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Game Theory Framework to Evaluate Nuclear Deterrence

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GAME THEORY FRAMEWORK TO EVALUATE NUCLEAR DETERRENCE

THESIS

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Abstract

This research game theory framework evaluates the resilience of nuclear deterrence options between two players. We use lexicographic prioritization to value four priorities of political, military, economic, and civilian casualties. The value order may be varied. We demonstrate our approach with six player choices of no nuclear strike, demonstration, counterforce, tactical military, economic, or countervalue strike. We use game theory to construct and analyze the resulting damage matrix. We conclude that credible deterrence requires having at least equivalent offensive damage capabilities.

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Michael A Cevallos.

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GAME THEORY FRAMEWORK TO EVALUATE NUCLEAR DETERRENCE

I. Introduction

In the introduction, we provide background on historical and current nuclear strategy. We then provide insight into theories that motivated those strategies. We then proceed to describe possible shortcomings of the current nuclear deterrence framework and state the goal of this paper.

Deterrence is achieved by threatening retaliation to ensure their adversary does not take a specific action. For nuclear war, each side wants to deter their opponent from using a nuclear weapon. The threat needs to be credible. A threat that inflicts significantly more damage than the attack is not credible because it is not commensurate with damage received and would escalate the conflict. Similarly, a response that inflicts significantly less damage than the attack leaves the attacker in a better relative position; hence, a weak response may not deter an opponent.

In this thesis, we propose a game theory framework that enables the evaluation of nuclear attack options for both an initial attacker and responder across four criteria (military, political, economic, and casualties). We demonstrate the framework on a few scenarios, which indicate that inability to respond at specific levels of damage inhibit deterrence in some circumstances. However, the framework may be applied in diverse scenarios or with different prioritized goals.

1.1 Background

Counterforce is the nuclear strategy that targets an opponent's offensive nuclear attack capability (Schneider, 2004). Targets include nuclear armaments, leadership, and C3 (command, control, communication) systems. Counterforce achieves two objectives. First, counterforce limits the scope of the entire engagement to predominantly military targets. Second, if executed effectively, one or both nations' nuclear capabilities would be crippled to prevent continued nuclear strikes. The foundation of nuclear deterrence is based on the threat of retaliation. If a country loses its capability to retaliate, it becomes vulnerable to further attacks or intimidation. In contrast, the country with surviving nuclear capabilities would have a much stronger political and military position.

The nation that "wins" the war has the most robust, hard-to-target nuclear capabilities and accurate, powerful offensive capabilities in the current framework. Deterrence through counterforce led Russia and the United States (U.S.) to invest heavily in many launch sites, hardened facilities, and mobile launch platforms, such as submarines. As of 2021, there were 13,800 nuclear warheads publicly declared. Though much lower than the peak estimated 70,000 warheads in the 1980s, the current inventory presents a much more difficult military position to navigate (Kelsey, 2021). Today's arsenal is much more sophisticated, with diverse delivery methods; Russia and China continuously innovate and develop nuclear arsenals.

1.2 Problem Statement

The diversity of nuclear delivery methods is the cornerstone of modern nuclear deterrence. Hardened missile sites, advances in stealth aircraft, and the presence of mobile nuclear delivery systems on trucks, trains, and submarines along with local and area defenses make a counterforce attack to destroy all the adversary's nuclear weapons unlikely to be successful. Hence, adversaries are deterred by the potential retaliation within the counterforce framework.

Deterrence from a counterforce attack is achieved when one country can prevent any adversary from destroying a significant portion of its nuclear capabilities. The deterrence concept is that the attacked country will retaliate in a devastating way if they have any remaining nuclear weapons. However, deterrence from nuclear attacks other than counterforce should be considered. Two potential shortcomings in the counterforce deterrence framework may be exploited.

The first potential weakness to deterrence based on counterforce is the inability to guarantee a successful counterforce attack. The U.S and Russia have spent the last 50 years developing resilient command, control, and communication (C3) systems, new launch systems, and hardening launch sites to ensure that their offensive nuclear capabilities are resilient to a counterforce attack. A nation that does not fear a counterforce strike against it has the freedom to explore alternatives with its nuclear arsenal.

The second weakness of counterforce is that it may not be credible in deterring the use of a limited number of nuclear weapons. We see the result of this with Russia. As seen in the 2018 Nuclear Posture Review (Office of the Secretary of Defense, 2018), Russia has spent the last decade aggressively enhancing its non-strategic nuclear arsenal. The threat of massive counterforce retaliation might not achieve the deterrence necessary to prevent the use of tactical nuclear weapons in a regional conflict. What retaliatory threat is credible, in both ability to execute it and commensurate to the situation, to achieve deterrence? Determining a viable retaliation is particularly difficult against an adversary with a significant strategic nuclear arsenal. Gallagher and Cevallos (2021) discuss that using one or two nuclear weapons striking political and economic targets does not significantly change the deterrence from counterforce attacks between major nuclear powers. These two developments pose a challenge that requires thinking beyond counterforce to achieve nuclear deterrence.

The primary goal of this research effort is to provide a nuclear response framework that supports national leaders in evaluating nuclear deterrence while grounded in current and future technological and political realities. The purpose of creating this new framework is not to encourage or motivate employment of nuclear weapons. This framework exists to enhance deterrence against any potential nuclear weapon use.

This paper does not seek to identify and analyze any specific scenario or current global event. To achieve this goal, we analyze if relying solely on counterforce strategies has any shortcomings.

II. Literature Review

In these two sections, we review game theory and several deterrence strategies.

2.1 Game Theory

2.1.1 Normal and Extensive Form Games

Normal form games describe a type of game that can be represented by a matrix. The matrix represents the strategies and corresponding payouts for each player. Normal Form games can be easily described with a matrix but can only represent static games where both players make a decision at the same time, without knowing their opponent's move.

Extensive form games allow for the player's turn order to be incorporated. Extensive form games are represented with game trees. These trees incorporate the sequencing of each player's choice at each turn. As a result of this added information in the system, the representation becomes more complex.

2.1.3 Payoff Matrix

A payoff matrix is a tool used in game theory to represent all the possible outcomes that can occur between two players when they make a decision. It displays the results in the form of a table that displays the gain or loss that each player accrues given both players' decisions. A payoff matrix is an effective tool for decision makers because it lets them see outcomes based on both players' choices (Kevin & Shoham, 2008). In this paper, we use a damaged matrix, where each pair of values represent the amount of damage that each player takes. We construct a payoff matrix that shows the penalties that each player incurs given the damage matrix for the scenario and the player's nuclear strategy. Each player seeks to minimize their penalty taken instead of maximizing a payoff.

