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Statistically Defensible Wind Tunnel Models

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THESIS

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Abstract

Wind tunnels are used to test scale-model air frames in order to collect aerodynamic data. The Subsonic Aerodynamic Research Laboratory (SARL) Wind Tunnel is a low speed wind tunnel located at Wright-Patterson Air Force Base. The SARL Wind Tunnel team approached AFIT for assistance in creating statistically defensible models for the conditions inside the wind tunnel. During a wind tunnel test, pressure sensors cannot be placed at the test model. Instead, pressure is measured by a pitot probe permanently mounted in the corner of the test chamber. The pressure at the model location is predicted from the measurements taken by this pitot probe. This thesis analyzes the models used previously to predict pressure and creates new models using more rigorous statistical methods. These new models have a high prediction accuracy and follow all the necessary assumptions to ensure accuracy for the SARL wind tunnel team and their customers.

Dedicated to my brother.

Acknowledgements

Foremost, I would like to express my sincere gratitude to my advisor, Lt Col Brooks, the SARL team, and all of my friends and family that have helped me get here.

Timothy A. Roche

Table of Contents

Page

List of Figures

List of Tables

STATISTICALLY DEFENSIBLE WIND TUNNEL MODELS

I. Introduction

1.1 Motivation and Background

The Subsonic Aerodynamic Research Laboratory (SARL) Wind Tunnel is a low speed wind tunnel located at Wright-Patterson Air Force Base. Low speed wind tunnels operate at wind speeds between MACH 0 and MACH 0.5. They test scale models of aircraft, air foils or other models, providing aerodynamic data on the model. The conditions around the test model are measured, and measurements are taken from sensors within the model, to determine lift, drag, air resistance, and other important properties. [1]

Pressure is an important measurement during a test, as it is used in calculating many of the important properties. While testing, pressure is measured using a pitot probe mounted in the corner of the wind tunnel test chamber. A pitot probe, also known as a pitot-static probe, is a common tool for measuring pressure and air speed. The probe measures total (or stagnation) and static pressure, which is then used to calculate other measurements such as dynamic pressure and airspeed [2]. The conditions at this mounted probe do not directly match the conditions in the center of the wind tunnel where the test model is located. Our goal is to predict the conditions in the center based off measurements from the mounted pitot probe.

1.2 Problem Statement

During a test, the conditions inside the test chamber are measured using the pitot probe mounted in the wind tunnel. However, the measurements taken at this location may not be equivalent to the conditions at the model being tested. In the past, the SARL team used a simple linear fit model to predict the conditions at the test location.

The SARL Wind Tunnel team asked for a "statistically defensible" method of predicting the conditions at the test location based off measurements taken by the mounted pitot probe. There are three different response variables, total pressure, static pressure, and dynamic pressure, requiring a model for each.

Currently, a simple linear fit model is used to estimate the pressures at the test site, based off the measurements taken at the mounted probe. For this thesis, a number of other factors were tested using standard regression techniques in an attempt to create a more accurate and defensible model. The potential factors include different measurements from the pitot probe as well as several environmental factors.

1.3 Research Objectives

The primary goal for this thesis is to test the models used previously by the SARL team, and to create "statistically defensible" models using more advanced regression techniques. The SARL team is also interested in modeling and predicting error in the future, therefore, the residual terms from the linear regression models are of particular interest. The SARL team also asked to investigate the effect, if any, external weather conditions (temperature, wind speed, wind direction, etc...) have on the pressures in the wind tunnel.

1.4 Organization of the Thesis

This thesis is structured as follows: first, the methodology behind the data collection and regression techniques are discussed; next, the models used by the SARL team in the past are examined; and lastly, new models are created using more rigorous regression techniques.

II. Methodology

2.1 Overview

This chapter discusses the data, as well as the data collection process. A brief explanation of linear regression and the statistical process used in this thesis is also provided. Also covered are methods used to evaluate linear regression models, as well as important assumptions the models must meet.

The SARL team asked for "statistically defensible" models. Linear regression is a very common and widely used statistical process. By applying this common method, and assuring the assumptions are met for the models created, the models will meet the requirement of being statistically defensible.

All analyses was performed using JMP v15.0, and an α of 0.01 was used for statistical inference. A validation set of approximately 40% of the data was randomly selected for validation of the model. The same validation set was used for each model.

2.2 Data Collection

The data for model building came from a calibration test performed following the replacement of filter screens within the SARL wind tunnel over the summer of 2020. A true randomized design of experiment could not be accomplished as the only factor controlled by the wind tunnel team is the speed of the turbine, meaning the factors of interest could not be set manually. Further, the wind tunnel equipment does not allow for a quick change in speed. Due to the combination of these two limitations, the SARL team was not able to perform a true randomized design for the data collection.

The methodology used for data collection was to collect data as the wind tunnel accelerated from 0 MACH to 0.5 MACH. The speed of the wind tunnel would be increased in increments of 0.05 MACH and a set of data points would be collected. The speed would then be held constant and another set of data points at that speed was collected after five minutes, and then again after ten minutes. After finishing at each MACH level, the tunnel would accelerate to the next MACH level and the process would repeat. Data were also collected as the tunnel decelerated after reaching 0.5 MACH. Only one data point was taken at each MACH level during deceleration. Ten separate tests were performed, and a total of 393 data points were collected.

In some of the later tests, the time between measurements was shortened to four minutes. This allowed the wind tunnel team to run the test twice per day, as they had been been unable to test for a full week due to weather conditions. The SARL team does not believe this change had any affect on the data collected.

During these calibration tests, pressure sensors were placed on the center platform of the test chamber where the test model would be placed during an actual test. These pressure sensors measured total, static and dynamic pressure at the test platform. There are two measurements for dynamic pressure, MR1 for MACH 0 to MACH 0.23, and MR2 for MACH 0.23 to MACH 0.5. These measurements are used as the response variable for the regression model, and are summarized in Table 1.

Table 1. Response Variable Characteristics

Variable	Description	Mean	StDev	Min	Max
$P0_{PRB}$	Total Pressure	14.3511	0.0650	14.1963	14.4455
PS_{PRB}	Static Pressure	13.6394	0.6698	12.1344	14.4460
$Q_{PRB}1PSID$	Dynamic Pressure MR1	0.1106	0.1824	0.0000	1.0944
$Q_{PRB}2PSID$	Dynamic Pressure MR2	0.7126	0.6617	-0.0004100	2.1283

The analysis in this thesis focuses on the predictor variables summarized in Table 2. The SARL team performs many corrections and calculations on the data collected in these tests. For this thesis, only the measured data was used.

The predictor variables include measurements for total static, and dynamic pres-

Table 2. Predictor Variable Characteristics

Variable	Description	Mean	StDev	Min	Max
$P0TS$	Pitot Total Pressure	14.3514	0.0652	14.1990	14.4483
$PREF$	Tunnel Reference Pressure	14.6956	0.002880	14.6905	14.7013
PS_{TPRB}	Uncorrected Static Pressure	13.6167	0.6822	12.0908	14.4392
Q_{TPRB}	Uncorrected Dynamic Pressure	0.7285	0.6742	-0.0001000	2.1674
ATM	Reference Total Pressure	14.3570	0.06473	14.1960	14.4470
$PPT4$	Reference Total Pressure	14.4611	0.06405	14.2960	14.5560
$T0TS$	Tunnel Total Temperature	520.1534	9.2715	499.8000	536.1400
$DewPt$	Dew Point	500.5000	15.0304	469.0000	529.0000
WS	Wind Speed	16.3507	4.6669	3.3756	32.0684

sure. A number of reference pressures are also measured, as well as the temperature within the tunnel. External conditions, dew point and wind speed, are also included in the analysis.

