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Stochastic Programming Design for West Africa Logistics Networks

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STOCHASTIC PROGRAMMING DESIGN FOR WEST AFRICA LOGISTIC NETWORKS

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Abstract

The USAFRICOM is studying methods to increase the resiliency of its West African Logistic Network, a hub-and-spoke network design responsible for sustaining long term humanitarian and security missions in the West Africa region. This paper employs a two-stage stochastic programming network design modeled on the WALN that builds a flexible supply chain capable of responding to periodic disruptions while maintaining peak resiliency. Elements such as cost, probability, and event-based disruption are integrated into the model to mirror challenges the WALN faces. We demonstrate that incorporating resilient based response mechanism provide a 90% reduction in cost compared to meeting the logistical challenges covered with a naive approach.

To my parents, thank you for bestowing your time, love and wisdom throughout my life. To my sister, thank you for your cheerful spirit and sense of humor. Your collective support enables me to strive for ever greater heights.

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STOCHASTIC PROGRAMMING DESIGN FOR WEST AFRICA LOGISTIC NETWORKS

I. Introduction

Military operations take place in unstable locations that frequently require dynamic changes in resource allocation to respond to fluctuations in demand and delivery channel capacities. Regular channels have become so dispersed yet interconnected that managing them is a non-trivial exercise in bookkeeping. Logisticians frame these connections as supply networks and devise algorithms to maximize the flow of goods at minimal cost. However, conventional strategies, such as the standard min-cost network flow framework, are incapable of addressing resiliency metrics for network disruptions. Stochastic programming provides a means to model these complex interactions in the face of many potential setbacks [9, 16, 31, 41, 44].

1.1 Background

The United States (U.S.) military uses logistic networks to conduct humanitarian missions in countries where low economic development combined with high levels of political, social, and environmental disruptions can impede any supply network(s). We observe one or more of these conditions in West Africa [11] (See Table 1). In 2008 the U.S. Africa Command (USAFRICOM) established its operations in West Africa in order to provide "access and partnerships, protection, mission command, intelligence, and sustainment" in the West Africa subregion [37]. To this end, USAFRICOM faces a number of regional challenges, including terrorism, military conflicts, natural disasters, and pandemics [52].

General Townsend, Commander of USAFRICOM, speaks about how these challenges negatively influence the economic and political growth of African countries. He outlines that political and economic stability in Africa is an American objective, stating that the continent is currently at crossroads concerning international commerce and trade [51]. West Africa is predominately made of functional democracies with a growing economy. Some states have formed societies such as the Economic Community of West African States to encourage further economic growth. However, this growth is matched by the parallel development and expansion of violent extremist organizations (VEOs). Since 2018, VEOs have increased violence by 250% in Burkina Faso, Mali, and Western Niger [51]. As a result, the region is destabilizing and faces issues with which the local security cannot cope.

Helping West Africa develop as a stable region requires USAFRICOM to design a systems capable of withstanding or responding to disruptions. There are key differences between withstanding a disruption and responding to one. A system capable of withstanding disruptions is defined as *robust*. A measure of a network's robustness is how well the network maintains the same level of service despite a disruption. In contrast, responding to disruption taps into properties traditionally defined as *resiliency*. Explicit definitions for resiliency vary in literature. We follow the pattern of Gu et al. [19], Hutchison and Sterbenz [22], Mensah and Merkuryev [33], and Tipper [48] in defining resiliency as the ability to restore a network to its original or optimal levels of service after a disruption (see [2, 26, 32, 35, 55] for alternate definitions that indicate

Table 1. West African Countries of Interest

Burkino Faso	Cote	D'Ivoire	Togo
Benin	Cameroon	Central African Republic	Chad
Gabon	Ghana	Guinea	Guinea Bissau
Liberia	Mali	Mauritania	Niger
Nigeria	Senegal	Sierra Leone	

resiliency and robustness are synonymous). Studies that use more limited definitions often mention the network's speed of recovery in tandem, labelled "restoration ability" as something separate from the ability to recover from damage, and state the need for both in a resilient system. Resiliency and restoration are often seen together, since both require an initial network configuration to adapt based on fluctuations in supplies, demand, and capacities.

In particular, scenarios that increase demand tend to minimize the number of nodes available for rerouting. This in turn, lowers arc availability and leads to situations where network service may drop to critical levels. Adapting to situations such as these reinforces the importance of resiliency in system design [2, 8, 48, 54].

1.2 Motivation, Region Status, and Future Direction

Supporting West Africa's development is a strategic maneuver to deal with the actions of adversarial state actors. Both the U.S. and China primarily focus political and military efforts on the Asian Pacific theater. However, gaining a geopolitical advantage is not confined to this area. General Kosinski, USAFRICOM Director of Logistics, points out that China and Russia have been investing into African countries for years [43]. These relationships resulted in China's first non-domestic base, and are continuing to drive other economic and political transactions. The U.S. can cultivate relationships with Africa for its own economic and political benefits. Bolstering American presence and influence in Africa through actions such as seeking preferred trading status or providing other support would be mutually beneficial. General Kosinski mentions that supporting local security to provide a stable geopolitical environment could improve both African economic development and trade with American businesses [20]. He explains that supporting local security is indirectly supporting the development of the West Africa Logistic Network (WALN). The WALN is a “hub-and-spoke approach to logistics that use C-17s and C-130s” to get both troops and African partners the support they need [43].

The WALN interconnects the interior of West Africa. These connections are leveraged to move people and goods from one place of commerce to another. The trade agreements necessary to maintain the network in its current form requires coordination from 20 countries [56]. Management of this structure is currently aided by US efforts [10, 56].

The value of the WALN was recently demonstrated when the U.S. State Department was unable to transport 4,000 pounds of medical supplies from Manchester, U.K. to the U.S. Embassy in Accra, Ghana. USAFRICOM responded by re-routing the shipment of supplies from RAF Mildenhall, U.K. to Ramstein Air Base (Germany), and then through the WALN to Niamey (Niger), and finally Accra (Ghana) [20]. The resulting goods flow was routed as designed by the hub-and-spoke arrangement [40].

The hub-and-spoke method is the current network design to distribute goods throughout the WALN. While this efficient structure achieves extremely low operational costs [10], it carries significant operational risk in adverse environments such as West Africa. We iterate through different analytical methods to potentially able to assess these risks.

1.3 Problem Statement and Approach

Problem

The ability of a hub-and-spoke design used by the WALN to respond to long term dynamic changes is dubious. Cox, Smith, Breitbach, Baker, and Rebeiz identify in their paper a need for a network design able to handle the day to day needs of the network while maintaining the flexibility to deal with logistical challenges imposed by the West African environment [11]. Cox et al. built a scenario-based robust optimization supply chain network design capable of responding to those logistical challenges that corrects for the short comings of a basic hub-and-spoke design [11]. However, their model falls short on several self-identified fronts, two of which we hope to address in this thesis. Cox et al. recognize their model fails to implement a shipping cost based on the distance an item travels. Incorporating a distance-based cost may change the structure of the recommended backups in the paper. The other shortcoming is resiliency is measured on a node to node basis. Attempts to maximize instances of resiliency may lead to inefficient routing suggestions that decrease the logistic network's overall resiliency.

Cox et al. suggest that a two-stage stochastic programming (SP) technique can help design a WALN both resilient and robust while also addressing the shortcomings in their mathematical model [11]. The premise is a stochastic model employs the scenario-based design that allows for the accurate portrayal of challenges unique to the WALN, while inserting elements that represent the logistical challenges in a dynamic way. The elements the model must address are: (a) cost for transshipment node creation, (b) distance-based cost, (c) capacity limits based on city and the method of travel, as well as (d) random demand fluctuations in comparison to the robust model covered by their research.

Reframe of the Problem

Developing a model that can overcome the issues arising from the WALN's hub-and-spoke design while avoiding issues present in Cox et al. paper is a worthwhile area of research. We frame this research as the task of finding a resilient network design for the WALN that, when subjected to specific disruptions according to stated probabilities, restores the flow of goods to an established baseline level at minimal cost. Accomplishing this task requires understanding what techniques exist for both modeling and examining a network that can be modified to include resiliency.

Potential Measures of Resiliency in the Problem

Traditionally, resiliency includes delaying disruptions or minimizing the amount of time a break in service occurs. The earliest resiliency measures involved counting "errors" i.e. instances where demand was not met [2]. This single-dimensional response metric limited researchers' ability to compare solution algorithms across various network topologies [22]. In contrast, modern resilient network design studies use a two-dimensional paradigm by measuring both the severity (quantity of unmet demand) and probability (number of arc or path failures) [18, 54]. Using this framework permits assessment of section-specific network failures, leading to deeper characterization of the network's vulnerabilities. Our stochastic programming model integrates a failure's severity and probability into a probability-weighted minimum cost network flow problem objective function for meeting all flow demands under each scenario. By incorporating both the scope and severity of potential disruptions, the network design formulation is able to evaluate trade-offs between designing around potential issues or responding to them directly as needed [54].

Simulation Approach

Through the use of simulation, researchers can obtain a deep understanding of how and why a system behaves for specific scenarios. A simulation paradigm performs a comparative evaluation on candidate network designs that are chosen a-priori. Generating all the candidates to evaluate can be a challenge. If the candidate network designs have n topologies, and c modeling environments, there is a need for nC models to fully scope the problem [45]. The computational strain presented by a challenging environment can be non trivial as well [45]. After the candidate network designs are chosen and generated, the evaluation considers how each design performs on a variety of scenarios. The design with the best aggregated performance is chosen. In contrast, our approach seeks to shape an optimal network design based on threat profiles, including probability and severity [15].

Potential Solution

Our approach to identifying a resilient design is based on two-stage stochastic modeling, as recommended by Cox et al. [11] and Hosseini et al. [21]. This approach contrasts that of a simulation-based method. The two-stage SP is appealing as introducing new parameters takes minor design modification. One modification is adding fixed costs for arc construction that represents the use or creation of backup paths. Another modification that increases realism is allowing both shortfalls and excess supplies at the various nodes. This design choice was partly motivated by a focus on resiliency over robustness in network.

Our two-stage SP design in general is straightforward in terms of what each stage addresses. The first stage seeks to establish a set of baseline flows on the network. After choosing the baseline flows, the model allows for one of several scenarios to emerge. These scenarios represent various disruption events, which may require abstract mappings [17]. After the disruption, the model allows for a second stage response decision that seeks to restore the network flow at a minimal cost. The response options usually consist of restoring or replacing nodes or arcs, which are readily modeled in our construct using a fixed-cost integer programming approach.

1.4 Preview

In Chapter 2, we review a variety of approaches in the literature to the problem of resilient network design. Chapter 3 presents our two-stage stochastic programming approach to this problem. It will address the model constructed, the decision variables, and the structures of the networks represented along with the design and results. In Chapter 4 we present a case study of our approach, applied to a problem of supply chains in the WALN. We describe the problem scenario and consider key results. Chapter 5 concludes by presenting future research options.

II. Literature Review

This literature review covers techniques used by researchers to overcome the challenges associated with resilient network design. Resilient design requires the ability to address random arc elimination and the resulting supply shortage. The chapter is divided into summaries of different methods used by researchers to measure, understand, or support resilient network design. The first section reviews how supporting resilient network design can be based on backup path path selection. However, the ability to select the optimal path(s) for a LP depends on the ability to understand and manipulate a network's arc-centric or path-centric formulation. We cover the benefits and disadvantages of both formulations. We expand from there to review several researchers techniques for building or assessing the resiliency of a design by reviewing column generation, bi-objective formulation, network utilization rate, hub-and-spoke design, *ad-hoc* design, and culminate with the two-stage stochastic programming design this paper implements in the search for feasible and resilient designs.

2.1 Back Up Paths: Creating Reserve Capacity in the Logistic Network

Backup paths are the pre-designated sets of nodes and arcs that provide a means to reroute the flow in a network. Resilient network designers may use backup paths to meet demand while adapting to the new conditions. Backup paths are designed based on the premise that survivability increases with redundant capacity. Researchers attempt to measured excess capacity and quantify the amount needed to meet desired service level thresholds. If the excess capacity in a network is high enough even a diminishing overall capacity can accommodate any demand. Allocating this excess capacity in an efficient manner to minimize the overall amount needed is the next challenge. This builds to the examination of backup paths as a way to increase or maintain efficiency with diminishing excess capacity. Defining all the possible backup paths in a network from one node to another is a difficult exercise, since the number of potential backup paths grows factorially with the size of the network [1]. Invalidating those paths with a brute force method is impractical. Restricting the number of paths tested based on predefined criteria is considered a more feasible approach. Below we list ways to create backup paths that address this issue and reinforce the network's integrity. Path column generation as a way to to evaluate backup paths in network designs is covered as well. Backup paths created in a temporal manner are reviewed as well.

2.1.1 Resilient Designs using Path-centric Formulations

Arcs vs. Paths

In formulating network design problems, researchers generally use either an arc-centric or path-centric formulation [12, 57]. The standard network design formulation uses individual arcs flows as decision variables [57]. The benefit of representing flow with arcs is based on limitations in the number of DVs. Since there is one DV per arc, the number of DVs is quadratic in the number of nodes in the graph. Handling quadratic expansion of DVs is manageable with linear programming. The disadvantage of arc representation comes from difficulties filtering arc combinations. Every single feasible or non-feasible solution must provide a value for every single decision variable. Enumerating the solutions, or finding every possible combination of values for DVs to take on (i.e. creating paths) is an exponentially difficult task. If the values are continuous, the number of paths could be infinite [13].