We choose to leverage a normal form payoff matrix to represent an extensive form game for two reasons. First, the game is finite, only considering the first move of each player. Second, the responding player has perfect and complete information since they are able to see the previous player's action. This allows us to represent the extensive form using a normal form payoff matrix without losing critical information.

2.1.4 Pareto Optimal

A Pareto optimal outcome in a payoff matrix exists when no change can lead to the improvement for at least one of the players without the other player outcome worsening. A Pareto dominated result is one where at least one player still improves without the other taking any loss. A game can have multiple Pareto solutions.

2.1.5 Nash Equilibrium

The Nash equilibrium exists when neither player in a game can increase his or her payoff by unilaterally choosing another option in response to their opponent's choice. Every game has at least one Nash equilibrium if mixed strategies are allowed. Without mixed strategies, a Nash equilibrium is not guaranteed. It is important to note that the Nash equilibrium is not always Pareto optimal. This is because the Nash equilibrium seeks to increase the payoff of each individual Player and not the total value. A payoff matrix helps to determine what the dominant strategy is and if a Nash equilibrium exists

Subgame perfect equilibrium, SPE, is a Nash equilibrium that is found in extensive form games. It differs from a Nash equilibrium because it looks for Player 1's best choice given that Player 2 will give their best response.

2.2 Nuclear Deterrence Theories

This section discusses minimum deterrence, proportional deterrence, countervalue, and counterforce.

2.2.1 Minimum Deterrence

Minimum deterrence theory is a defensive framework constructed by RAND in the 1950s (Wohlsetters, 1958). The theory states that a small nuclear arsenal is all that is required to deter aggressor states. A small nuclear arsenal is effective because it has the capacity to threaten unacceptable damage to any potential aggressors. According to Brodie (1959), a nuclear arsenal exists to win and end a war decisively. In contrast, minimum deterrence contends that a nation with significantly weaker military power can maintain a robust defensive posture with only a small but effective nuclear force. A country with a minimum deterrence policy must be willing to retaliate with a countervalue approach, targeting population and industrial centers, regardless of the threats or potential gains an opponent can truly make. Eric Schlosser sums up the shortcoming of minimum deterrence as such: "It can only kill millions of enemy civilians after the United States has already been attacked" (Schlosser, 2014).

Countervalue was the nuclear military policy adopted by the U.S. at the beginning of the Cold War in response to the Soviet military threat of Europe (Feiveson, 1999). Countervalue policy targets cities and other major population centers. Countervalue was adopted because it was the only strategy that technology could support during the early phases of the Cold War. Mutually Assured Destruction (MAD) is the result of both sides implementing a countervalue strategy. The Soviets and the U.S had three technological limitations that necessitated targeting countervalue targets.

First was the lack of intelligence, surveillance, and reconnaissance (ISR) available to both sides. Until the development of specialized reconnaissance, such as the Lockheed U-2, which began in 1955, and satellite technology, the U.S. operated with very little intelligence regarding Soviet military forces. Insufficient ISR eliminated targeting military forces or critical military infrastructure as an option. In contrast, cities were large, known targets that could not be hidden or moved. The second limitation was the vulnerability that aircraft had while in route and launching nuclear strikes. The USSR and the U.S. needed to launch many aircraft, hoping that sufficient aircraft would penetrate defenses. The third limitation was accuracy. The lack of an accurate air-to-ground targeting system required many targeting attempts to hit their targets effectively. Countervalue strategy did not depend on which side attacked first because both entities would suffer from catastrophic damage regardless of who attacked first. Countervalue would eventually be replaced by counterforce as ISR and targeting capabilities improved.

2.2.3 Counterforce

Counterforce is a nuclear military doctrine where a country strikes its opponent's nuclear/military capabilities and leadership. It was developed after the MAD strategy of the

1950s with the hope of avoiding cities and limiting the destructive potential that MAD would have on both the Soviet Union and the U.S if deterrence failed. Unlike MAD, a first strike is incentivized when both players execute counterforce strategies because the initial attack limits the ability of their opponent to respond.

If neither player is in conflict, the solution is not to attack. However, suppose there is a possibility of war. In that case, counterforce strategy can incentivize players to strike first if the retaliatory strike is less than the damage taken in a conflict (Karl, Castillo, Forrest, Negeenpegahi, & Brian, 2006). A first strike is incentivized because a well-executed pre-emptive counterforce attack by Player 1 weakens Player 2's capability to respond. A first strike puts the attacked player in a situation where they cannot effectively strike back.

Similarly, Player 2 is better off attacking first. If either player thinks that conflict is imminent the damage when not attacking is greater than the risk of being attacked first and not being able to adequately respond. Hence, both sides attempt to attack first. Though the damage is lower than in MAD, a counterforce policy is more likely to result in nuclear use as the collateral damage from nuclear strikes is lower. The winner is the country that attacks first if the counterforce attack is effective.

As a result, the strategic plan of nuclear powers has been to increase the difficulty of having their nuclear capabilities targeted so that a retaliatory strike can maintain deterrence (Karl, Castillo, Forrest, Negeenpegahi, & Brian, 2006). Three effective ways countries have improved the survivability of their arsenal is to harden launch sites and create mobile launch sites that are difficult to locate, track and target. Robust and difficult-to-target nuclear capabilities negate the effectiveness of an enemy's initial counterforce strike. Additionally, local and area defenses contribute to the survivability of a country's nuclear arsenal. The result of improving the survivability of nuclear weapons is that neither side can confidently execute a counterforce attack. In our methodology section, we consider other aspects of nuclear strikes.

2.2.4 Proportional Deterrence

Proportional deterrence policy deters opponents by inflicting a cost that exceeds any potential gain that an opponent could acquire by using either conventional military forces or nuclear weapons (Wiitala, 2016) . The capabilities required to maintain deterrence with this policy differ greatly from traditional minimum deterrence. A proportional deterrence policy requires a much more robust nuclear arsenal, capable of a wide spectrum of responses that can change depending on the opponent and the situation (Keir & Daryl, 2009). In contrast, minimum deterrence only requires a small, survivable, and capable arsenal that could threaten massive retaliatory damage, regardless of the damage inflicted or potential gains by the aggressor state.

The understanding of minimum deterrence has shifted because the prevailing view on nuclear weapons has changed. In the article "Remembrance of Things Past," U.S. military scholars explain how nuclear weapons are viewed as much more than simply more devastating weapons. They are political tools that are not always suited on the modern battlefield (James, B, & Gary, 2010). However, this is not a viewpoint shared by other nations. In "Nukes We need," Lieber and Press (2017) state that traditional minimum deterrence does not deter relatively weak states from using nuclear weapons in a limited capacity, such as denying access to enemies' allied bases. Russian Scholar Fedorov (2002) backs this idea up, stating that limited use can deter and de-escalate conflicts against a superior force in favor of the weaker force without risking escalation. This puts countries that rely solely on minimum deterrence at a disadvantage because countries may resort to using nuclear weapons if they believe it is in their interest and do not believe there is a credible response to their actions (Wiitala, 2016).