Pressure is measured in PSI, wind speed in feet per second, and temperature is measured in Rankine, the imperial equivalent of Kelvin. The SARL team discovered an error in reference total pressure (PPT4), and at their request, it was not included in any analysis.

2.3 Simple Linear Regression

Previously, the SARL team had used a simple linear regression model to accomplish the task of modeling the pressures at the test platform. Simple linear regression is a technique in which one variable, the response variable (normally denoted as Y), is estimated based off another variable, the predictor variable (often denoted as X). The overall goal of a linear regression model is to predict the value of Y for any given value of X. The regression process is performed using a set of collected data, and results in a formula for the simple linear regression model of the form:

$$Y_i = \beta_0 + \beta_1 * X_i + \epsilon_i, \tag{1}$$

where Y_i and X_i are the corresponding values for the i_{th} observed values of Y and X respectively, β_0 and β_1 are the parameters for the y-intercept an slope of the equation respectively, ϵ_i is the random unknown error for the i_{th} observation, for i=1,2,..,n. The prediction equation for a Simple Linear Regression Model is:

$$\hat{Y} = \hat{\beta_0} + \hat{\beta_1} * X, \tag{2}$$

where $\hat{\beta_0}$ and $\hat{\beta_1}$ are the estimates for the parameters β_0 and β_1, and $\hat{Y}$ is the predicted value for the response variable given a value of predictor variable X. The prediction equation is used to estimate the value of the response variable for any given values within the range of the predictor variable. [3]

2.4 Multiple Linear Regression

The new models created in this thesis were built using multiple linear regression. Multiple linear regression has the same premise as simple linear regression, except more than one predictor is used. The general formula for multiple linear regression is:

$$Y_i = \beta_0 + \beta_1 * X_{i1} + \beta_2 * X_{i2} + ... + \beta_K * X_{iK} + \epsilon_i, \tag{3}$$

where Y_i, β_0, and ϵ_i are the same as given in equation 1. The main difference between equation 1 and 3 is that there are up to k possible predictor variables, denoted such that each β_k is the coefficient for the corresponding predictor X_k. The prediction equation for multiple linear regression is:

$$\hat{Y} = \hat{\beta_0} + \hat{\beta_1} * X_1 + \hat{\beta_2} * X_2 + ... + \hat{\beta_K} * X_K \tag{4}$$

where $\hat{\beta_0}$,...,$\hat{\beta_K}$ are the estimates for the parameters β_0,...,β_K, and $\hat{Y}$ is the predicted value for the response variable given values for each predictor X_1,...,X_k. [3]

A technique known as backwards elimination was applied to fit the multiple linear regression models for this thesis. Backwards elimination is a step-wise selection process which considers all the candidate predictors. Then, one by one the predictors with the least significance are removed until all remaining predictors in the model are significant. Significance is determined by comparing the p-value associated with a particular predictor to a pre-selected level of significance denoted by α. A predictor with a p-value smaller than α is said to be significant. Least significance is determined by the magnitudes of the p-values that are larger than α; larger in this regards corresponds to less significant. [3]

These methods are used to fulfill the SARL team's requirement that the models be statistically defensible. Regression is a standard and well studied statistical method, and adhering to this process will provide a statistically rigorous model for the SARL team.

2.5 Model Evaluation

This thesis uses the correlation of determination, R^2, to evaluate the model adequacy and usefulness. The R^2 is an indicator of the strength of a model, measured on a scale between zero and one. Essentially, it is a measures of how well the hypothesized model fits the data. It accounts for the percentage of the total variation explained by the model. The R^2 is calculated as follows:

$$R^2 = 1 - \frac{SSE}{SST}, \tag{5}$$

where, SSE is error sum of squares and SST is the total sum of squares defined as follows:

$$SSE = \sum (Y_i - \hat{Y}_i)^2, \tag{6}$$

$$SST = \sum (Y_i - \bar{Y})^2, \tag{7}$$

where $\bar{Y}$ is the mean of the response variables, Y_i.

An $R^2 = 1$ is an indication that the predictor variables explain all the variation in the response variable, while an $R^2 = 0$ is an indication that the predictor variables are not related to the response variable, and cannot be used in the prediction. The higher the R^2 the more accurate the model, in other words, the closer the predicted value ($\hat{Y}_i$) is to the actual value (Y_i) for all i. [3]

The R^2_{adj} is also used in this thesis. The R^2_{adj} has the same interpretation as the R^2, however, the R^2 will always increase as predictors are added to the model. The R^2_{adj} modifies the R^2 to account for additional predictors by penalizing the R^2 value if the included predictors do not contribute to better explaining the response variable. The equation for R^2_{adj} is:

$$R^2_{adj} = 1 - \frac{(1 - R^2)(N - 1)}{N - K - 1}, \tag{8}$$

where N is the number of observations and K is the number of predictors included in the model. [3]

Lack of fit is a test of whether or not the model fits the data well. In order to do a formal lack-of-fit test, there must be multiple observations with the same measurement for each predictor. For this thesis, the requirements to perform a formal lack-of-fit test were not met. Instead, lack of fit is assessed informally via visual inspection. Lack of fit normally manifests as a pattern, such as a curve, in residual (defined in Equation 11) vs. predicted plots. Having lack of fit in a model usually indicates a variable is missing from the model or that the variables in the model are misspecified. [3]

This thesis also uses mean squared prediction error (MSPE) as a test for model accuracy. The data is randomly divided into a training set and a validation set. The models are build using only the training set, and then tested on the validation set. The SSE for the validation set (SSE_{pred}) is calculated using the same formula shown in Equation 6, but only using the data from the validation set to calculate $\hat{y}_i$. The formula for MSPE is:

$$MSPE = \frac{SSE_{pred}}{n*}, \tag{9}$$

where n* is the number of data points in the validation set. The MSPE is then compared to the model MSE, which for comparison purposes is calculated as follows:

$$MSE = \frac{SSE}{n}, \tag{10}$$

where MSE is normally calculated by dividing SSE by $n - k - 1$. For this thesis the square root of the MSPE and MSE (denoted as RMSPE and RMSE) is used instead of the MSPE and MSE. Using a validation set in this way allows verification that the predictions for the training and validation set are equally accurate.

2.6 Assumption Diagnostics

The error terms, or residuals (e_i), for a linear regression model are calculated as follows:

$$e_i = Y_i - \hat{Y}_i. \tag{11}$$

Linear regression models must meet some assumptions regarding residuals in order to meet the “statistically defensible” requirement. The three main assumptions are normality, independence, and constant variance.

Of the three, the most important assumption is that the residuals are independent of one another. If the residuals are not independent, this is often due to autocorrelation and often manifests as patterns in the residuals based off the order the data is collected in. If autocorrelation is present, the estimated coefficients are inefficient and the prediction equation could be ineffective. This leads to issues in the estimation of variance, which in turn affects all other statistical tests. For this thesis, the Durbin-Watson test is used to test for autocorrelation. The Durbin-Watson test statistic is compared to an upper and lower bound which depends on α, K, and N. If the test statistic is above the upper bound, it is concluded there is no autocorrelation, below the lower bound means there is autocorrelation, and between the two bounds, the test is inconclusive. The Cochrane-Orcutt procedure is used to remedy autocorrelation. [3]

Another important assumption of linear regression is normality. This is the assumption that the residuals follow a normal distribution. However, linear regression is robust against minor departures from normality. Regardless, it is best if the residuals are approximately normally distributed. The Anderson-Darling Goodness-of-Fit test is used to test for normality. Also, normal quantile plots and histograms of the residuals are useful visualization tools when exploring this assumption. If the shape of the histogram resembles the normal curve, and the residuals when graphed on the normal quantile plot are close to linear along the diagonal line, it is indication that the residuals are normally distributed, and deviations from these are indications of non-normality. [3]

Constant variance, also known as homoscedasticity, is the assumption that the variance of the residuals is constant for all observations. Heteroscedasticity, or non-constant variance, is when the variance of the residuals is different for different values of the response variables. Ideally, the variance would be the same for all levels of the response variable. Departures from constant variance would make error prediction difficult, as the distribution of the error terms would be different depending on the values of the variables. The Breusch-Pagan test is used in this thesis to test for constant variance. Also, a residuals vs. the predicted plot is useful visualization tools when exploring this assumption as well. [3]

III. Results and Analysis

This chapter is organized into three sections, one for each response variable explored: total pressure, static pressure, and dynamic pressure. Each section covers the previous methodology used by the SARL team to create their initial prediction model, new model development, remedial measures, and results for each response variable.

