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Defensive Counterair for Air Base Air Defense using Directed Energy and Kinetic Energy Weapons

THESIS

Andrew S. Wilson, CPT, USA

AFIT-ENS-MS-21-M-194

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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THESIS

Presented to the Faculty
Department of Operational Sciences
Graduate School of Engineering and Management
Air Force Institute of Technology
Air University
Air Education and Training Command
in Partial Fulfillment of the Requirements for the
Degree of Master of Science in Operations Research

Andrew S. Wilson, BS
CPT, USA

March 25, 2021

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DEFENSIVE COUNTERAIR FOR AIR BASE AIR DEFENSE USING DIRECTED ENERGY AND KINETIC ENERGY WEAPONS

THESIS

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AFIT-ENS-MS-21-M-194

Abstract

Air bases are a vital component to the ability of a military to project forces. The attainment of key locations to emplace, or maintain air bases provides a strategic advantage to the countries that have access to it. Air bases are a means to quickly deploy forces globally to respond to adversarial aggression. Because of their strategic importance, air bases maintained by the United States and its allies are subject to persistent threat in the current geopolitical climate. Countries such as China and Russia seek to mar the political influence and security provided by the United States. As indicated in the *Joint Operating Environment 2035* [31], rival states may execute advanced multi-layered attacks with long-range cruise and ballistic missiles to maintain standoff from the U.S. and allied forces. The United States is placing focus on modernizing the missile defense capability to implement a layered missile defense system to disrupt the theater cruise missile threat [25].

Both ground and air assets comprise a layered missile defense system. Combat air patrols conducted by fighter aircraft provide air defense for air bases by establishing standoff from the base, as well as maneuverability to quickly react to incoming cruise missile threats. A fighter is typically armed with a variety of weapons, but in a defensive counterair operation, it will most likely have a mixture of air intercept missiles (kinetic energy weapons) that can engage cruise missiles from short range to beyond visual range. The Department of Defense has plans to introduce a directed energy weapon that is capable of shooting down cruise missiles [17]. The Air Force Research Laboratory has established a road map to employ directed energy weapons on fighters [1]. High energy lasers provide the advantage of being cost effective as compared to their kinetic energy counterparts, but they are not a replacement to the

current air intercept missiles; they will augment the current capabilities.

This research seeks to determine the optimal positioning and maneuvering of a fixed number of fighter aircraft in a combat air patrol for defensive counterair to maximally and effectively attrit the inbound cruise missile threat to a stationary high value asset. The problem formulation identifies multiple objectives for the model: minimize the distance traveled for the fighters, minimize the time utilized to engage the incoming cruise missiles, and maximize the number of cruise missiles destroyed. The model is a mobile routing problem with a mobile demand, where the demand is the set of incoming cruise missiles that the fighter seeks to engage.

The ε-constraint method is used as the multi-objective optimization technique to determine the full set of Pareto efficient solutions. The objective functions respectively related to the time allotted for missile engagements and the total number of cruise missiles destroyed inform the ε-constraints because they are integer-valued. Both ε-constraint bounds are varied over all possible values, and the problem is iteratively solved to minimize the distance traveled by each fighter. The ε-constraint method yields the entire Pareto frontier of solutions for the representative scenario.

The ε-constraint method determined efficient solutions, but it is not a viable method for an actively mobile facility to implement within the specified time period. Therefore, another multi-objective optimization technique is utilized that is efficiently solved within the time threshold. An *a priori* hierarchical multi-objective method called lexicographic optimization is used to determine if there is a ranking of objectives that solved more efficiently. Lexicographic optimization is used to solve the model for each objective sequentially. Lexicographic optimization that maximized the number of cruise missiles destroyed, minimized the time allotted for missile engagements, and finally minimized the fighter distance traveled solves within the time threshold, 98% of the time in the representative scenario. The lexicographic method is also evaluated using a single fighter routing over the network and incrementally varying the number of incoming cruise missiles. A single fighter was able to engage up to four incoming cruise missiles while still making the routing decisions within the predetermined time threshold.

This research is dedicated to my wife, and our two kids.

Acknowledgements

Without the support of Dr. Brian Lunday, this research would not have been possible. His incredible tenacity and patience were key to the development of the framework for this study, as well as for my education and mentoring.

I especially want to express my gratitude to my wife. She has devoted her time and energy to holding down the home front while I spent long days in my educational endeavors. Her unwavering love and support has helped me through this journey.

Andrew S. Wilson

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DEFENSIVE COUNTERAIR FOR AIR BASE AIR DEFENSE USING DIRECTED ENERGY AND KINETIC ENERGY WEAPONS

I. Introduction

"Deliver performance at the speed of relevance. Success no longer goes to the country that develops a new technology first, but rather to the one that better integrates it and adapts its way of fighting." [25]

This chapter describes the motivation for this research. Section 1.1 addresses the strategic importance of air base air defense (ABAD). Section 1.2 defines the problem statement. The research objectives are outlined in Section 1.3. Section 1.4 provides a brief explanation of the following chapters within this thesis.

1.1 Motivation and Background

"Air bases are a determining factor in the success of air operations" [5]. Air bases, either friendly or adversary, are a means to project air power. A wide dispersion of airfields creates a decentralized target and enables forces to maintain air presence almost constantly and over a wide area [5]. However, the availability of viable air bases has decreased in the changing geopolitical climate, as fewer nations allow enduring United States (U.S.) presence within their borders. Despite partner nations not allowing enduring bases, they do have airfields available to U.S. forces during contingencies [18]. Operation Desert Shield is an example of a large-scale deployment of Aerospace Expeditionary Forces (AEF) that displayed the potential for global force projection [30]. Figure 1 shows the overseas basing support in 2005 for the Global War on Terrorism established with multiple coalition partners. In the contemporary operating environment, the ability to project power globally, and rapidly is increasingly important with the lack of permanent basing, especially against near peer threats such as Russia and China [30].

Figure 1. Overseas basing supporting Global War on Terrorism in 2005 [30]

The Air Force relies on the "global network of fixed facilities" for force generation. The United States Air Force (USAF) "creates operational effects from bases" [38]. Depending on their locations, air bases have the potential to have strategic impacts. Moreover, their disablement can be detrimental to a strategic advantage. "An enduring USAF presence in key regions is foundational to deter aggression, reassure partner nations, and–if necessary–fight wars" [38]. The occupation of a few key locations provides coverage over airspace into potential adversarial territory. The selection of air bases is extremely important in contingency operations, and the appropriate selection can posture U.S. forces for force projection and threat deterrence [28].

Due to the importance of air bases to the security of the United States and its allies, they are constantly at risk of attack from potential adversaries such as China and Russia who "challenge American power, influence, and interests, attempting to erode American security" [36]. It is evident that America is not immune to a number of threats from long-term strategic competitors such as China and Russia, as well as rogue regimes from countries such as North Korea and Iran [25]. China has recently had substantial economic and industrial growth which may result in the desire to gain global power through the implementation of new nuclear doctrine, and Russia is expected to modernize its current, "land, air, and sea-based intercontinental nuclear forces" [31]. Adversaries will concentrate on establishing evolving missile technologies, and expanding the range of strike capabilities to stimy the ability of U.S. forces to project power. The enemy will target the air bases that the U.S. relies on for force projection [31]. The *Joint Operating Environment 2035* describes how the U.S. could be threatened by a number of states that form an active coalition to coordinate resistance that impairs the ability of the U.S. to project power. Potential adversaries that are outside of the range of American power projection can heavily influence the actions of regional proximate countries, even through the use of violence and coercion [31]. Rival states may conduct coordinated attacks against major airports and possibly military bases to disrupt the ability of the U.S. to "generate, deploy, and maintain the Joint Force" [31]. Conflicts in the future will be comprised of recurrent efforts by adversaries to disrupt power projection capabilities. Attacks may occur from significant distances either based from home, or through use of the global commons for military purposes. The global commons are spaces available to all countries without ownership, such as international areas in the oceans or outer space [31].

