
13)	.212034	.000000
14)	.124727	.000000
15)	.124727	.000000
16)	-.000003	.000000
17)	.000002	.000000
18)	.124727	.000000
19)	.124727	.000000
20)	-.000003	.441055
21)	.000002	.558945
22)	.079126	.000000
23)	.000000	.906480
24)	.000000	-.118514

Budget = 0.75

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .763263

VARIABLE	VALUE	REDUCED COST
R1	.621565	.000000
X2	.000000	.038051
X3	.300000	-.043203
X4	.000000	.037928
X5	.200000	-.024337
X6	.400000	-.034249
X7	.000000	.038051
X8	.169663	.000000
X9	.054564	.000000
V	.763263	.000000
R2	.524086	.000000
R3	.524086	.000000
R5	.384831	.000000
R6	.524086	.000000
R7	.524086	.000000
R8	.384827	.000000
R4	.384827	.000000
R9	.384831	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.120196
Y4	.000000	.120196
Y5	.000000	.120196
Y6	.000000	.120196
Y7	.000000	.120196
Y8	.000000	.003656
Y9	.000000	.000000
Y2	.000000	.120196

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-1.000000
2)	-.000008	-.089538
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.507727
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.492273
10)	.000000	.492273
11)	.000000	.507727
12)	.000000	.000000
13)	.236737	.000000
14)	.139258	.000000
15)	.139258	.000000
16)	-.000001	.000000
17)	.000003	.000000
18)	.139258	.000000
19)	.139258	.000000
20)	-.000001	.492273
21)	.000003	.507727
22)	.183503	.000000
23)	.000000	.883460
24)	.000000	-.120196

Budget = 1.00

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .750331

VARIABLE	VALUE	REDUCED COST
R1	.731689	.000000
X2	.000000	.008503
X3	.300000	-.005979
X4	.000000	.009185
X5	.200000	-.001936
X6	.400000	-.004581
X7	.000000	.008503
X8	.364024	.000000
X9	.245122	.000000
V	.750331	.000000
R2	.628881	.000000
R3	.628881	.000000
R5	.482012	.000000
R6	.628881	.000000
R7	.628881	.000000
R8	.482012	.000000
R4	.482012	.000000
R9	.482012	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.125139
Y4	.000000	.125139
Y5	.000000	.125139
Y6	.000000	.125139
Y7	.000000	.125139
Y8	.000000	.000602
Y9	.000000	.000000
Y2	.000000	.125139

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-1.000000
2)	.000000	-.013984
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.501206
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.498794
10)	.000000	.498794
11)	.000000	.501206
12)	.000000	.000000
13)	.249669	.000000
14)	.146861	.000000
15)	.146861	.000000
16)	-.000008	.000000
17)	-.000008	.000000
18)	.146861	.000000
19)	.146861	.000000
20)	-.000008	.498794
21)	-.000008	.501206
22)	.293626	.000000
23)	.000000	.875463
24)	.000000	-.125139

Budget = 1.25

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .958983

VARIABLE	VALUE	REDUCED COST
R1	.998163	.000000
X2	.700000	-.199129
X3	.000000	.457142
X4	.500000	-.048511
X5	.157146	.000000
X6	.400000	-.003211
X7	.000000	.195918
X8	.357139	.000000
X9	.300000	-.042917
V	.958983	.000000
R2	.965754	.000000
R3	.998163	.000000
R5	.957139	.000000
R6	.965754	.000000
R7	.998163	.000000
R8	.957146	.000000
R4	.957146	.000000
R9	.957139	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.023187
Y4	.000000	.023187
Y5	.000000	.023187
Y6	.000000	.023187
Y7	.000000	.023187
Y8	.000000	.000000
Y9	.000000	.005354
Y2	.000000	.023187

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-1.000000
2)	.000000	.653060
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.434690
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.565310
10)	.000000	.565310
11)	.000000	.434690
12)	.000000	.000000
13)	.041017	.000000
14)	.008608	.000000
15)	.041017	.000000
16)	.000000	.000000
17)	-.000007	.000000
18)	.008608	.000000
19)	.041017	.000000
20)	.000000	.565310
21)	-.000007	.434690
22)	.560101	.000000
23)	.000000	.982167
24)	.000000	-.023187

Budget = 1.50

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) 1.000000

VARIABLE	VALUE	REDUCED COST
R1	1.000000	.000000
X2	.700000	.000000
X3	.271429	.000000
X4	.500000	-.708571
X5	.200000	.000000
X6	.400000	.000000
X7	.000000	.000000
X8	.400000	-.708571
X9	.300000	.000000
V	1.000000	.000000
R2	1.000000	.000000
R3	1.000000	.000000
R5	1.000000	.000000
R6	1.000000	.000000
R7	1.000000	.000000
R8	1.000000	.000000
R4	1.000000	.000000
R9	1.000000	.000000
R0	.438062	.000000
Y1	1.000000	.000000
Y3	.000000	.000000
Y4	.000000	.000000
Y5	.000000	.000000
Y6	.000000	.000000
Y7	.000000	.000000
Y8	.000000	.000000
Y9	.000000	.000000
Y2	.000000	.000000

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-1.000000
2)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.000000
7)	.000000	1.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.000000
11)	.000000	.000000
12)	.000000	.000000
13)	.000000	.000000
14)	.000000	.000000
15)	.000000	.000000
16)	.000000	.000000
17)	.000000	.000000
18)	.000000	1.000000
19)	.000000	.000000
20)	.000000	.000000
21)	.000000	.000000
22)	.561938	.000000
23)	.000000	1.000000
24)	.000000	.000000

Appendix D. *Model 3 GINO Output for Sample Network*

Budget = 0.00
SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .868368

VARIABLE	VALUE	REDUCED COST
R1	.438062	.000000
X2	.000000	.096742
X3	.000000	.050208
X4	.000000	.024214
X5	.000000	.089854
X6	.000000	.048371
X7	.000000	.117152
X8	.000000	.020179
X9	.000000	.102690
V	.868368	.000000
R2	.382320	.000000
R3	.370560	.000000
R5	.300000	.000000
R6	.382320	.000000
R7	.370560	.000000
R8	.197232	.000000
R4	.197232	.000000
R9	.300000	.000000
R0	.438062	.000000
Y1	.000000	.043877
Y3	.000000	.043877
Y4	.453424	.000000
Y5	.046576	.000000
Y6	.000000	.043877
Y7	.000000	.043877
Y8	.453424	.000000
Y9	.046576	.000000
Y2	.000000	.043877

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.805175
2)	.000000	.000000
4)	.000000	.000000

5)	.000000	.000000
6)	.000000	.606012
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.099582
10)	.000000	.199163
11)	.000000	.303006
12)	.000000	.000000
13)	.131632	.000000
14)	.075890	.000000
15)	.064130	.000000
16)	.000000	.182192
17)	.000000	.317808
18)	.075890	.000000
19)	.064130	.000000
20)	.000000	.182192
21)	.000000	.317808
22)	.000000	.000000
23)	.000000	.868368

Budget = 0.10

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .864073

VARIABLE	VALUE	REDUCED COST
R1	.478673	.000000
X2	.000000	.056196
X3	.069589	.000000
X4	.000000	.017892
X5	.000000	.028490
X6	.000000	.003138
X7	.000000	.082962
X8	.085479	.000000
X9	.000000	.044445
V	.864073	.000000
R2	.427718	.000000
R3	.408992	.000000
R5	.342740	.000000
R6	.427718	.000000
R7	.408992	.000000
R8	.206819	.000000
R4	.206819	.000000
R9	.342740	.000000
R0	.438062	.000000
Y1	.000000	.061583
Y3	.000000	.061583
Y4	.500000	.000000
Y5	.000000	.000000
Y6	.000000	.061583
Y7	.000000	.061583
Y8	.500000	.000000
Y9	.000000	.048818
Y2	.000000	.061583

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.773472
2)	.000000	-.055467
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.546943
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.113264
10)	.000000	.226528
11)	.000000	.093907
12)	.000000	.000000
13)	.135927	.000000
14)	.084971	.000000
15)	.066246	.000000
16)	.000000	.226528
17)	-.000006	.453036
18)	.084971	.000000
19)	.066246	.000000
20)	.000000	.226528
21)	-.000006	.093907
22)	.040611	.000000
23)	.000000	.864069