3.1 Total Pressure Prediction

3.1.1 Previous Model

The first part of the analysis was to test the original model used by the SARL team. A simple linear regression model was used, with the total pressure measurement from the pitot probe (P0TS) as the predictor, and the reading from the central pressure sensor ($P0_{PRB}$) as the response. Since ultimately a new model was going to be created, no remedial measures were performed on this model, and the residual analysis will be brief. This model was explored to get an understanding of what had been done previously, and to set a baseline for comparison to the new model. After fitting a linear regression to the model building data set ,the prediction equation that resulted was:

$$P0\hat{_{PRB}} = 0.0001325 + 1.0011 * P0TS. \tag{12}$$

This model has an R^2 of 0.9988 for the training set and 0.9986 for the validation set.

There is also a clear autocorrelation issue seen from the pattern in the residual vs. observation graph in Figure 1, confirmed by a Durbin-Watson test. The Durbin-Watson test statistic is 0.3341, well below the lower bound of 1.571, indicating there is autocorrelation.

The residuals do not appear to be normally distributed, as shown in Figure 2, and this was confirmed with an Anderson-Darling Goodness-of-Fit test with a p-value less than 0.0001. There also appears to be non-constant variance, from the residual vs. predicted graph in Figure 3. The Breusch-Pagan test for constant variance depends on independence, and as there is autocorrelation, the results are not reliable and was not performed for this model.

Overall, this model did not meet the necessary assumptions to be labeled as statistically defensible, and a new model was created.

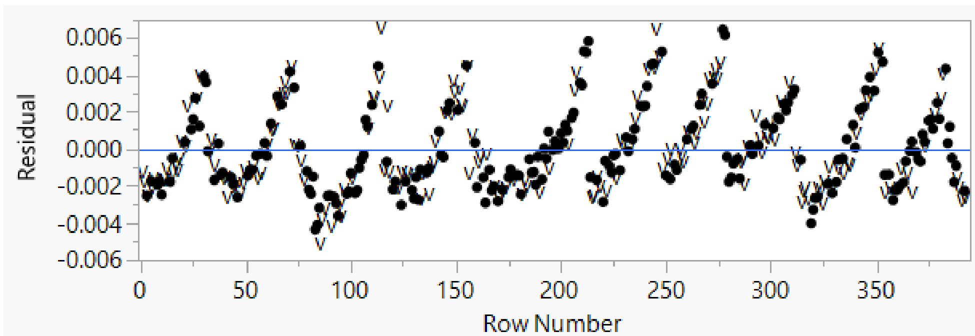

Figure 1. Total Pressure Simple Model: Residual vs. Observation

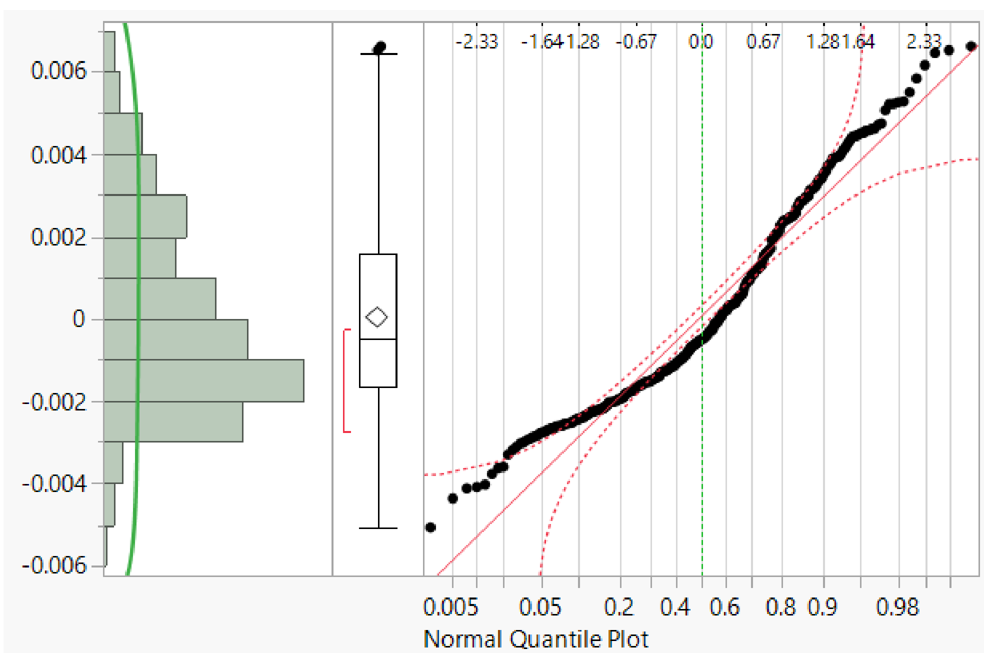

Figure 2. Total Pressure Simple Model: Residual Distribution

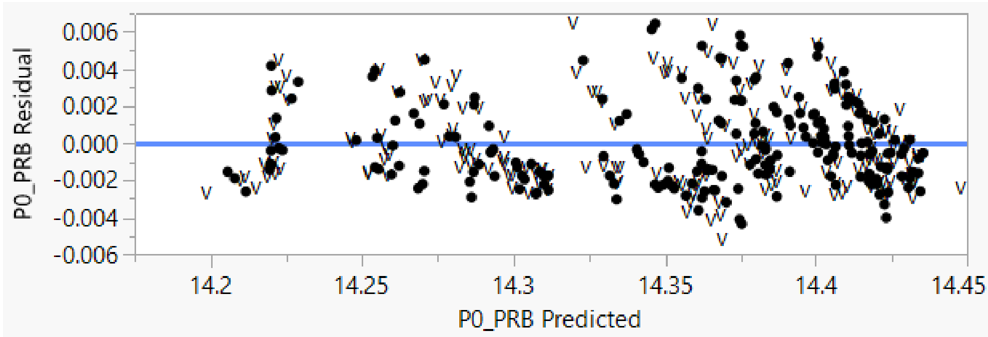

Figure 3. Total Pressure Simple Model: Residual vs. Predicted

3.1.2 New Model Development

A backwards elimination technique was used to build the new model. Starting with a full model including all the predictor variables (see Table 2 for list of predictors), the variables with the highest p-values were removed one at a time until all remaining predictors were significant at alpha = 0.01. The three remaining predictors were P0TS, Reference Total Pressure (ATM), and Tunnel Total Temperature (T0TS). The prediction equation for this new model is:

$$\hat{P0_{PRB}} = 0.04730 + 0.6865 * P0TS + 0.3117 * ATM - 0.00004622 * T0TS. \quad (13)$$

The R^2_{adj} of this model is 0.9998 for both the training and validation set, an increase from the original model. The RMSPE and RMSE for this model were 0.00078 and 0.00090, indicating the model fits the validation set well and has a high prediction accuracy.