The threats to America have evolved to be combined across all domains with increasing speeds and even directly threatening the homeland [25]. Given the current geopolitical climate, the *National Security Strategy* [36] indicates many other rival nations are building an array of arsenals including advanced missiles, some of which can reach the continental United States from overseas. The technology for weapons is rapidly evolving, including adversaries' long-range strike capability. Future delivery of weapons includes global strike assets from adversaries such as long-range cruise missiles and an array of other weapons with the aim of penetrating the United States' defensive systems [31]. Adversaries may use anti-access/area denial strategies employing a single or coordinated attack with aircraft and/or missiles to target the United States' power projection capabilities such as airfields [32]. Rival states can use advanced, multi-layered air defenses, long-range cruise and ballistic missiles, as well as other domains, to threaten overseas air bases, and maintain standoff from the U.S. and its allies [31]. China and Russia are investing in precision-guided conventional cruise and ballistic missiles that are accurate enough to destroy hardened facilities, and they will be able to inflict great damage on high value assets critical to the joint and combined forces. The Indo-Pacific regional missile threat map shown in Figure 2 depicts the diverse missile arsenal China possesses, and the significant threat it poses to the U.S. and its allies operating in the Indo-Pacific region [22]. A large salvo of highly accurate cruise missiles poses a substantial threat to air bases that can overwhelm proximity missile defense systems and, in turn, disrupt the ability to project air power and provide air support during conflict [27].

The *National Defense Strategy* [25] advocates missile defense as a key capability to modernize. Air bases can employ ground or air based weapons to defend against cruise missiles, and, due to the constant advancement of technologies, air bases will require a "layered missile defense system...against missile attacks" [36]. The character of war is constantly changing with rapid developments in technology such as advanced computing, directed energy, and hypersonic technologies. These technologies will enable the nation to win wars in the future, but they are also increasingly

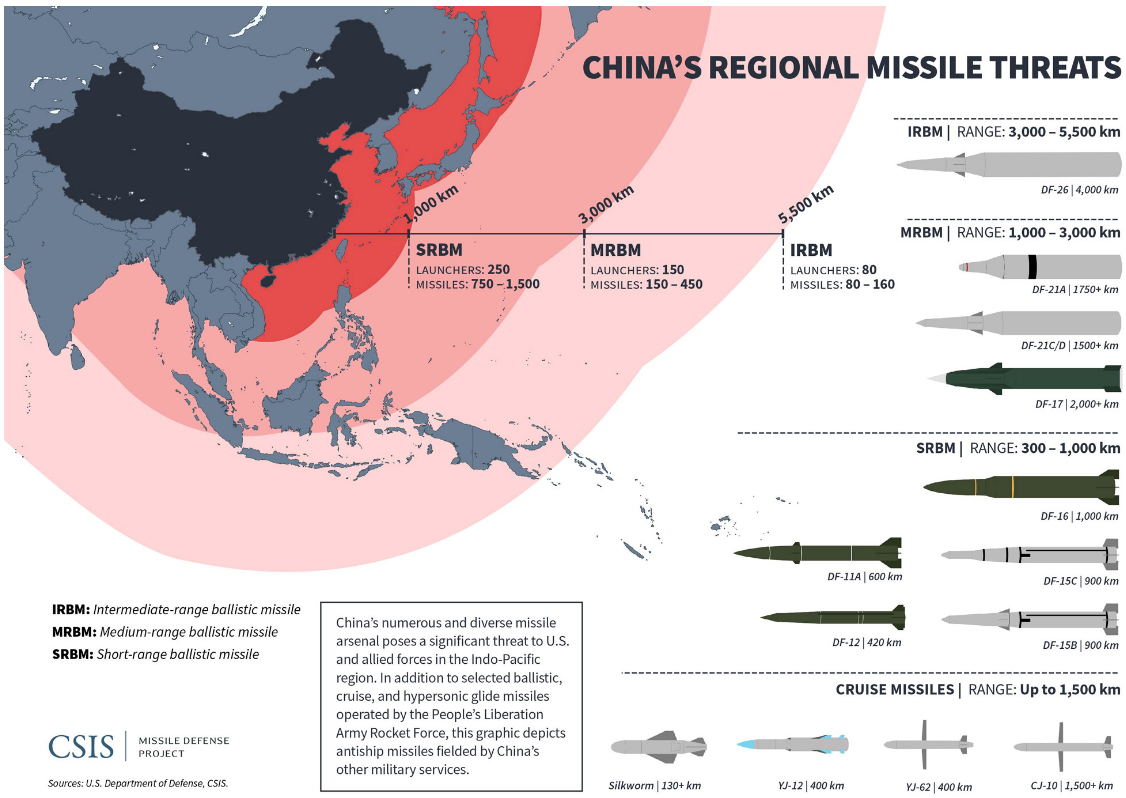

Figure 2. China's missile arsenal that poses a significant threat to the U.S. and allied forces in the Indo-Pacific region [22]

available to rival states and non-state actors from the commercial sector [25]. Due to the accessibility of technology to adversaries, the United States is bolstering the air and missile defense capabilities, especially against the "projected ballistic and cruise missile threats" [36]. The Department of Defense is actively investing in systems for air defense such as the "Patriot, the Terminal High-Altitude Area Defense (THAAD) system, and sea-based SM-3 missiles to shoot down ballistic and cruise missiles" [27]. To withstand enemy anti-access systems on the infrastructure of forward-based U.S. forces and its allies, the network of air bases must be hardened, adequately dispersed, and contain active defensive capabilities against cruise missiles [27].

Defense against cruise missiles can be comprised of either kinetic or directed energy weapons. Green [19] discussed the optimal static placement of ground-based high energy lasers to provide the maximum coverage in defense of high value assets such as air bases. Many of the missile defense systems air bases employ are ground-based and positioned relatively nearby, thus they may not provide adequate coverage to completely destroy a cruise missile threat, especially from a salvo of cruise missiles. These systems are still important as a component of a layered missile defense system that provides multiple opportunities to engage an incoming threat. Fighter aircraft are another component of the layered missile defense system that provide a tactical advantage against the cruise missile threat; because they provide standoff from the asset being defended, they can be positioned in an area to intercept a plausible threat, and they are quickly maneuverable to a location to engage an incoming cruise missile.

The combination of air and ground systems is "Defensive counterair (DCA) [which] encompasses air and missile defense (AMD),...[and] is direct (active and passive) defensive actions taken to destroy, nullify, or reduce the effectiveness of hostile air and BM [ballistic missile] threats against friendly forces and assets" [32]. An array of weapons, sensors, communication systems, and command and control platforms comprise DCA operations to protect friendly assets from air and cruise missile threats [32]. Fighter aircraft can be used for both offensive and defensive counterair operations in a combat air patrol (CAP) for high value asset protection, such as an air base. The CAP is a method of assigning two to four fighters as a barrier to protect multiple assets, or to protect a specific asset which can be stationary or airborne [32]. The fighter aircraft are usually armed with weapons designed for air-to-air engagements, and aircraft can also be outfitted with radars and beyond visual range weapons systems enabling them to engage multiple threats simultaneously [32]. The typical air-to-air armament in the Air Force fighter aircraft arsenal consists of two types of air intercept missiles (AIMs): AIM-9 Sidewinders, and AIM-120 AMRAAMs [9]. The AIM-9 Sidewinder is a short range, supersonic, heat-seeking missile with a range of over ten miles. The supersonic AIM-120 advanced medium-range air-to-air missile (AMRAAM), by contrast, has beyond visual range capability with a range greater than 20 miles [9].

The Air Force has plans to test high-energy lasers on its aircraft for surface-to-air and air-to-air missile engagements [26]. Therefore, fighters may eventually have the ability to be outfitted with a directed energy weapon system. The directed energy weapon capability of a 300-kilowatt laser weapon for “the Army, Air Force, and Navy..., one powerful enough to shoot down cruise missiles” [17] is projected to be available in 2022. A significant advantage of directed energy weapons is their operation at the speed of light which makes targeting and tracking an incoming threat more efficient [26]. Directed energy weapons have already been tested and put into utilization such as the 30-kilowatt Laser Weapons System (LaWS) on the USS Ponce located in the Persian Gulf. The LaWS can disable a drone in “less than two seconds” by focusing on a vital spot of the target [16]. A second advantage of directed energy weapons is their low cost relative to kinetic energy weapons. The cost per shot for the

30-kilowatt LaWS is $0.59 [16] compared to a unit cost of $386,000 for the AIM-120 AMRAAM [3] and $603,817 for the AIM-9 [4].