Choosing to use paths to represent the networks comes with both benefits and challenges. The path representation of arc flow allows for a clear representation of different collection of arcs as unique decision variables. This provides a more holistic understanding than arc-centric formulations of which flows are needed in a resilient network design. However, while the number of arcs in a path is limited the number of nodes minus one, the number of arc combinations to construct a path grows exponentially with the number of nodes. Enumerating all the decision variables is a near impossible task. The benefit of path-centric formulation comes from inserting each path as a DV into a LP that can be solved for a single useful path. Desrosiers and Lübbecke point out that converting the arcs to convex combinations of path flows allows us to disregard some amount of infeasible paths during enumeration [12]. Using path flow formulations delivers the additional advantage of the ability to differentiate network paths.

Bi-objective Formulation

Bi-objective Formulation allows for a model to purse two objectives that may come into conflict. Tomaszewski, Pióro, and Żotkiewicz suggest a path-centric formulation for designing and assessing resilient network designs using a bi-objective construct [49]. The first objective is to minimize flow cost, as usual. The second objective is to maximize link and node differences. This second goal supports reinforcing network integrity since it reduces the chances of one destroyed link crippling the network. Removing single points of failure is vital to improving resiliency. Marti, Velarde, and Duarte provide huerstic algorithms to solve this problem [30]. The resulting solutions provide decision-makers a means to balance flow costs with resiliency.

Computational Challenges

Path column generation reduces the number of decision variables evaluated in the exponentially growing problem set to only those that improve the solution. The path flow formulation allows us to generate attractive path flows (i.e. columns), so that at any point in the algorithm we only consider the most promising arcs and add additional arcs as needed in an iterative manner to construct feasible paths [12]. Through this method combinations of paths are created. These combinations are evaluated as relaxed LPs to ensure convex combination multipliers are integers. Essentially, only the paths that consist of minimal-cost flows are constructed. Dzida, Zagożdźon, Pióro, Śliwiński, and Ogryczak use column generation to solve their resilient network design problem [15]. Researchers have also applied column generation to other similar problems such as the vehicle routing problem, demonstrating the wide applicability of this technique [47]. Desrosiers and Lübbecke [12], Dzida et al. [15], and Tomaszewski et al. [49] advocate column generation as more efficient computationally than standard approaches. They consider the only reason to prefer standard methods over column generation is the subjective criteria involved in selecting the sub-set of variables for the master problem. However, even with the computational efficiencies of column generation, some problem instances still cannot be solved in a reasonable amount of time. In these cases decision makers must balance the quality of an incumbent solution with the cost of finding a better solution.

2.1.2 Network Utilization Rate

For dense networks (i.e., many arcs) the number of alternate routes makes it computationally infeasible to evaluate the design for resiliency using path-centric formulations, even with column generation techniques. Network utilization approaches answer this need by focusing, not on the existence of back-up paths, but on the amount of residual capacity in the network. The set-up treats flow restoration as a two part act. The first act distributes arc flow throughout the network to meet projected demand. Then an event occurs to eliminate some arc capacities which triggers the second act. The second act redistributes arc flow to meet all realized demand and infeasible flow. The ratio of arcs used (i.e. having positive flow) to the full network size provides an indicator to how stressed the network is. The network utilization ratio is the amount of planned for arcs in the first act plus the amount of unplanned arcs used in the second act compared to the full network size in the first act. The lower the utilization rate, the better the network is at handling demand. In this way, the focus shifts from path cost to network utilization rate of all the arcs created and maintained to deal with setbacks [58].

Xiong and Mason highlight the dual nature of residual capacity, describing it as a cost in terms of "global versus failure-oriented reconfiguration, path versus link restoration, and state-dependent versus state-independent restoration" [58]. In each of these comparisons the latter represents problem solving on a local scale. Solving problem on a local scale leads to myopic decisions that fail to improve the system's overall response to failure(s). This approach is visible in Vaghani and Lung's work which depicts a network dependent on switches to automatically re-route dropped network traffic (demand). That switch enables the network to be highly responsive to failure. However an all-switch network needs spare capacity at every switch to adequately respond to all potential points of failure. The immediate response mechanism leads to inefficient resource allocation compared to the efficiency of a controller. The controller is a mechanism that can extend the response time of a switch, thereby pushing arc restoration to a more opportune moment. The controller can also transfer the responsibility for restoring traffic to another switch. These two abilities harkon to state-dependent restoration tactics that delay the quickest response in favor of a more coordinated systematic outlook. The controller represents problem solving on a global scale. The controller supports a resilient system by considering both the immediate and future health of the network. The capacity in a controller based network needs enough capacity to delay re-routing traffic at a few key nodes. With careful selection of controller placement along with well chosen backup paths, the system can respond to failure better with less overall capacity in the network [53].

Vajanapoom, Tipper, and Akavipat expand on the network utilization approach by adding a fixed “arc activation” cost to represent repair costs. Using risk profiles, they evaluate the likelihood and impact to network utilization and resiliency if the repair cost for damaged components exceeds a cost-effectiveness threshold parameter, and those components are discarded from the network [54]. These approaches seek the best systematic reaction strategies to network disruption. Iraschko, MacGregor, and Grover focus more on the amount of spare capacity in the system rather than its cost [23]. They iterate through different bands of upper and lower spare capacity to find the amount necessary to meet time-based restrictions as well as maintain robustness at the desired utilization rate [23].

At the opposite end of the spectrum, trying to minimize residual capacity, *ad-hoc* networks prioritize immediate responses to disruptions by reestablishing the flow of goods immediately. *Ad-hoc* networks are examined as a way to build or rebuild primary paths at the last moment. This concept was first introduced when Toyota sought to minimize factory inventory on hand and, by extension, their company overhead [34]. *Ad-hoc* networks choose flows to minimize potential action costs, subject to current funding and capacity limit. This enables them to rapidly expand in order to meet changing demands. Marina and Das present a novel approach to resilient network design that combines *ad-hoc* principles with a link-disjoint path approach based on node sequencing [29]. Becker, Beber, Windt, and Hütt analyze *ad-hoc* design with simulated logistic networks, noting that *ad-hoc* networks provide low wait times, and that this ability to respond in a resilient manner improves with the network connectivity [7]. However, several drawbacks do exist with *ad-hoc* network designs:

1. they cannot evaluate the trade-off between current tactical demand and future

strategic demand [7].

2. they are highly dependent on reliable supply chains. Kannan and Tan argue that an *ad-hoc* network's resilience is more indicative of a robust supply chain design than of a resilient distribution network [24].

3. the amount of data (goods) flowing through a network must be relatively small compared to the potential amount of goods to be transferred [17]. This observation parallels the notions indicated by Xiong and Mason [58] that measuring spare capacity is a good proxy measure for resiliency.

4. *ad-hoc* networks require tightly controlled environments to work well, because success of the technique is based on continuous improvement in both the distribution and transferal of goods [28].

The relationship between *ad-hoc* networks and a low utilization rate can be conflicting. The highly dependable environments *ad-hoc* networks need to work indicate a high level of excess capacity in the network. This corresponds to a low utilization rate which can be seen as positive sign of resiliency. However, a low utilization can indicate the system is not efficient as well. This depends on whether unused but readily available arcs in the *ad-hoc* network require more resources than previously given to be used. A network that can freely use all arcs may have a low utilization rate yet retain efficiency as overall resource consumption is low. However, if there is a cost for arc use, a low utilization rate can be a warning sign of a network with too much built up infrastructure which limits the flexibility needed for resiliency.

2.2 Resiliency in Hub-and-Spoke Networks

The hub-and-spoke network design mirrors centralized main and subsidiary control points in a geographically localized fashion. The hub-and-spoke design is very common in air distribution networks because of its efficiencies, and because initial operating costs tend to be lower with the proper hub selection [59]. This streamlined structure tends to scale well by focusing on a few high capacity routes [3], but the efficiencies and scalability come at a cost of resiliency [25]. An, Zhang, and Zeng explain that "single disruptions often resonate network-wide," crippling or removing the capability of the network to continue operating. These network degradations are often followed by considerable economic losses for the network owner [3].

Many researchers attempt to address this issue using alternative hubs and routes to build resiliency [3, 4, 25, 36, 39, 59]. For example, An et al. research multiple allocation hub-and-spoke designs. They note this method of dealing with disruption is reliant on a small number of hubs. They observe that backup hub models can sufficient raise resiliency of the network to a high degree, but acknowledge their model does not consider hub congestion and the corresponding hub availability problem [3]. Zhalechian et al. address the lack of resiliency in the traditional hub-and-spoke problems through careful selection and fortification of key hubs. They propose a bi-objective two-stage SP hub-and-spoke model which provides a resilient response to unexpected disruptions [59]. Torkestani, Seyedhosseini, Makui, and Shahanaghi pivot away from fortifying hubs and favor location based tactics, relying on Monte-Carlo simulations to understand the best hub and edge location to install a resilient network as is [50]. Torkestani et al. envision a hub-and-spoke resilient network design capable of expanding and contracting as necessary. Careful placement of the hub(s) and its edges can maximize material flow stability in multi-modal hub designs under network uncertainties [50]. Tapia echoes that hub placement in a resilient network is an important design element [46]. O'Kelly takes new direction from both the endurance based models by An et al. [3], Zhalechian et al. [59] and location based models such as Torkestani et al. [50], Tapia [46]. O'Kelly reviews a resiliency metric that compares the possible number of interconnections in a network to the subset what remains after a disruption [36]. O'Kelly ties a large number of remaining connections to greater recovery ability, but considers backup secondary main hubs within the scope of practical based on price [36].

2.3 Two Stage Stochastic Design: Looking beyond a single event

Our approach to resilient network design is built on a two-stage stochastic programming framework. This approach follows the example and recommendations of Cox et al. [11] and Kristianto et al. [27]. Two-state SP can effectively capture the action and reaction mechanisms of networks responding to unknown demands and disruptions. Byeon, Hentenryck, Bent, and Nagarajan cover how reaction mechanisms can be based on re-activation of arcs that are triggered by specific ranges of demand inside a resilient network design formulation using a two-stage stochastic program [9]. They describe two-stage SPs as useful for testing across different potential disasters to find allowable levels of costs and dependencies inside the network. Byeon et al. improve the performance of resilient network mechanisms across multiple scenarios by integrating branch-and-price techniques into their solution approach [9]. Sadghiani, Torabi, and Sahebjamnia reinforce the need for scenario based network models, declaring that characterizing networks without scenarios makes the decision maker susceptible to fallacies about the redundancy and resiliency of the network. They consider understanding the level of uncertainty about robustness in the network important enough to include fuzzy parameters for supply and demand [41]. They exhort that a two-stage design is the practical method to address uncertain cost, demand and supply scenarios. Both Byeon et al. [9] and Sadghiani et al. [41] assert that misunderstandings and or false information will compound into wrong objective value projections. Similarly, Smith, Schaefer, and Yen researche a model that penalizes in-correct assumptions about supply and discuss general conclusions about information transfer in stochastic networks [44].

Marufuzzaman, Eksioglu, and Huang apply a two-stage stochastic model to a supply chain. They leverage the timing of the decision variables in their network design to prove it outperformed the deterministic equivalent [31]. Their two-stage stochastic model was compared to a static policy model. The objective value for the stochastic model, after subjecting the first stage constraints to Lagrangian relaxation to establish the upper bound for the cost of the problem, outperformed the deterministic model in speed when the problem was expanded to a large scale [31]. This problem demonstrated the malleability of the two-stage stochastic programming to real world scenarios. Franca, Jones, Richards, and Carlson pursue real world goals as well using a multi-objective stochastic model to explore network under uncertainty for issues other than cost such as quality control [16]. They state two-stage SPs provide a suitable way to compare goals and evaluate the system simultaneously. Franca et al. propose the two-stage SP is capable of minimizing disruption and evaluating risk in terms of gain while obtaining a systemic view of the network [16].

III. Methodology

3.1 Overview

This chapter covers the modelling techniques needed to construct the two-stage stochastic program crafted for this thesis. The chapter covers Min-Cost Network Flow Problem notation followed by how to model disruption in the network. The chapter then concludes with the sets, DVs, parameters, and formulation for the two-stage SP modelled in this thesis.

3.2 Min-Cost Network Flow Problem (MCNFP)

The core of our problem formulations comes from variations on the min-cost network flow problem. Most networks that need to move items from a supply node to a demand node can be modelled as a min-cost network flow problem. An arc is a connection from one node to another. When that connection is one-way it is called a directed arc. Every arc has an associated cost that must be paid for each item that goes through it. All of the items traveling along the arcs available to them to reach their end states is referred to as the flow of goods. The min-cost problem is flowing all of the goods to their end state at the lowest possible cost.

3.2.1 Formulation

Let there be n cities and m routes between the cities. For each node $n \in N$, there is an associated flow parameter, f_i (see [6]). All demands are placed on an entry node and all supplies on an exit node. Every city that supplies material is classified as: supply node if $-\infty < f_i < 0$, transshipment node if $f_i = 0$, and demand node if $0 < f_i < \infty$. The set of possible nodes is then N with the connections determined by set A. The connections between each city form a directed network that is modeled as a graph $G = (N, A)$.