As with the previous model, there is autocorrelation for this model, with a Durbin-Watson test statistic of 1.2245, below the lower bound of 1.571. Since autocorrelation may influence the tests for normality and constant variance, these tests will be performed after the autocorrelation is resolved via the Cochrane-Orcutt Procedure.

3.1.3 Remedial Measures

The Cochrane-Orcutt procedure was used to remedy the autocorrelation issue. After one iteration of the Cochrane-Orcutt procedure, the model passed the Durbin-Watson test, with test statistic 2.2381, above the upper bound of 1.779, and it can be concluded there is no longer autocorrelation. The residual vs. observation graph post Cochrane-Orcutt is shown in Figure 4.

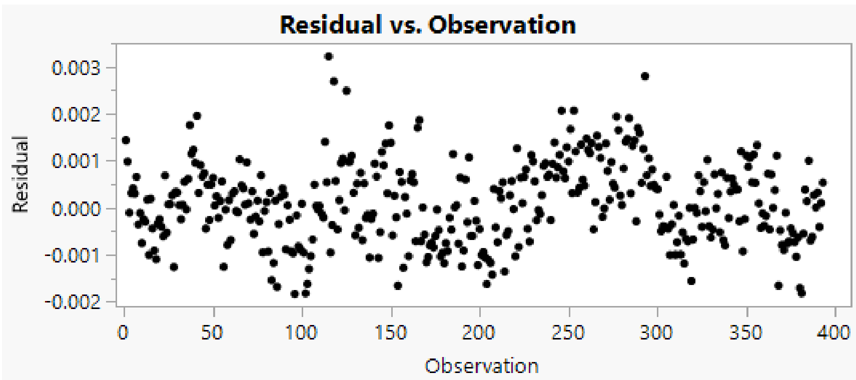

Figure 4. Total Pressure Final Model: Residual vs. Observation

3.1.4 New Model: Total Pressure

The final model produced after the remedial measure was performed is:

$$\hat{P0}_{PRB} = 0.04139 + 0.6836 * P0TS + 0.3151 * ATM - 0.00004572 * T0TS. \quad (14)$$

The coefficients have been transformed to account for the autocorrelation. The final R^2_{adj} for the model was 0.9998 for both the training and validation sets, an improvement over the original model. The RMSPE is 0.000788, compared to the RMSE of 0.000899, indicating the model fits the validation set well and has a high prediction accuracy.

The residuals pass the Anderson-Darling Goodness-of-Fit Test with a p-value of 0.2870. Figure 5 shows the histogram and normal quantile plot of the residuals.

From the residual vs. predicted graph in Figure 6, the model does not appear to suffer from drastic non-constant variance. The model passes the Breusch-Pagan test for constant variance with a p-value of 0.0131. Additionally, there is no clear funneling in the residual vs. predicted graph. Therefore, the assumption of constant variance is met for this model.

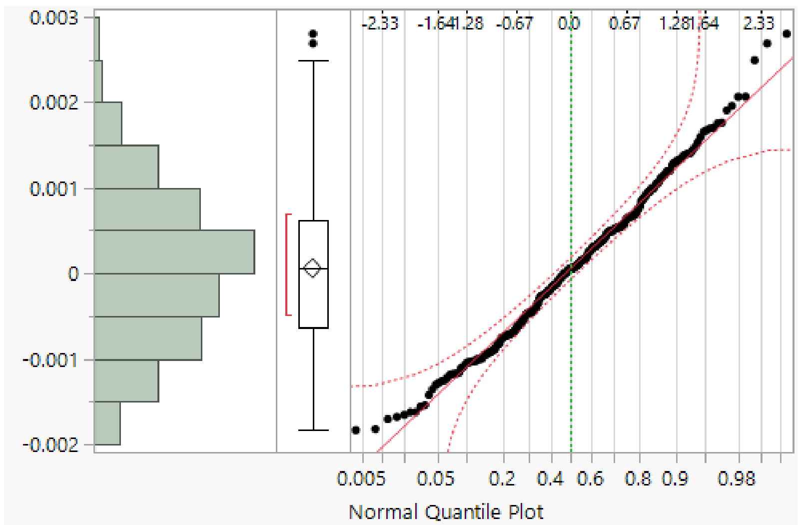

Figure 5. Total Pressure Final Model: Residual Distribution

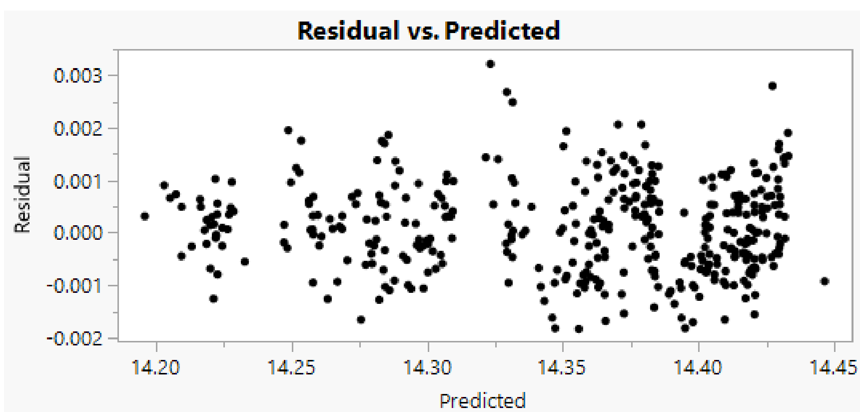

Figure 6. Total Pressure Final Model: Residual vs. Predicted

This new model is overall a better model than what has been used previously. The model has a high prediction accuracy, and despite the small non-constant variance issue, this model will allow for much better error prediction for the SARL team in the future.

3.2 Static Pressure Prediction

3.2.1 Previous Model

The analysis for static pressure began with a recreation of the methodology used previously by the SARL team. A simple linear regression model with with the static pressure measurement from the pitot probe (PS_{TPRB}) as the predictor, and the static pressure reading from the central sensor (PS_{PRB}) as the response. The formula for the simple linear regression was estimated to be:

$$\hat{PS_{PRB}} = 0.2738 + 0.9816 * PS_{TPRB}. \tag{15}$$

This model had a strong fit, with an R^2 of 0.99998 and a RMSPE of 0.00317. However, this model suffers from autocorrelation, failing the Durbin-Watson test with a test statistic of 0.2807, well below the lower bound of 1.571. This autocorrelation can be seen in the patterns evident in the residual vs. observation graph in Figure 7.

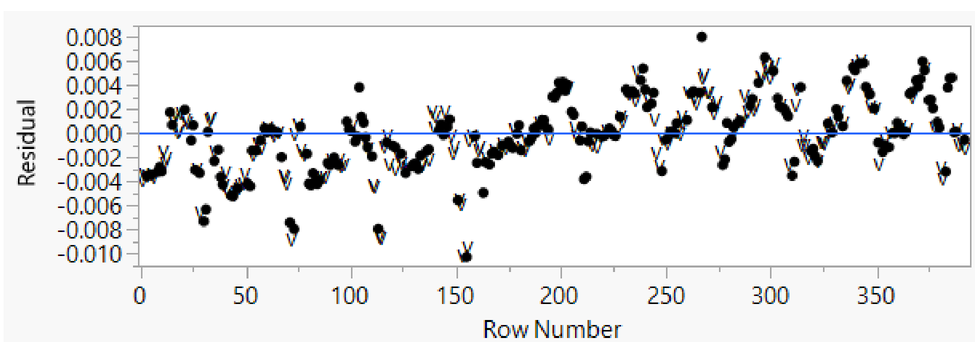

Figure 7. Static Pressure Simple Model: Residual vs. Observation

As with total pressure, a new model will ultimately be created and residual analysis is not thoroughly explored. However, there is a clear lack of fit in this model as seen in the curvature in the residual vs. predicted graph in Figure 8. This indicates

(PS_{TPRB}) and (PS_{PRB}) may not have a linear relationship and (PS_{PRB}) may be better described by a polynomial or other function of (PS_{TPRB}).