Budget = 0.2

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .858754

VARIABLE	VALUE	REDUCED COST
R1	.504205	.000000
X2	.000000	.048671
X3	.177788	.000000
X4	.000000	.018859
X5	.000000	.030914
X6	.000000	.001467
X7	.000000	.088842
X8	.125914	.000000
X9	.000000	.046220
V	.858754	.000000
R2	.456901	.000000
R3	.427171	.000000
R5	.362957	.000000
R6	.456901	.000000
R7	.427171	.000000
R8	.221724	.000000
R4	.221724	.000000
R9	.362957	.000000
R0	.438062	.000000
Y1	.000000	.065363
Y3	.000000	.065363
Y4	.499976	.000000
Y5	.000000	.054845
Y6	.000000	.065363
Y7	.000000	.065363
Y8	.499976	.000000
Y9	.000049	.000000
Y2	.000000	.065363

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.768599
2)	.000000	-.050820
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.537199
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.115700
10)	.000000	.231401
11)	.000000	.462732
12)	.000000	.000000
13)	.141246	.000000
14)	.093942	.000000
15)	.064212	.000000
16)	-.000001	.231389
17)	-.000002	.074467
18)	.093942	.000000
19)	.064212	.000000
20)	-.000001	.231389
21)	.000005	.462755
22)	.066142	.000000
23)	.000000	.858756

Budget = 0.30

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .853901

VARIABLE	VALUE	REDUCED COST
R1	.528899	.000000
X2	.000000	.041726
X3	.286638	.000000
X4	.000000	.019370
X5	.000000	.032547
X6	.000000	.000045
X7	.000000	.092621
X8	.165589	.000000
X9	.000000	.047110
V	.853901	.000000
R2	.485100	.000000
R3	.445009	.000000
R5	.382795	.000000
R6	.485100	.000000
R7	.445009	.000000
R8	.236719	.000000
R4	.236719	.000000
R9	.382795	.000000
R0	.438062	.000000
Y1	.000000	.069038
Y3	.000000	.069038
Y4	.499976	.000000
Y5	.000000	.061016
Y6	.000000	.069038
Y7	.000000	.069038
Y8	.499976	.000000
Y9	.000049	.000000
Y2	.000000	.069038

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.763704
2)	.000000	-.046261
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.527407
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.118148
10)	.000000	.236296
11)	.000000	.472500
12)	.000000	.000000
13)	.146099	.000000
14)	.102299	.000000
15)	.062209	.000000
16)	.000002	.236285
17)	-.000006	.054907
18)	.102299	.000000
19)	.062209	.000000
20)	.000002	.236285
21)	.000002	.472523
22)	.090837	.000000
23)	.000000	.853902

Budget = 0.4

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .849512

VARIABLE	VALUE	REDUCED COST
R1	.552718	.000000
X2	.000000	.046855
X3	.300000	-.001083
X4	.000000	.019390
X5	.000000	.033569
X6	.112220	.000000
X7	.000000	.081117
X8	.204446	.000000
X9	.000000	.047294
V	.849512	.000000
R2	.502650	.000000
R3	.473749	.000000
R5	.402223	.000000
R6	.502650	.000000
R7	.473749	.000000
R8	.251757	.000000
R4	.251757	.000000
R9	.402223	.000000
R0	.438062	.000000
Y1	.000000	.072549
Y3	.000000	.072549
Y4	.499976	.000000
Y5	.000000	.067159
Y6	.000000	.072549
Y7	.000000	.072549
Y8	.499976	.000000
Y9	.000049	.000000
Y2	.000000	.072549

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.758930
2)	.000000	-.041670
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.517860
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.120535
10)	.000000	.241070
11)	.000000	.482047
12)	.000000	.000000
13)	.150488	.000000
14)	.100420	.000000
15)	.071519	.000000
16)	.000000	.241058
17)	-.000007	.035813
18)	.100420	.000000
19)	.071519	.000000
20)	.000000	.241058
21)	.000000	.482070
22)	.114655	.000000
23)	.000000	.849512

Budget = 0.50

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .845562

VARIABLE	VALUE	REDUCED COST
R1	.575728	.000000
X2	.000000	.051545
X3	.300000	-.002203
X4	.000000	.019069
X5	.000000	.033654
X6	.240766	.000000
X7	.000000	.068520
X8	.242567	.000000
X9	.000000	.046457
V	.845562	.000000
R2	.518508	.000000
R3	.503027	.000000
R5	.421284	.000000
R6	.518508	.000000
R7	.503027	.000000
R8	.266874	.000000
R4	.266874	.000000
R9	.421284	.000000
R0	.438062	.000000
Y1	.000000	.075974
Y3	.000000	.075974
Y4	.499993	.000000
Y5	.000041	.000000
Y6	.000000	.075974
Y7	.000000	.075974
Y8	.499966	.000000
Y9	.000000	.073486
Y2	.000000	.075974

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.754004
2)	.000000	-.037310
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.508007
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.123001
10)	.000000	.245996
11)	.000000	.016111
12)	.000000	.000000
13)	.154438	.000000
14)	.097218	.000000
15)	.081737	.000000
16)	.000009	.245986
17)	.000000	.491917
18)	.097218	.000000
19)	.081737	.000000
20)	.000001	.245986
21)	-.000006	.016111
22)	.137666	.000000
23)	.000000	.845564

Budget = 0.75

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .835106

VARIABLE	VALUE	REDUCED COST
R1	.650091	.000000
X2	.000000	.027900
X3	.300000	-.016469
X4	.000000	.027058
X5	.097208	.000000
X6	.400000	-.012397
X7	.000000	.027900
X8	.370396	.000000
X9	.000000	.013250
V	.835106	.000000
R2	.582194	.000000
R3	.582194	.000000
R5	.485198	.000000
R6	.582194	.000000
R7	.582194	.000000
R8	.320303	.000000
R4	.320303	.000000
R9	.485198	.000000
R0	.438062	.000000
Y1	.000000	.092211
Y3	.000000	.092211
Y4	.500000	.000000
Y5	.000000	.092211
Y6	.000000	.092211
Y7	.000000	.092211
Y8	.500000	.000000
Y9	.000000	.019529
Y2	.000000	.092211

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.720392
2)	-.000004	-.040721
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.440783
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.139804
10)	.000000	.279608
11)	.000000	.440783
12)	.000000	.000000
13)	.164894	.000000
14)	.096997	.000000
15)	.096997	.000000
16)	.000000	.279608
17)	.000001	.000000
18)	.096997	.000000
19)	.096997	.000000
20)	.000000	.279608
21)	.000001	.440783
22)	.212028	.000000
23)	.000000	.835107

Budget = 1.00

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .900307

VARIABLE	VALUE	REDUCED COST
R1	.970387	.000000
X2	.000000	.056469
X3	.117072	.000000
X4	.500000	-.006136
X5	.107038	.000007
X6	.000000	.151406
X7	.700000	-.074437
X8	.270698	.000000
X9	.300000	-.007010
V	.900307	.000000
R2	.966526	.000000
R3	.891809	.000000
R5	.870698	.000000
R6	.966526	.000000
R7	.891809	.000000
R8	.770982	.000000
R4	.770982	.000000
R9	.870698	.000000
R0	.438062	.000000
Y1	.000000	.068318
Y3	.000000	.068318
Y4	.500000	.000000
Y5	.000000	.068318
Y6	.000000	.068318
Y7	.000000	.068318
Y8	.500000	.000000
Y9	.000000	.036937
Y2	.000000	.068318

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.657392
2)	.000000	.273714
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.314783
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.171304
10)	.000000	.342608
11)	.000000	.314783
12)	.000000	.000000
13)	.099693	.000000
14)	.095831	.000000
15)	.021114	.000000
16)	-.000010	.342608
17)	.000003	.000000
18)	.095831	.000000
19)	.021114	.000000
20)	-.000010	.342608
21)	.000003	.314783
22)	.532325	.000000
23)	.000000	.900301

Budget = 1.25

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .987214

VARIABLE	VALUE	REDUCED COST
R1	.999660	.000000
X2	.000000	.118937
X3	.268396	.000000
X4	.500000	-.038606
X5	.200000	-.013216
X6	.000000	.248050
X7	.700000	-.158357
X8	.386871	.000000
X9	.300000	-.055123
V	.987214	.000000
R2	.999585	.000000
R3	.989234	.000000
R5	.986871	.000000
R6	.999585	.000000
R7	.989234	.000000
R8	.974085	.000000
R4	.974085	.000000
R9	.986871	.000000
R0	.438062	.000000
Y1	.000000	.009362
Y3	.000000	.009362
Y4	.500000	.000000
Y5	.000000	.009362
Y6	.000000	.009362
Y7	.000000	.009362
Y8	.500000	.000000
Y9	.000000	.005936
Y2	.000000	.009362

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.633935
2)	.000000	.419069
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.267870
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.183032
10)	.000000	.366065
11)	.000000	.267870
12)	.000000	.000000
13)	.012786	.000000
14)	.012712	.000000
15)	.002361	.000000
16)	-.000001	.366065
17)	-.000002	.000000
18)	.012712	.000000
19)	.002361	.000000
20)	-.000001	.366065
21)	-.000002	.267870
22)	.561597	.000000
23)	.000000	.987212