Proportional deterrence contends to deter the use of any nuclear weapons requires the capability to respond with capabilities that scale from small yield precision strikes all the way to large yield strategic attacks (George & Vaddi, 2021).

III. Methodology

To explore a variety of new attack options and strategies in our framework, we need to determine six model aspects:

1. Mathematical model
2. National Objectives
3. Decision Attack or Response Options
4. Nuclear "targets" used
5. Game theory assumptions
6. Scenario

The subsequent sections address each of these aspects of our framework.

3.1 Lexicographic Prioritization

We seek to build a framework that can achieve three objectives. First, our framework should take into account the complexity involved in finding a new nuclear strategy. Second, it should be flexible and quickly adjustable so that it can adapt to current and future scenarios. Lastly, the framework should offer decision-makers a clear and rational process to give inputs and understand the results. When dealing with the consequences of nuclear weapons and their use, decision-makers must clearly understand the process they are using to construct policy.

Decision-makers have prioritized goals and seek the strategy that best aligns with their goals. For this reason, we apply lexicographic (pre-emptive) prioritization to optimize for each of the decision maker's objectives in order of importance (Neelavathi, 2015). LLGP has been used to model conventional forces (Anderson, et al., 2020), nuclear war (Cullenbine, Gallagher,

& Moore, 2003), along with military budgets and plans (Nestico, Pav, Campbell, Tama, & Gallagher, 2022; Ledwith, Hufstetler, & Gallagher, 2021). Lexicographical prioritization prevents the need to determine objective weights and allows what would have been constraints, such as casualties, to become part of the objective to solve for (Ledwith, Hufstetler, & Gallagher, 2021).

3.2 Objectives

For our demonstrations, we assume that the decision-makers in charge of nuclear policy seek to balance political, military, economic, and civilian casualty-related objectives. In the first two cases the analyst sequentially minimizes the political, military, economic, and civilian damage received by both countries (Gallagher & Cevallos, 2021). Our framework enables the priority order of these objectives to be varied. In our discussion, we evaluate the model first under the assumption that both countries have the same priority order. These objectives represent the fundamental objectives a nation must keep in order to remain functional.

3.2.1 Political

Political objective represents the solvency and legitimacy that the governing apparatus has over a country. Political legitimacy can be threatened either domestically due to unpopular actions or by external powers that threaten to delegitimize the ruling government with sanctions and embargoes.

3.2.2 Military

This objective represents the capacity for a nation to defend itself. In the analysis, any loss in military force makes defending the country and exerting external pressure more difficult.

3.2.3 Economic

Economic measures are the economic consequences of choosing any one of the options. Unlike political or military, which can have multiple ways of measuring relative cost, economic objectives are evaluated in dollars.

3.2.4 Civilian

The objective of the civilian category is the most straightforward and measures the number of civilian lives lost due to choosing any of the available actions.

3.3 Decisions: Attack or Response Options

Each country has six mutually exclusive attack options to choose from. The first is to make no nuclear strike and attempt to resolve conflicts through other means. It is the most critical decision variable to compare other options against. If the cost of any other decision is better than taking no nuclear strike, deterrence fails. The second option is to strike at enemies' nuclear and conventional capabilities in line with the current counterforce strategy. The third option is for a country to conduct a nuclear demonstration. This action confirms a country's capacity and implied willingness to use nuclear weapons. The fourth is to target a country's conventional military forces with a tactical nuclear strike. These decision variables were chosen because they have previously been considered viable strategic options by nuclear powers (Shamberg, 2001). The fifth option is to target an opponent's critical economic infrastructure, such as a port, in a manner that limits civilian casualties to the extent possible. The last attack option is to attack countervalue targets.

We test whether counterforce offers sufficient deterrence against these other five attack options. These decision variables are binary and mutually exclusive. All scenarios will have two players. The decision variables for Player 1 and Player 2 are represented in Table 1 as P_{ij}. The first number represents the player. The second number represents the decision.

Table 1 Decision Variables for Players 1 and Player 2

P_{ij}	Decision Variables
P_{i1}	No Nuclear Strike
P_{i2}	Counterforce Target
P_{i3}	Nuclear Demonstration
P_{i4}	Tactical Strike
P_{i5}	Economic/Infrastructure
P_{i6}	Countervalue (Cities)

3.4 Damage Matrix

In Lexicographical prioritization, every category is given a numerical value corresponding to an amount of damage. The damage of each category type is assigned a corresponding value between zero and five depending on the amount of damage received. We use these categories because damage in one category is not commensurate with damage in another category. Table 2 Damage Levels depicts the damage matrix levels by categories. We use "No change" since some level of damage may already be occurring in the situation.

Table 2 Damage Levels

damage Categories	Damage Levels by National Objectives					
	0	1	2	3	4	5
Political	No change	Sports and diplomatic sanctions	Economic sanctions/military sanctions	Civil unrest	Loss of government control	End of regime
Military	No change	Local military efforts delayed < 1 day	Regional military efforts damaged delayed, C3*down 1-3 days	Regional offensive capabilities, C3 down 3-10 days	Regional offensive capabilities, C3 down 10-30 days	Military completely immobilized, offensive capabilities destroyed, targets down >30 days
Economic	No change	Regional economy suffers < 10%	Regional economy suffers 10-25%	Market recession/ global and national supply chains interrupted 10%-25 %/ regional economy suffers 25-50%	Market depression / global and national supply chains interrupted 25-50 %/regional economy suffers 50%-90%	Sustained market collapse/ global and national supply chains greatly interrupted (decrease> 50%)/ regional economy destroyed > 90%
Casualties	No change	<5,000	5,000-10,000	10,000-50,000	50,000-100,000	>100,000

* COMMAND CONTROL COMMUNICATION (C3) SYSTEMS

A particular action may result in damage in more than one category.

Table 4 shows the notional level of damage by category both players received depending on the decision variables the chosen. For example, if either player chooses "no nuclear strike," as shown in the first row of Table 2 Damage Levels

Table 4, the opponent receives no damage in any category. Similarly, if Player 2 attacks against counterforce targets, Player 1 receives Level 2 damage in political, level 3 in the military, level 1 in economic, and level 1 in casualties. There are 5 damage tables used to calculate every damage matrix. They consist of the damager Player 1 does against Player 2 for every action, the damage Player 2 does against Player 1, the damage that both players inflict on themselves for choosing an action, and lastly a current damage table that calculates the cost Player 1 begins the scenario with.