The MCNFP formulation is:

$$\text{Min} \sum_{(i,j)\in A} c_{ij} x_{ij} \tag{1a}$$

$$s.t. \sum_{j:(i,j)\in A} x_{ij} - \sum_{j:(j,i)\in A} x_{ji} = f_i \quad \forall i \in N \tag{1b}$$

$$0 \leq x_{ij} \leq u_{ij} \quad \forall (i,j) \in A \tag{1c}$$

The amount of items (x_{ij}) that flow from one node to another (1b) while minimizing cost (1a) to not exceed or fall short of the number of items required for transportation (1c) must equal demand (f_i). The possible nodes an item can be at, i, is determined by the options given as set, N.

Table 2. MCNFP Sets

Notation	Description
$N = \{1 \dots n\}$	is set of all nodes
$A \subseteq N^2$	is the set of directed arcs, where $\lvert A \rvert = m$

Table 3. MCNFP Parameters

Notation	Description
$f_i \in \mathbb{Z}$	is the demand of each node $i \in N$
$\sum_{i\in N} f_i = 0$	is the total supply $f_i > 0$ and demand $f_i < 0$ at every node $i \in N$ balancing out
$c_{ij} \geq 0$	is the shipping cost per unit of material over arc $(i, j) \in A$
$u_{ij} = M$	is the capacity of arc $(i, j) \in A$ set at a sufficiently large value M to not restrict flow

Example of a Notional MCNFP

We use Figure 1 to demonstrate how a notional network would look. This network sets node 1 as the source node ($f_1 = -2$) and node 4 as the demand node ($f_4 = 2$). Nodes 2 and 3 are transhippment nodes, having neither supply nor demand ($f_2 = f_3 = 0$). We list some parameters and potential solutions in Table 4. In Table 4, set values i and j are placed next to parameter c_{ij} and a list of potential DV values. The DV x_{ij} is given a superscript number to indicate its a possible solution, and a * to indicate its the optimal solution.

Table 4. MCNFP Notional Example Parameters and Solutions

i	j	c_{ij}	x_{ij}^1	x_{ij}^2	x_{ij}^*
1	2	1	0	1.5	2
1	3	7	2	0.5	0
2	4	5	0	1.5	2
3	4	1	2	0.5	0

In Figure 1 the flow of material can follow several paths. Column x_{ij}^1 from Table 4 corresponds to taking all 2 units of supply at f_1 and moving them through nodes 1-3-4 to meet demand at f_4. This ordered set of nodes, 1-3-4, is referred to as a path. The objective value for this solution (path) is $\sum_{i \in \{1,3\}} \sum_{j \in \{3,4\}} c_{ij} x_{ij} = 16$. This solution is then compared to column x_{ij}^2 which splits the flow with 75% of the supply at f_1 traveling path 1-2-4 and the remaining 25% traveling path 1-3-4. The objective value for this split flow is equivalent to $\sum_{i \in \{1,2,3\}} \sum_{j \in \{2,3,4\}} c_{ij} x_{ij} = 13$. Since the goal of a MCNFP is to minimize cost this represents an improvement in value. However, when we solve this problem as a LP via the simplex algorithm, the objective value corresponding to x_{ij}^* obtained is $\sum_{i \in \{1,2\}} \sum_{j \in \{2,4\}} c_{ij} x_{ij} = 12$. We know analytically this is the optimal solution in for the network flows.

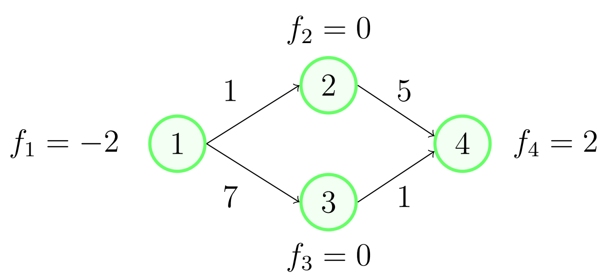

Figure 1. Notional Network Flow

3.2.2 Min-Cost Assumptions

Formulation (1) has assumptions consistent with all linear programming problems:

1. Proportionality

   Definition: This property states the contribution of the objective function from each decision variable is proportional to the value of the decision variable [57]. This property extends to the "contribution of each variable to the left side of each constraint as being proportional to the value of the variable" as well [57].

   Impact: Every unit of material of the same type transferred across an equivalent distance exhibits the same cost. The benefits or penalties assigned on a per-item basis do not grow or diminish with economies of scale.

2. Additivity

   Definition: This property states the contribution to the objective function for any variable is independent of the values of the other decision variables [57]. This property extends to the "contribution of a variable to the left-hand side of each constraint as independent of the values of the variable" [57].

   Impact: (Exogenous/overhead costs) - The order or mix of goods used will not affect the objective function value and neither will the ratio of one arc flow to another. This order does not affect the addition or subtraction of terms in the constraints as well.

3. Divisibility

   Definition: This property states that each decision variable is allowed to assume fractional values [57].

   Impact: (Continuous goods flow (vs. discrete aircraft)) - The amount of goods flowing in the network operates on a continuous scale.

4. Determinism

   Definition: This property states each parameter (objective function coefficient, right-hand side, and technological coefficient) is known with certainty [57].

   Impact: The model converges on the same optimal solution when the same starting parameters and network sets are used.

3.3 Modeling disruptions

By proper selection of the parameters, we can represent the conditions of destroyed nodes, disrupted arcs, and changed demands using the MCNFP.

3.3.1 Arc disruptions

The arc disruption represents real world disruptions to supply and transportation routes. Any good traveling from one location to another must have a conduit enabling its transmission. Real world conduits can be shipping lanes, air routes, railroads, etc. These conduits, which must endure real world restrictions on how many items can transit at one time, correspond to u_{ij}, presented in (1c). Many events that diminishes the amount of goods allowed to flow are accompanied by a smaller u_{ij}. For this problem set the constraint $x_{ij} \geq 0$ could be added to emphasize the non-negativity constraint employed. This is done for convenience as a negative x_{ij} value would represent reverse flow on a directed arc, converting that directed arc into an un-directed arc.

Example of a Disrupted Arc in a MCNFP

The demands of the nodes, ($f_1 = -2, f_2 = f_3 = 0, f_4 = 2$), are kept equivalent to the notional network in section 3.2.1. The pertinent info to described this disrupted network is listed in Table 5. The optimal solution has changed from that for the notional network and the other solutions given may not be feasible.

Table 5. MCNFP with Disrupted Arc Example Parameters and Solutions

i	j	c_{ij}	u_{ij}	x^*_{ij}	x^1_{ij}	x^2_{ij}
1	2	1	0	0	1.5	2
1	3	7	M	2	0.5	0
2	4	5	M	0	1.5	2
3	4	1	M	2	0.5	0

For our example, Figure 2 draws a line through the directed arc $(1, 2)$. If the arc is completely shutdown (i.e., $u_{12} = 0$), then all goods must now flow through path 1-3-4. The positive flow on arc $x_{1,2}$ is no longer allowed. We demonstrate in section 3.2.1 with our notional network example that the objective value for forcing all flow through path 1-3-4 is 16. This solution is now both the only feasible option and the optimal one as well (x^*_{ij}). This value is worse than the 12 units of cost obtained by the optimal solution to the undisturbed network (x^2_{ij}), which now represents an unfeasible path.

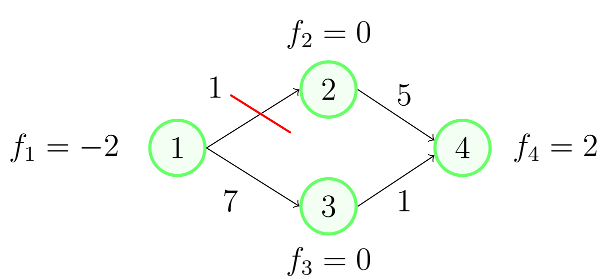

Figure 2. Disrupted Arc

3.3.2 Node disruption

The node disruption represents an inability to gain access to a particular location or state. One real world counterpart would be a city going on lockdown due to political maneuvers by incumbent or rebel forces. Another would be self-regulation if a region is deemed too dangerous for visitation. A less dramatic instance would be the local train tracks breaking down, which reduces the amount of goods allowed to flow out of the city. In all cases, the amount of goods allowed to flow through the city is reduced and needs to be represented in the model. This is accomplished through node splitting, where every disrupted node, i, is replaced with a pair of nodes, i_a and i_b. This arrangement allows for an intercity arc to represent the new restrictions on how many goods can transition through a city.

All inbound arcs to node i are redirected to node i_a, and all outbound arcs from node node i are adjusted to originate from node i_b instead. Finally we connect i_a and i_b with the new arc, (i_a, i_b). This new graph has one more arc and edge than the original graph. Setting $C_{i_a,i_b} = 0$ and $u_{i_a,i_b} = \infty$ gives us a formulation equivalent to the original graph. Alternativelty, if we wish to model a reduced processing capacity at node i, then we can set $u_{i_a,i_b} < \infty$. If we set $u_{i_a,i_b} = 0$, this represents the case where node i is completely shut down and no flow can pass through the node.

Example of a Disrupted Node in a MCNFP

This example's network is based on the notional network presented in section 3.2.1, but it demonstrates the loss of node 2. The inability to travel through node 2 is represented by the red line in Figure 3. The relevant set and parameter values are recorded in Table 6.

Figure 3 shows disrupted node 2 split into two parts. In this configuration, our optimal solution from section 3.2.1 is infeasible because 1-2-4 becomes 1-2a-2b-4 and

Table 6. MCNFP with Disrupted Node Example Parameters and Solutions

i	j	c_{ij}	u_{ij}	x^*_{ij}	x^1_{ij}	x^2_{ij}
1	2a	1	0	0	1.5	2
1	3	7	M	2	0.5	0
2a	2b	0	0	0	1.5	2
2b	4	5	M	0	1.5	2
3	4	1	M	2	0.5	0

we would require $x_{2a,2b} = 2 > 0$. This solution is infeasible to (1c) if $u_{2_a,2_b} = 0$. The new optimal solution (as in 3.3.1) uses path 1-3-4 and the DV values are listed in column x^*_{ij}. The optimal objective function value is 16.

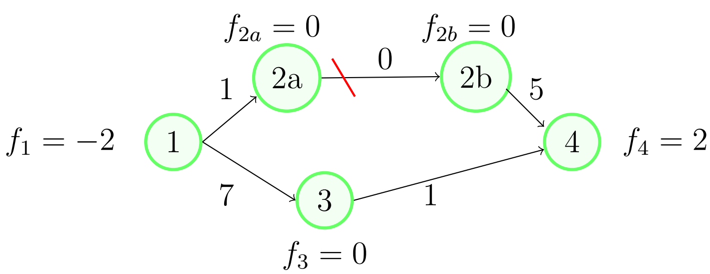

Figure 3. Disrupted Node

3.3.3 Supply/Demand changes

For some disruptions, parameters such as supply magnitude and demand magnitude shift as well. Unexpected bounties or travesties can change the supply and demand for items such as food and medicine. When this occurs the previous solutions may become infeasible. Previously optimal solutions may become sub-optimal. In this environment the allocation of goods must be recalculated in order to maintain an efficient logistics system.

Modified MCNFP Supply and Demand Values

This example's network is based on the notional network presented in section 3.2.1, but it demonstrates the re-allocated flow after changing supply and demand values. The differences in supply in and demand can be seen by comparing Figure 1 and Figure 4. The source node supply increases by 1 ($f_1 = -3$) while transshipment node 3 is converted to a demand node ($f_3 = 1$). Nodes 2 and 4 remain the same. The relevant set and parameter values are recorded in Table 7.

Table 7. MCNFP with Changed Demand Example Parameters and Solutions

i	j	c_{ij}	x_{ij}^1	x_{ij}^2	x_{ij}^3	x_{ij}^*
1	2	1	0	1.5	2	0
1	3	7	2	0.5	0	3
2	4	5	0	1.5	2	0
3	4	1	2	0.5	0	2

With the change in supply and demand as depicted in Figure 7, all the previous solutions for the network are infeasible as they fail to meet demand at node 3. The 2 supply units previously assigned is not enough to meet the 3 units of demand needed for this situation. As such, the model needs to be re-solved. The new optimal solution routes 1 unit of goods through arc (1,3) to satisfy demand at f_3 due to (1b). The other two units are routed through path 1-2-4. The objective value for this optimal solution is $\sum_{i\in\{1,2,3\}} \sum_{j\in\{2,3,4\}} c_{ij}x_{ij} = 19$. That represents a performance 7 units worse than the notional MCNF in section 3.2.1.

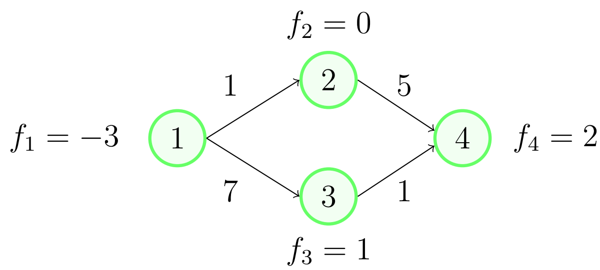

Figure 4. New Demand Network Flow

3.3.4 Scenario set

Networks scenarios are combinations of parameters that dictate what type of disruption, the magnitude of the disruption, and how many disruptions the network will face. All disruptions in this thesis are modelled as an arc or node disruption which has ramifications on cost, capacities, and demands in the network. Any given scenario could have one or more arc disruptions, node disruptions, or numerous accounts of both. Each combination of parameters (i.e., costs, capacities, and demands) represent a unique scenario. The second stage of the two-stage stochastic programming defines the possible scenarios (ω) with the finite set Ω. Each scenario is assigned a probability (p_ω). Let ω index over our scenarios, and $\Omega = \{\omega\}$ is the collection of all scenarios. An inverse build of this portfolio would be to look at a disruptive environment and classify each event conducive to the mayhem as a different ω event. In either use case, $\omega \in \Omega$ is a useful index method for disruptions.