Even though the technology for directed energy weapons is evolving rapidly, making the prospect of deploying these weapons on multiple platforms inevitable, directed energy "will not replace kinetic weapons" [26]. Instead, they will more likely augment the current weapons. A fighter-mounted laser with the power to defeat a cruise missile is not available yet. The most challenging attributes for directed energy weapons on an aircraft are still the "size, weight, and power efficiency requirements" [17]. Fighter aircraft are also subject to limitations. There are only a finite number of fighter aircraft that can be tasked for the DCA CAP mission at any given time. Each air base has a limited capacity for aircraft. Furthermore, a fleet of fighters is subject to both scheduled and unscheduled maintenance which limits the number of mission capable aircraft at any given time. A fighter burns a significant amount of fuel during a sortie adding to the cost of a mission. Depending on the weapon system, the aircraft may have to be oriented along a specific vector to engage an incoming air-to-air threat. The range of the engagement will also be limited by the weapon selected.

1.2 Problem Statement

This research seeks to determine the optimal positioning and maneuvering of a fixed number of fighter aircraft in a combat air patrol for defensive counterair to maximally and effectively attrit the inbound cruise missile threat to a stationary high value asset.

1.3 Research Objectives

To more adequately scope the problem, the following research objectives will be addressed in this thesis.

Objective 1.

Fighter aircraft are highly maneuverable, and therefore can assume a response path to engage an incoming cruise missile attack. The first research objective is to determine an optimal route for an aircraft to traverse such that the weapons systems can most effectively engage the cruise missiles. This objective will be achieved through the use of a two-dimensional Cartesian plane.

Objective 2.

The cost of a single AIM is higher than that of a discharge from a high energy laser, (HEL). The HEL, on the other hand, is sensitive to the direction of engagement and the time required to engage. A fighter aircraft will have both weapon types available. With these trade-offs, the second objective determines which weapons to leverage in a given engagement to both minimize cost and maximize effectiveness.

1.4 Organization of the Thesis

The remainder of this thesis is organized as follows. Chapter II reviews literature pertinent to location theory, agent routing problems, and the combination thereof. Chapter III introduces the mathematical programming formulation to address the underlying problem, as well as the accompanying solution methodology used in testing. Chapter IV identifies a representative test instance and examines the efficacy of the proposed model and solution methodology, and it examines several instance variations to gain meaningful, practical insights. Chapter V summarizes the major outcomes of the research and provides recommendations for future research.

II. Literature Review

This chapter provides a review of relevant literature to the defensive counterair problem. Section 2.1 reviews the literature related to maximal covering location problems and their formulation. Section 2.2 reviews literature related to vehicle routing to provide coverage of demands over a time period.

2.1 Related Covering Problems

The maximal covering location problem is a class of optimization problems that seeks to determine an optimal location of a set of facilities to satisfy demands. Toregas et al. [35] analyzed the placement of emergency services to cover the demand of the local population in *The Location of Emergency Service Facilities*. The emergency service in the study could apply to fire stations, emergency medical services, or even public services such as libraries. However, this thesis applies the location of emergency facilities to the DCA problem by setting the combat air patrol as a set number of facilities and the incoming cruise missiles as the demand population requiring the service.

Church and ReVelle [11] formulated the maximal covering location problem, which is an optimization that seeks to minimize the population left uncovered given a set number of facilities. Church and ReVelle [11] designed and tested a Greedy Adding with Substitution algorithm that allows facilities to be relocated if that improves the coverage (for one facility, the algorithm will return the optimal solution). This method may be useful in rapid locating of one or multiple combat air patrols to provide coverage with a mandatory closeness constraint.

Karatas et al. [24] assessed the maximal coverage of a multi-criteria objective using p-median problems (p-MP) and p-center problems (p-CP). "The p-MP aims to find the locations of p facilities among a number of candidate locations such that the total weighted distance between all demands and their nearest facilities is minimized" [24]. The p-MP is useful in DCA because a set of missile facilities targeting an air base could be known through military intelligence assets. The proximity of missile facilities combined with early detection systems and the expected time to react can be used as a weight for facilities. The p-CP is used to find locations that minimize the maximum distance from the facility to all demand sites [24]. This study specifically aimed to evaluate multiple performance criteria such as requiring backup coverage for each demand node. This method is applicable since a number of threats could occur close together which would require multiple engagements either from the same aircraft or from multiple aircraft.

In their book *Location Covering Models*, Church and Murray [10] explain that, in a location covering problem, the term *cover* can be described in many ways. The location covering models use a range of values for facilities to develop trade-offs in other constraints such as resources. In this thesis, cover is defined as maximum effective range from the aircraft to a missile. This study falls into the category of location models that "maximize coverage subject to budgetary or resource constraints" [10].

2.2 The Routing Problem

This section reviews the literature pertaining to the complicating requirement to use a dynamic model to determine the path of travel for DCA aircraft to provide optimal coverage that attrits the most inbound cruise missiles. Given a detected threat, an aircraft may have to maneuver into a position from which it can effectively engage a missile. The optimal maneuver path is the one that requires the least time, while also providing an ample enough time to engage the target. The shortest path problem is covered in many textbooks [2, 7]. Multiple algorithms are provided for networks having various topologies, including the well-known Dijkstra's Algorithm for directed acyclic networks. The features of each of these algorithms are provided as a comparison for running times. The original implementation of Dijkstra's algorithm "achieves the best available running time for dense networks" [2]. The minimum runtime of an algorithm will be critical in routing an aircraft during an engagement, although this research may favor direct solution of mathematical programs prior to exploring dynamic programming approaches or heuristic solution methods.

The Mobile Facility Routing Problem (MFRP) is similar to a Maximal Coverage Location Problem, but instead of fixed sites the MFRP has facilities that can relocate over a continuous time set [21]. Halper and Raghavan [21] used the MFRP with a set of event points that have instantaneous demand functions. The facilities are routed over a set of nodes, each of which allows a co-located facility to cover a subset of the event points.

Another variant of the routing problem involves not only the routing of the facility to provide maximum coverage, but also the mobile demand. Pugliese et al. [29] discussed the use of flying drones to cover a set of targets based on a line-of-sight coverage constraint. The line-of-sight is defined via a circle corresponding to a fixed, Euclidean-distance radius emanating from a point where the unmanned aerial vehicle is located. The targets are mobile and must be persistently monitored to remain covered. The objective function seeks to both maximize coverage and minimize the operating costs of multiple aircraft. The multi-objective routing problem from Pugliese et al. [29] provides a practical application to DCA routing because there are only a limited number of resources available for CAPs to cover cruise missile threats, and the costs from weapon engagements are a significant factor. This work provides key insights into the definition of coverage for the DCA problem.

The journal article "*The Flying Warehouse Delivery System: A Quantitative Ap-*

proach for the Optimal Operation Policy of Airborne Fulfillment Center" [23], by Jeong et al., entailed determining optimal scheduling of a system called the airborne fulfilment center (AFC) patented by Amazon. The AFC is a large aerial system that can carry a significant amount of high demand items. The AFC employs unmanned aerial vehicles (UAVs) that can service demand from Amazon Prime customers. The article provided a baseline mathematical model that is possibly new to the field of optimization. Many problems exist where the objective is to service demand nodes within a network and return to a static facility, which are extensions of the traveling salesman problem (TSP). Also, vehicle routing problems for multiple vehicles that return to a depot are referred as the TSP with multiple servers. *The Flying Warehouse Delivery System* can be described as a mobile routing problem of the AFCs to provide the maximum coverage of the demand nodes with further mobile routing of UAVs. This study showed that the system can be dynamic with potential changing constraints. This problem formulation is applicable to the mobile routing of an aircraft, that delivers missiles, to demand nodes, such as adversarial cruise missile threats.

Ghasemi and Bashiri [6] examined another aspect of the routing problem with a "two-stage stochastic selective-covering-inventory-routing (SCIR) model". The SCIR model is used to analyze the system for blood collection and inventory management under uncertainty. This study assumed blood demand is stochastic. The authors also indicated a maximum capacity in the bloodmobiles, which can be directly applied to a limited weapons capacity. Because the expected routes of incoming cruise missiles can be modeled stochastically, an approach similar to the SCIR model provides an extension of this research beyond robust optimization.