Table 3 Damage Levels to Self for Initial Attacker

	Damage to Attacker (Player 1 or 2)			
Decision Variables (Attack Option)	**Political**	**Military**	**Economic**	**Casualties**
No Nuclear Strike	0	0	0	0
Counterforce Targets	2	1	0	0
Nuclear Demonstration	1	0	0	0
Tactical Strike	2	1	0	0
Economic/Infrastructure	3	1	0	0
Countervalue	2	1	0	0

Table 4 Damage Levels to Opponent

	Damage to Opponent (Player 1 or 2)			
Decision Variables	**Political**	**Military**	**Economic**	**Casualties**
No Nuclear Strike	0	0	0	0
Counterforce Targets	2	3	1	3
Nuclear Demonstration	1	0	0	0
Tactical Strike	0	2	0	0
Economic/Infrastructure	3	0	3	1
Countervalue	5	0	5	5

Table 5 Starting Damage Player 1 and 2

Value	Category
Political	0
Military	0
Economic	0
Casualties	0

Table 5 is the amount of damage that Player 1 and 2 starts the scenario with. Table 6 is the priority ranking list used with the larger the number, the higher the ranking. The values being powers of 10 result in the lexicographical ranking since no damage from one category can cause as much damage as any higher category. Other priorities may be used in this framework. In our framework demonstrations, we evaluate the damage level of an attack as the sum of the individual damage levels from Tables 3, 4, and 5 across each category. It is then multiplied by the priorities shown in 6. This gives us the total damage in each category for every player.

Table 6 Priority Order Ranking

Value	Category
1	Political
10	Military
100	Economic
1000	Casualties

We define a proportional response to be when Player 2 can respond to Player 1 with a strike of equivalent damage to Player 1. The equivalent is defined as having damage in the same region, as seen in Table 6. The lexicographic ordering is achieved by multiplying with priority rankings that have an order of magnitude difference. The resulting sum products found in the damage matrix will always be significantly greater than damage from a lower priority. This value is then translated to a lexicographical value to see better equivalent responses based on damage. 'A' represents the smallest damage a player can receive, and 'G' represents the largest damage**.** We use the damage matrix to build the penalty matrix.

Table 6 Numerical to Lexicographic Representation

Numerical Value	Lexicographic Value
0-9	A
10-19	B
20-99	C
100-999	D
1000-2999	E
3000-4999	F
>5000	G

3.5 Penalty Matrix

The damage matrix is a useful tool to identify the resulting damage in a two-sequence exchange. However, it does not offer any insight into what each player should do. The damage matrix translates into a penalty matrix to which we can apply game theory. Player 1 and Player 2's objective is to minimize their respective penalties. The penalty for Player 1 is the same as the damage received from the damage matrix. Hence, Player 1 is minimizing damage to their country. The penalty value for Player 2 is the difference between the damage of Player 1 and 2. Therefore, Player 2's objective to have a commensurate damage deemed both sufficient for deterrence and yet credible. Player 2 is operating under the proportional deterrence strategy.

3.6 Strategic Frameworks and their Imposed Constraints on Best Responses

For each scenario, there will be one damage matrix and three payoff matrixes that correspond with the three nuclear deterrence theories covered, proportional deterrence, minimum deterrence, and counterforce. Each of these three frameworks has its own set of response constraints that yield different Nash equilibria. The proportional deterrence constraint states that any response by Player 2 to Player 1 must not exceed the lexicographical damage value that Player 2 received from Player 1. This is due to the non-credibility inherit in escalatory moves. The minimum deterrence constraints require that Player 2 can only respond to aggression with either no action or a countervalue attack. Counterforce constraints only allow for either no action or a counterforce response to any aggression.

3.7 Assumptions

We analyze these scenarios assuming that both players have perfect and complete information and can only choose one course of action for each scenario. The game is assumed to be an extensive form game. We look for the subgame perfect equilibrium, which is defined as Player 1's best choice, given that Player 2 chooses the best response. Each player can only choose one strategy, and since the decision can only be made one time, there are no mixed strategies. We also evaluate only one round of an attack and a response. We analyze what Player 2's best options are to create deterrence against Player 1.

Another vital assumption we are making for all three scenarios is that both players have successfully guarded themselves from enemy counterforce attacks. This is due to improved hardening, mobility and defense systems. This will allow the Player 2 to respond with their forces relatively intact. It is unlikely that any nation will survive with no damage. However, if we assume both players to have large arsenals (1000 +), then as long as the majority of the attacked arsenal and deployment systems survive, the balance of power will not be impacted in any significant way.

3.8 Scenarios

The scenarios compare two adversaries. Each attack option is an installation or target type. Each alliance is limited to six possible options, from which they may only pick one. We evaluate one round of nuclear exchanges. The options are to 1) no nuclear strike, 2) launch a counterforce attack, 3) nuclear demonstration, 4) tactical strike against military targets, 5) economic/ infrastructure target, and 6) countervalue targets. Each action has four types of associated costs, which are types of damage. These costs are political, military, economic, and civilian casualties. The decision variables will be label with a P and two numbers; the first number indicates the player (1 or 2), and the second number indicates which of the six options is selected.

We assume that both countries may inflict equivalent damage as they receive each for each objective category and attack type. If two countries suffer equal economic damage in terms of monetary value, the relative difference in their economy sizes can make the comparable damage suffered feel drastically different. We do not account for these economic differences. The players may represent countries or coalitions. We also assume that only two players are involved, and no other countries interfere. We evaluate three different scenarios. These scenarios determine the damage parameters for each country's decision variable.

IV. Results

4.1 Scenario 1: Status Quo

This status quo scenario represents two players who are currently in a state of equilibrium where neither player is damaging the other. As a result, the first scenario compares two adversaries that have identical cost functions with zero damage for no nuclear strike.

4.1.1 Damage Matrix

The resulting damage matrix for Player 1 and Player 2 is seen in Figure 1 Status Quo Damage Matrix. The cells, which represent combinations of attacks by Player 1 and responses by Player 2, are color-coded to reflect the damage of Player 2's response relative to the damage inflicted by Player 1's attack. Gray cells indicate an escalatory response since it is much more severe than the attack. These more severe responses may lack credibility if Player 2 is perceived as not wanting to escalate the nuclear war. Yellow cells represent weak responses. Weak responses are options that do significantly less damage to Player 1 than Player 1 inflicted on Player 2. The responding player may evaluate these as insufficient to deter since the original attacker ends in a better relative position. The green highlighted cells represent decision variables that are considered to be of equivalent levels of damage and represent credible deterrence options under proportional deterrence theory. The damage values apply to all three frameworks.