Budget = 1.50

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .999998

VARIABLE	VALUE	REDUCED COST
R1	1.000000	.000000
X2	.000000	-.000002
X3	.299995	-.354998
X4	.500000	.000002
X5	.200000	-.499997
X6	.366673	.000000
X7	.700000	-.354997
X8	.400000	.000003
X9	.300000	-.499997
V	.999998	.000000
R2	1.000000	.000000
R3	1.000000	.000000
R5	1.000000	.000000
R6	1.000000	.000000
R7	1.000000	.000000
R8	.999996	.000000
R4	.999996	.000000
R9	1.000000	.000000
R0	.438062	.000000
Y1	.000000	.000000
Y3	.000000	.000000
Y4	.500000	-.000004
Y5	.000000	.000000
Y6	.000000	.000000
Y7	.000000	.000000
Y8	.500000	.000000
Y9	.000000	.000000
Y2	.000000	.000000

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.500000
2)	.000000	.000001
4)	.000000	.000000
5)	.000000	.000000

6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.000000
10)	.000000	.500000
11)	.000000	.000000
12)	.000000	.000000
13)	.000002	.000000
14)	.000002	.000000
15)	.000002	.000000
16)	.000000	1.000000
17)	.000002	.000000
18)	.000002	.000000
19)	.000002	.000000
20)	.000000	.000000
21)	.000002	.000000
22)	.561938	.000000
23)	.000000	.999996

Appendix E: *Model 4 GINO Output for Sample Network*

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .838465

VARIABLE	VALUE	REDUCED COST
R1	.623068	.000000
X2	.700000	.000000
X3	.300000	.000000
X4	.000000	.110065
X5	.000000	.000000
X6	.007132	.000000
X7	.252403	.000000
X8	.000000	.091721
X9	.000000	.000000
V	.838465	.000000
R2	.516542	.000000
R3	.537996	.000000
R5	.300000	.000000
R6	.516542	.000000
R7	.537996	.000000
R8	.461526	.000000
R4	.461526	.000000
R9	.300000	.000000
R0	.438062	.000000
Y1	.000000	.074554
Y3	.000000	.074554
Y4	.000000	.000000
Y5	.499992	.000000
Y6	.000000	.074554
Y7	.000000	.074554
Y8	.000000	.062124
Y9	.500008	.000000
Y2	.000000	.074554

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.769231
2)	.000000	.000000
4)	.000000	.000000

5)	.000000	.000000
6)	.000000	.230769
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.076947
10)	.000000	.538462
11)	.000000	.115383
12)	.000000	.000000
13)	.161535	.000000
14)	.055009	.000000
15)	.076462	.000000
16)	-.000007	.461515
17)	-.000002	.230769
18)	.055009	.000000
19)	.076462	.000000
20)	-.000007	.076947
21)	.000003	.230769
22)	.185006	.000000
23)	.000000	.838462

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .835117

VARIABLE	VALUE	REDUCED COST
R1	.649999	.000000
X2	.700000	.000000
X3	.202732	.000000
X4	.033722	.000000
X5	.000000	.000000
X6	.000000	.000000
X7	.437422	.000000
X8	.000000	.000000
X9	.000000	.000000
V	.835117	.000000
R2	.573643	.000000
R3	.548635	.000000
R5	.320233	.000000
R6	.573643	.000000
R7	.548635	.000000
R8	.485116	.000000
R4	.485116	.000000
R9	.320233	.000000
R0	.438062	.000000
Y1	.000000	.099320
Y3	.000000	.099320
Y4	.000000	.099320
Y5	.500000	.000000
Y6	.000000	.099320
Y7	.000000	.099320
Y8	.000000	.033757
Y9	.500000	.000000
Y2	.000000	.099320

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.584954
2)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.301184
7)	.000000	.000000

8)	.000000	.000000
9)	.000000	.397632
10)	.000000	.397632
11)	.000000	.150592
12)	.000000	.000000
13)	.164883	.000000
14)	.088528	.000000
15)	.063520	.000000
16)	.000000	.000000
17)	.000001	.301184
18)	.088528	.000000
19)	.063520	.000000
20)	.000000	.397632
21)	.000001	.301184
22)	-.000001	-.113862
23)	.000000	.835117

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .830507

VARIABLE	VALUE	REDUCED COST
R1	.700000	.000000
X2	.652261	.000000
X3	.001788	.000000
X4	.101689	.000000
X5	.000000	.000000
X6	.000000	.000000
X7	.700000	.000000
X8	.000000	.000000
X9	.060322	.000000
V	.830507	.000000
R2	.633776	.000000
R3	.583082	.000000
R5	.361013	.000000
R6	.633776	.000000
R7	.583082	.000000
R8	.530506	.000000
R4	.530506	.000000
R9	.361013	.000000
R0	.438062	.000000
Y1	.000000	.100858
Y3	.000000	.100858
Y4	.000000	.100858
Y5	.500000	.000000
Y6	.000000	.100858
Y7	.000000	.100858
Y8	.000000	.032224
Y9	.500000	.000000
Y2	.000000	.100858

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.633724
2)	.000000	.000000
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.297529
7)	.000000	.000000

8)	.000000	.000000
9)	.000000	.404941
10)	.000000	.404941
11)	.000000	.148765
12)	.000000	.000000
13)	.169493	.000000
14)	.103270	.000000
15)	.052575	.000000
16)	.000000	.000000
17)	.000000	.297529
18)	.103270	.000000
19)	.052575	.000000
20)	.000000	.404941
21)	.000000	.297529
22)	.000000	-.068747
23)	.000000	.830507

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS:

OBJECTIVE FUNCTION VALUE

3) .815050

VARIABLE	VALUE	REDUCED COST
R1	.749991	.000000
X2	.157445	.000000
X3	.000000	.038032
X4	.247446	.000000
X5	.000000	.009366
X6	.000000	.062813
X7	.700000	-.005522
X8	.022223	-.000106
X9	.151014	.000000
V	.815050	.000000
R2	.720005	.000000
R3	.565034	.000000
R5	.465078	.000000
R6	.720005	.000000
R7	.565034	.000000
R8	.532626	.000000
R4	.532626	.000000
R9	.465078	.000000
R0	.438062	.000000
Y1	.000000	.052957
Y3	.000000	.052957
Y4	.149166	.000000
Y5	.350834	.000000
Y6	.000000	.052957
Y7	.000000	.026879
Y8	.149166	.000000
Y9	.350834	.000000
Y2	.000000	.052957

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	-.470764
2)	.000000	.169180
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.241322
7)	.000000	.000000
8)	.000000	.140996

9)	.000000	.207290
10)	.000000	.414580
11)	.000000	.120661
12)	.000000	.000000
13)	.184950	.000000
14)	.154963	.000000
15)	-.000007	.000000
16)	.000008	.243631
17)	-.000006	.185871
18)	.154963	.000000
19)	-.000007	.140996
20)	.000008	.243631
21)	-.000006	.185871
22)	-.000009	-.326133
23)	.000000	.815051

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS:

OBJECTIVE FUNCTION VALUE

3) .820755

VARIABLE	VALUE	REDUCED COST
R1	.799990	.000000
X2	.000000	.123038
X3	.000000	.098875
X4	.377600	-.000030
X5	.000000	.110479
X6	.000000	.373122
X7	.700000	-.062353
X8	.131397	.000000
X9	.031945	.000000
V	.820755	.000000
R2	.788666	.000000
R3	.679621	.000000
R5	.641874	.000000
R6	.788666	.000000
R7	.679621	.000000
R8	.441510	.000000
R4	.441510	.000000
R9	.641874	.000000
R0	.438062	.000000
Y1	.000000	.179240
Y3	.000000	.179240
Y4	.500000	.000000
Y5	.000000	.179240
Y6	.000000	.179240
Y7	.000000	.179240
Y8	.500000	.000000
Y9	.000000	.179240
Y2	.000000	.179240

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	.847671
2)	.000000	.692450
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000

9)	.000000	.250000
10)	.000000	.500000
11)	.000000	.000000
12)	.000000	.000000
13)	.179245	.000000
14)	.167921	.000000
15)	.058875	.000000
16)	.000005	.500000
17)	.021129	.000000
18)	.167921	.000000
19)	.058875	.000000
20)	.000005	.500000
21)	.021129	.000000
22)	-.000010	-1.347671
23)	.000000	.820760