3.4 Two-Stage SP Intro

3.4.1 Generic Two-Stage Stochastic Programming Formulation

Shapiro, Dentcheva, and Ruszczyński provide an excellent primer on the SP modeling paradigm [42]. Their lecture gives the essential formulation as:

$$\min_{x \in \mathbb{R}^n} c^\top x + \mathbb{E}[Q(x, \xi)] \tag{2a}$$

$$\text{s.t. } Ax = b \tag{2b}$$

$$x \geq 0 \in \mathbb{R}^n \tag{2c}$$

$$\text{where } Q(x, \xi) = \min_{y \in \mathbb{R}^m} q^\top y \tag{3a}$$

$$\text{s.t. } Tx + Wy = h \tag{3b}$$

$$y \geq 0 \in \mathbb{R}^m \tag{3c}$$

$$\text{where } \xi = (q, h, T, W) \tag{3d}$$

The first stage is an action mechanism responding to demand b. The action meets this presumed demand using decision variable x, which is a scalar vector. The cost for making those decision is $c^\top x$. While the first stage actions have quantifiable effects (A) to improve the objective function value, there is a limit to what can be done (b) as (2b) implies. The first stage in isolation could be solved as a LP for which several techniques exist in the literature [1, 57]. However, this formulation is complicated by some event causing uncertainty in both the environment and actual demand.

The first stage will end once the decision variable x is determined. Accordingly the first stage represents planned actions taken during a pre-event temporal period. Knowledge of exactly which event will occur is non-existent. The future is considered a spectrum of scenarios with associated probabilities. These probabilities may be derived from historic logs or logical deduction. In order to make an optimal first-stage, the decision maker must consider each possible scenario and the resources needed to attain the optimal solution for that scenario. This consideration should be weighted by probability for each scenario. The second stage provides a different set of parameters for every scenario presented. The scenarios are described by $\xi = (q, h, T, W)$. Responding to the two-stage SP requires a unique mixture of first stage decisions x and second stage decisions y. Future responses to the scenarios are bound both by limits imposed by pre-event decisions, T, and the second-stage demand h. The cost for the second-stage decision vector y is $q^\top y$.

When given a diverse set of scenarios, the two-stage SP records the expected value of these optimal second stage solutions based on the distribution associated with ξ. In this lecture $Q(x, \xi)$ is the cost of the best recourse decision given the first stage decisions, x, and the scenario ξ. The expected value, taken over all scenarios, $\mathbb{E}[Q(x, \xi)]$, is added to the first stage objective (2a).

3.4.2 Two-Stage Stochastic Programming Benefits

This two-stage approach has a number of advantages over a single-stage network formulation approach. In our application, the first stage for the two-stage SP incorporates a set of initial or baseline network flow decisions. The random elements of the second stage are parameters that reflect the supply and demand magnitude(s). Our second stage decision variables (i.e., recourse decisions) are the re-routing of material through the network to meet demand at the lowest possible cost.

The first benefit of this two-stage SP is bypassing the limits of a single stage approach. In a single stage approach, one might consider multiple scenarios independently, but this limits the ability to find solutions that work well under a wide range of scenarios. Often the solutions provided are feasible for a few scenarios, but per-form poorly or are infeasible in many other scenarios. Alternatively, the single stage approach may be configured to provide a single robust solution that seeks feasibility under any scenario, but these solutions tend to be overly conservative — sacrificing either cost or performance in favor of robustness. In contrast, the two-stage approach allows us to both find a set of Stage 1 flows that balance across all scenarios according to their respective probabilities, and to use Stage 2 decisions to ensure feasibility and desired performance in each scenario.

Another benefit is the ability to compare recourse cost across multiple scenarios in direct relation to each scenario's probability. Re-scoping the LP to each predicted scenario gives specific solutions to specific instances but provides zero guards against providing solution sets that are possible for a small subset of scenarios and impossible for the rest. Building solution sets that are always feasible is inherently dealing with the uncertainty in the network, referred to as perturbations in some literature, which provides an account of possible events and actions ([31, 38]).

3.5 Recourse decisions

The recourse decision is the new direction of action after the new information is obtained. The two-stage stochastic programming covers two time periods, the pre-event and the post-event. New information arrives during the event that changes the demands and other parameters given in the pre-event stage to those set for the post-event stage. The real world inferences we are seeking require assumptions about trends from pre-event to post-event changes. Once such thing is growing cost. The event that will eventually occur is considered mostly detrimental to the network. The degraded environment usually raises the cost of every decision. Paying a premium may be required to gain the ability to make certain decisions. Other detrimental trends are parameters such as demand only getting larger or shifting location. The recourse decisions in the model are the primary way to address these changes.

3.5.1 Recourse flows

The recourse flows are the change in arc flow to redirect flow from projected demand in the pre-event period to the actual demand in the post-event period. While our first set of actions responding to planned demand and supply points, our recourse decisions, $(y_{ij}^{\omega,+}, y_{ij}^{\omega,-})$, are responding to the actual situation. For example, securing a contract to ship rice to a particular town guides our pre-determined action. Learning that a town has cancelled their order is new information. Shipping the rice to an adjacent city willing to buy out the contract is the new direction. This aspect is captured by setting the arc cost in a new environment, c_{ij}^{y+} as higher than the original arc cost c_{ij}^{x}. Some of the cost can be recouped by never acting on the first stage decisions, but not all. This reclaimed cost is represented by c_{ij}^{y-} in the model. The relationship of our two-stage parameters for flow cost is summarized $0 \leq c_{ij}^{y-} \leq c_{ij}^{x} \leq c_{ij}^{y+}$ $\forall(i,j) \in A$. This final action of pulling the original order and sending it to a new location is represented by the decision variables $y_{ij}^{\omega,-}$ and $y_{ij}^{\omega,+}$ respectively. These new flows are subject to equal flow constraints as represented by

$$\sum_{i:(i,j)\in A} (x_{ij} + y_{ij}^{+} - y_{ij}^{-}) - \sum_{j:(j,k)\in A} (x_{jk} + y_{jk}^{+} - y_{jk}^{-}) = f_j \tag{4a}$$

The first stage flow into node j as x_{ij} plus any augmented flow, y_{ij}^{+}, must equal the flow out, x_{jk}, and the augmented outflow y_{jk}^{-} plus any demand at node j represented as f_j. The flow in the first or second stage can be withdraw to satisfy the equal flow constraints, with these withdrawn flows being y_{ij}^{-} and y_{ij}^{+} respectively.

Example of MCNFP Recourse Flows

The demands of the nodes, ($f_1 = -3, f_2 = 0, f_3 = 1, f_4 = 2$), are kept equivalent to the example network in section 3.3.3. The pertinent info to describe the recourse process for this network is given in Table 8. The changes necessary to obtain an optimal solution are illustrated through Figures 5-7.

Table 8. MCNFP Recourse Flows' Example Parameters and Solutions

i	j	c_{ij}^{x}	$c_{ij}^{y,-}$	$c_{ij}^{y,+}$	x_{ij}^{1}	x_{ij}^{*}	y_{ij}^{-}	y_{ij}^{+}
1	2	1	1	2	2	2	0	0
1	3	7	1	8	1	0	1	1
2	4	5	1	6	2	0	0	0
3	4	1	1	2	0	0	0	0

In Figure 5 the un-disrupted network has an optimal value of 19 due to using the same optimal solution given in section 3.3.3. The DV values for this solution are listed as x_{ij}^{1}. However, we then gain new information about the network which causes arc (1,3)'s capacity to become zero. In Figure 6 you can see this represented by the red line restricting all flow through arc (1,3). Our previously optimal solution given by x_{ij}^{1} is no longer feasible as $x_{1,3}$ no longer allows positive flow. To satisfy (4a) we must retract the one unit of flow on arc (1,3) given in stage one. This is done by setting $y_{1,3}^{-} = 1$ which takes out stage one arc flow. With the previous objective solution no longer feasible, our next best solution is to only route 2 units of flow through path 1-2-4. However, by (4a) not meeting demand at node 3 makes this solution infeasible. In response, we then augment flow from node 1 to node 3 by using an external arc that still allows positive flow albeit at a higher cost. This sequence of choices is illustrated by Figure 7. These set of choices necessary to to make these decisions are given by columns x_{ij}^{*}, y_{ij}^{-}, and y_{ij}^{+}. The corresponding objective value is 28 which is the cost for the optimal decision to respond to the situation.

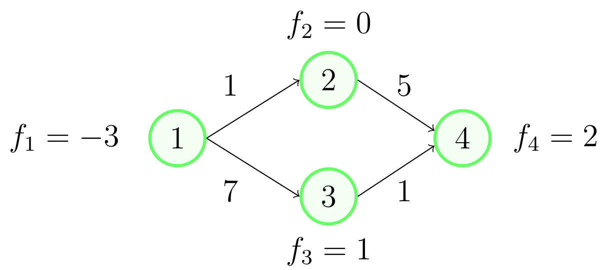

Figure 5. Network Recourse Flow: Baseline

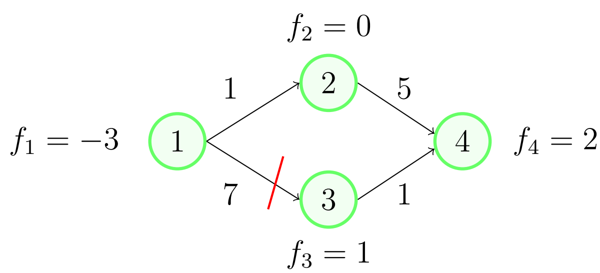

Figure 6. Network Recourse Flow: Disruption

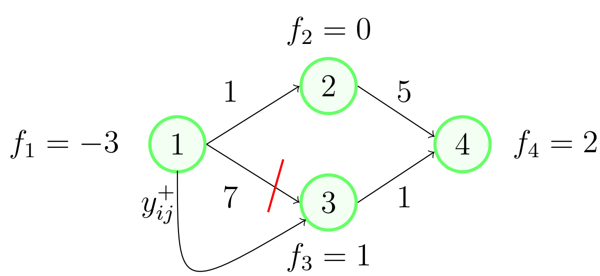

Figure 7. Network Recourse Flow: Restoration

3.5.2 Arc Activation

Arc activation is the decision to pay for the ability to use recourse flows. That decision is represented by a_{ij}^{ω}. Rerouting any item to a new location requires understanding the new distance and disseminating that information to the transport team. This change is assumed to cause some inconvenience, lost time, and other resources. To represent the negative aspect of demand transferal an activation cost, (c_{ij}^{a}), is assigned to any change. Whether that change be in demand magnitude or location.

The same goes for changes in supply.

Example of Arc Activation

For example, lets use the exact same network as Figure 7 in section 3.5.1. The red line in Figure 8 indicates arc flow on arc (1,3) is no longer possible. It is still possible to take the path 1-2-4, but like before this fails to meet demand at node 3. We still have external arc (1,3) for recourse flow as an option. However, this external arc is considered broken but repairable, as represented by the dashed line. In order to establish positive flow on this arc, we must pay an activation (or repair) cost, c^a_{13}. We set $c^a_{13} = 5$. As the arc flows in the system must balance out, the 5 unit premium must be paid in order to restore arc (1,3). The previous optimal objective value for with an unbroken arc was 28 units. However, once we pay the 5 unit premium the network is repaired as seen in Figure 9. Since re-allocating the flow can only cause worse objective function values, the flow distribution remains the same. With all else constant after paying the premium the new optimal objective function value is $28 + 5 = 33$.

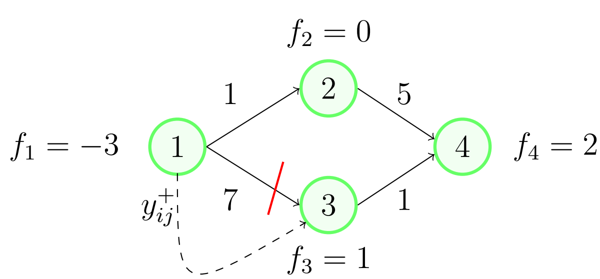

Figure 8. Disrupted Arc not Restored

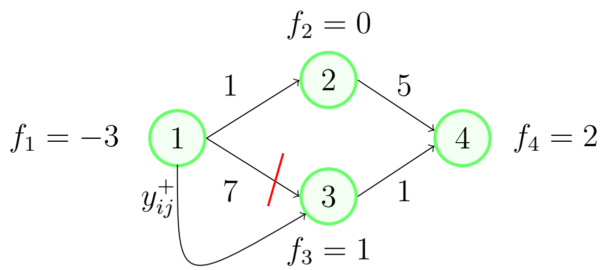

Figure 9. Restored Disrupted Arc

3.5.3 Slack Variables

Every problem set must have a supply greater than or equal to demand, or some method of compensating for the gap. That gap is the slack in the system, s_i. That slack has two parts: the unmet demand (called a shortage) and too much supply (called excess). Our model divides the slack into two decision variables, one for shortages, $s_i^{\omega,-}$ and and one for excesses $s_i^{\omega,+}$. Both shortages and excesses are penalized on a per-item basis by $q_i^{\omega,-}$ and $q_i^{\omega,+}$ respectively. Using slack gives us the ability to assess whether meeting a particular demand is worth the resources needed. We can demonstrate this comparison by modifying (4a) to become

$$\sum_{i:(i,j)\in A} (x_{ij} + y_{ij}^+ - y_{ij}^-) - \sum_{j:(j,k)\in A} (x_{jk} + y_{jk}^+ - y_{jk}^-) = f_j - s_j^- + s_j^+ \tag{5a}$$

All of the conditions that applied before with one modification. The demand at f_j can be modulated by paying a shortfall s_j^- or excess s_j^+ penalty.