	No Nuclear Strike P21		Counterforce P22		Nuclear Demonstration P23		Tactical Strike P24		Economic/Infrastructure P25		Countervalue P26	
	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2
No Nuclear Strike P11	A	A	F	B	A	A	C	B	E	B	G	B
Counterforce P12	B	F	F	F	B	F	C	F	E	F	G	F
Nuclear Demo P13	A	A	F	B	A	A	C	B	E	B	G	B
Tactical Strike P14	B	C	F	C	B	C	C	C	E	C	G	C
Econ Strike P15	B	E	F	E	B	E	C	E	E	E	G	E
Countervalue P16	B	G	F	G	B	G	C	G	E	G	G	G

Figure 1 Status Quo Damage Matrix

The damages, shown in Figure 1, indicate that the counterforce option for Player 1, P12, is ineffective in that it does not decrease the damage that Player 2 can inflict upon Player 1. This implication is due to the hardening, mobility, and defenses of Player 2's forces. The framework does not depend upon this assumption. An alternative assessment on the effectiveness of a counterforce strike and hence the possible response may be incorporated.

Figure 1 has eight response combinations that are deterrent under a proportional deterrent framework. As the name suggests, the deterrent options against Player 1are equivalent responses by Player 2. The only exception is with nuclear demonstration, which can be responded to with option P21. We build on the damage matrix to construct the following three penalty matrices as game theory payoff matrices.

4.2.2 Penalty Matrices

We use Figure 1 to construct the following three penalty matrices. We then compare the resulting Nash equilibria to determine which framework has the most credible deterrence effect. White and blue cells represent the feasible response sets. The blue cells represent the Nash equilibrium for each framework. Dark grey cells represent responses that are not in the feasible region. Under proportional deterrence escalatory responses are not feasible. Under minimum and counterforce deterrence any option besides the respective policy is not feasible.

Proportional Deterrence

Figure 2 displays the penalty matrix under a proportional deterrence framework. Proportional deterrence allows the responding Player 2 to respond with anything as long as it is not considered escalatory. We shaded the non-feasible responses dark gray. Escalatory responses are non-feasible in a proportional deterrence framework.

	No nuclear strike P21		Counterforce P22		Nuclear Demo P23		Tactical Nuke P24		Econ Strike P25		Countervalue P26	
	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2
No nuclear strike P11	0	0	3132	-3120	0	1	20	-8	1303	-1290	5505	-5490
Counterforce P12	12	3120	3144	0	12	3121	32	3112	1315	1830	5517	-2370
Nuclear Demo P13	1	-1	3133	-3121	1	0	21	-9	1304	-1291	5506	-5491
Tactical Nuke P14	12	8	3144	-3112	12	9	32	0	1315	-1282	5517	-5482
Econ Strike P15	13	1290	3145	-1830	13	1291	33	1282	1316	0	5518	-4200
Countervalue P16	15	5490	3147	2370	15	5491	35	5482	1318	4200	5520	0

Figure 2 Scenario 1 Status Quo--Proportional Deterrence Penalty Matrix

A proportional deterrence framework favors responding with the same type of attack. Player 2's penalty is calculated by subtracting Player 2 damage from Player 1. The closer the damage values of Player 2 and Player 1, the smaller the difference in penalty values. Since Player 2 is minimizing their penalty, we see that Player 2 favors responding in kind. The Nash equilibrium results become a subset of the proportional deterrence set in the damage matrix.

In this framework, Nash equilibrium solutions are seen as both credible and having sufficient deterrence against Player 1 by Player 2. Figure 2 shows a Nash equilibrium solution for every option except for P13, a nuclear demonstration. This means that Player 1 is not deterred from using nuclear demonstration against Player 2 since Player 2 has no credible response. However, if Player 2 lacks the capability to for responding in kind, such as no tactical nuclear weapons or small yield weapons for an economic strike, the associated column(s) should be removed. In those cases, Player 2 may not have a credible deterrence for those types of attack.

Minimum Deterrence

This minimum deterrence matrix, shown in Figure 3, is constructed under the assumption that Player 2 is operating under a minimum deterrence framework. Hence, Player 2 deters by an overwhelming retaliation of striking the attacker’s cities. As a result, the only option for Player 2 as a response is P26, countervalue strike, for every scenario except as a response to P11, no nuclear strike. Player 2 is only allowed to respond to attacks by Player 1 because the scenario assumes both players are not in conflict. Player 1 is allowed to use every option they have.

	No nuclear strike P21		Counterforce P22		Nuclear Demo P23		Tactical Nuke P24		Econ Strike P25		Countervalue P26	
	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2
No nuclear strike P11	0	0	3132	-3120	0	1	20	-8	1303	-1290	5505	-5490
Counterforce P12	12	3120	3144	0	12	3121	32	3112	1315	1830	5517	-2370
Nuclear Demo P13	1	-1	3133	-3121	1	0	21	-9	1304	-1291	5506	-5491
Tactical Nuke P14	12	8	3144	-3112	12	9	32	0	1315	-1282	5517	-5482
Econ Strike P15	13	1290	3145	-1830	13	1291	33	1282	1316	0	5518	-4200
Countervalue P16	15	5490	3147	2370	15	5491	35	5482	1318	4200	5520	0

Figure 3 Scenario 1 Status Quo-- Minimum Deterrence Penalty Matrix

The minimum deterrence framework yields drastically different results from a proportional deterrence framework. For Player 2, options P22 through P25 are all non-feasible because minimum deterrence states that a countervalue response always causes sufficient deterrence against Player 1. The resulting penalty matrix has two Nash equilibria, (P11, P21) and (P13, P26).

The second Nash equilibrium occurs when Player 1 choses a nuclear demonstration. This is a result of the binary decision Player 2 has to take to all of Player 1's options. What threshold of damage requires an overwhelming response? With the current damage and priority values in this matrix, the answer is a nuclear demonstration, P13. However, the solution changes depending on every Player 2's damage matrices and priority order.

The ultimate conclusion of minimum deterrence requires inaction until a minimum threshold of damage is taken, at which point a counterforce attack is executed. This has two weakness. First, it offers no deterrence options for any actions that are under that specific damage threshold. Second, if Player 2 is forced to retaliate with countervalue, that strike will most likely escalate the situation even more since Player 1 receives considerably more damage. Lack of viable responses remains regardless of the damage threshold used for Player 1 and 2. If the second Nash equilibrium point had been (P16, P26), then there would have been not credible responses by Player 2 for actions P12 through p15.