2.3 Conclusion of the Literature Review

The literature examined from this chapter provides a set of viable models, heuristics, and algorithms that can inform the models in this thesis. The different approaches can be applied, each with a standard of performance to determine its actual usefulness in a real-world application. The models to follow in this thesis will successively build from the Maximal Coverage Location Problem of a static system, the shortest path of engagement for a combat air patrol, the optimal routing of an aircraft to provide the maximum coverage of the highest demand, and the routing of multiple aircraft to provide coverage with redundancy.

III. Modeling and Solution Methodology

This chapter develops the methodology used for the optimization of the positioning and routing of combat air patrols to provide air base air defense. Section 3.1 presents the assumptions required to model the problem. Section 3.2 contains the formulation of the model. Finally, Section 3.3 describes how the problem will be solved during instance testing.

3.1 Modeling Assumptions

The model requires several assumptions to be formulated using linear programming. The assumptions fall into three distinct categories: generalizations about how a set of fighters can route over the network, deterministic assumptions of the path a set of cruise missiles can traverse, and how the two entity types interact within the network.

Each engagement will be observed over a set time period. The time period can be further subdivided into discrete, equal duration time periods. The aircraft routing and missile engagement decisions are made at the beginning of each period. The possible locations of any given aircraft at a discrete time period are finite, and, furthermore, they may be approximated by a discrete tessellation of the continuous space. Within the tessellation, the aircraft may only travel to adjacent points from one time period to the next. All fighters are assumed to travel at a constant speed, which is required to approximate the time it takes each fighter to travel between two adjacent locations. The travel time between adjacent points is an integer valued number. Requiring integrality between adjacent points is not a severely restrictive assumption since rational numbers at a given, bounded level of precision can be converted to integers via multiplying them by a sufficiently large number. Therefore, if higher resolution is required for the route analysis of a group of fighters, shorter duration time periods may be considered. The vectoring of fighters is instantaneous at the beginning of a time period. A fighter may change direction from its current location to any of the adjacent points in the tessellation, even the point from which it just arrived, while maintaining the magnitude of its velocity from the previous time period along the new vector. Moreover, a group of fighters may travel in a tight formation which can be generalized to a single point in the discrete tessellation. Pilots are aware of any adjacent, friendly aircraft, and can maneuver accordingly to avoid collision, as necessary. Therefore, if a fighter needs to reroute to or from a point near the position of another friendly fighter aircraft, it will avoid colliding with any other adjacent aircraft.

Upon a cruise missile launch event, the launch site of the cruise missile is detectable almost instantaneously. Once the missile has reached its cruising altitude, the path along which it is traveling may be used to estimate the target of the cruise missile. Thus, the origin and target of each incoming cruise missile are deterministic. Moreover, a cruise missile travels directly to its target; it does not deviate or conduct evasive maneuvers to avoid contact. Therefore, the velocity at which a cruise missile travels is constant throughout the period of engagement, and can be measured at each time interval to make the necessary aircraft routing and cruise missile engagement decisions.

A fighter with a given weapon system can destroy a cruise missile if the distance from the current location of the fighter to the missile is less than the maximum effective range of the weapon. As required, to account for the launch and travel time of kinetic energy weapons, or the accumulation of irradiance for directed energy weapons, the maximum effective range can be reduced to yield a conservative engagement plan. Missile engagements (i.e. the selection and launch of a fighter's missile) occur instantaneously from a fighter's position. After the fighter has engaged the cruise missile, the fighter assumes the incoming cruise missile will be destroyed. Directly following the engagement event, a fighter may redirect to follow on cruise missile threats under the same assumptions stated previously for the fighter. Thus, the engagement event, routing decision, and re-vectoring occur simultaneously.

These assumptions enable the formulation of the model as a linear program with network flows over a discrete time period.

3.2 Model Formulation

This section provides the formulation of the defensive counterair problem using optimization techniques that were informed by the literature review in Chapter II. The model is based on a combination of the mobile facility routing problem, and the maximum cover location problem. The objectives of the model are that each fighter traverses the least distance, they engage all cruise missiles within the least possible time, and the maximum possible number of cruise missiles are destroyed. The model formulation develops the sets used for the system, the parameters of the network, and the decision variables which are used to determine the solution of an instance.

3.2.1 Set Notation

- T: The set of discrete time increments at which decisions are made, indexed by t, where $T = \{0, 1, ..., |T|\}$.
- N: The set of nodes within the network indexed on i or j, alternatively, and which represent the possible locations of fighter aircraft, and cruise missiles at any time.
- A: The set of undirected arcs in the network indexed by (i, j), $i, j \in N, i \neq j$.

- $G(N, A)$: The complete network composed of nodes N and arcs A.
- F: The set of fighters traversing the network, indexed by f.
- M: The set of cruise missiles traversing the network, indexed by m.
- W: The set of weapons that can be used in an engagement of a cruise missile, indexed by w.

3.2.2 Parameters

- d_{ij}: The distance from node i to node j on the arc (i, j) in the induced network.
- τ_{ij}: The time required for a fighter to traverse arc (i, j). This parameter is informed by s_f, the speed at which a fighter f is traveling.
- a_{imtw}: A parameter equal to 1 if a fighter at node i can cover (i.e., destroy) missile m at time t with weapon w, and 0 otherwise. This parameter is informed by the following instance-related data. First, it considers the distance between node i and the location of missile m at time t. The missile's location is determined via (1) the location and time at which missile m is detected, (2) the target of missile m, and (3) the speed, s_m, at which missile m is traveling. Whether a fighter with a weapon w at node i can destroy that missile results from the maximum coverage radius of weapon type w.
- b_{fit}: A parameter equal to 1 if fighter f "enters" the network for maneuvering at location i, -1 if the fighter terminates maneuvering at location i during time t, and 0 otherwise.
- u_{fw}: The upper bound of the number of engagements a fighter f can conduct with weapon type w. This parameter is limited by the number of kinetic energy weapons that a fighter is capable of carrying., or by the amount of time it takes

for a directed energy weapon to reach the appropriate power level to effectively engage a cruise missile.

- υ_w: The upper bound on the number of engagements a weapon can conduct at a given time.

- δ_w: The amount of time required between engagements for weapon type w. This parameter is determined by the amount of time needed to acquire a follow-on target after a given engagement (for either kinetic energy or directed energy weapons), or the amount of time it takes for a directed energy weapon to reach the appropriate power level to effectively engage a cruise missile after an engagement, whichever is more.

3.2.3 Decision Variables

- x_{fijt}: A binary decision variable equal to 1 if fighter f begins traversing arc (i, j) at time t, 0 otherwise.

- β_{fit}: A binary decision variable equal to 1 if fighter f is at node i at time t, 0 otherwise.

- y_{fmtw}: A binary decision variable equal to 1 if fighter f engages missile m with weapon w at time t, 0 otherwise.

- z_m: A binary decision variable equal to 1 if missile m is engaged, 0 otherwise

3.2.4 Formulation

The sets, parameters, and decision variables for the network are developed, and now can be implemented in the model formulation. Recall, this research seeks to determine optimal positioning and routing of fighter aircraft to effectively attrit the inbound cruise missile threat. Thus, the objective function and constraints of the fighter mobile routing problem **P1** are:

$$\mathbf{P1}: \min_{\boldsymbol{x},\boldsymbol{\beta},\boldsymbol{y},\boldsymbol{z}} \quad (f_1(\boldsymbol{x}), f_2(\boldsymbol{y}), f_3(\boldsymbol{z})) \tag{1}$$

$$\text{s.t.} \quad f_1(\boldsymbol{x}) = \sum_{f\in F}\sum_{(i,j)\in A}\sum_{t\in T} d_{ij}x_{fijt}, \tag{2}$$

$$f_2(\boldsymbol{y}) = \sum_{f\in F}\sum_{m\in M}\sum_{t\in T}\sum_{w\in W} t y_{fmtw}, \tag{3}$$

$$f_3(\boldsymbol{z}) = -\sum_{m\in M} z_m, \tag{4}$$

$$\sum_{j:(i,j)\in A} x_{fijt} - \sum_{j:(j,i)\in A} x_{fji(t-\tau_{ji})} \leq b_{fit},\ \forall\ f\in F,\ i\in N,\ t\in T, \tag{5}$$