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS:
OBJECTIVE FUNCTION VALUE

3) .754704

VARIABLE	VALUE	REDUCED COST
R1	.849990	.000000
X2	.000000	.115702
X3	.000000	.252107
X4	.458485	-.000115
X5	.000000	.270208
X6	.000000	.377242
X7	.504531	.000000
X8	.198997	.000000
X9	.000000	.156719
V	.754704	.000000
R2	.839679	.000000
R3	.789431	.000000
R5	.765827	.000000
R6	.839679	.000000
R7	.789431	.000000
R8	.359409	.000000
R4	.359409	.000000
R9	.765827	.000000
R0	.438062	.000000
Y1	.000000	.245291
Y3	.000000	.245291
Y4	.500000	.000000
Y5	.000000	.245291
Y6	.000000	.245291
Y7	.000000	.245291
Y8	.500000	.000000
Y9	.000000	.245291
Y2	.000000	.245291

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	.693578
2)	.000000	.709758
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000
8)	.000000	.000000
9)	.000000	.250000

10)	.000000	.500000
11)	.000000	.000000
12)	.000000	.000000
13)	.245296	.000000
14)	.234985	.000000
15)	.184736	.000000
16)	.000005	.500000
17)	.161132	.000000
18)	.234985	.000000
19)	.184736	.000000
20)	.000005	.500000
21)	.161132	.000000
22)	-.000010	-1.193578
23)	.000000	.754709

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .699877

VARIABLE	VALUE	REDUCED COST
R1	.900000	.000000
X2	.000000	.075393
X3	.000000	.268046
X4	.500000	-.008827
X5	.000000	.277641
X6	.000000	.315153
X7	.318948	.000000
X8	.257193	.000000
X9	.000000	.185182
V	.699877	.000000
R2	.891842	.000000
R3	.871588	.000000
R5	.857193	.000000
R6	.891842	.000000
R7	.871588	.000000
R8	.299755	.000000
R4	.299755	.000000
R9	.857193	.000000
R0	.438062	.000000
Y1	.000000	.300123
Y3	.000000	.300123
Y4	.500000	.000000
Y5	.000000	.300123
Y6	.000000	.300123
Y7	.000000	.300123
Y8	.500000	.000000
Y9	.000000	.300123
Y2	.000000	.300123

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	.528304
2)	.000000	.616571
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000

8)	.000000	.000000
9)	.000000	.250000
10)	.000000	.500000
11)	.000000	.000000
12)	.000000	.000000
13)	.300123	.000000
14)	.291965	.000000
15)	.271711	.000000
16)	.000000	.500000
17)	.257316	.000000
18)	.291965	.000000
19)	.271711	.000000
20)	.000000	.500000
21)	.257316	.000000
22)	.000000	-1.028304
23)	.000000	.699877

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .650427

VARIABLE	VALUE	REDUCED COST
R1	.949999	.000000
X2	.000000	.050511
X3	.000000	.284104
X4	.500000	-.034071
X5	.000000	.288316
X6	.000000	.280983
X7	.166821	.000000
X8	.333256	.000000
X9	.000000	.207729
V	.650427	.000000
R2	.945470	.000000
R3	.939984	.000000
R5	.933256	.000000
R6	.945470	.000000
R7	.939984	.000000
R8	.250855	.000000
R4	.250855	.000000
R9	.933256	.000000
R0	.438062	.000000
Y1	.000000	.349572
Y3	.000000	.349572
Y4	.500000	.000000
Y5	.000000	.349572
Y6	.000000	.349572
Y7	.000000	.349572
Y8	.500000	.000000
Y9	.000000	.349572
Y2	.000000	.349572

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	.455145
2)	.000000	.568282
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000

8)	.000000	.000000
9)	.000000	.250000
10)	.000000	.500000
11)	.000000	.000000
12)	.000000	.000000
13)	.349573	.000000
14)	.345043	.000000
15)	.339557	.000000
16)	.000000	.500000
17)	.332830	.000000
18)	.345043	.000000
19)	.339557	.000000
20)	.000000	.500000
21)	.332830	.000000
22)	-.000001	-.955145
23)	.000000	.650428

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

3) .603973

VARIABLE	VALUE	REDUCED COST
R1	1.000000	.000000
X2	.000000	.031920
X3	.000000	.298480
X4	.500000	-.053573
X5	.000000	.298620
X6	.000000	.257040
X7	.033333	.000000
X8	.400000	.000000
X9	.000000	.226480
V	.603973	.000000
R2	1.000000	.000000
R3	1.000000	.000000
R5	1.000000	.000000
R6	1.000000	.000000
R7	1.000000	.000000
R8	.207947	.000000
R4	.207947	.000000
R9	1.000000	.000000
R0	.438062	.000000
Y1	.000000	.396027
Y3	.000000	.396027
Y4	.500000	.000000
Y5	.000000	.396027
Y6	.000000	.396027
Y7	.000000	.396027
Y8	.500000	.000000
Y9	.000000	.396027
Y2	.000000	.396027

ROW	SLACK OR SURPLUS	PRICE
1)	.000000	.405831
2)	.000000	.535733
4)	.000000	.000000
5)	.000000	.000000
6)	.000000	.000000
7)	.000000	.000000

8)	.000000	.000000
9)	.000000	.250000
10)	.000000	.500000
11)	.000000	.000000
12)	.000000	.000000
13)	.396027	.000000
14)	.396027	.000000
15)	.396027	.000000
16)	.000000	.500000
17)	.396027	.000000
18)	.396027	.000000
19)	.396027	.000000
20)	.000000	.500000
21)	.396027	.000000
22)	.000000	-.905831
23)	.000000	.603973

Appendix F: *Model 5 Results for Sample Network*

R1	Model 5	Model 4	V(B=0.512
0.438062	0.75917	0.868367	
0.475			
0.5			
0.525			
0.55			
0.57574	0.765	0.85	
0.6			
0.625	0.767543	0.838	
0.65	0.76	0.84	
0.675			
0.7	0.752278	0.830505	
0.725			
0.75	0.7	0.815	
0.775			
0.8	0.641509	0.820755	0.829
0.825			
0.85	0.509409	0.754704	0.7637
0.875			
0.9	0.399755	0.699877	0.707
0.925			
0.95	0.30087	0.650427	0.6575
0.975			
1	0.207947	0.603973	0.610676