Example of Slack Variables

We continue with the network in Figure 8 from section 3.5.2 to demonstrate the flexibility provided by allowing slack variables. As presented in Figure 10, we again face the challenge of routing goods across arc (1,3) at the price of $c^a_{ij} = 5$. However, unlike before we now have the ability to modulate demand at node 3, by paying the slack price $q_3 = 1$ given in Table 9. However we now have one excess unit of supply in the network at node 1. We can route that unit across arc (1,2) and pay $c_{1,2} = 1$ unit of cost for transfer and $q^+_2 = 1$ unit of cost to leave that unit at node 2 as excess. The cost to remove demand at node 3, move supply from node 1, and leave the supply at node 2 as excess gives a cost of $1 + 1 + 1 = 3$. If we pay shortfall and excess penalties instead of paying a 5 unit premium to restore arc (1,3), we save $5 - 3 = 2$ units of cost improving the objection function value to 31 units.

Table 9. MCNFP Example Slack Penalties

i	q^-_i	q^+_i
1	1	1
2	1	1
3	1	1
4	2	2

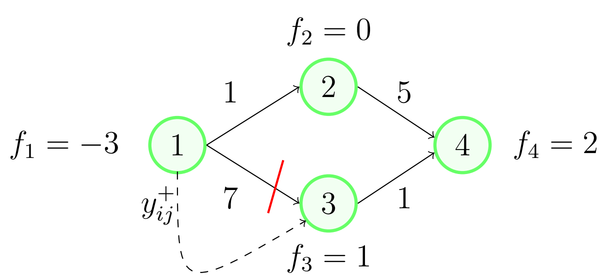

Figure 10. Disrupted Arc not Restored

3.6 Definition

To assign material between the cities in the first stage, a scalar variable x_{ij} is used for each $(i, j) \in A$. Note that if $0 < x_{ij} < \infty$ the material flows from one port to another. If $x = 0$, no material flows. If $x_{ij} > 0$ some amount of material is deemed prudent to ship despite whatever conditions may occur given the scenarios probabilities distribution p_ω. The ability to augment or reconsider the assigned material flow is governed by second stage decisions $y_{ij}^{\omega,+}$ and $y_{ij}^{\omega,-}$.

The augmented flow at a higher expenditure allowed for materials to be shipped at a subsequent period in time. Note that if $0 < y_{ij}^{\omega,+} < \infty$, then augmented material flows from first stage decisions. If $y_{ij}^{\omega,+} = 0$ then no additional material flows.

The withholding of material at some non-zero expenditure rate allowed for reassignment of first stage goods. This withholding is represented by $y_{ij}^{\omega,-}$. If $0 < y_{ij}^{\omega,-} < \infty$ then some material is rescinded from first stage decisions. If $y_{ij}^{\omega,-} = 0$, then there is no material flow reduction.

Actual supply or demand for the final amount of goods may be subject to some shortage $s_i^{\omega,-}$ or excess $s_i^{\omega,+}$.

All of these flows are subject to capacity limits u_{ij}^ω.

Table 10. Sets

Notation	Description
$N = \{1 \dots n\}$	is set of all nodes
$A \subseteq N^2$	is the set of directed arcs, where $\|A\| = m$
$\Omega = \{1 \dots \omega\}$	is the set of scenarios

Table 11. Decision Variables

Notation	Description
$x_{ij} \geq 0$	is the stage 1 amount of material shipped over arc $(i,j) \in A$
$y_{ij}^{\omega,+} \geq 0$	is the stage 2 decision for scenario ω in Ω giving the amount of additional material shipped over arc $(i,j) \in A$
$y_{ij}^{\omega,-} \geq 0$	is the stage 2 decision for scenario ω in Ω giving the amount of material not shipped over arc $(i,j) \in A$
$s_i^{\omega,+} \geq 0$	is the stage 2 supply excess at node $i \in N$ under scenario $\omega \in \Omega$
$s_i^{\omega,-} \geq 0$	is the stage 2 supply shortfall at node $i \in N$ under scenario $\omega \in \Omega$
$a_{ij}^{\omega} \in \{0,1\}$	is the stage 2 binary decision representing the choice to "activate" arc $(i,j) \in A$.

Table 12. Parameters

Notation	Description
$c_{ij}^{x} \geq 0$	is the shipping cost per unit of material over arc $(i,j) \in A$ for the stage 1 decision
$c_{ij}^{y+} > c_{ij}^{x}$	is the shipping cost per unit of additional material over arc $(i,j) \in A$ under scenario $\omega \in \Omega$ during Stage 2
$c_{ij}^{y-} \geq 0$	is the shipping savings per unit of material not shipped over arc $(i,j) \in A$ under scenario $\omega \in \Omega$ in Stage 2
$c_{ij}^{a} \geq 0$	is the installation cost for missing or destroyed arcs $(i,j) \in A$
$f_i^{\omega} \in \mathbb{Z}$	is the demand of each node i in N under scenario $\omega \in \Omega$
$p_{\omega} \in [0,1]$	is the probability scenario $\omega \in \Omega$ occurs, where $\sum_{\omega\in\Omega} p_{\omega} = 1$
$q_i^{\omega,-}$	is the penalty for when a shortfall of goods occur at node $i \in N$ under scenario $\omega \in \Omega$
$q_i^{\omega,+}$	is the penalty for excess goods at node $i \in N$ under scenario $\omega \in \Omega$
$u_{ij}^{\omega} \geq 0$	is the capacity of arc $(i,j) \in A$ under scenario $\omega \in \Omega$
$u_{ij}^{a} \geq 0$	is the added capacity on arc (i,j) when $a_{ij} = 1$

$$\text{Min} \sum_{(i,j)\in A} c^x_{ij} x_{ij} \tag{6a}$$

$$+\sum_{\omega\in\Omega} p_\omega \tag{6b}$$

$$\left(\sum_{(i,j)\in A} c^{y+}_{ij} y^{\omega,+}_{ij} - c^{y-}_{ij} y^{\omega,-}_{ij} \right. \tag{6c}$$

$$+\sum_{i\in N} (q^{\omega,+}_i s^{\omega,+}_i + q^{\omega,-}_i s^{\omega,-}_i) \tag{6d}$$

$$\left. + \sum_{(i,j)\setminus A} c^a_{ij} a^\omega_{ij} \right) \tag{6e}$$

s.t.

$$a^0_{ij} = y^{0,-}_{ij} = y^{0,+}_{ij} = 0 \qquad \forall (i,j) \in A \tag{7a}$$

$$s^{0,-}_i = s^{0,+}_i = 0 \qquad \forall i \in N \tag{7b}$$

$$\sum_{i:(i,j)\in A} (x_{ij} + y^{\omega,+}_{ij} - y^{\omega,-}_{ij}) - \sum_{j:(j,k)\in A} (x_{jk} + y^{\omega,+}_{jk} - y^{\omega,-}_{jk}) = \tag{8a}$$

$$f^\omega_i - s^{\omega,-}_i + s^{\omega,+}_i, \quad \forall i \in N,\ \omega \in \Omega \tag{8b}$$

$$s^{\omega,+}_i \geq 0,\ s^{\omega,-}_i \geq 0 \qquad \forall i \in N,\ \omega \in \Omega \tag{9a}$$

$$a^\omega_{ij} \in \{0,1\} \qquad \forall (i,j) \in A,\ \omega \in \Omega \tag{9b}$$

$$0 \leq x_{ij} + y^{\omega,+}_{ij} - y^{\omega,-}_{ij} \leq u^\omega_{ij} + u^a_{ij} a^\omega_{ij}, \qquad \forall (i,j) \in A,\ \omega \in \Omega \tag{10a}$$

$$0 \leq x_{ij} - y^{\omega,-}_{ij} \leq u^\omega_{ij}, \qquad \forall (i,j) \in A,\ \omega \in \Omega \tag{10b}$$

$$y^{\omega,+}_{ij} \leq M a^\omega_{ij} \qquad \forall (i,j) \in A \tag{10c}$$

$$0 \leq x_{ij} \leq u^0_{ij} \qquad \forall (i,j) \in A \qquad (10d)$$

The objective function minimizes the cost for the flow of goods in the first stage with the first term (6a). After the first stage one of several scenarios occur and some scenarios present major setbacks.

The second term, (6b), reflects the branching possibilities over all scenarios $\omega \in \Omega$. Lines (6c) - (6e) give the cost contribution from flow redistribution, shortfalls or excess, and arc activation / rebuilding, respectively, under the given scenario.

Constraints (7a) to (10d) restrict all recourse decision variables to 0 for scenario 0. We do this because scenario $\omega = 0$ is the baseline scenario, where there is no recourse opportunity.

To ensure the proper flow of goods, constraints (8a) and (8b) balance the amount of goods flowing into, out of, and kept by a given node based on node supply or demand in each scenario. Slack variables offset imbalances in supply and demand to guarantee equilibrium. The original slack variable(s) (s^ω_i) were converted to deviational variables for benefits covered by Winston [57].

Constraints (9a) enforces the non-negativity of the slack variables.

Each scenario limits the amount of material shipped on arc $(i, j) \in A$ by the baseline capacity u^0_{ij} in (10a). Constraint (10a) also affords the opportunity to augment the capacity by u^a_{ij}.

IV. Testing, Results, and Analysis

4.1 Overview

Our method of creating a resilient network design to represent the West Africa Logistics Network (WALN) involved a computational approach to the problem. We created a two-stage stochastic model dependent on probability based events to analyze the resiliency-focused objective. The goals of the model are to create a program capable of: taking real world events and modelling them at a high level, taking into account the frequency of those events, and responding in a cost efficient manner to those events.

This chapter first identifies the network characteristics and assumptions unique to the WALN. This is followed by a review of the scenarios created for this problem. This chapter then covers the results which displays the final network flows and exposes some trends in the data. The last part presents network measures listed in Table 22 followed by a summary of the impact this two-stage SP can have.

4.2 Experiment Data Set

4.2.1 Network Topology

We tested our formulation using a network of seven cities (i.e. nodes; see Table 13). The seven cities are the Tier 1 hub cities select by Baker [5] as nodes requiring a unique demands [11]. Baker narrowed down the cities most capable of shifting large volumes of goods through her hub-and-spoke model and identified them as Tier 1 cities. Since her facility location formulation does not account for disruptions, her hub-and-spoke solution design sacrifices resiliency in order to improve efficiency [5]. We will seek to improve Baker's work by starting from the Tier 1 cities and incorporating resiliency into the model using our two-stage formulation. The WALN flows goods from supply points in the North American and European continents to these cities inside West Africa. From these cities, the supplies are moved within the theater using C-17s to the forward operating locations.

Any route that exceeds 2600 KM is prevented from being used in the baseline scenario to represent transportation and aircraft operation limits of C-17. This limit was imposed for demonstration purposes to indicate that transportation range is a factor for consideration. This limitation prevents arcs from Dakar and Accra to cities east of Niamey. Only C-17's were considered in this thesis and as such the arc cost are based on flying cost per KM estimates for C-17's [46].

Notably, in Table 13, we use two network nodes to represent the city Niamey. This is because in one of our scenarios (scenario 1) the city of Niamey becomes unavailable for transshipment flows. To allow for this scenario we use the principles outlined in section 3.3.2 to create two nodes: Niamey-in (which collects all inbound arcs to Niamey) and Niamey-out (which collects all outbound arcs from Niamey). We then connect these nodes with an intercity arc (see Table 13). The shutdown of Niamey is represented by restricting flow across the Niamey intercity arc.

Changing the network design from a hub-and-spoke to an arc based design allows for model designation of arc flow. The dense graph representation of the network allows the model to respond flexibly. Unlike the hub-and-spoke design employed by Baker [5], the recourse arc flow is not fixed in one direction between any two cities. Baker's thesis prioritizes sending goods from Tier 1 hub cities to less active cities. Our design pursues a self directed flow of goods in order to understand more about possible network designs.