$$\beta_{fit} = \sum_{j:(j,i)\in A} x_{fji(t-\tau_{ji})},\ \forall\ f\in F,\ i\in N,\ t\in T, \tag{6}$$

$$y_{fmtw} \leq \sum_{i\in N} a_{imtw}\beta_{fit},\ \forall\ f\in F,\ m\in M,\ t\in T,\ w\in W, \tag{7}$$

$$\sum_{m\in M}\sum_{t\in T} y_{fmtw} \leq u_{fw},\ \forall\ f\in F,\ w\in W, \tag{8}$$

$$\sum_{m\in M}\sum_{\hat{t}=t}^{t+\delta_w+1} y_{fm\hat{t}w} \leq v_w,\ \forall\ f\in F,\ t\in T,\ w\in W, \tag{9}$$

$$\sum_{m\in M}\sum_{w\in W} y_{fmtw} \leq 1,\ \forall\ f\in F,\ t\in T, \tag{10}$$

$$z_m = \sum_{f\in F}\sum_{t\in T}\sum_{w\in W} y_{fmtw},\ \forall\ m\in M, \tag{11}$$

$$x_{fijt} \in \{0,1\},\ \forall\ (i,j)\in A,\ f\in F,\ t\in T, \tag{12}$$

$$\beta_{fit} \in \{0,1\},\ \forall\ i\in N,\ f\in F,\ t\in T, \tag{13}$$

$$y_{fmtw} \in \{0,1\},\ \forall\ f\in F,\ m\in M,\ t\in T,\ w\in W, \tag{14}$$

$$z_m \in \{0,1\},\ \forall\ m\in M. \tag{15}$$

The objective function (1) minimizes the distance traveled by the set of fighters, F, across the network $G(N, A)$, while also minimizing the time, t, allocated to each missile engagement, and maximizes the number of missiles engaged. The function, $f_1(\boldsymbol{x})$, in equation (2) is the total distance traveled based on the decision variables x_{fijt} selected. Equation (3) is a function of the decision variables y_{fmtw}, $f_2(\boldsymbol{y})$, that accounts for the time spent on the missile engagements. A missile engagement is indicated by z_m, thus, equation (4) is the sum of those engagements multiplied by -1. The objective function, $f_3(\boldsymbol{z})$, in P1 is a minimization, but its actual objective is to maximize the number of missiles engaged. Maximizing the number of missiles engaged is equivalent to minimizing the negative summation of the number of missiles engaged.

Constraints (5) ensure conservation of flow at each node. The first term in (5) is the outflow from node i, and the second term is the inflow to node i. Since $b_{fit} \in \{-1, 0, 1\}$, $b_{fit} = -1$ implies there is a demand for a unit of flow to node i, $b_{fit} = 0$ implies there is neither a fighter located at node i, nor is there a demand requirement, and $b_{fit} = 1$ implies there is a fighter at node i able to flow out to an adjacent node j. Constraints (6) uses the first term from (5), the outflow from node i, to determine if a fighter f is located at node i at time t. If there is a fighter available at node i, and time t, denoted by β_{fit}, then constraints (7) enforce an engagement of missile m with fighter f at time t to only occur if the missile is within the coverage radius of the specific weapon type w from that node by parameter a_{imtw}. Constraints (8) enforce upper bounds on the number of engagements, y_{fmtw}, to the number of kinetic energy weapons a fighter can carry, or the number of directed energy weapon engagements a fighter can conduct in a given time period. Constraints (9) limit the number of weapons of type w that can be used at a specific time, and ensure the time required between engagements depending on δ_w. The constraints in (10) limit the types of weapons that can be used in an engagement by a fighter at a time period. Constraints (11) are indicator variables for whether or not a missile m is engaged. Constraints (12)-(15) require the decision variables to be binary.

3.3 Solution Methodology

The model formulation P1 is a multi-objective optimization problem for the functions $f_1(\boldsymbol{x})$, $f_2(\boldsymbol{y})$, and $f_3(\boldsymbol{z})$. A multi-objective optimization problem can be solved using a variety of techniques, where *solve* consists of finding non-dominated solutions on the Pareto frontier. The Weighted Sum Method may not provide all Pareto optimal solutions depending on the constraint space. In contrast, the *a posteriori* ε-constraint Method for multi-objective optimization is well suited for solving instances of problem P1. Additionally, the ε-constraint Method guarantees Pareto optimal solutions is a feasible solution if the objective constraints are binding, given appropriate increments of ε-values for the corresponding, bounded objective function values [8]. Therefore P1 is reformulated to minimize $f_1(\boldsymbol{x})$, and bound the objective functions $f_2(\boldsymbol{y})$ and $f_3(\boldsymbol{z})$ with ε-constraints in **P2**, subject to constraints (5)-(15) from P1:

$$\mathbf{P2}: \min_{\boldsymbol{x},\boldsymbol{\beta},\boldsymbol{y},\boldsymbol{z}} \sum_{f\in F} \sum_{(i,j)\in A} \sum_{t\in T} d_{ij} x_{fijt}, \tag{16}$$

$$\text{s.t.} \sum_{f\in F} \sum_{m\in M} \sum_{t\in T} \sum_{w\in W} t y_{fmtw} \leq \varepsilon_2, \tag{17}$$

$$\sum_{m\in M} z_m \geq \varepsilon_3. \tag{18}$$

The objective function, Equation (16) in P2, is equivalent to minimizing $f_1(\boldsymbol{x})$ in P1. Constraint (17) ensures the time allotted for missile engagements remains within a certain threshold, ε_2. Since both t and y_{fmtw} are greater than or equal to zero, ε_2 must also be greater than or equal to zero. However, $\varepsilon_2 = 0$ is trivial because it implies any engagements must occur at time period t_0, implying an engagement at time t_0 is only possible if any missile is within range of at least one of a given fighter's weapons at the initialization of the model. Constraint (18) is derived from $f_3(\boldsymbol{z})$ in Equation (4). Since each z_m is binary, the value of ε_3 must be greater than or equal to zero. For any $\varepsilon_3 \geq 0$, the total number of missiles engaged, $\sum_{m \in M} z_m$, is at least as great in magnitude as ε_3. If $\varepsilon_3 = 0$ in Equation (18), then no missile must be engaged during the model.

If either of the ε-constraints in Equations (17) and (18) are not binding, then each non-binding ε-value can be adjusted until it yields a binding constraint to identify an efficient solution. If both ε-constraints are binding, then both ε_2 and ε_3 can be incremented over different values (as long as they remain binding) to determine alternative efficient solutions. The set of alternative, non-dominated solutions comprise a Pareto frontier. The Pareto frontier enables the decision-maker to assess tradeoffs between the multiple objectives.

The General Algebraic Modeling System (GAMS), version 30.2.0, is the software used for formulating the model and testing instances in this research. GAMS has the ability to call other solvers. CPLEX is a commercial solver for linear, nonlinear, and mixed-integer optimization problems. GAMS with CPLEX solver (version 12.10.0.0) invoked is used for the testing, results, and analysis presented in Chapter IV.

IV. Testing, Results, and Analysis

Chapter IV validates the model formulation developed in Chapter III, and expands it to several instances to gather meaningful insights. Section 4.1 verifies the functionality of the model, and validates it using simple instances. Section 4.2 evaluates problem specific data through accurate parameterization in multiple test instances. The chapter concludes with a sensitivity analysis to determine an efficient frontier of the multi-objective optimization problem in Section 4.3.

4.1 Model Validation

The problem formulation used to validate the model is P2. P2 is a multi-objective optimization problem which uses the ε-constraint Method to appropriately bound the decision variables y_{fmtw} and z_m in Equations (17) and (18) respectively. A discrete square tesselation is synthesized to allow for movement and tracking of a set of fighters within the two-dimensional space. The test instances use a square tesselation consisting of 25 nodes, with five rows of five nodes each. The nodes are numbered from 0 to 24 with node 0 situated at the origin of the xy-plane, and node 24 located at the point corresponding to coordinates $(4, 4)$. The distance between adjacent nodes either horizontally or vertically is set to one unit. The distance between diagonally adjacent nodes is $\sqrt{2}$, by the Pythagorean Theorem. Travel is only permitted to adjacent nodes. Fighters may only enter or travel in the network, $G(N, A)$, as defined in the model formulation.