The residuals for this model appear normally distributed, as shown in the histogram and normal quantile graph in Figure 9. Normality is confirmed with an Anderson-Darling Goodness-of Fit test, passing with p-value of 0.014.

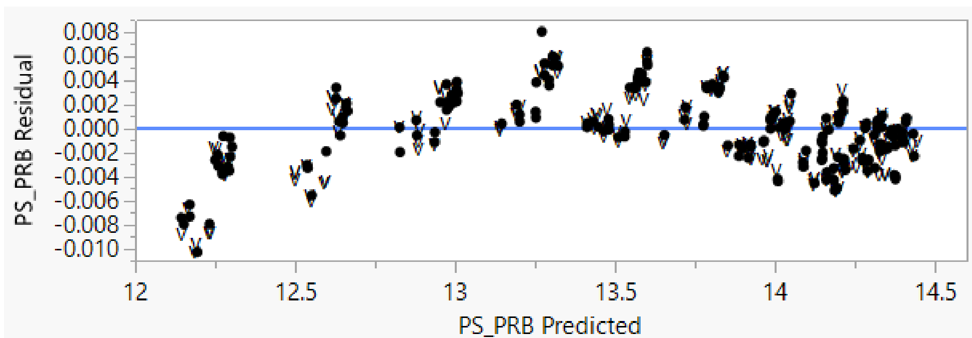

Figure 8. Static Pressure Simple Model: Residual vs. Predicted

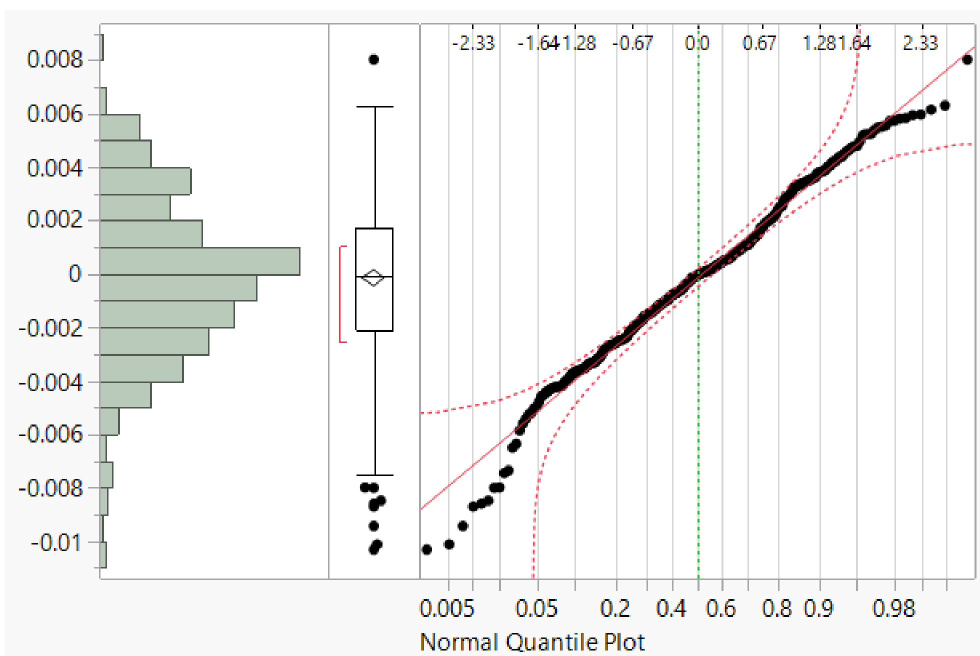

Figure 9. Static Pressure Simple Model: Residual Distribution

3.2.2 New Model Development

It was clear from the simple linear model that a higher order term for PS_{TPRB} should be considered in the new model to resolve the lack of fit issue. The backward elimination method was used once again, and the final model included: PS_{TPRB}, $(PS_{TPRB})^2$, and tunnel reference pressure (ATM), as well as two external weather conditions, dew point (DEW) and wind speed (WS). Other polynomial terms were also explored and were not significant. The formula for this new model was:

$$\begin{aligned} \hat{PS_{PRB}} &= -0.9931 + 1.1211 * PS_{TPRB} - 0.005225 * PS^2_{TPRB} \\ &+ 0.02502 * ATM - 0.00006623 * WS - 0.00003978 * DEW. \end{aligned} \tag{16}$$

The R^2_{adj} for the new model was 0.999997, with a RMSPE of 0.00115, improving on the original. The lack of fit issue from the simple linear regression model seems to have been resolved, based on the residual vs. predicted plot in Figure 10. However, this model fails the Durbin-Watson test for autocorrelation with a test statistic of 0.7190, below the lower bound of 1.571, with the residual vs observation plot shown in Figure 11. The rest of the assumptions will be tested after the autocorrelation is resolved.

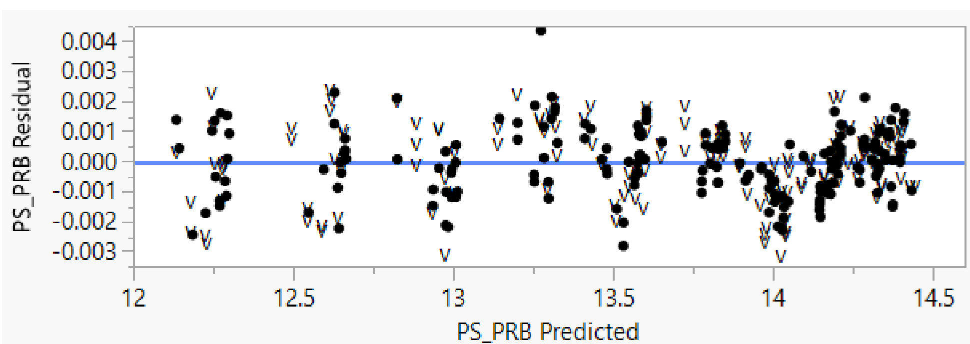

Figure 10. Static Pressure New Model: Residual vs. Predicted

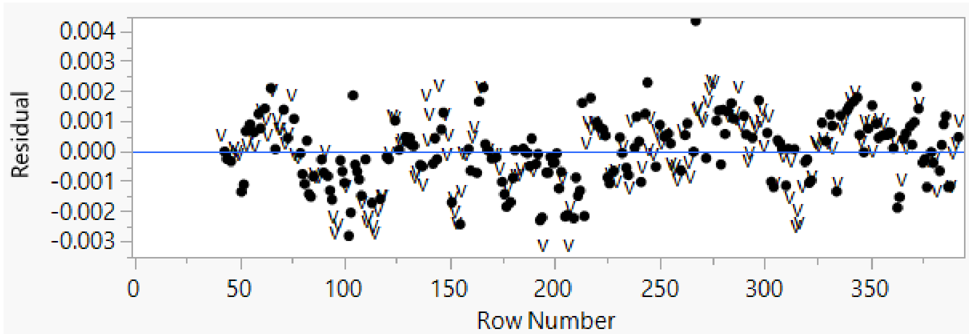

Figure 11. Static Pressure New Model: Residual vs. Observation

3.2.3 Remedial Measures

The Cochrane-Orcutt procedure was performed to fix the autocorrelation. After one iteration the autocorrelation was resolved, with a Durbin-Watson test statistic of 2.1806, above the upper bound of 1.779. The residual vs. observation graph for the transformed model is shown in Figure 12.