Appendix G. *GINO Input for Network B Models*

MODEL to prove importance (or lack thereof):

```
1) MAX= V ;
2) X62 = ( .04 + X3 ) * ( 1.7 + X6 + X7 + X8 - ( .5 + X6 ) * ( .6 + X7
  ) - ( 1.1 + X6 + X7 - ( .5 + X6 ) * ( .6 + X7 ) * ( .6 + X8 ) ) ) ;
3) X69 = ( .04 + X3 ) * ( 1.2 + X7 + X8 - ( .6 + X7 ) * ( .6 + X8 ) ) ;
4) X42 = 1.3 + X4 + X5 - ( .8 + X4 ) * ( .5 + X5 ) ;
5) R0 = ( .07 + X1 ) * U + ( .93 - X1 ) * D ;
6) .5 * X1 + .5 * X3 + .8 * X4 + .5 * X5 + .5 * X6 + .6 * X7 + .6 * X8
  + X9 + X10 + X11 + .7 * X12 < .00 ;
7) XBU = X62 + X42 - X62 * X42 + X11 - X11 * ( X62 + X42 - X62 * X42 ) ;
8) D = ( .7 + X12 ) * ( .52 + X9 ) * ( .548 + X10 ) + DL - DL * ( .7 +
  X12 ) * ( .52 + X9 ) * ( .548 + X10 ) ;
9) DL = X69 + X5 + .5 - X69 * ( .5 + X5 ) ;
10) Y1 + Y2 + Y3 + Y4 + Y5 + Y6 + Y7 + Y8 + Y9 + Y10 + Y11 + Y12 = 1 ;
11) V < 1 - ( 1 - Y1 ) * ( R0 - D ) ;
12) X102 = .87 * ( .7 + X12 ) * ( .52 + X9 ) ;
13) X1021 = ( .7 + X12 ) * ( .52 + X9 ) * ( .548 + X10 ) ;
14) V < 1 - ( 1 - Y2 ) * ( - 1 ) * ( ( .07 + X1 ) * ( X69 + X11 - X69 *
  X11 + .03 * ( X102 + .6 - .6 * X102 ) + .97 * X102 ) + ( .93 + X1 )
  * ( X1021 + X69 - X1021 * X69 ) ) - ( 1 - Y2 ) * R0 ;
15) U = .03 * ( XBU + .6 - .6 * XBU + X102 - X102 * ( XBU + .6 - .6 *
  XBU ) ) + .97 * ( XBU + X102 - XBU * X102 ) ;
16) V < 1 - ( 1 - Y9 ) * ( R0 - ( ( .07 + X1 ) * ( .03 * ( XBU + .6 -
  XBU * .6 ) + .97 * XBU ) + ( .93 - X1 ) * DL ) ) ;
17) V < 1 - ( 1 - Y12 ) * ( R0 - ( ( .07 + X1 ) * ( .03 * ( XBU + .6 -
  XBU * .6 ) + .97 * XBU ) + ( .93 - X1 ) * DL ) ) ;
18) V < 1 - ( 1 - Y10 ) * ( R0 - ( ( .07 + X1 ) * U + ( .93 - X1 ) * DL
  ) ) ;
19) R0 > .5 ;
20) U3 = X102 + X11 - X102 * X11 + X42 - X42 * ( X102 + X11 - X102 * X11
  ) ;
21) V < 1 - ( 1 - Y3 ) * ( R0 - ( .07 + X1 ) * ( U3 + .03 * .6 ) + ( .93
  - X1 ) * ( X1021 + X5 + .5 - ( .5 + X5 ) * X1021 ) ) ;
22) V < 1 - ( 1 - Y11 ) * ( R0 - ( ( .07 + X1 ) * ( .03 * ( X62 + X42 -
  X62 * X42 + ( .6 + X102 - .6 * X102 ) - ( X42 + X62 - X42 * X62 ) *
  ( .6 + X102 - .6 * X102 ) ) - .97 * ( X62 + X42 - X62 * X42 + X102 -
  X102 * ( X62 + X24 - X62 * X42 ) ) ) + ( .93 - X1 ) * D ) ) ;
23) V < 1 - ( 1 - Y4 ) * ( R0 - ( ( .03 * UU4 + ( .5 + X5 ) - UU4 * ( .5
  + X5 ) + .97 * ( X69 + X102 - X69 * X102 ) ) * ( .07 + X1 ) + ( .93
  - X1 ) * D ) ) ;
```

```
24) UU4 = X11 + .6 - .6 * X11 + X69 - X69 * ( X11 + .6 - X11 * .6 ) +
   X102 - X102 * ( X11 + .6 - X11 * .6 + X69 - X69 * ( X11 + .6 - X11 *
   .6 ) ) ;
END

SLB    X3     .000000
SUB    X3     .960000
SLB    X6     .000000
SUB    X6     .500000
SLB    X7     .000000
SUB    X7     .400000
SLB    X8     .000000
SUB    X8     .400000
SLB    X4     .000000
SUB    X4     .200000
SLB    X5     .000000
SUB    X5     .500000
SLB    X1     .000000
SUB    X1     .930000
SLB    X9     .000000
SUB    X9     .480000
SLB    X10    .000000
SUB    X10    .552000
SLB    X11    .000000
SUB    X11    1.000000
SLB    X12    .000000
SUB    X12    .300000
SLB    Y1     .000000
SLB    Y2     .000000
SLB    Y3     .000000
SLB    Y4     .000000
SLB    Y5     .000000
SLB    Y6     .000000
SLB    Y7     .000000
SLB    Y8     .000000
SLB    Y9     .000000
SLB    Y10    .000000
SLB    Y11    .000000
SLB    Y12    .000000

LEAVE
```

Complete Model (Unimportant components fixed):

```
1) MAX= R0 ;
2) X62 = ( .04 + X3 ) * ( 1.7 + X6 + X7 + X8 - ( .5 + X6 ) * ( .6 + X7
   ) - ( 1.1 + X6 + X7 - ( .5 + X6 ) * ( .6 + X7 ) * ( .6 + X8 ) ) ) ;
3) X69 = ( .04 + X3 ) * ( 1.2 + X7 + X8 - ( .6 + X7 ) * ( .6 + X8 ) ) ;
4) X42 = 1.3 + X4 + X5 - ( .8 + X4 ) * ( .5 + X5 ) ;
5) R0 = ( .07 + X1 ) * U + ( .93 - X1 ) * D ;
6) .5 * X1 + .5 * X3 + .8 * X4 + .5 * X5 + .5 * X6 + .6 * X7 + .6 * X8
   < .00 ;
7) Y1 + Y2 + Y3 + Y4 + Y5 + Y6 + Y7 + Y8 + 0.00 = 1 ;
8) D = .7 + X5 - .2 * ( X5 + .5 ) + X69 - X69 * ( .7 + X5 - .2 * ( X5 +
   .5 ) ) ;
9) U = .03 * ( .73 + ( X62 + X42 - X62 * X42 ) - .73 * ( X62 + X42 -
   X62 * X42 ) ) + .97 * ( .32 + X62 + X42 - X62 * X42 - .32 * ( X62 +
   X42 - X62 * X42 ) ) ;
10) V < 1 - ( 1 - Y1 ) * ( R0 - D ) ;
11) V < 1 - ( 1 - Y2 ) * ( R0 - ( .07 + X1 ) * ( .03 * ( .73 + X62 - .73
   * X62 ) + .97 * ( .32 + X62 - .32 * X62 ) ) - ( .93 - X1 ) * ( .2 +
   X69 - .2 * X69 ) ) ;
12) V < 1 - ( 1 - Y3 ) * ( R0 - ( .07 + X1 ) * ( .03 * ( .73 + X42 - .73
   * X42 ) + .97 * ( .32 + X42 - .32 + X42 ) ) - ( .93 - X1 ) * ( .7 +
   X5 - .2 * ( X5 + .5 ) ) ) ;
13) V < 1 - ( 1 - Y4 ) * ( R0 - ( .07 + X1 ) * ( .03 * ( .73 + X62 - .73
   * X62 + .5 + X5 - ( .5 + X5 ) * ( .73 + X62 - .73 * X62 ) ) + .97 *
   ( .32 + X69 - .32 * X69 + .5 + X5 - ( .5 + X5 ) * ( .32 + X69 - .32
   * X69 ) ) ) - ( .93 - X1 ) * D ) ;
14) V < 1 - ( 1 - Y5 ) * ( R0 - ( .07 + X1 ) * ( .03 * ( .73 + X62 - .73
   * X62 ) + .97 * ( .32 + X62 - .32 * X62 ) ) - ( .93 - X1 ) * ( X69 +
   .2 - .2 * X69 ) ) ;
15) V < 1 - ( 1 - Y6 ) * ( R0 - ( .07 + X1 ) * .03 * ( X69 + X42 - X69 *
   X42 + .73 - .73 * ( X69 + X42 - X69 * X42 ) ) + .97 * ( X69 + X42 -
   X69 * X42 + .32 - .32 * ( X69 + X42 - X69 * X42 ) ) - ( .93 - X1 ) *
   D ) ;
END

SLB    X3     .000000
SUB    X3     .960000
SLB    X6     .000000
SUB    X6     .500000
SLB    X7     .000000
SUB    X7     .400000
SLB    X8     .000000
```

SUB	X8	.400000
SLB	X4	.000000
SUB	X4	.200000
SLB	X5	.000000
SUB	X5	.500000
SLB	X1	.000000
SUB	X1	.930000
SLB	Y1	.000000
SLB	Y2	.000000
SLB	Y3	.000000
SLB	Y4	.000000

LEAVE

Appendix H. *GINO Output to test importance*

1) MAX= V ;

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS:

OBJECTIVE FUNCTION VALUE

1) .889617

VARIABLE	VALUE	REDUCED COST
R0	.775643	.000000
X62	.148658	.000000
X3	.269703	.000000
X6	.000000	.603044
X7	.000000	.633853
X8	.000000	.641217
X69	.260151	.000000
X42	.900000	.000000
X4	.000000	1.032710
X5	.000000	.964810
X1	.230317	.000000
U	.942873	.000000
D	.703865	.000000
X9	.000000	.907518
X10	.000000	1.043265
X11	-.000010	1.050157
X12	.000000	.622366
XBU	.914865	.000000
DL	.630075	.000000
Y1	.000000	.110762
Y2	.121289	.000428
Y3	.878711	.000000
Y4	.000000	.110762
Y5	.000000	.110762
Y6	.000000	.110762
Y7	.000000	.110762
Y8	.000000	.110762
Y9	.000000	.110762
Y10	.000000	.110762
Y11	.000000	.110762
Y12	.000000	.110762
V	.889617	.000000
X102	.316680	.000000

X1021	.199472	.000000
U3	.931667	.000000
UU4	.797776	.000000
X24	5.651591	.000000

ROW	SLACK OR SURPLUS	PRICE
2)	.000000	-.015850
3)	.000000	.728369
4)	.000000	-.131910
5)	.000000	-.786526
6)	.000000	1.208444
7)	.000000	-.158500
8)	.000000	-.550319
9)	.000000	-.440546
10)	.000000	.110762
11)	.038605	.000000
12)	.000000	.208298
13)	.000000	.657364
14)	-.000004	.878292
15)	.000000	-.236207
16)	.050802	.000000
17)	.050802	.000000
18)	.058753	.000000
19)	.275643	.000000
20)	.000000	.004433
21)	.000001	.121708
22)	.000001	.000000
23)	.008811	.000000
24)	.000000	.000000