Missiles are modeled as having an origin and a destination, assuming a straight line path of travel. By contrast, missiles are not necessarily restricted to travel in the network, $G(N, A)$, as the fighters are. However, the missiles' flight path may direct them through, or near, the network which the fighters are patrolling. The time of missile launch and speed are known, thus the location of the missile can be determined at any time along the missile path. The destination of each missile is the high-value asset (i.e., air base) for which the set of fighters is providing an echelon of protection via defensive counterair operations. The triggering event for fighters to begin missile targeting is the detection of adversary missiles. By assumption, the fighters are in a specific known loiter position upon initialization of the model. Four distinct test instances ensure the proper functionality of the optimization model under varying circumstances.

4.1.1 Test Instance 1: Uncapacitated Fighter Engagement

The first test instance is depicted in Figure 3. The test instance is composed of two fighters. Fighter 1 is located at node 4 corresponding to the point $(4, 0)$, and Fighter 2 is located at node 20, point $(0, 4)$. The fighters can travel a speed of one unit per time period, and engage targets within 1.5 units. Neither of the fighters have a limit to the number of engagements they can conduct. The ε-constraints for Equations (17) and (18) are set so that the problem is not infeasible, but also so the solution is not trivial. For Test Instance 1, the maximum time alloted is $\varepsilon_2 = 5$, and the minimum number of missiles engaged is $\varepsilon_3 = 2$.

The origins of the missiles are indicated by the red circles in Figure 3. In Test Instance 1, the location of Missile 1 is set to node 1 at coordinates $(1, 0)$, and the location of Missile 2 is set to node 5 at coordinates $(0, 1)$. The fact that both missiles are collocated with a node in $G(N, A)$ is immaterial to the problem, as will be shown in Test Instance 2. Each missile travels 0.5 units per time period. Both Missile 1, and Missile 2 are targeting a single air base, which is modeled as being collocated with node 24. From each missile's origin, the total flight time required to reach the destination, node 24, is 10 time periods. The flight path of each missile is indicated by the red lines emanating from the missile origins, and directed toward the destination air base at coordinates $(4, 4)$. The location of each missile is shown by the red, evenly-dispersed tick marks along the red lines that are labeled by the time period at which the missile will be for each given tick mark.

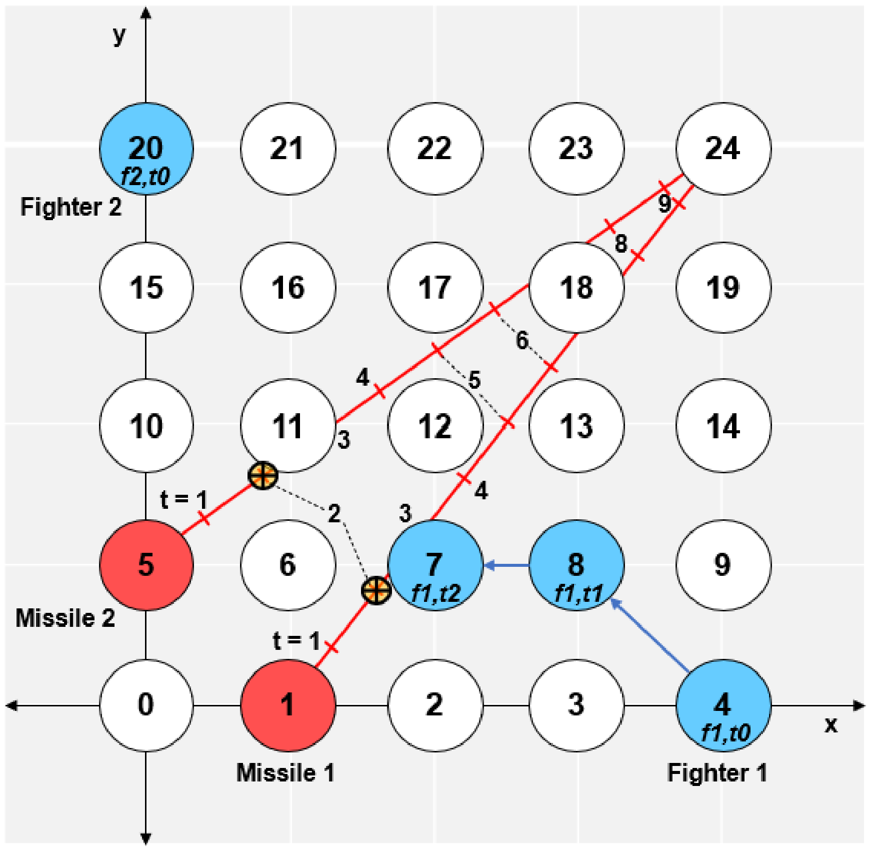

Figure 3. Model Test Instance 1: Uncapacitated single fighter utilization

The fighters' locations throughout the engagement are shown in blue. The initial positions of Fighter 1 and Fighter 2 shown at time t_0 are nodes 4 and 20, respectively. Fighter 2 does not route beyond node 20. Fighter 1, however, travels along the path $4 - 8 - 7$ in $G(N, A)$. The route of Fighter 1 is shown as directed arcs, depicted by blue arrows in Figure 3. At time t_2, Fighter 1 is at node 7. From node 7, Fighter 1 engages both Missile 1 and Missile 2, as depicted by the yellow highlighted cross hairs along the missiles' flight paths. Both missile engagements occur simultaneously at t_2.

The optimal objective value for this instance is the total distance traveled from Equation (16), which is approximately 2.41 units. Both missiles are destroyed at t_2, thus Constraint (17) yields $\sum_{f \in F} \sum_{m \in M} \sum_{t \in T} \sum_{w \in W} t y_{fmtw} = 4 \leq 5 = \varepsilon_2$, and Constraint (18) yields $\sum_{m \in M} z_m = 2 \geq 2 = \varepsilon_3$, implying both missiles are engaged. To attain an efficient solution, ε_2 could be adjusted to $\varepsilon_2 = 4$. If $\varepsilon_2 < 4$, up to one missile engagement could occur, violating Constraint (18), and thereby making the problem infeasible. It follows, that adjusting the values of ε_2 and ε_3 could yield alternate efficient solutions, given the problem is still feasible.

Since there is no limit on how many missiles a fighter can engage, and the objective seeks to minimize the distance traveled, the optimal solution path only involves routing Fighter 1 to engage both missiles. Fighter 2 does not incur any distance cost onto the objective value, and does not route through the network $G(N, A)$. Fighter 1, likewise, disengages from the fight directly following time t_2. The parameter, b_{fit}, is the supply or demand level for the conservation of flow Constraints (5). The fighters, by design, must either route through the network to neutralize a threat, or exit the network through a demand node if no further missiles can be engaged. The demand node is determined by the optimal routing of a given fighter. Fighter 2 enters and leaves the network at node 20, and at time period t_0. Since Fighter 2's flow is balanced, $b_{2,20,0} = 0$, despite it being set to 1 for the instance specific data. Having a lower b_{fit} parameter is allowed since Constraints (5) are not equality constraints. By contrast, the supply of Fighter 1 at the initialization of the model is $b_{1,4,0} = 1$. The demand for Fighter 1 occurs at node 7, and at time period t_2, thus $b_{1,7,2} = -1$. The aircraft routing and missile engagement decisions are made at the beginning of each time period, therefore, Fighter 1 engages both missiles and ceases routing at node 7.

4.1.2 Test Instance 2: Uncapacitated Fighter Engagement Utilizing Two Aircraft

The utilization of a single fighter to engage two missiles makes sense from an optimization perspective. However, to validate whether the model will implement both fighters in an engagement, the parameters are adjusted. In Test Instance 2, as depicted in Figure 4, the starting positions for Fighter 1 and Missile 1 are maintained from Test Instance 1 at coordinates $(4, 0)$ and $(1, 0)$, respectively. Fighter 2 is relocated to node 22. The origin of Missile 2 is set to coordinates $(-1, 3)$. The Missile 2 coordinates do not correspond to a node, which will verify whether a missile that is merely passing in the vicinity of a set of nodes in $G(N, A)$ can be engaged by a fighter. If the missile origin, destination, and speed are available, the problem of routing a fighter is feasible given enough time, and provided the missile flight path falls within the engagement range of at least one fighter. The speed for both missiles in Test Instance 2 is maintained at 0.5 units per time period, and the speed of both fighters is one unit per time period, similar to Test Instance 1. The weapon engagement range of each fighter is adjusted to one unit, meaning a missile must be within a one unit radius of a fighter for a fighter to accurately engage it. There is not a limit to the number of engagements for either fighter.