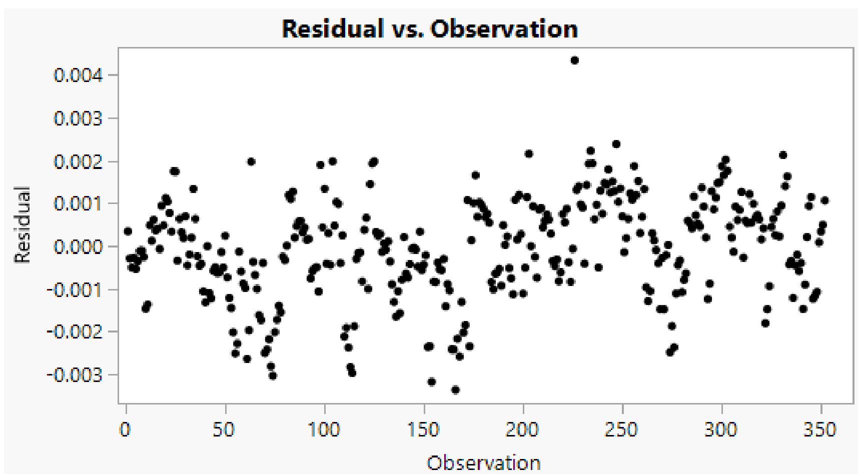

Figure 12. Static Pressure Final Model: Residual vs. Observation

3.2.4 New Model: Static Pressure

The final model for static pressure post Cochrane-Orcutt procedure is:

$$\begin{aligned} \hat{PS}_{PRB} = &-0.9470 + 1.1186 * PS_{TPRB} - 0.005138 * PS_{TPRB}^2 \\ &+ 0.02314 * ATM - 0.00004366 * WS - 0.00004585 * DEW. \end{aligned} \tag{17}$$

The R_{adj}^2 for this model is 0.999997, and the RMSPE is 0.001123. For comparison, the RMSE of the model is 0.001123 indicating the model fits the validation set well.

Now that the autocorrelation has been resolved, tests for normality and constant variance can be reliably performed. The model passes the Anderson-Darling Goodness-of-Fit test with a p-value of 0.0130. The histogram and normal quantile plot of the residuals is shown in Figure 13.

The model also fails the Breusch-Pagan test for constant variance with a p-value less than 0.0001. However, from the residual vs. predicted graph in figure 14, there is no clear indication of non-constant variance and the constant variance assumption is accepted.

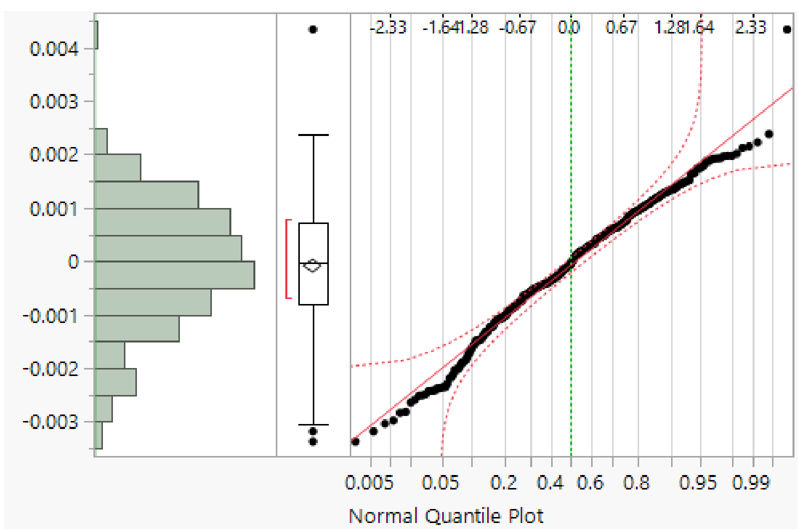

Figure 13. Static Pressure Final Model: Residual Distribution

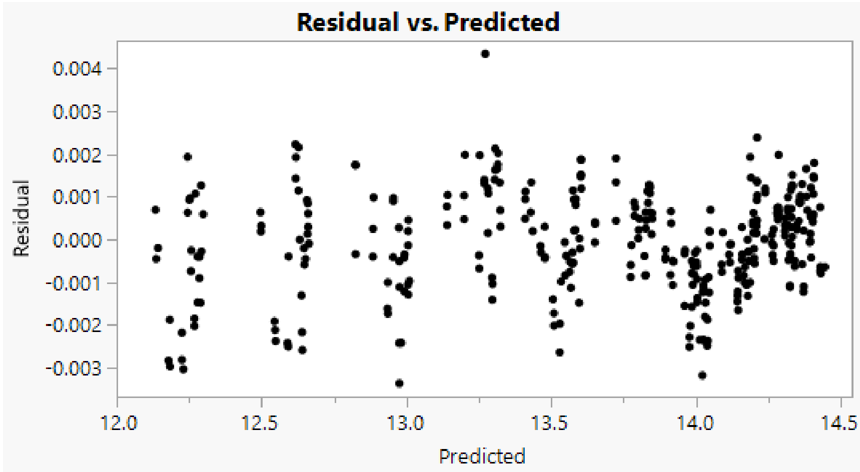

Figure 14. Static Pressure Final Model: Residual vs. Predicted

3.3 Dynamic Pressure Prediction

The process for dynamic pressure is more complicated than total or static pressure. In the past, the SARL team calculated dynamic pressure by divided the data into two MACH ranges: MR1 and MR2, and using different calculations for each range. MR1 is between MACH 0 and MACH 0.23 and MR2 is from MACH 0.23 to MACH 0.5. Therefore, different formulation is used on each of the MACH ranges.

3.3.1 MACH Range Decomposition

There was no variable for measured dynamic pressure, the only response variable available was corrected dynamic pressure. Instead there are two sensors that measure for MR1 and MR2. Rather than exactly replicating the SARL team's process initially, a simple linear regression for corrected dynamic pressure was created. The resulting formula was not a recreation of the SARL team's previous method and was only used to gain insight into the problem. Thus the model itself was not of much concern, but the interesting take away was the lack of fit observed. Figure 15 is the residual vs. predicted graph for the simple dynamic pressure model. As evident by the patterns in this graph, this model suffers from both lack of fit and non-constant variance. The curvature in the model suggests that a higher order model may be more appropriate for this data, opposed to a straight-line model. This graph also appears to have a clear polarization regarding the variance. The predicted values ≤ 0.5 appears to have a linear lack of fit and those > 0.5 appears to have a curved lack of fit. Overall, the residual vs. predicted graph in Figure 15 depicts a lack of fit, but also presents a dichotomy of the residuals.

In order to explore the lack of fit further, a polynomial model was created and Figure 16 is the residual vs. predicted graph for that model. As suspected, the higher-order fit appears to be a more appropriate model. However, this plot shows the presence of the dichotomous residuals remains. The divide in the dichotomy corresponds with the two MACH ranges used by the SARL team: MR1 and MR2. In response to this, two different models were created for dynamic pressure, one for each MACH range. This approach is similar to the SARL team's methodology.