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS:

OBJECTIVE FUNCTION VALUE

1) .800589

VARIABLE	VALUE	REDUCED COST
R0	.799998	.000000
X62	.106950	.000000
X3	.182812	.000000
X6	.000000	1.113667
X7	.000000	1.219571
X8	.000000	1.207543
X69	.187162	.000000
X42	.919506	.000000
X4	.000000	1.614949
X5	.097529	-.010338
X1	.219659	.000000
U	.951763	.000000
D	.738112	.000000
X9	.000000	1.652362
X10	.000000	1.814745
X11	.000000	2.039619
X12	.000000	1.132176
XBU	.928115	.000000
DL	.672856	.000000
Y1	.000000	.199407
Y2	.198901	.000000
Y3	.801099	.000000
Y4	.000000	.199407
Y5	.000000	.199407
Y6	.000000	.199407
Y7	.000000	.199407
Y8	.000000	.199407
Y9	.000000	.199407
Y10	.000000	.199407
Y11	.000000	.199407
Y12	.000000	.199407
V	.800589	.000000
X102	.316680	.000000
X1021	.199472	.000000
U3	.944997	.000000
UU4	.777829	.000000

X24	4.683695	.000000

ROW	SLACK OR SURPLUS	PRICE
2)	.000000	.035986
3)	.000000	1.302940
4)	.000000	.407085
5)	.000000	2.300126
6)	.000000	2.223486
7)	.000000	.447068
8)	.000000	1.633874
9)	.000000	1.307962
10)	.000000	.199407
11)	.137525	.000000
12)	.000000	.230499
13)	.000000	.588404
14)	.000004	.801098
15)	.000000	.666252
16)	.146582	.000000
17)	.146582	.000000
18)	.153057	.000000
19)	-.000002	-2.981446
20)	.000000	.011459
21)	.000005	.198902
22)	.000003	.000000
23)	.093847	.000000
24)	.000000	.000000

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

1) .599821

VARIABLE	VALUE	REDUCED COST
R0	.900000	.000000
X62	.019200	.000000
X3	.000000	.016844
X6	.000000	.555682
X7	.000000	.656583
X8	.000000	.656296
X69	.033600	.000000
X42	.968059	.000000
X4	.000000	.863365
X5	.340296	.000000
X1	.159704	.000000
U	.978979	.000000
D	.876448	.000000
X9	.000000	.799846
X10	.000000	.884340
X11	.000000	.996739
X12	.000000	.546545
XBU	.968672	.000000
DL	.845662	.000000
Y1	.000000	.400180
Y2	.297976	.000000
Y3	.702024	.000000
Y4	.000000	.400180
Y5	.000000	.400180
Y6	.000000	.400180
Y7	.000000	.400180
Y8	.000000	.400180
Y9	.000000	.400180
Y10	.000000	.400180
Y11	.000000	.400180
Y12	.000000	.400180
V	.599821	.000000
X102	.316680	.000000
X1021	.199472	.000000
U3	.978174	.000000
UU4	.735856	.000000

X24	4.695383	.000000

ROW	SLACK OR SURPLUS	PRICE
2)	.000000	.004777
3)	.000000	.638689
4)	.000000	.160633
5)	.000000	.970361
6)	.000000	1.111273
7)	.000000	.149568
8)	.000000	.747465
9)	.000000	.598367
10)	.000000	.400180
11)	.376627	.000000
12)	.000000	.118678
13)	.000000	.508079
14)	-.000001	.702023
15)	.000000	.222896
16)	.374227	.000000
17)	.374227	.000000
18)	.376465	.000000
19)	.000000	-1.551988
20)	.000000	.020395
21)	.000000	.297977
22)	.227094	.000000
23)	.307036	.000000
24)	.000000	.000000

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

1) .458507

VARIABLE	VALUE	REDUCED COST
R0	1.000000	.000000
X62	.019200	.000000
X3	.000000	.123447
X6	.000000	.467051
X7	.000000	.553916
X8	.000000	.553916
X69	.033600	.000000
X42	1.000000	.000000
X4	.000000	.747281
X5	.500000	.000000
X1	.000000	.000000
U	1.000000	.000000
D	1.000000	.000000
X9	.000000	.739932
X10	.000000	.769803
X11	.000000	.900128
X12	.000000	.509631
XBU	1.000000	.000000
DL	1.000000	.000000
Y1	.000000	.541496
Y2	.291328	.000000
Y3	.708672	.000000
Y4	.000000	.541496
Y5	.000000	.541496
Y6	.000000	.541496
Y7	.000000	.541496
Y8	.000000	.541496
Y9	.000000	.541496
Y10	.000000	.541496
Y11	.000000	.541496
Y12	.000000	.541496
V	.458507	.000000
X102	.316680	.000000
X1021	.199472	.000000
U3	1.000000	.000000
UU4	.735856	.000000

X24	4.695383	.000000

ROW	SLACK OR SURPLUS	PRICE
2)	.000000	.000000
3)	.000000	.409053
4)	.000000	.037531
5)	.000000	.726543
6)	.000000	.934102
7)	.000000	.034127
8)	.000000	.675685
9)	.000000	.540905
10)	.000000	.541496
11)	.541493	.000000
12)	.000000	.034523
13)	.000000	.451370
14)	.000000	.708676
15)	.000000	.050858
16)	.541493	.000000
17)	.541493	.000000
18)	.541493	.000000
19)	.000000	-1.313633
20)	.000000	.005941
21)	-.000010	.291324
22)	.485153	.000000
23)	.514590	.000000
24)	.000000	.000000

Appendix I. *GINO Output for Network B*

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE

1) .790206

VARIABLE	VALUE	REDUCED COST
R0	.793277	.000000
X62	.259200	.000000
X3	.500000	.000000
X6	.000000	.269522
X7	.000000	.257790
X8	.000000	.230584
X69	.453600	.000000
X42	.900000	.000000
X4	.000000	.442494
X5	.000000	.598047
X1	.000000	.466995
U	.950537	.000000
D	.781440	.000000
Y1	.000000	.104897
Y2	.105062	.000000
Y3	.000000	.104897
Y4	.000000	.104897
Y5	.105062	.000000
Y6	.789876	.000000
Y7	.000000	.104897
Y8	.000000	.104897
V	.790206	.000000

ROW	SLACK OR SURPLUS	PRICE
2)	.000000	.033587
3)	.000000	.296485
4)	.000000	-.036445
5)	.000000	-.822990
6)	.000000	.530339
7)	.000000	.104897
8)	.000000	-.744850
9)	.000000	-.057609
10)	.197957	.000000
11)	.000000	.447469
12)	.098780	.000000
13)	.200432	.000000
14)	.000000	.447469
15)	.000000	.105062

16)	.293277	.000000

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE
1) .559629

VARIABLE	VALUE	REDUCED COST
R0	.899998	.000000
X62	.095643	.000000
X3	.159257	.000000
X6	.000000	.474923
X7	.000000	.525812
X8	.000000	.516925
X69	.167376	.000000
X42	.968148	.000000
X4	.000000	.761195
X5	.340743	.000000
X1	.000000	.457012
U	.980767	.000000
D	.893919	.000000
Y1	.000000	.220184
Y2	.216064	.000000
Y3	.000000	.220184
Y4	.000000	.220184
Y5	.216064	.000000
Y6	.567873	.000000
Y7	.000000	.220184
Y8	.000000	.220184
V	.559629	.000000

ROW	SLACK OR SURPLUS	PRICE			
2)	.000000	.029734			
3)	.000000	.546701			
4)	.000000	-.022563			
5)	.000000	.678280			
6)	.000000	.947002			
7)	.000000	.220184			
8)	.000000	.717631			
9)	.000000	.047480	10)	.434291	.000000
11)	.000003	.391969	12)	.485441	.000000
13)	.435513	.000000	14)	.000003	.391969
15)	.000000	.216063	16)	-.000002	-1.386203

SOLUTION STATUS: OPTIMAL TO TOLERANCES. DUAL CONDITIONS: SATISFIED.