The missile flight paths of Missiles 1 and 2 are shown as the red lines in Figure 4. Missile 2 is not in $G(N, A)$, therefore the red circle indicating its origin is not enumerated as a node. As in Test Instance 1, the location of each missile along its respective flight path is shown as a tick mark labeled by the time of occurrence. Both missiles are targeting the high value asset located at node 24.

Figure 4 depicts the engagements of Fighters 1 and 2. Fighter 1 is routed along the path $4 - 3 - 2$. Fighter 1 engages Missile 1 at time t_2 from node 2, and ceases routing through $G(N, A)$. Fighter 2 is routed along the path $22 - 21 - 20$. Fighter 2

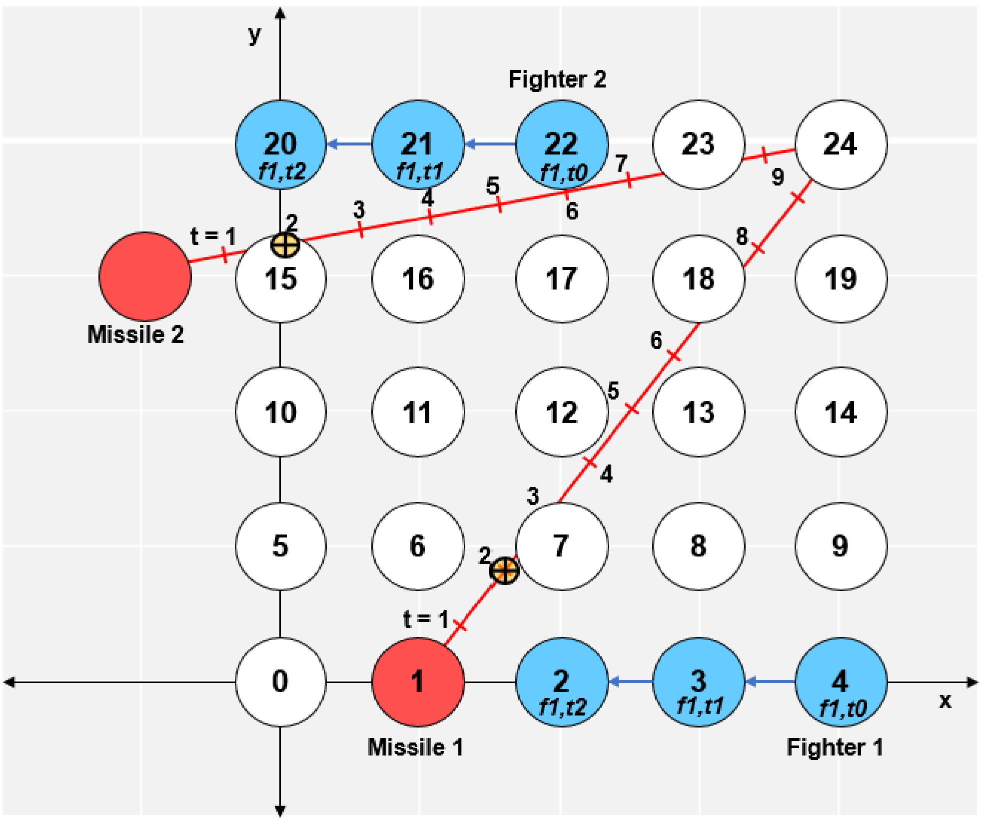

Figure 4. Model Test Instance 2: Uncapacitated two fighter utilization

engages Missile 2 at time t_2 from node 20.

The optimal objective value of Test Instance 2 is a total distance of 4 units. The ε-constraints are maintained from Test Instance 1, with $(\varepsilon_2, \varepsilon_3) = (5, 2)$. Since both missiles are engaged at time period t_2, constraint (17) is satisfied at $4 \leq 5$, and constraint (18) is binding at 2.

4.1.3 Test Instance 3: Capacitated Fighter Engagement

In Test Instances 1 and 2, if a missile falls within a certain range of a fighter, the fighter can engage it. Test Instance 3 introduces the concept of not only a weapon, but also an upper bound on the number of engagements a given weapon system can conduct. The upper bound of a weapon can be either time-based or capability-based. Some weapons may only be able to be fired a certain number of times for a given time period, which is an example of a time-based limit on weapon capacity. The more common type of limit on weapon capacity is capability-based. For instance, a fighter cannot fire more missiles than it can carry. In Test Instance 3, the capability-based weapon capacity limit is introduced. For Test Instance 1, Figure 3 showed an instance wherein only one fighter is utilized. Test Instance 3 uses the same instance data as Test Instance 1, but it alters the weapon types and capacities. If the weapon capacity limits are introduced, the optimal aircraft routing solution for Test Instance 3 should vary from that recommended for Test Instance 1, given the model is formulated correctly.

The instance specific data for Test Instance 3 is depicted in Figure 5. Fighters 1 and 2 are positioned at nodes 2 and 20, respectively. Missile 1 is detected at node 1, and Missile 2 is detected at node 5. Missiles 1 and 2 have a high value asset destination located at node 24. Similar to Test Instances 1 and 2, the missile flight paths are shown as the red lines directed from the missile origins to the destination node 24.

Each fighter has 2 weapon types, referred to here as Weapon 1 and Weapon 2. Weapons 1 and 2 have the same performance characteristics, regardless of the fighter on which they are mounted. However, the remaining capacity of each weapon type varies depending on how many engagements a given fighter has conducted. Weapon 1 on Fighters 1 and 2 has a capacity of 1, and Weapon 2 has a capacity of 0 on all fighters. For Test Instance 3, the weapon engagement range of both Weapon 1 and Weapon 2 is 1.5 units. The goal of Test Instance 3 is to validate that both fighters will engage the missiles given limited weapon capacities as compared to the single-fighter engagement in Test Instance 1.