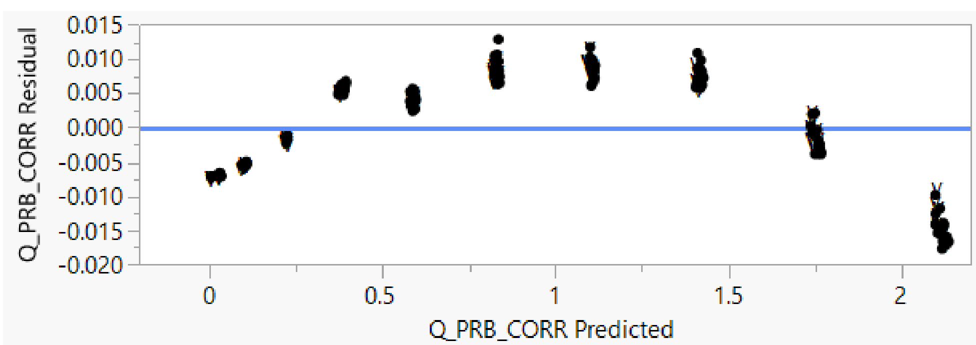

Figure 15. Lack of Fit in Simple Model

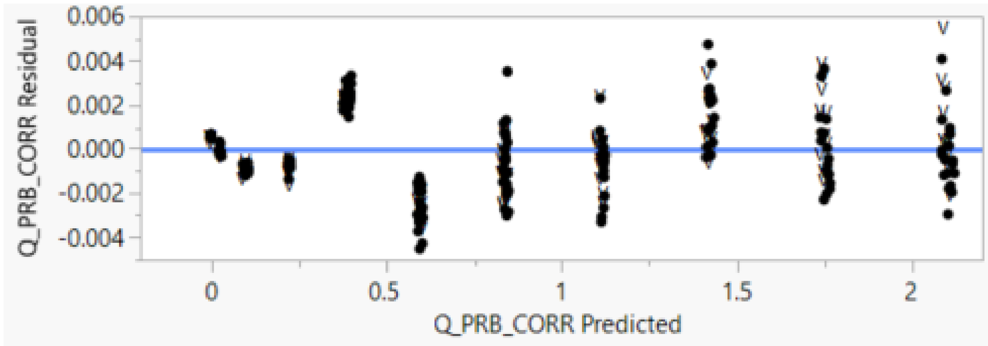

Figure 16. Lack of Fit in Polynomial Model

3.3.2 New Model: Dynamic Pressure

In response to the difference between MR1 and MR2, the data was partitioned accordingly and a model was created for each range.

MACH Range 1

The response variable for this model was $Q_{PRB}1PSID$, and the final model for MACH Range 1 contains static pressure (PS_{TPRB}) and total pressure ($P0TS$). The model originally suffered from autocorrelation and the Cochrane-Orcutt procedure was used once again. The prediction equation post Cochrane-Orcutt procedure for dynamic pressure in MACH Range 1 is:

$$Q_{PRB}\hat{1}PSID = 0.001133 + 0.9761 * P0TS - 0.9768 * PS_{TPRB}. \tag{18}$$

The R^2_{adj} for this model is 0.999998 for both data sets, with a RMSPE of 0.000222. This RMSPE, when compared to the RMSE for the model of 0.000226, indicates the model fits the validation data well and has a high prediction accuracy.

The autocorrelation was resolved using the Cochrane-Orcutt procedure, and the test statistic for the Durbin-Watson test was 2.0805, above the upper bound of 1.765. The Residual vs. Observation graph is shown in Figure 17. The residuals are normally distributed, with p-value of 0.052 for the Anderson-Darling Goodness-of-Fit test. The histogram and normal quantile plot of the residuals is shown in Figure 18. The curved lack of fit issue observed earlier has been fixed, however there may still be a small lack of fit seen in the residual vs. predicted chart in Figure 19. Regardless, there is nothing drastic observed in the residual vs. predicted graph, and the constant variance assumption is accepted for this model.

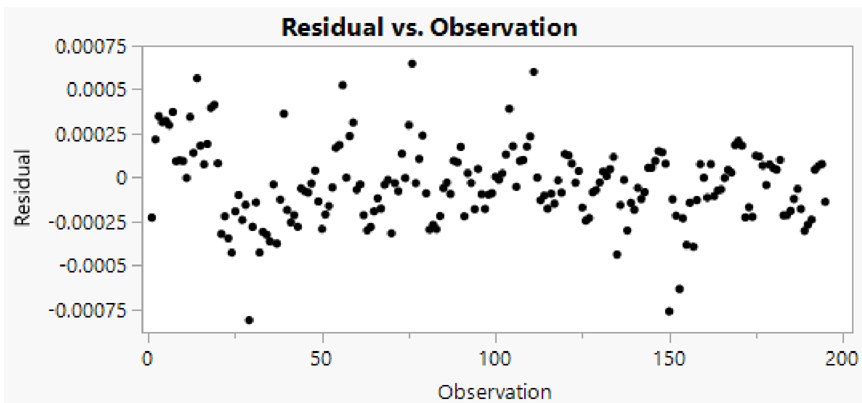

Figure 17. Dynamic Pressure MR1: Residual vs. Observation

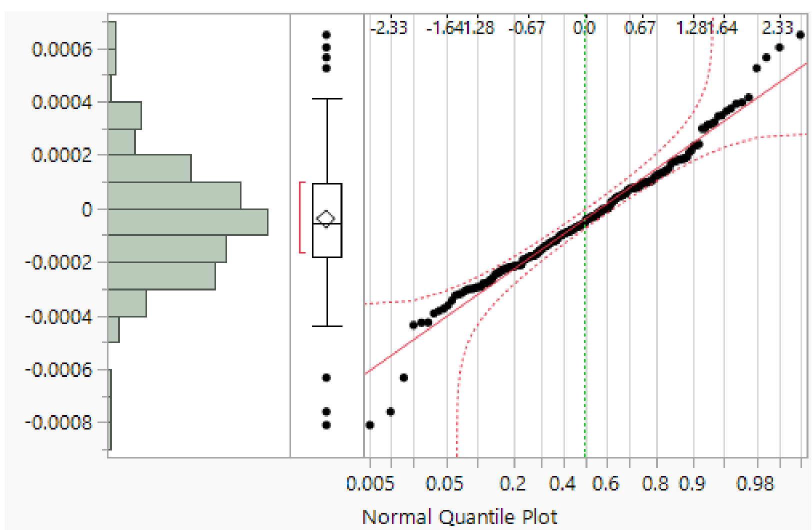

Figure 18. Dynamic Pressure MR1: Residual Distribution

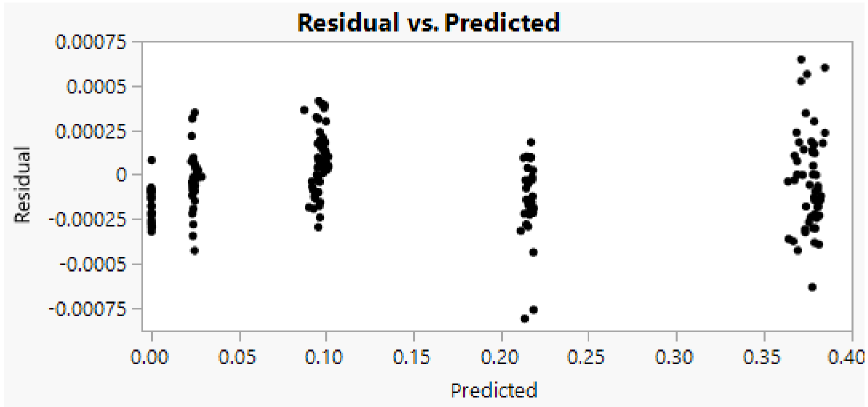

Figure 19. Dynamic Pressure MR1: Residual vs. Predicted

MACH Range 2

The response variable for the MR2 model was $Q_{PRB}2PSID$, and the final model for MACH Range 2 contains $P0TS$, $DewPt$, PS_{TPRB}, $PREF$, and Q_{TPRB}, as well as several interactions. The prediction equation for MACH Range 2 is:

$$\begin{aligned} Q_{PRB}\hat{2}PSID = -2.8471 + 1.4310 * P0TS - 0.002066 * DewPt \\ - 1.4144 * PS_{TPRB} + 0.2197 * PREF + 1.5776 * Q_{TPRB} \\ + 0.006411 * Q_{TPRB}^2 + 2.5059 * DewPt^2 - 0.00003136 * PS_{TPRB} * DewPt \\ - 0.1335 * PREF * Q_{TPRB} - 0.004965 * Q_{TPRB} * P0TS. \end{aligned} \tag{19}$$