OBJECTIVE FUNCTION VALUE
1) .441091

VARIABLE	VALUE	REDUCED COST			
R0	1.000000	.000000			
X62	.019200	.000000			
X3	.000000	.000000			
X6	.000000	.345946			
X7	.000000	.408690			
X8	.000000	.407193			
X69	.033600	.000000			
X42	1.000000	.000000			
X4	.000000	.553129			
X5	.500000	.000000			
X1	.000000	.423208			
U	1.000000	.000000			
D	1.000000	.000000			
Y1	.000000	.279455			
Y2	.269250	.000000			
Y3	.000000	.279455			
Y4	.000000	.279455			
Y5	.269250	.000000			
Y6	.461500	.000000			
Y7	.000000	.279455			
Y8	.000000	.279455			
V	.441091	.000000			
U11	1.000000	.000000	U12	1.000000	.000000
U21	1.000000	.000000	U22	1.000000	.000000
X666	.032000	.000000	X665	.043880	.000000

ROW	SLACK OR SURPLUS	PRICE			
2)	.000000	.024958	3)	.000000	.397293
4)	.000000	-.075980	5)	.000000	.356958
6)	.000000	.691412	7)	.000000	.279455
8)	.000000	.466812	9)	.000000	.024987
10)	.558909	.000000	11)	.000000	.365375
12)	.626809	.000000	13)	.558909	.000000
14)	.000000	.365375	15)	.000000	.269250
16)	.000000	-1.035944	17)	.628909	.000000
18)	.558909	.000000			

Appendix J. FUZZYFTA Input and Output

```
   FUZZY FAULT TREE ANALYSIS
        INFUZZ.DAT
&PARAMS
 NUMG=12,IOFLAG = 1,N1=0,M1=0
/
GATE DESCRIPTION SECTION
 NO.  NAME     DESCRIPTION
  1   TOP    TOP EVENT,  A, B & C INPUTS
  2    A    AND GATE,  A1, A2, A3 & A4 INPUTS
  3    B    AND GATE,  B1, B2, A3 & A4 INPUTS
  4    C    AND GATE, C1 & C2 INPUTS
  5   A1    FIRST BOTTOM EVENT TO AND GATE A
  6   A2    SECOND BOTTOM EVENT TO AND GATE A
  7   A3    THIRD BOTTOM EVENT TO AND GATE A
  8   A4    FOURTH BOTTOM EVENT TO AND GATE A
  9   B1    FIRST BOTTOM EVENT TO AND GATE B
 10   B2    SECOND BOTTOM EVENT TO AND GATE B
 11   C1    FIRST BOTTOM EVENT TO AND GATE C
 12   C2    SECOND BOTTOM EVENT TO AND GATE C
LOGIC SECTION
 NO.  NAME   TYPE  N,G  INPUT GATES
  1   TOP    OR    3    2  3  4
  2    A    AND    2    5  6  7  8
  3    B    AND    2    7  8  9 10
  4    C    AND    4   11 12
  5   A1     BE    0
  6   A2     BE    0
  7   A3     BE    0
  8   A4     BE    0
  9   B1     BE    0
 10   B2     BE    0
 11   C1     BE    0
 12   C2     BE    0
BOTTOM EVENT PROBABILITY SECTION
 NO.  NAME  IPROB  QL      PL      PR      QR
  5   A1    1  0.1200  0.1500  0.2500  0.2700
  6   A2    1  0.2500  0.2800  0.3300  0.3600
  7   B1    1  0.2000  0.2200  0.2700  0.3000
  8   B2    1  0.0500  0.1000  0.2000  0.2500
  9   C1    1  0.0500  0.1000  0.2000  0.2500
 10   C2    1  0.0500  0.1000  0.2000  0.2500
 11   C3    1  0.0500  0.1000  0.2000  0.2500
 12   C4    1  0.0500  0.1000  0.2000  0.2500
```

12- FEB-1997

12:52

FUZZY FAULT TREE ANALYSIS
INFUZZ.DAT

FUZZY LOGIC FAILURE PROBABILITY RANGE OF TOP EVENT

Q,L	P,L	P,R	Q,R
.184E+00	.337E+00	.473E+00	.566E+00

FUZZY LOGIC PROBABILITY RANGE OF ALL GATES
(BOTTOM EVENTS NOT INCLUDED)

NO.	NAME	Q,L	P,L	P,R	Q,R
1	TOP	.184E+00	.337E+00	.473E+00	.566E+00
3	B	.405E-01	.145E+00	.216E+00	.282E+00
6	B2	.220E+00	.712E+00	.812E+00	.100E+01
7	B3	.614E+00	.638E+00	.634E+00	.641E+00

FUZZY LOGIC IMPORTANCE FACTORS

NO.	NAME	IMP. FACTOR	RANK
4	C	.7692E+00	1
5	B1	.4629E+00	2
9	C2	.4292E+00	3
10	C3	.4265E+00	4
2	A	.7513E-01	5
12	C5	.7223E-02	6
8	C1	.7180E-02	7
11	C4	.6597E-02	8

POINT ESTIMATE FAILURE PROBABILITY OF TOP EVENT

FAILURE PROBABILITY = .406E+00

POINT ESTIMATE PROBABILITY OF ALL GATES
(BOTTOM EVENTS NOT INCLUDED)

NO.	NAME	PROBABILITY
1	TOP	.406E+00
3	B	.179E+00
6	B2	.762E+00
7	B3	.636E+00

Appendix K. Linear Program For Sample Network

```
MAX    V
SUBJECT TO
    2)  V - 0.01 Y2 <=  0.99
    3)  V - 0.12 Y3 <=  0.88
    4)  V - 0.63 Y4 <=  0.37
    5)  V - 0.16 Y5 <=  0.84
    6)  V - 0.01 Y6 <=  0.99
    7)  V - 0.4 Y7 <=  0.6
    8)  V - 0.43 Y8 <=  0.57
    9)  V - 0.16 Y9 <=  0.84
   10)  Y2 + Y3 + Y4 + Y5 + Y6 + Y7 + Y8 + Y9 =   1
END

LP OPTIMUM FOUND AT STEP      5

     OBJECTIVE FUNCTION VALUE

     1)      .6881278

 VARIABLE        VALUE          REDUCED COST
      V         .688128         .000000
     Y2         .000000         .155936
     Y3         .000000         .155936
     Y4         .504965         .000000
     Y5         .000000         .155936
     Y6         .000000         .155936
     Y7         .220319         .000000
     Y8         .274716         .000000
     Y9         .000000         .155936


     ROW   SLACK OR SURPLUS    DUAL PRICES
      2)        .301872         .000000
      3)        .191872         .000000
      4)        .000000         .247518
      5)        .151872         .000000
      6)        .301872         .000000
      7)        .000000         .389840
      8)        .000000         .362642
      9)        .151872         .000000
     10)        .000000         .155936

NO. ITERATIONS=       5

RANGES IN WHICH THE BASIS IS UNCHANGED:

                       OBJ COEFFICIENT RANGES
 VARIABLE        CURRENT       ALLOWABLE       ALLOWABLE
                  COEF          INCREASE        DECREASE
      V         1.000000        INFINITY        1.000000
```

Y2	.000000	.155936	INFINITY
Y3	.000000	.155936	INFINITY
Y4	.000000	.207229	.630000
Y5	.000000	.155936	INFINITY
Y6	.000000	.155936	INFINITY
Y7	.000000	.255566	.400000
Y8	.000000	.244660	.430000
Y9	.000000	.155936	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	.990000	INFINITY	.301872
3	.880000	INFINITY	.191872
4	.370000	.422771	.356046
5	.840000	INFINITY	.151872
6	.990000	INFINITY	.301872
7	.600000	.144434	.303016
8	.570000	.185340	.243016
9	.840000	INFINITY	.151872
10	1.000000	.973938	.565153

Appendix L. Linear Program For Network B

```
MAX     V
SUBJECT TO
     2)  V - 0.01 Y1 <=  0.99
     3)  V - 0.225 Y2 <=  0.775
     4)  V - 0.17 Y3 <=  0.83
     5)  V - 0.205 Y5 <=  0.795
     6)  V - 0.05 Y7 <=  0.95
     7)  V - 0.05 Y8 <=  0.95
     8)  Y1 + Y2 + Y3 + Y5 + Y7 + Y8 =   1
END
```

LP OPTIMUM FOUND AT STEP 5

OBJECTIVE FUNCTION VALUE

1) .8684630

VARIABLE	VALUE	REDUCED COST
V	.868463	.000000
Y1	.000000	.065769
Y2	.415391	.000000
Y3	.226253	.000000
Y5	.358356	.000000
Y7	.000000	.065769
Y8	.000000	.065769

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	.121537	.000000
3)	.000000	.292304
4)	.000000	.386874
5)	.000000	.320822
6)	.081537	.000000
7)	.081537	.000000
8)	.000000	.065769

NO. ITERATIONS= 5

RANGES IN WHICH THE BASIS IS UNCHANGED:

	OBJ COEFFICIENT RANGES		
VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
V	1.000000	INFINITY	1.000000
Y1	.000000	.065769	INFINITY
Y2	.000000	.092933	.225000
Y3	.000000	.107267	.170000
Y5	.000000	.096835	.205000
Y7	.000000	.065769	INFINITY

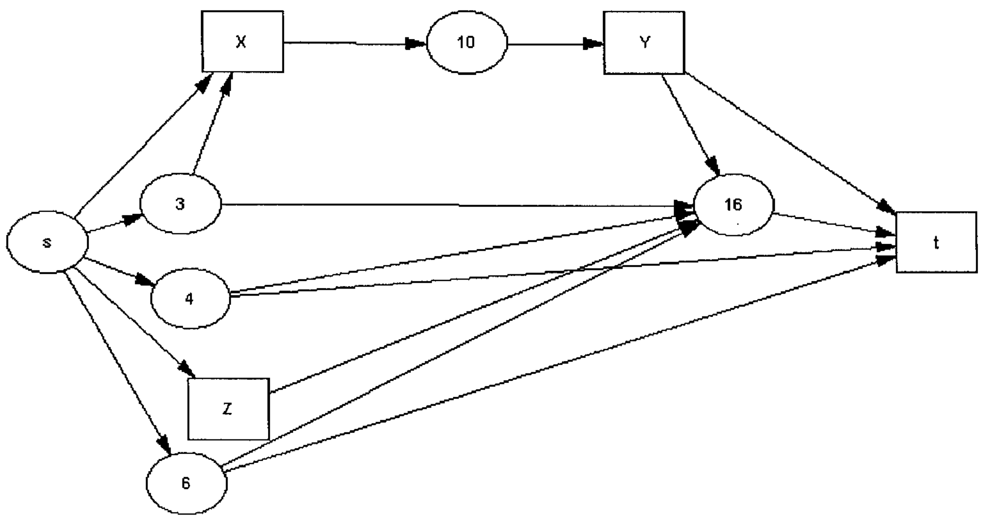
Y8	.000000	.065769	INFINITY

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	.990000	INFINITY	.121537
3	.775000	.132067	.131585
4	.830000	.062733	.189889
5	.795000	.108165	.119889
6	.950000	INFINITY	.081537
7	.950000	INFINITY	.081537
8	1.000000	1.239758	.584824

Appendix M. Network B and Reduction Algorithm

1. Identify points between which pure parallel and pure series configuration exists or between which factoring will quickly yield a result. For Network B, from *Node 10* to the sink is one set of points (call this set of components *Y*), between *node 16* and the sink is another, between the source and *Node 10* another (call this set of components *X*), and from the source through *Node 5* to *Node 16* another (call this set *Z*). Reduce the components between the points using basic reduction formulas, and redraw the network with these reductions reflected as aggregated "supernodes". Note that the reliability of these supernodes depends on whether node 3 (for supernode X) and node 16 (for supernode Y) are up or down.

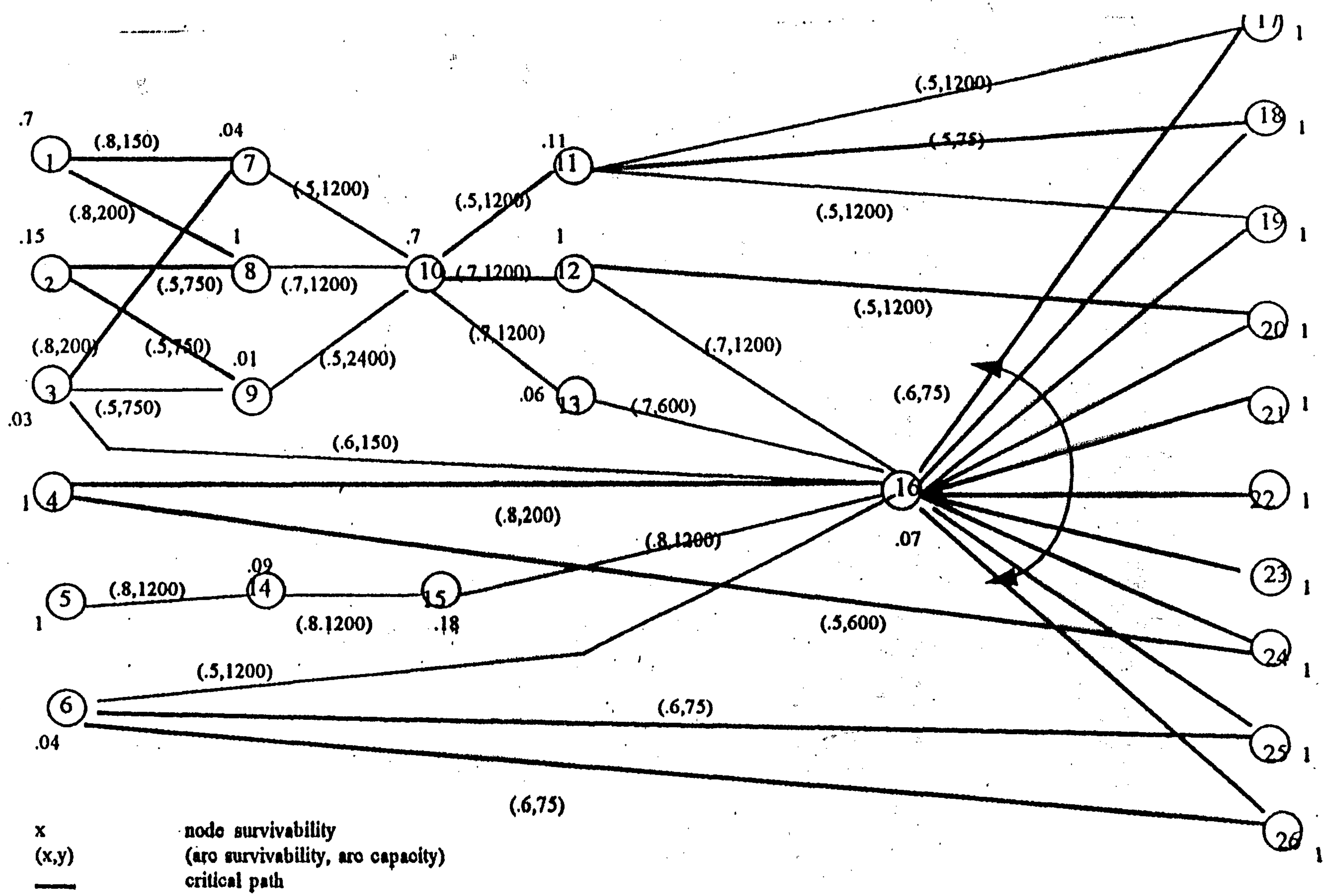


Network B after aggregation

2. With this simpler network, calculate the probability by factoring. For Network B, Factor on Node 16 first. Thus, reliability = Probability (s connected to t) = r_{16} * Probability (s connected to t | 16 is working) + (1-r_{16})*Probability (s connected to t | 16 is not working). When node 16 is not working, network B reliability can be calculated without further factoring since all of the paths from *s* to *t* are disjoint.

3. Given the previous node is up, factor again. For Network B, this was the last factoring required (factor on Node 3). For more complex networks, repeat this step until no more factoring is required.

4. If necessary, factor given the nodes are down. For Network B, each node factored upon yielded closed form solutions when the nodes were down.


x node survivability
(x,y) (arc survivability, arc capacity)
critical path

Vita

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He graduated from Milton High School in Milton, Florida, in 1985. Captain Lyle earned a Bachelor of Science degree in Aerospace Engineering from the University of Alabama in 1989 and a Master of Art degree in Mathematics from Eastern New Mexico University in 1992. Before arriving at AFIT in September 1995, Captain Lyle was the Base Chief of Services at Noervenich AB, GE (1994-1995), the Emergency Action Training Officer and Assistant Operations Officer for the 7502 Munitions Support Squadron at Noervenich AB, GE (1993), and the Squadron Section Commander for the 428th Fighter Squadron at Cannon AFB, NM (1990-1992). Before beginning active duty in February, 1990, he was a Mechanical Engineer for the Warheads Branch of the Air Force Armament Laboratory (1989). He has also taught college courses in Physics and Algebra for the Clovis Community College (1990), Eastern New Mexico University (1991), and the University of Maryland (1993-1994).

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This research optimized two measures of network security by hardening components and improving their reliability. A common measure of effectiveness (MOE) for networks is statistical reliability, which ignores the effects of hostile actions. A new MOE which includes hostile actions was developed. Both measures require component reliability functions, derived using fault trees. Fuzzy Logic and Monte Carlo simulation were used to quantify uncertainty. Results from the model are compared to traditional Risk Assessment results.

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