Table 13. Network Demand by City

Node	**Country**	**City (i)**	f_i^0	f_i^1	f_i^2
A	Burkina Faso	Ouagadougou	10	0	0
B	Cameroon	Garoua	0	0	0
C	Chad	N'Djamena	0	8	13
D	Ghana	Accra	-20	-20	11
E	Niger	Agadez	14	16	-5
F	Niger	Niamey-in	0	0	0
G	Niger	Niamey-out	0	0	0
H	Senegal	Dakar	-4	-4	-19

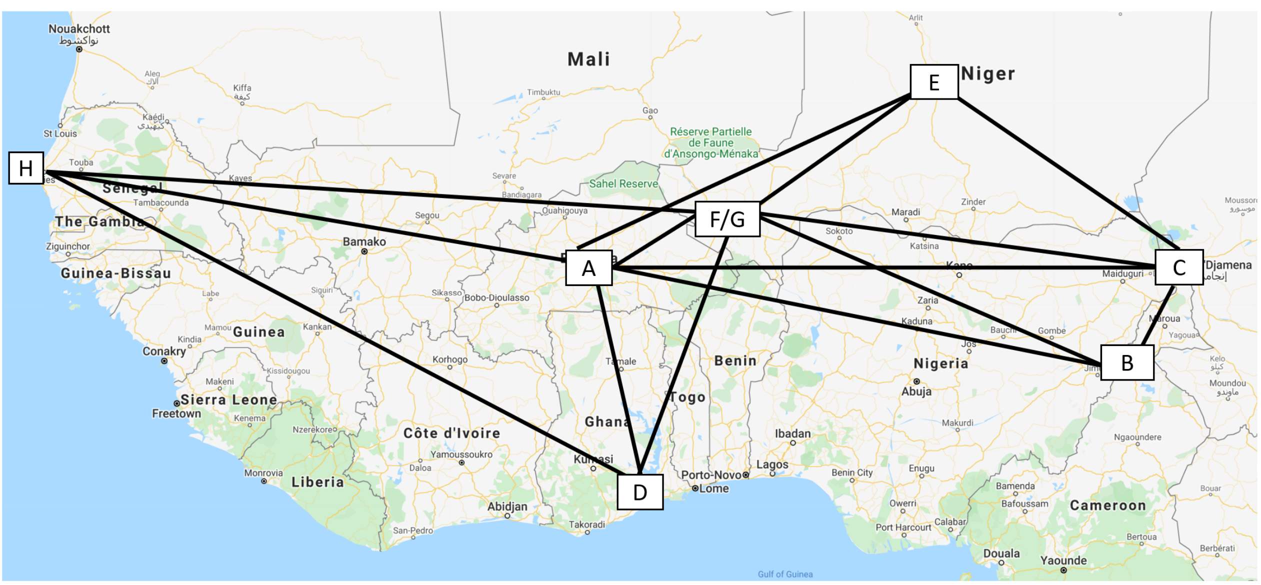

Figure 11. Network Outline

4.2.2 Scenarios/ Parameters

This section covers assumptions and observations about scenarios followed by a review of the parameters that compose them. The end of this section is analysis of how the network performed when subject to these scenarios.

Assumptions/Limitations

Real world wear and tear will be disregard in this model. Similarly, we assume the transportation vehicle will never have a reduced carrying ability. Finally, we assume the amount of miles travelled for the delivery of all goods will not run into lifetime mileage limits. These assumptions parallel those assumptions pertinent to all LPs. The lack of real world wear and tear supports proportionality as the scale of time needed or volume of goods delivered will not impact the cost in a non-linear fashion. Removing concerns about lifetime mileage limits the sporadic and unpredictable failures (in relatively small time-volume windows) in the system which violate the LP assumption of determinism.

Parameters Independent of ω

The first stage DVs, $c^x_{ij}, c^{y+}_{ij}, c^{y-}_{ij}$, and c^a_{ij} are all independent of ω. The Stage 1 flow cost c^x_{ij} is the distance between the cities i and j, with ranges between 400 and 1600 KM. Our example uses the Matlab generated distances based on geographic coordinates that are rounded to the nearest whole number (See Appendix 5.2). The distances in c^x_{ij} are multiplied by 1.2 to create c^{y+}_{ij} (See Appendix 5.2). In a similar fashion, we set $c^{y-}_{ij} = 0.5c^x_{ij}$ for all $(i, j) \in A$. We set $c^a_{ij} = 1000$ for each $(i, j) \in A$ to signify there are barriers to augmented flows in the model that are significant but not prohibitive.

The second stage DV's as well as $u^\omega_{ij}, f^\omega_i, q^{\omega,+}_i, q^{\omega,-}_i$, and p_ω are all set based on scenario. All the parameters given are listed in Table 12,

Scenario 0

Scenario $\omega = 0$ is our baseline scenario. It represents the optimal state of the network, and it should represent the network's operating state most of the time. This scenario retains full arc capacity throughout the network which is set at $u_{ij}^0 = 50$ for every arc $(i, j) \in A$.

Determining the location of supply and demand nodes was a two fold process. In Baker [5], the port Accra is one of the dominant supply points with respect to delivering goods to the interior of West Africa, and in a similar manner is chosen as a supply point for this scenario. From this point on, we deemed it prudent to create a flow conducive to the left to right flow presented in textbooks and other documents [1, 42]. We hold this pattern by generally placing supply nodes in the west and demands nodes in the east. This scenario places 20 units of supply at Accra (Node D), 4 units of supply at Dakar (Node H), along with 10 units of demand at Ouagadougou (Node A) and 14 units of demand at Agadez (Node E). The placement of these supplies and demand can be seen in Figure 12 and the values are summarized in Table 13.

While shortfall and excess are not allowed in the baseline scenario due to $s_i^{0,-} = s_i^{0,+} = 0$ via (7b), the values for $q_i^{0,-}$ and $q_i^{0,+}$ initialize all the following scenario shortfall and excess penalties. The vectors for $q_i^{0,-}$ and $q_i^{0,+}$ were randomly generated. The shortfall and excess penalties for other scenarios was based on multiplying the baseline penalty by $q_i^{\omega,-} = q_i^{0,-}(1 + \frac{\omega+1}{6})$ and $q_i^{\omega,+} = q_i^{0,+}(1 + \frac{\omega+1}{6})$. The exact values used are outlined in Appendix section 5.2.

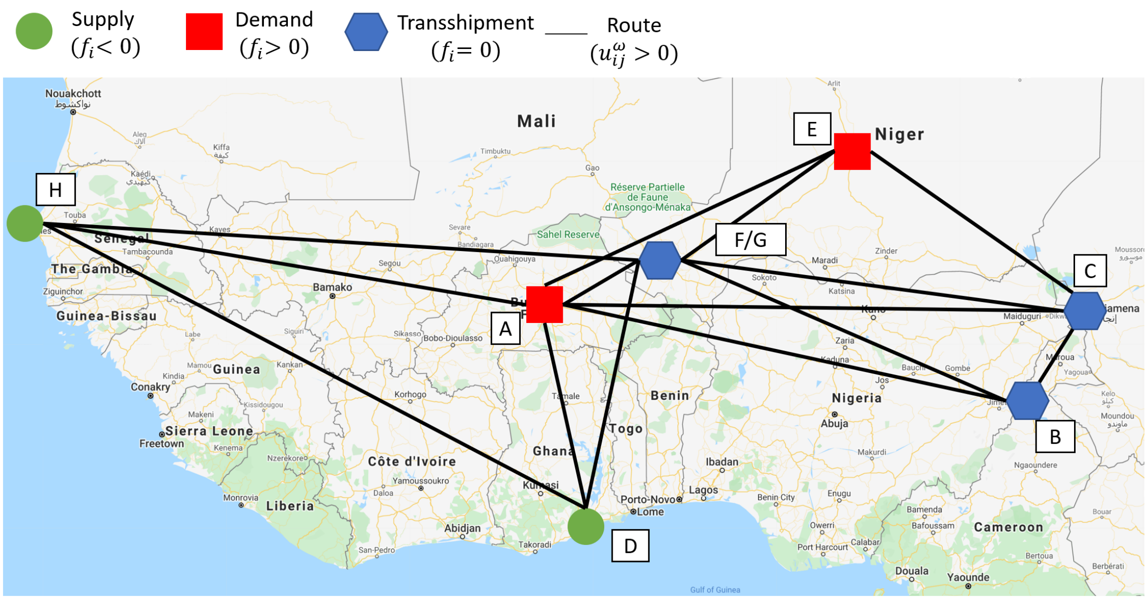

Figure 12. Scenario 0: Baseline

Scenario 1

Our next scenario, where $\omega = 1$, corresponds to the complete shut-down of Niamey (Node FG) as a disrupted node. A political protest that blocks the main roads can represent the sort of challenges capable of shutting down a city from a logistics perspective. The penalties for excess ($q_i^{1,+}$) or shortage ($q_i^{1,-}$) are 16.7% higher than the baseline values to represent the potential for the protest to spread causes widespread hoarding. To represent the loss of the primary methods of travel the related arcs capacity u_{FG}^1 is set to zero (See Table 14). Changes in demand as well as nodes force more intricate parts of the model to be used. The model deals with Niamey's complete shutdown, where Niamey's intercity arc capacity is set to zero. Other starting conditions change as well. The supply nodes do not change as Accra (Node D) provides 20 units of supply and Dakar (Node H) supplies 4 units, however the demand nodes change with demand at Agadez (Node E) growing to 16 units and the second demand shifts from Ouagadougou (Node A) to N'Djamena (Node C) for 8 units. The placement of these supplies and demand can be seen in Figure 13.

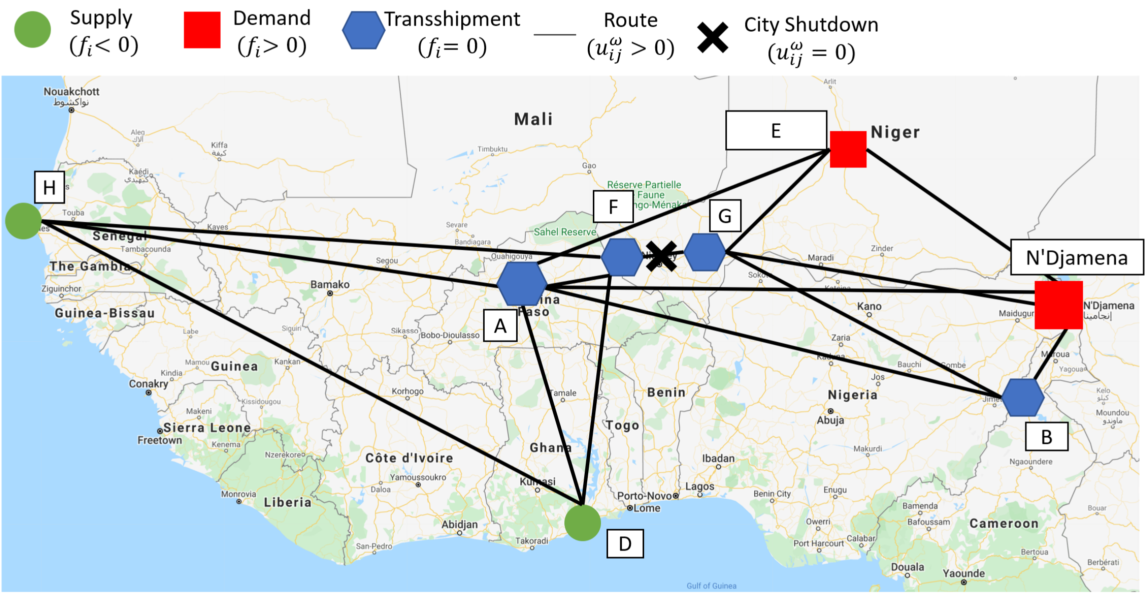

Figure 13. Scenario 1: Niamey Shutdown

Scenarios 2

The last scenario, where $\omega = 2$, completely shuts-down the arc between Dakar (Node H) and Ouagadougou (Node A) as an arc disruption. This scale of disruption could be due to interstate hostilities that both block off the most direct routes between Dakar (Node H) and Niamey (Node FG) while raise the price of goods in the surrounding areas. The penalties for excess $(q_i^{2,+})$ or shortage$(q_i^{2,-})$ is 33% more than the baseline, which reflects the extra security precautions everyone takes. This change in capacity caused by the hostilities is recorded in Table 14. The setup for this scenario was also constructed to show how radical shifts in supply and demand magnitudes are still viable. Changes in the demand and supply locations makes direct comparison to the previous two scenarios harder, but still provides insight into the network's design. The supply at Dakar (Node H) increases to 19 units and Agadez (Node E) gains a supply of 5 units. Accra (Node D) demands 11 units of material and N'Djamena (Node C) demands 13. The placement location of these supplies and demand can be seen in Figure 14.