The R^2_{adj} for this model is 0.9999992 for both the validation and training data. The RMSPE is 0.0004255, compared to the model RMSE of 0.0004303, indicates the model fits the validation data well and has a high prediction accuracy. The Durbin-Watson test statistic for this model was 1.4584. This test statistic falls between the lower and upper bound (1.335 and 1.765 respectively) for the Durbin-Watson test criteria, meaning the test is inconclusive and a visual inspection will be used. From the residual vs. observation graph shown in Figure 20, there are no clear signs of autocorrelation, and for this model the independence assumption is accepted.

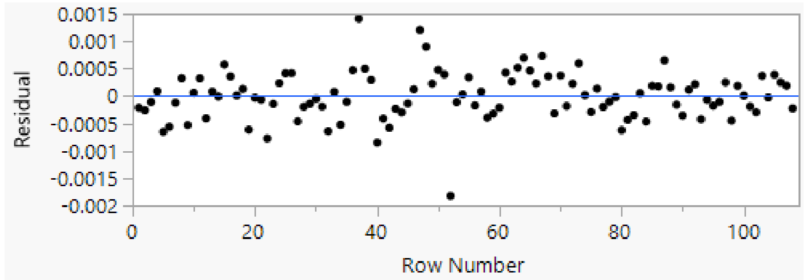

Figure 20. Dynamic Pressure MR2: Residual vs. Observation

The model also passed the Anderson-Darling Goodness-of-Fit test with a p-value of 0.2650. Figure 21 shows the histogram and normal quantile plot for the residuals of this model.

For the constant variance assumption, this model fails the Breusch-Pagan test with p-value less than 0.0001. However, as with other models in this thesis, there is nothing overly concerning in the residual vs. predicted graph in Figure 22. For this thesis, the constant variance assumption is accepted for this model.

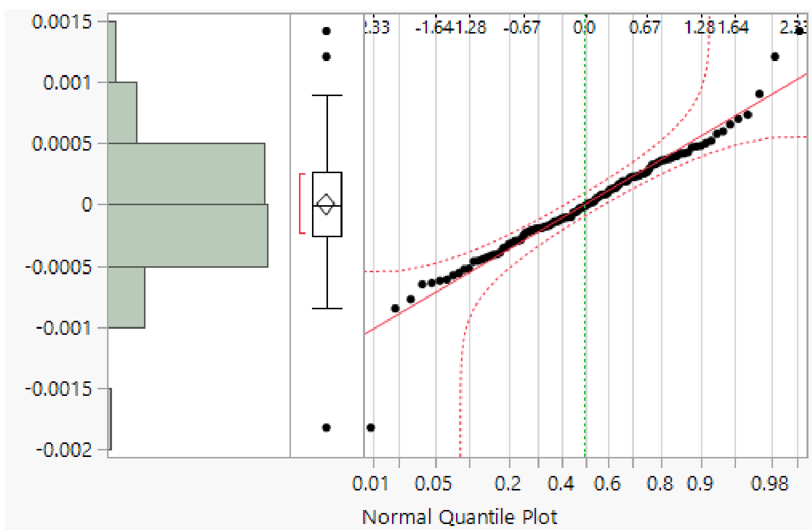

Figure 21. Dynamic Pressure MR2: Residual Distribution

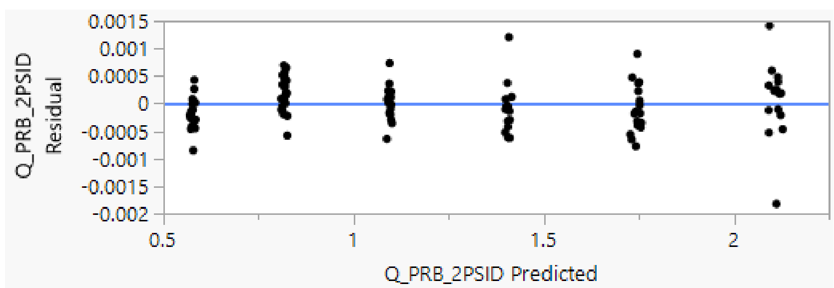

Figure 22. Dynamic Pressure MR2: Residual vs. Predicted

IV. Conclusions and Recommendations

4.1 Conclusions

The methods used by the SARL team in the past had a high prediction accuracy and were suitable for their needs. However, the SARL team's desire for statistically defensible models and accurate error prediction led to the creation of new models created with statistical rigor.

Four strong models have been created in this thesis, one each for total and static pressure, and two for dynamic pressure, one for each MACH range. These models were created using standard regression techniques, and despite a pervading issue with non-constant variance, the models meet all the necessary assumptions. Every model failed the Breusch-Pagan test for constant variance. The Breusch-Pagan test can be unreliable when the R^2 of the model is high and the residuals are small, which is the case for every model in this thesis. [3] In response to this limitation, a visual inspection of the residual vs predicted graphs was performed to check for non-constant variance. Based on these visual inspections, there is reasonable assurance that the constant variance assumptions are acceptable. Based on the criteria laid out in this thesis, all four models meet the criteria to be labeled statistically defensible.

The analysis has also answered another of the SARL team's questions. Environmental factors such as wind speed and dew point are significant. The static pressure and dynamic pressure models both have environmental factors in the formulation. These factors should continue to be collected and utilized where appropriate.

4.2 Recommendations

It is the recommendation of this thesis that the SARL team use the models created herein for future tests. These models have a high accuracy and by accepting the necessary assumptions, estimates for error can easily be calculated. The analysis in this thesis allows the SARL team to tell their customers with confidence that their methodology is statistically rigorous. These models can be used to give highly accurate predictions, as well as confidence intervals for the error terms.

It is also recommended that the SARL team monitor external weather conditions as these measurements affect the conditions inside the wind tunnel and are needed for the prediction models.

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14. ABSTRACT

Windd tunnels are used to test scale-model air frames in order to collect aerodynamic data. The Subsonic Aerodynamic Research Laboratory (SARL) Wind Tunnel is a low speed wind tunnel located at Wright-Patterson Air Force Base. The SARL Wind Tunnel team approached AFIT for assistance in creating statistically defensible models for the conditions inside the wind tunnel. During a wind tunnel test, pressure sensors cannot be placed at the test model. Instead, pressure is measured by a pitot probe permanently mounted in the corner of the test chamber. The pressure at the model location is predicted from the measurements taken by this pitot probe. This thesis analyzes the models used previously to predict pressure and creates new models using more rigorous statistical methods. These new models have a high prediction accuracy and follow all the necessary assumptions to ensure accuracy for the SARL wind tunnel team and their customers.

15. SUBJECT TERMS

SARL, Wind Tunnel, Pitot Probe, Linear Regression

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