Counterforce Deterrence

The counterforce penalty matrix, shown in Figure 4, has several similarities with the minimum deterrence penalty matrix. Player 2's strategy removes options P23 through P26. Since both nations are not in conflict, Player 2 can only respond to actions P12 through P16. Similar to the minimum deterrence matrix, this penalty matrix has two Nash equilibrium. The Nash equilibriums for this solution set are (P11, P21) and (P13, P22). Like the previous two penalty matrices, the Nash equilibrium is still (P11, P21). This holds true for Player 1 and 2 for response set of Player 2 other than P21. Similar to minimum deterrence, counterforce only offers two points of credible deterrence, where the response inflicts damage commensurate with the attack, (P11, P21) and (P12, P22) with response in kind.

	No nuclear strike P21		Counterforce P22		Nuclear Demo P23		Tactical Nuke P24		Econ Strike P25		Countervalue P26	
	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2
No nuclear strike P11	0	0	3132	-3120	0	1	20	-8	1303	-1290	5505	-5490
Counterforce P12	12	3120	3144	0	12	3121	32	3112	1315	1830	5517	-2370
Nuclear Demo P13	1	-1	3133	-3121	1	0	21	-9	1304	-1291	5506	-5491
Tactical Nuke P14	12	8	3144	-3112	12	9	32	0	1315	-1282	5517	-5482
Econ Strike P15	13	1290	3145	-1830	13	1291	33	1282	1316	0	5518	-4200
Countervalue P16	15	5490	3147	2370	15	5491	35	5482	1318	4200	5520	0

Figure 4 Scenario 1 Status Quo-- Penalty Matrix Counterforce Deterrence

4.3.3 Scenario 1 Status Quo Results

All three penalty matrices had the same Nash equilibria solution of (P11, P21). However, they differ in the number Nash equilibrium solutions present. Proportional deterrence has five Nash equilibrium solutions. Minimum and counterforce only have two. Responses by Player 2 that do not have a Nash equilibrium indicates that there is no credible deterrence against that option against Player 1.

The increased number of Nash equilibrium solutions in proportional deterrence allows Player 2 to respond to the actions of Player 1 credibly and enhances deterrence across almost every option that Player 1 can make. Proportional deterrence is the only framework that shows Player 2 has no good response to Player 1 conducting a nuclear demonstration. This is in stark contrast to the other frameworks, which automatically responded to nuclear demonstrations with their respective attacks. Under minimum deterrence and counterforce, Player 2 only has one Nash equilibrium to one of Player 1's moves, which is the case of responding in kind. Every other action Player 1 can take may not be deterred by Player 2.

4.2 Scenario 2: Conflict with Mirrored Priority Order

Both players use the same priorities as the previous scenario. This scenario examines when the two players are already in a non-nuclear conflict. As such, Player 1 is receiving damage through political, economic, or conventional military attacks. When an adversary is at a disadvantage, they are more likely to search for options that can improve their position. We represent this in the second scenario by changing the cost matrix of Player 1. Now even when Player 2 does not conduct a nuclear strike, Player 1 receives damage in the damage matrix. This represents two adversaries in a state of conflict where Player 1 is currently taking damage, either due to shifting political landscape, increased military presence, or economic wars, and is seeking to improve its relative position. For this scenario, we gave Player 1 a political and military damage value of 20.

4.2.1 Damage Matrix

The damage matrix for the second scenario differs from the first due to the differing cost parameter used to calculate damage. In this scenario, Player 1 takes damage by not executing a nuclear strike, such as being under sanctions or losing a conventional war. They are now incentivized to look in the damage matrix to see whether they can choose an option that can decrease the amount of damage being taken.

	No nuclear strike P21		Counterforce P22		Nuclear Demonstration P23		Tactical Strike P24		Economic/Infrastructure P25		Countervalue P26	
	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2
No nuclear strike P1	C	A	F	B	C	A	C	B	E	B	G	B
Counterforce P12	C	F	F	F	C	F	C	F	E	F	G	F
Nuclear Demo P13	C	A	F	B	C	A	C	B	E	B	G	B
Tactical Nuke P14	C	C	F	C	C	C	C	C	E	C	G	C
Econ Strike P15	C	E	F	E	C	E	C	E	E	E	G	E
Countervalue P16	C	G	F	G	C	G	C	G	E	G	G	G

Figure 5 Conflict and Mirrored Values Damage Matrix

Unlike the Status Quo Damage Matrix in Figure 4, Figure 5 shows Player 2 does not have proportional responses across every action Player 1 makes. Having an entire row be gray means that Player 1 has not done any action that warrants a change action from Player 2. For example, if Player 1 chose action P13, Player 2 would not have an action that would not be perceived by Player 1 (and the world community) as not escalatory. This is because the scenario is running under the assumption that even action P21, Player 2's current non-nuclear course of action, is causing damage to Player 1.

4.2.2 Penalty Matrices

Proportional Deterrence

The introduction of current damage to Player 1 does not drastically affect the penalty matrices. The Nash equilibrium responses for all the frameworks remain the same with the exception of the proportional deterrence matrix which has one less Nash equilibrium than in the previous scenario. The specific values of the Nash equilibrium are slightly different, but not enough to change any other Nash equilibrium locations.

	No Nuclear Strike P21		Counterforce P22		Nuclear Demo P23		Tactical Nuke P24		Econ Strike P25		Countervalue P26	
	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2
No Nuclear Strike P1	55	-55	3187	-3175	55	-54	75	-63	1358	-1345	5560	-5545
Counterforce P12	67	3065	3199	-55	67	3066	87	3057	1370	1775	5572	-2425
Nuclear Demo P13	56	-56	3188	-3176	56	-55	76	-64	1359	-1346	5561	-5546
Tactical Nuke P14	67	-47	3199	-3167	67	-46	87	-55	1370	-1337	5572	-5537
Econ Strike P15	68	1235	3200	-1885	68	1236	88	1227	1371	-55	5573	-4255
Countervalue P16	70	5435	3202	2315	70	5436	90	5427	1373	4145	5575	-55

Figure 6 Scenario 2 Proportional Deterrence Penalty Matrix

The Nash equilibrium solution for the proportional deterrence framework is still (P12, P22). However, now they are skewed in favor of Player 2. Player 1 has a higher penalty than in the previous scenario, and Player 2 has a negative penalty. The Nash equilibrium points remain as mirror responses back by Player 2, with the exception of P13 and P15. Both of these options lack a Nash equilibrium. Option P13 has no good responds by Player 2 since the entire row is escalatory. Therefore, the proper response for Player 2 is to not change their course of action. If Player 1 chooses option P15, Player 2 has options that are non-escalatory, but without a Nash equilibrium. Therefore Player 2 will choose the option with the smallest penalty for themselves that is not escalatory. This is to retaliate with an economic strike.

The decreasing amount of Nash equilibria represents a decrease in deterrence. Under proportional deterrence theory, the deterrent response is to respond with an equivalent strike.