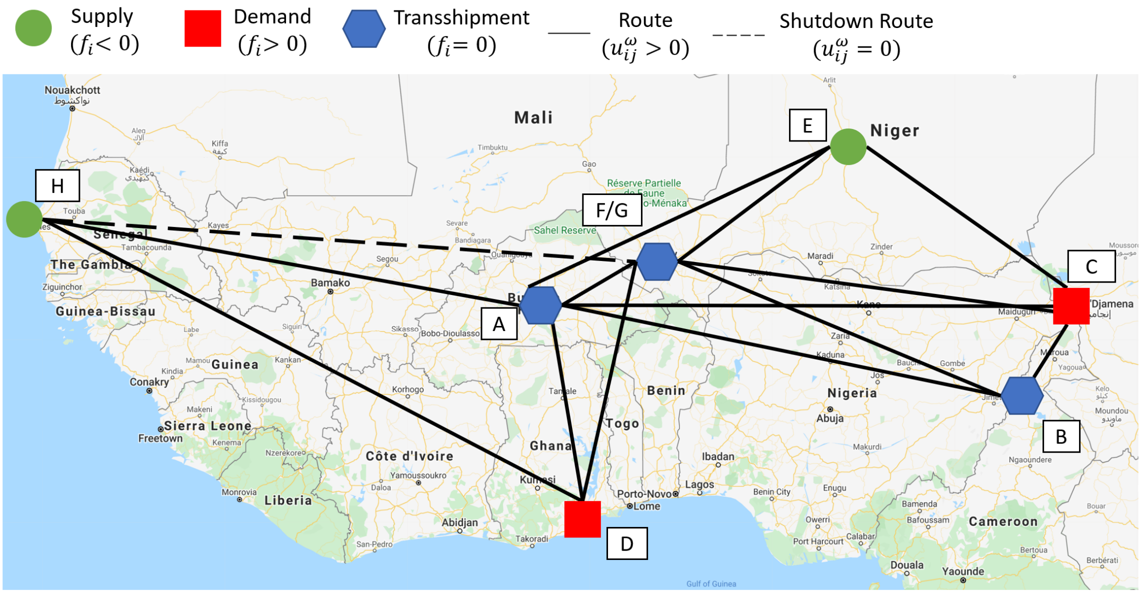

Figure 14. Scenario 2: Dakar-Niamey Route Shutdown

Table 14 is a concise summary of the arc capacity changes listed in the scenarios above. The ability to only partially shutdown a city or route is not demonstrated in this thesis, however that option is possible and the reduced capacity value rather than the zeros representing complete shutdown below would be present in the table below.

Table 14. Scenario Based Disruption

ω	routes /nodes reduced	corresponding arcs (i-j)	new capacity (u^{ω}_{ij})
0	None	(Baseline)	
1	Niamey Shutdown	F − G	0
2	Dakar−Niamey Reduced	H − F	0

4.2.3 Probability Sets

Probability shapes the risk posed by a given set of disruptions. The larger the disruption and the more probable it is, the more resources a resilient strategy must secure to prepare for potential disaster. Conversely, lower frequencies or disruptive potential for events allows the model to pursue better objective function values. A probability set is a group of scenarios, ω, and there associated probabilities. We consider several probability sets containing scenarios $\{0, 1, 2\}$ where each probability set assigns different probabilities to the scenarios. This will demonstrate the impact of having one scenario more prevalent than others. Our first probability set is considered a fair assessment of the risk distribution over the scenarios, with $p_0 = 70\%$ for the baseline scenario, $p_1 = 20\%$ for the scenario where Niamey is shut down, and $p_2 = 10\%$ gives the liklihood that the Dakar−Niamey route closes. These probabilities, as well as similar probabilities for five more probability sets, are summarized in Table 15.

Table 15. Scenario Probability Sets

	p_ω		
Set Number	p_1	p_2	p_3
I	0.7	0.2	0.1
II	0.2	0.7	0.1
III	0.1	0.2	0.7
IV	1.0	0	0
V	0	1.0	0
VI	0	0	1.0

4.3 Results

4.3.1 Scenario Solutions

In our study, we found that our optimal flow decisions were insensitive to the scenario probabilities. That is, all probability sets used the same recourse flows. The first stage flows are necessarily the same by constraints (7a) and (7b) since all probability sets use the same baseline scenario.

In this section we present an optimal set of flows for every scenario. We represent the final flows, including both first and second stage flow variables; i.e., $x_{ij} + y_{ij}^{\omega,+} - y_{ij}^{\omega,-}$ $\forall (i, j) \in A$. We present the values in a matrix form, where the arc source (i) is given by the row, and the arc sink (j) is given by the column. Our optimal second-stage slack variables are also the same for all probability sets. They are given in Table 20 and Table 21.

Scenario 0

The baseline scenario's ($\omega = 0$) results can be interpreted as meeting the demands presented in Table 13 through the node to node pairings read from the matrix in Table 16. Only 14 of the 20 units of material available at Accra flow to Niamey, while the other 6 units flow to Ouagadougou. From Niamey the 14 units flow to and fulfill all demand at Agadez. All 4 units of material at Dakar flow to Ouagadougou, fulfilling the demand at Ouagadougou.

Table 16. Baseline Network Final Flow (Scenario 0)

City Name	A	B	C	D	E	F	G	H
Ouagadougou(A)								
Garoua(B)								
N'Djamena(C)								
Accra(D)	**6**					**14**		
Agadez(E)								
Niamey-in (F)							**14**	
Niamey-out (G)					**14**			
Dakar(H)	**4**							

Scenario 1

In the scenario concerning the disruption of Niamey ($\omega = 1$), the decision to restore the intercity arc provides the problem setup. This problem setup is solved by reconstructing the Niamey intercity arc as it improves the objective function more than any other alternative. This arc construction involves paying the c_{ij}^{a} penalty, which is set to 1000 for all arcs in a_{ij}^{ω}. The newly constructed direct arc is represented as a one in Table 17.

Once the original network is restored, routing the materials is the next step. The

Table 17. **Niamey Shutdown Bridge Construction (Scenario 1)**

City Name	A	B	C	D	E	F	G	H
Ouagadougou(A)								
Garoua(B)								
N'Djamena(C)								
Accra(D)								
Agadez(E)								
Niamey-in (F)							**1**	
Niamey-out (G)								
Dakar(H)								

supply points and demands have changed from the baseline scenario. The demand at Agadez increases by 2 units to 16 units while the demand at Ouagadougou simultaneously shifts to N'Djamena and decreases by 2 units to 8 units of demand. Similar to the baseline 14 units of material are sent from Accra through Niamey to Agadez. These 14 units are recorded as augmented flow since they were not possible in the first stage. Dakar sends 4 units to Ouagadougou. Accra also routes 6 units to Ouagadougou as well. However the similarity to the baseline scenario stops here. Only 2 units of the 10 available at Ouagadougou are shipped to Agadez. The last 8 units are left in the network as excess They are stored in the $s_i^{\omega,-}$ and $s_i^{\omega,+}$ decision variables as necessary (see Table 21). The last 8 units of demand at N'Djamena are met by locally sourced (generated at and shipped from) materials from Garoua.

Table 18. **Niamey Final Flow (Scenario 1)**

City Name	A	B	C	D	E	F	G	H
Ouagadougou(A)					**2**			
Garoua(B)			**8**					
N'Djamena(C)								
Accra(D)	**6**					**14**		
Agadez(E)								
Niamey-in (F)							**14**	
Niamey-out (G)					**14**			
Dakar(H)	**4**							

Scenario 2

Disrupting the Dakar−Niamey route in scenario $\omega = 2$ forces all 5 units from Agadez to meet some of the demand in N'Djamena. Niamey sources the other 8 units through Agadez to fulfill all 13 units of demand at N'Djamena. The 4 units sent to Ouagadougou from Dakar during the first stage are rerouted to meet demand Accra. Then another 7 units are shipped directly to Accra from Dakar to meet all 11 units of demand at Accra. The 8 units remaining at Dakar are counted as excess.

Table 19. Dakar−Niamey Route Shutdown Final Flow (Scenario 2)

City Name	A	B	C	D	E	F	G	H
Ouagadougou(A)				**4**				
Garoua(B)								
N'Djamena(C)								
Accra(D)								
Agadez(E)			**13**					
Niamey-in (F)								
Niamey-out (G)					**8**			
Dakar(H)	**4**			**7**				

Shortage and Excess

The corresponding shortages and excesses for all scenarios covered are captured in Table 20 and Table 21. In all scenarios the shortage and excess was held to zero for the baseline scenario.

Examined Scenario Responses

Looking at commonalities between the scenario solutions, we observe that having two central nodes, Niamey and Ouagadougou, is crucial to forming a resilient network. Across all probability sets Niamey is used in both scenarios $\omega = 0$ and $\omega = 1$. The baseline scenario routes flows through Niamey without impedance. Scenario $\omega = 1$ restores Niamey as the optimal solution to reducing that scenario's cost. Scenario

Table 20. Shortage by Scenario $s_i^{\omega,-}$

	ω	
City Name	1	2
Ouagadougou	0	0
Garoua	0	0
N'Djamena	0	0
Accra	0	0
Agadez	0	0
Niamey-in	0	0
Niamey-out	0	8
Dakar	0	0

Table 21. Excess by Scenario $s_i^{\omega,+}$

	ω	
City Name	1	2
Ouagadougou	8	0
Garoua	0	0
N'Djamena	0	0
Accra	0	0
Agadez	0	0
Niamey-in	0	0
Niamey-out	0	0
Dakar	0	8

$\omega = 2$ deals with the shutdown of the route Dakar to Niamey by shipping some materials through Ouagadougou which is also centrally located in WALN like Niamey.

Relative cost magnitudes between the aggregated cost associated with distance and the collective cost associated with slack drove the solutions found. The aggregated distance based cost include c_{ij}^{x} and c_{ij}^{y+} offset by c_{ij}^{y-}. The collective cost associated with slack, are shortage ($q_i^{\omega,-}$) and excess penalties ($q_i^{\omega,+}$). The breaking point between whether the distance or slack cost degraded the objection function value more depends on which group's summed magnitude is smaller. Both cost linearly scale on a per unit basis. The per unit condition is based on both parameters linearly scaling with the amount of material shipped, making this comparison independent of volume flowing through the system to a certain limit. This observation breaks if more supply than demand or vice versa is present in the initial state of the system. A visual description of this relationship is provided in the appendix, section 5.2.

4.3.2 Solution metrics

We examine the quality of the solution using three different measures. The first measurement deals with cost and the other two deal with ratios pertinent to the network in a two-stage SP. Each measurement provides some insights about the network's ability to handle disruptions.

Objection Function Value

Interpreting the objective function values provides a way to understand the network's reaction to risk. Table 22 displays the probability sets by number, objective function value to respond to that set of probability, and the monetary value associated with that objective value based on $1585 United State Dollars (USD) per unit of travel distance. The monetary equivalent of the objective function values range in cost from 5.5 million ($M) for the baseline scenario alone to the 14.9 $M needed to prepare for the Dakar−Niamey route disruption. This is a wide range needs to be tailored to the decision maker's preference for balancing cost and risk. With multiple probability sets the decision maker can see the impact to the budget as certain risks become more prominent in the environment. This metric highlights the importance of knowing the frequency of risk in the network. If we take probability set one as the most likely risk distribution, making a proposal for 7.8 $M which is approximately two million more dollars than the basic operating cost can be justified. That request for funds is eleven million dollars less than preparing for the worst case scenario at all cost.

Table 22. Scenario Probability Sets

	p_ω				
Set Number	p_1	p_2	p_3	obj value	USD
I	0.7	0.2	0.1	43551	6.903 $M
II	0.2	0.7	0.1	56884	9.016 $M
III	0.1	0.2	0.7	77747	12.323 $M
IV	1.0	0	0	34650	5.492 $M
V	0	1.0	0	49425	7.834 $M
VI	0	0	1.0	94106	14.916 $M

Utilization Rate

The utilization rate is an indirect measure of how the network design protects against risk. This metric is the ratio of arcs with positive flow to the total number of arcs (ignoring arcs within split cities). The formula for calculating this ratio is below:

$$\text{Utilization Rate} = \frac{|\{(i,j) \in A : x_{ij} + y_{ij}^{\omega,+} > 0\}|}{|A|^2} \tag{11a}$$

For our purposes the amounts of cities in the baseline will be used as the number of nodes to count. The seven cities covered (see Figure 11) gives 49 potential arcs to use. The utilization ratio provides a general gauge of what amount of the network is unused and will not attempt to correct for flow-restricted arcs based on design assumptions. Using the definition of effectiveness as excess capacity in the network, the utilization rate quantifies effectiveness by the rationale that unused arcs represent potential remaining capacity to deal with additional risks. A ratio near zero indicates high excess capacity and resilience. A ratio of one means the network is saturated and all remaining demand must be registered as shortage. The ratios for every probability set and scenario is presented in Table 23

Table 23. Utilization Rate

	ω		
Set Number	0	1	2
I	0.102	0.184	0.184
II	0.102	0.184	0.184
III	0.102	0.184	0.184
IV	0.102		
V		0.184	
VI			0.184

The stability of the utilization rate as in scenario $\omega = 1$ is a result of the zero'd baseline slack constraints presented in chapter 3. This stability carries over into the other utilization ratios as the baseline scenario utilization ratio is the lower boundary for utilization ratios. The small range of values for the rates for a given scenario across all probability set number pinpoints how the network design controls resource management. In particular the lack of difference in utilization from probability set number 2 to 3 despite facing disparate challenges illustrates the designs coping mechanism to minimize the amount of arcs in use. The 0.082 difference in utilization rate between probability set number I and probability set number IV for scenario $\omega = 1$ illustrates how little excess capacity the network gives up despite the Niamey shutdown probability growing from a 10% risk to a 100% risk, meaning the network handles the risk presented in this research well. The lack of increase in utilization rate between probability set II and VI highlights how the network is probability insensitive.

Magnitude of Used Network

Table 24 displays the proportion of directed arcs used in the first stage versus the second stage by probability set and scenario. This proportion of arcs provides insight to whether the network needs to grow as it mitigates risk. A ratio of one signifies the network's usage is consistent. This ratio increases as use more of the network in stage 2.

$$\text{Network Growth} = \frac{|\{(i,j) \in A : x_{ij} + y_{ij}^{\omega,+} > 0\}|}{|\{(i,j) \in A : x_{ij} > 0\}|} \tag{12a}$$

Iterating through probability set I, the first stage's solution uses 5 arcs. As there was no need for a recourse reaction in the baseline scenario, the original 5 arcs are used in stage two so the network growth rate is 1. The 5 arcs needed for stage one holds true for probability sets 1-3. For scenario $\omega = 1$ the arc representing Niamey was destroyed. As this arc is restored the total number of arcs returns to 5, so the size of the network is conserved. For scenario $\omega = 2$, the route to Niamey from Dakar was destroyed and a new arc, Dakar to Ouagadougou, is used instead. This counts as growing the network. The number of arcs used is now 6 as the 5 original arcs plus the new arc adds to six. The fraction $\frac{6}{5}$ is placed into Table 24 as 1.2.

Table 24. Magnitude of Network Ratio

	ω		
Set Number	0	1	2
I	1	1	1.2
II	1	1	1.2
III	1	1	1.2
IV	1		
V		1	
VI			1.2

As the scenarios become more disruptive, the network is expected to grow. A more piecemeal network may contradict that pattern. The data in Table 24 for probability set VI, scenario $\omega = 2$ seems to shrink. This is due the probability concentrating on scenario $\omega = 2$ alone which requires 6 arcs to solve in the first stage. The second stage network solution replaces the 6 arcs with 4 arcs to meet some demand and leaves the rest unmet as shortage.

4.4 Summary

The two-stage SP is a useful technique to characterize and assess a resilient network design. A resilient network design responds to disruptions in a cost effective manner with small changes to the excess capacity. The cost of responding to demands in this model ranged from 5$M to 15$M, a 300% increase. Yet the utilization rate of the network ranged from 0.102to 0.184, which is less than 200% increase. The relatively small changes in absolute value of utilization median rate indicates the model is capable of offsetting individual events with minimal increases in cost. The stability of the utilization rate through the different sets indicate the model itself is probability insensitive. In tandem with consistent utilization rates the network grew at most to 140% of its original size despite facing two very unique disruptions. This model based on the two-stage SP technique used simulated values that represented the WALN network to the greatest extent feasible. Replicating real world setups and events in the WALN and analyzing the monetary cost of risks present in this environment provides a basis for risk mitigation that can be presented to USAFRICOM.

V. Conclusions and Recommendations

5.1 Conclusions

A model capable of coordinating resilient responses to disruptions outperforms those limited to less flexible methods. This research investigated the underlying mechanisms of a resilient network. The network included seven major hubs of WALN and depicted disruptions to that network as well as adequate responses. Non-resilient designs fail to adapt to the challenging circumstances as demonstrated with probability set VI. The high cost (15$M) from only dealing with the worst case scenario in probability set VI shows how a single minded focus on one issue can cause the budget requirements to grow rapidly. Using a holistic view of both the magnitude of disruption and its probability creates budgets the more manageable budgets (7-15$M) found in probability sets I-III. A disruption free scenario such as probability set IV will cost less (5$M), but choosing this as the solution drops the realism aspect we pursued with this model.

Our computational experiments demonstrated that overall cost is sensitive to both geographic distance and the price of sourcing local materials. The three different scenarios provided a method of analyzing the network during normal operations, a node disruption, and a route capacity reduction with different frequency rates. The specific probability assigned to scenario one, two, and three inside the model were shifted through a rotating probability set to assess sensitivity to the expected distribution. The decision maker can consider strong repetitive elements present in the models signify key changeover points for financial decisions. Transitioning from scenario $\omega = 2$ to $\omega = 3$ caused large increases in overall cost, as circumnavigating Niamey city's shutdown, a small geographic incident, to circumnavigating the Dakar$-$Niamey route shutdown, a relatively bigger geographic incident, strongly influenced price swings over 100% from best to worst case scenario between probability set IV and VI.

5.2 Future Research

Expanding the realism of this model is to expand the scope of elements in this model or their granularity.

This model was limited to seven cities. While the scale of this model allows for comparison to one central hub-and-spoke designs, replicating this exercise with the 58 cities would allow for comparison against the designs proposed by Baker [5]. In a similar manner with regards to scale, currently the amount of disruptions are singular in regards to scenarios $\omega = 1$ and $\omega = 2$. Increasing the amount of disruptions encoded for demonstration would allow the flexibility aspect of the resilient design network to gain prominence. Another expansion would be to investigate multi-stage stochastic programming to incorporate a series of events on a scale greater than one step. The evolution of the system based on chain events can provide inferences on the long term system design.

Increasing the realism of the model pertains to its inner working as well. In this model, the transferal mechanism for arc flow is abstract. While we limit the volume of materials according to the capacity limits of a C-17, we fail to account for or limit the number of vehicles available to move materials, or the number of trips based on fuel accessible, both of which are pertinent real world constraints. Diversifying the transportation methods, by expanding the number of planes or pursuing new forms of travel such as ships and trucks, would be a possible direction of research. We expect adding this aspect to the model decision variables in their current form would be rather trivial. The corresponding constraints would be based on the level of detail the system designer pursues.

Shifting the focus of this research to different methods of resiliency might create opportunities to delve deeper into resilient network design. The present focus of this thesis's resilient network design is resolving arc disruption through cost-efficient and time-efficient methods. No effort is given to arc differentiation with respect to the types disruption a given arc is subject to. Installing a system of backups that contain arcs 'immune' to a particular types of risk requires exploring a new decision space between backup system diversity and the cost savings of uniformity. Downer investigates this premise by noting that creating backups without common cause failure points are inherently more resilient than similarly sized networks with identical unit (arc) replacement [14]. The initial cost of designing completely different system components must be balanced against long term cost savings, but the trade-off and potential savings offers a promising area of research. This provides more motivation to diversifying the methods of transportation beyond air to land and sea methods as few risks are crippling in all domains.

Future model designs could incorporate Design of Experiment factorial designs with cost and probability as factors to provide insights on cost and chance sensitivity. Other factors such as the maximum magnitude in change of either supply or demand change by percentage could also be assessed to provide limits to the differences between first stage expectations and second stage known supply and demand amounts. Further Design of Experiment changes could be juxtaposing the results obtained against a system with no constraints for the baseline scenario. Our results in this thesis was probability insensitive. Removing the constraints associated with (7a) and (7b) which force the network to meet minimum demand would allow the model to display outcomes more sensitive to event frequency.

Appendix: Parameter Generation and Insights

Ratio of Arc

Table 25 indicates the starting directed arcs in the network. Arcs removed in scenario 1 are also removed in all other scenarios.

Table 25. Ratio of Arcs Used

w	routes or nodes removed/ reduced	corresponding arcs
1	Dakar-Agadez	14-9
1	Dakar-Garoua	14-3
1	Dakar-N'djamena	14-5
1	Accra-Agadez	8-9
1	Accra-N'Djamena	8-5
2	Niamey	11-12
3	Dakar-Niamey	8-1

Distance between Arcs

The distance between cities shown in Figure 15 below was calculated by inputting the cities latitude and longitude coordinates in the Matlab great circle distance approximation function with the WiggsEllipsoid-84 model. The accuracy of the geographic distances are dependent on Google Maps latitude and longitude values. The distance matrix is the c_{ij}^{x}.

	Ouagadougou	Garoua	N'Djamena	Accra	Agadez	Niamey	Dakar
Ouagadougou	0	1.6648e+03	1.8033e+03	764.2302	1.1432e+03	415.1402	1.7452e+03
Garoua	1.6648e+03	0	360.3096	1.5537e+03	1.0294e+03	1.3137e+03	3.4100e+03
N'Djamena	1.8033e+03	360.3096	0	1.8245e+03	930.9431	1.4118e+03	3.5308e+03
Accra	764.2302	1.5537e+03	1.8245e+03	0	1.5419e+03	911.8073	2.1445e+03
Agadez	1.1432e+03	1.0294e+03	930.9431	1.5419e+03	0	736.9437	2.7361e+03
Niamey	415.1402	1.3137e+03	1.4118e+03	911.8073	736.9437	0	2.1192e+03
Dakar	1.7452e+03	3.4100e+03	3.5308e+03	2.1445e+03	2.7361e+03	2.1192e+03	0

Figure 15. Distance Matrix

Network Value or Distribution Cost

Calculation to find c_{ij}^{y+} and c_{ij}^{y-} cost

The cost for c_{ij}^{y-} is $0.5c_{ij}^{x}$. The cost for c_{ij}^{y+} involve more calculation. The formula is $c_{ij}^{y+} = c_{ij}^{x}(1.2)$ to indicate harsher environments raise the cost of everything on average. The c_{ij}^{y-} was limited to half of c_{ij}^{x} to convey that not doing something provides the same level of safety against missing information in all scenarios but has limited upside.

Calculation to find $q_i^{\omega,+}$ and $q_i^{\omega,-}$ cost

The values used to determine the penalties for not shipping a unit of demand or for removing the capacity barrier of a reduced arc are listed below. In this model the shortage and excess penalties $q_i^{\omega,-}$ and $q_i^{\omega,+}$ for every probability set and scenario are unique.

For the shortage penalty, $q_i^{\omega,-}$, the starting vector:

600	260	650	680	740	130	500

is multiplied by $(1 + \frac{\omega+1}{6})$ then rounded to the nearest whole number for each scenario. For example probability set one, scenario $\omega = 0$, the calculation is:

$$600(1 + \frac{0+1}{6}) = 700$$

That 700 is placed into the appropriate column and row as the example for probability set 1 in Table 26 below demonstrates.

The excess penalty, $q_i^{\omega,+}$, used the starting vector:

450	80	220	910	150	710	640

Table 26. Probability Set One: Shortage Penalty by Scenario $q_i^{\omega,-}$

	ω		
City Name	0	1	2
Ouagadougou	700	800	2700
Garoua	303	347	1170
N'Djamena	758	867	2925
Accra	793	907	3060
Agadez	863	987	3330
Niamey	152	173	585
Dakar	583	667	2250

and the process. The example below is for the probability set one.

Table 27. Probability Set One: Excess Penalty by Scenario $q_i^{\omega,+}$

	ω		
City Name	0	1	2
Ouagadougou	525	600	675
Garoua	93	107	120
N'Djamena	257	293	330
Accra	1062	1213	1365
Agadez	175	200	225
Niamey	828	947	1065
Dakar	747	853	960

The cost for constructing arcs in a_{ij}^{ω} is set at 1000 per arc.

Trade-off Cost

The trade-off in

1. the cost for fulfilling a demand through the first stage versus the second stage

2. the second stage choice between the alternate route cost and the shortage plus excess cost.

The excess and shortage cost move in tandem and the vertical component of this resultant vector is compared to the vertical component of the alternate route cost.

The smaller vertical vector improves the objective function the most and determines whether the node faces a shortage or not.

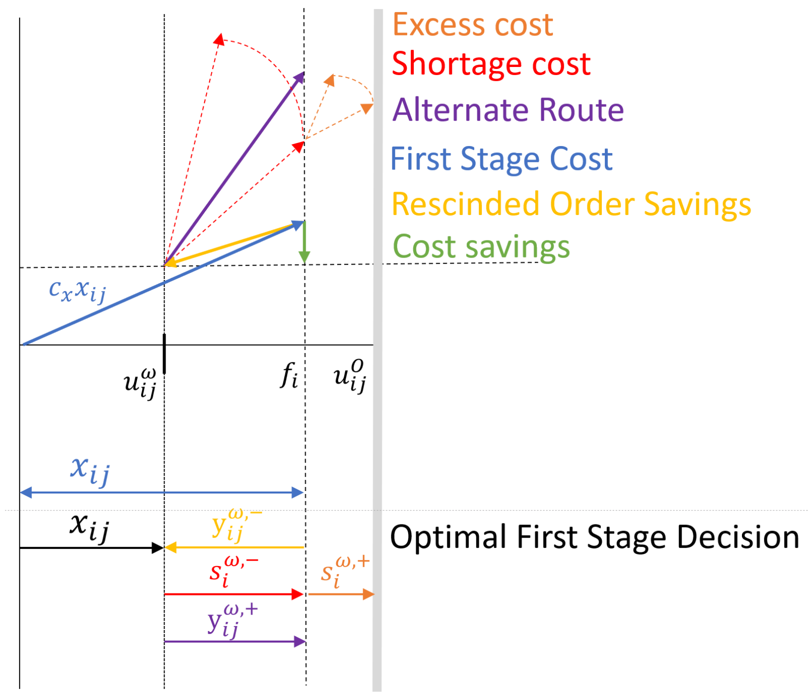

Figure 16. Trade Off between Alternate Route and Shortage Cost

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14. ABSTRACT
The USAFRICOM is studying methods to increase the resiliency of its West African Logistic Network, a hub-and-spoke network design responsible for sustaining long term humanitarian and security missions in the West Africa region. This paper employs a two-stage stochastic programming network design modeled on the WALN that builds a flexible supply chain capable of responding to periodic disruptions while maintaining peak resiliency. Elements such as cost, probability, and event-based disruption are integrated into the model to mirror challenges the WALN faces. We demonstrate that incorporating resilient based response mechanism provide a 90% reduction in cost compared to meeting the logistical challenges covered with a naive approach.

15. SUBJECT TERMS
bilevel programming, stochastic programming, path column generation

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