Minimum Deterrence and Counterforce Deterrence

The minimum deterrence and counterforce deterrence penalty matrices have the same structure as in Scenario 1. There are only two Nash equilibria, each in the same location as before. This represents a lack of credible deterrence across every other option that Player 1 makes for which Player 2 does not have a Nash equilibrium.

	No Nuclear Strike P21		Counterforce P22		Nuclear Demo P23		Tactical Nuke P24		Econ Strike P25		Countervalue P26	
	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2
No Nuclear Strike P1	55	-55	3187	-3175	55	-54	75	-63	1358	-1345	5560	-5545
Counterforce P12	67	3065	3199	-55	67	3066	87	3057	1370	1775	5572	-2425
Nuclear Demo P13	56	-56	3188	-3176	56	-55	76	-64	1359	-1346	5561	-5546
Tactical Nuke P14	67	-47	3199	-3167	67	-46	87	-55	1370	-1337	5572	-5537
Econ Strike P15	68	1235	3200	-1885	68	1236	88	1227	1371	-55	5573	-4255
Countervalue P16	70	5435	3202	2315	70	5436	90	5427	1373	4145	5575	-55

Figure 7 Scenario 2 Minimum Deterrence Penalty Matrix

	No Nuclear Strike P21		Counterforce P22		Nuclear Demo P23		Tactical Nuke P24		Econ Strike P25		Countervalue P26	
	P1	P2	P1	P2	P1	P2	P1	P2	P1	P2	P1	P2
No Nuclear Strike P1	55	-55	3187	-3175	55	-54	75	-63	1358	-1345	5560	-5545
Counterforce P12	67	3065	3199	-55	67	3066	87	3057	1370	1775	5572	-2425
Nuclear Demo P13	56	-56	3188	-3176	56	-55	76	-64	1359	-1346	5561	-5546
Tactical Nuke P14	67	-47	3199	-3167	67	-46	87	-55	1370	-1337	5572	-5537
Econ Strike P15	68	1235	3200	-1885	68	1236	88	1227	1371	-55	5573	-4255
Countervalue P16	70	5435	3202	2315	70	5436	90	5427	1373	4145	5575	-55

Figure 8 Scenario 2 Counterforce Deterrence Penalty Matrix

4.2.3 Scenario 2 Results

When two players are in conflict the deterrent effect of proportional deterrence decreases slightly. However, it still offers much greater response set than under minimum and counterforce frameworks of deterrence. These two frameworks only offered two Nash equilibrium Points to proportional deterrence's 4 Nash equilibrium points.

4.3 Scenario 3: Conflict with Differing Priority Order

This scenario is similar to scenario two. The only difference is that the priority order for each player differs from the other. Player 2 keeps the same priority order as the previous scenarios, however, Player 1's priority shifts to increase the importance of political and military, relative to economic impacts, as shown in Table 7. Each player will use the same damage level scale as in the previous two scenarios. The difference will be how each player calculates the final lexicographical damage value. This is because the priority order ranking used to calculate the damage value will be modified for Player 1. This will keep the damage value of each country consistent with to their own priorities. This scenario was created to test the limits of the framework; we explore what type of results when the priority order and damage were asymmetrical.

Table 7 Modified Priority Order Ranking Player 1

Value	Category
10	Political
100	Military
1	Economic
1000	Casualties

4.3.1 Damage Matrix

Unlike the previous two scenarios, this system examines two players that are currently in conflict and have different values. In this scenario, fewer sets of proportionally deterrence responses exist.

	No Nuclear Strike P21		Counterforce P22		Nuclear Demonstration P23		Tactical Strike P24		Economic/Infrastructure P25		Countervalue P26	
	P1	P2	P1	P2	P1	P2	P1	P2	P1	P2	P1	P2
No Nuclear Strike P1	D	A	F	B	D	A	D	B	E	B	G	B
Counterforce P12	D	F	F	F	D	F	D	F	E	F	G	F
Nuclear Demo P13	D	A	F	B	D	A	D	B	E	B	G	B
Tactical Nuke P14	D	C	F	C	D	C	D	C	E	C	G	C
Econ Strike P15	D	E	F	E	D	E	D	E	E	E	G	E
Countervalue P16	D	G	F	G	D	G	D	G	E	G	G	G

Figure 9 Damage Matrix with Conflict and Different Values

4.3.2 Penalty Matrices

The penalty matrices in scenario three are similar to the ones in Scenario 1 and 2. The change in priority order did not change the location of Nash equilibrium responses. However, it did reduce the number responses in a proportional deterrence framework to three. This is a result of more options, in this case P14, not having an adequate response by Player 2 that would not be seen as escalatory.

Proportional Deterrence

	No Nuclear Strike P21		Counterforce P22		Nuclear Demo P23		Tactical Nuke P24		Econ Strike P25		Countervalue P26	
	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2
No Nuclear Strike P11	550	-550	3871	-3859	550	-549	750	-738	1583	-1570	5605	-5590
Counterforce P12	670	2462	3991	-847	670	2463	870	2274	1703	1442	5725	-2578
Nuclear Demo P13	560	-560	3881	-3869	560	-559	760	-748	1593	-1580	5615	-5600
Tactical Nuke P14	670	-650	3991	-3959	670	-649	870	-838	1703	-1670	5725	-5690
Econ Strike P15	680	623	4001	-2686	680	624	880	435	1713	-397	5735	-4417
Countervalue P16	700	4805	4021	1496	700	4806	900	4617	1733	3785	5755	-235

Figure 10 Scenario 3: Proportional Deterrence Penalty Matrix

Figure 10 shows the proportional deterrence penalty matrix for scenario 3 with asymmetric priorities. Player 1 continues to minimize damage to their country as calculated by their value system. Player 2 is trying to minimize the difference in damage to the countries as assessed by their respective value (priority) structures.

In scenario 3, Player 2 does not have any deterrent effect against the use of tactical nuclear weapon that would not be perceived by Player 1 as an escalated response. Unlike P13, nuclear demonstration, P14 would cause harm to Player 2. This is a situation where solely relying on proportion deterrence would not have the necessary deterrent effect to prevent Player 1 from using nuclear weapons.

Minimum Deterrence and Counterforce Deterrence

The Penalty matrices for minimum and counterforce deterrence frameworks still have the same two Nash equilibrium points. The change in introduction of conflict and a change in values did not change resulting location of the despite the change in values. From this, we can surmise that the deterrent effect of both these strategies stays the same as in the previous two scenarios. Both of these policy frameworks rely on a single kind of response.

Minimum deterrence relies on the threat of escalation, and so it is not affected by the change in Player 1's priority order. Counterforce aims at crippling the opponent's offensive capability. Counterforce is does not rely on escalation, however is also not hindered by what might be perceived by Player 1 as escalation. Since neither of the frameworks fear escalation, the change in priority orders not change their conclusion.

	No Nuclear Strike P21		Counterforce P22		Nuclear Demo P23		Tactical Nuke P24		Econ Strike P25		Countervalue P26	
	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2
No Nuclear Strike P1	550	-550	3871	-3859	550	-549	750	-738	1583	-1570	5605	-5590
Counterforce P12	670	2462	3991	-847	670	2463	870	2274	1703	1442	5725	-2578
Nuclear Demo P13	560	-560	3881	-3869	560	-559	760	-748	1593	-1580	5615	-5600
Tactical Nuke P14	670	-650	3991	-3959	670	-649	870	-838	1703	-1670	5725	-5690
Econ Strike P15	680	623	4001	-2686	680	624	880	435	1713	-397	5735	-4417
Countervalue P16	700	4805	4021	1496	700	4806	900	4617	1733	3785	5755	-235

Figure 11 Scenario 3: Minimum Deterrence Penalty Matrix

	No Nuclear Strike P21		Counterforce P22		Nuclear Demo P23		Tactical Nuke P24		Econ Strike P25		Countervalue P26	
	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2	p1	p2
No Nuclear Strike P1	550	-550	3871	-3859	550	-549	750	-738	1583	-1570	5605	-5590
Counterforce P12	670	2462	3991	-847	670	2463	870	2274	1703	1442	5725	-2578
Nuclear Demo P13	560	-560	3881	-3869	560	-559	760	-748	1593	-1580	5615	-5600
Tactical Nuke P14	670	-650	3991	-3959	670	-649	870	-838	1703	-1670	5725	-5690
Econ Strike P15	680	623	4001	-2686	680	624	880	435	1713	-397	5735	-4417
Countervalue P16	700	4805	4021	1496	700	4806	900	4617	1733	3785	5755	-235

Figure 12 Scenario 3: Counterforce Deterrence Penalty matrix

4.3.3 Scenario 3 Results

In a Scenario with conflict and asymmetric values, the proportional deterrence framework fails to offer deterrence to any response that might be perceived as escalatory to Player 1. Minimum deterrence and counterforce deterrence are not affected by this same shortcoming. However, they are still hindered by the fact that they only have Nash equilibrium solutions. In comparison, the proportional deterrence had three Nash equilibrium solutions. This leaves proportion deterrence as having more credible deterrence against more options, but completely vulnerable to attacks that it cannot respond to in kind.

It is questionable how useful these results are since they mix two separate priority values. An alternate method that might garner better results is two create two entirely different penalty matrices. Within each matrix both players will use the same priorities. However, each matrix will use the priority values of either Player 1 or 2. If both matrices have the same set of Nash equilibrium points then the game retains similar deterrence as if they both had the same priority values system. However, if the matrices have Nash equilibriums then the system is unstable. Conflict is more likely to occur since each player might not have the deterrence they believe they do.

4.4 Analysis on Sufficient and Credible Deterrence.

In the minimum deterrence and counterforce deterrence frameworks, Player 1 receives similar penalty values down their respective columns. To Player 1 the small differences will be negligible if they fall within a certain range. At a first glance this demonstrates that Player 1 is deterred from any action due to every action carrying a large penalty. However, this ignores Players 2's penalty values. Player 2's penalty is calculated by the difference between the damage incurred by both players. Since both players are attempting to minimize the penalty this will lead to a rather large incentive for Player 2 to react to the smallest provocation by Player 1. So while Player 1 might see all penalties as equivalent, Player 2 sees large penalty difference among all the different actions.

This leads us to the concept of how truly sufficient and credible each action truly is. A negative penalty by Player 2 is highly sufficient at deterring Player 1 since the difference in damage is greater for Player 1 and 2. However, the more negative a number is the more escalatory the response is as well. This decreases the credibility of such a response. Similarly, a positive penalty values for Player 2 means that they have suffered more damage than Player 1.

This represents an insufficient response by Player 2. However, what has been lost in sufficiency it has gained in credibility. Player 2's penalty values are limited to only two columns in the counterforce and minimum deterrence penalty matrices for scenario 1 and 2. Within these columns Player 2 is constrained to options that range from very negative to positive. However what might appear as a perfect Nash equilibrium for Player 2, the most negative available response, might also be the least credible as it requires the most escalation.

Another issue with the penalty matrices arises in the counterforce matrix of scenario 1 and 2. Player 1 might not see a practical difference between any of the options as the penalty values for Player 1 are all very similar. However, the penalty value for Player 2 has a positive value when Player 1 choose P16. The large positive penalty value for Player 2 means that Player 2 has taken significantly more damage than Player 1. This presents a scary situation for Player 2, as this means that if Player 1 sees no practical difference in any of their offensive actions, they might as well choose the options that causes the most damage. Paradoxically, a counterforce doctrine might incentivize either massive escalation or a very limited attack. This is because a counterforce response to these actions carries either negative penalty value for Player 2, representing low credibility, or a high positive penalty, value which represent lack of sufficient deterrence.

V. Conclusions

All three deterrence options evaluated where created in different technological periods with vastly different goals. Minimum deterrence was created with the purpose of deterring the U.S and USSR from launching large offensives at each other after WWII. The technology at the time limited the number of available responses by both sides and were left with mass retaliation as the only credible response. The shift to counterforce deterrence occurred with the rise of advance ballistics, targeting and information networks which made accurate targeting possible. However, the existence of hardened, hidden and mobile nuclear weapons systems along with local and global defenses has once against changed the balance of power. Counterforce can no longer ensure adequate destruction to the opponent's offensive nuclear systems.

We have demonstrated that to maintain proper deterrence requires going beyond counterforce as a single one size response. Counterforce fails to account for nontraditional nuclear weapons attacks such as targeting economic centers or deployed tactically in a battlefield. Counterforce is a large commitment that could drastically escalate the conflict. In the proportional deterrence penalty matrices, we have demonstrated that there exists another policy that concisely offers more credible deterrence options while reducing the risk of escalating conflicts. However, even this nuclear policy falls short, as seen in Scenario 3. If both countries have asymmetric priority orders, then it may be impossible to respond without escalating the conflict.

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This research game theory framework evaluates the resilience of nuclear deterrence options between two players. We use lexicographic prioritization to value four priorities of political, military, economic, and civilian casualties. The value order may be varied. We demonstrate our approach with six player choices of no nuclear strike, demonstration, counterforce, tactical military, economic, or countervalue strike. We use game theory to construct and analyze the resulting damage matrix. We conclude that credible deterrence requires having at least equivalent offensive damage capabilities.

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