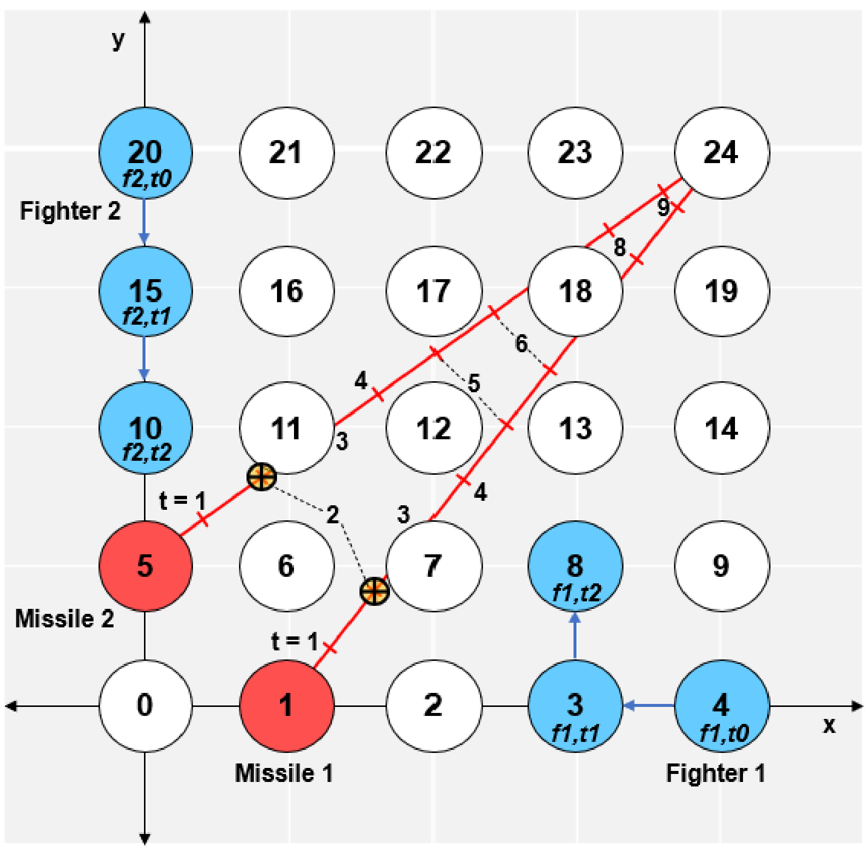

Figure 5. Model Test Instance 3: Capacitated two fighter utilization

The aircraft routing solution identified for Test Instance 3 is shown in Figure

5. Fighter 1 routes along the path $4-3-8$ shown with the blue directed arcs. Fighter 2 routes along the path $20-15-10$. Both Fighters 1 and 2 engage at time period t_2 from nodes 8 and 10, respectively. The engagements are determined by the decision variable y_{fmtw}, which is 0 for all $f \in F$, $m \in M$, $t \in T$, and $w \in W$ except for two engagements denoted by $y_{1,1,2,1} = y_{2,2,2,1} = 1$. The missile engagements are depicted as the yellow-highlighted cross hairs on the missile flight paths. Since Fighter 1 can only conduct a limited number of engagements, Fighter 2 is utilized to engage the second missile. The ε-constraints for Test Instance 3 are maintained at $(\varepsilon_2, \varepsilon_3) = (5, 2)$. The optimal objective value for Test Instance 3 is 4 distance units as compared to approximately 2.41 distance units from Test Instance 1. Test Instance 3 validates the model when using sets of weapons with engagement limitations.

4.1.4 Test Instance 4: Capacitated Fighter Engagement with Missile Target Time Offset

The previous test instances demonstrate how a set of fighters will engage a set of missiles with varying parameters to optimality. However, each test instance assumed the missiles arrive at the destination high value asset, node 24, at approximately the same time given they have not been engaged. As depicted in Figure 6, Test Instance 4 illustrates the routing decisions for a set of fighters wherein the expected missile arrival times to the destination node are offset.

Missiles 1 and 2 each travel at a speed of 0.5 distance units per time period. Missile 1 is detected at coordinates $(1, 0)$ and is targeting coordinates $(4, 4)$. At Missile 1's speed from node 1 to node 24, it will arrive to the high value asset at time period t_{10}, if not intercepted. Missile 2 is detected at coordinates $(-4, -2)$. According to Missile 2's speed and origin, it will arrive at coordinates $(4, 4)$, node 24, at time period t_{20}, if not intercepted. The time period labels of each missile are shown on the red tick marks along each missile path in Figure 6. The missiles travel along their respective paths at time intervals in ascending order. The origin of Missile 2 is not depicted in Figure 6, but the flight path of Missile 2 is shown. Notice, at time period t_{10}, Missile 1 could reach node 24, but Missile 2 would have only reached node 5.

Fighter 1 is positioned at coordinates $(4, 0)$, and Fighter 2 is positioned at coordinates $(0, 4)$. Each fighter can travel at a speed of one distance unit per time period. Fighters 1 and 2 have the same set of weapons, namely, Weapons 1 and 2. Each weapon has a range of 1.5 units, thus any missile within a radius of 1.5 units from the fighter can be engaged, provided the weapon has not reached its limit. Each Fighter has one unit of Weapon 1 to utilize in an engagement, and 0 units of Weapon 2.

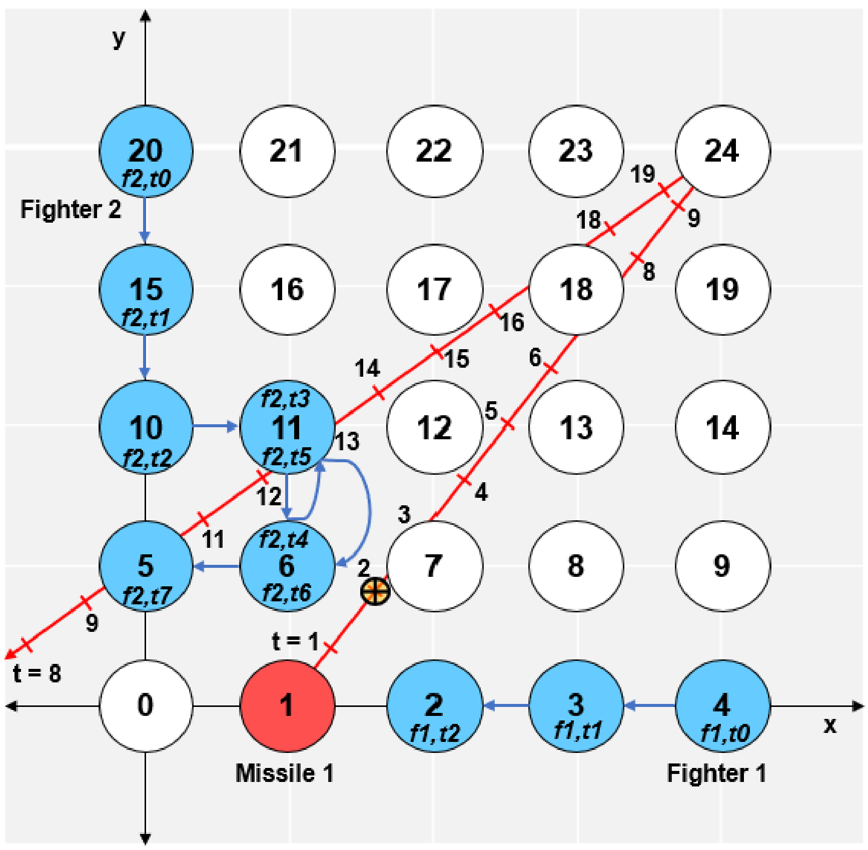

Figure 6. Model Test Instance 4: Capacitated two fighter utilization with missile target time offset

Test Instance 4 is the first instance where both missiles are not engaged at the same time. Since Missile 2 lies far outside of the coverage radius of either of the fighters for the first few time periods, the ε_2-constraint must be increased to make the engagement feasible. To allow for maneuverability of the fighters for the increased number of time periods, ε_2 is increased to 11. The ε_3-constraint is maintained at 2. The optimal objective value is 9 distance units traveled, with $f_2(\boldsymbol{y}) = 9 \leq 11$, and $f_3(\boldsymbol{z}) = 2 \geq 2$. The value of $f_2(\boldsymbol{y})$ shows that it was necessary to increase the level of ε_2.

Fighter 1 travels along the path $4-3-2$ with an engagement indicated by $y_{1,1,2,1} = 1$. After time period t_2, Fighter 1 departs the engagement since it can no longer engage a missile threat. Fighter 2 travels along the path $20-15-10-11-6-11-6-5$. Fighter 2 engages Missile 2 at time period t_7 with Weapon 1 as indicated by the decision variable $y_{2,2,7,1} = 1$. Fighter 2 loiters between nodes 11 and 6 to stay in position to engage Missile 2 when it gets within range. At time period t_7, Missile 2 is within range of Weapon 1 on Fighter 2 from node 5.

However, if $\varepsilon_2 < 9$, Fighter 2 would not have been able to engage Missile 2 while still satisfying Constraint (17). In turn, if Missile 2 cannot be engaged, Constraint (18) cannot be satisfied. Both ε-constraints in P2 must be satisfied to make the problem render a solution feasible. Furthermore, the binding ε-constraints comprise an efficient frontier.

4.2 Model Parameterization for a Representative Scenario

The model is functional on a small instance as shown in Section 4.1; however, to determine whether the granularity of the network can be increased to a more realistic instance, this section will introduce a representative scenario examined in Sections 4.3 and 4.4.

Geographically separated air bases require layered air defense to reliably counter incoming air threats. Fighter aircraft are a vital component of the layered air defense, but which platform is best suited for this mission? The demand for new fighter aircraft becomes increasingly important as the F-15Cs and F-15Ds reach the end of their serviceable lives. The actual demand for F-35s is outpacing production, but the advent of the F-15EX, a newer variant of the Strike Eagle series, provides a way for the Air Force to maintain fighter capacity given the "slow pace of F-35 acquisitions" [34]. The decision to invest in F-15EXs is from the Pentagon's Cost and Program Evaluation Office (CAPE). The F-15EX does not have the required stealth characteristics and sensor fusion to survive modern air defenses in contested airspace. "However, CAPE and Air Force officials see viable continuing mission for the F-15EX in homeland and air base defense, in maintaining no-fly zones where air defenses are limited or nonexistent, and in delivering standoff munitions" [34].

The F-15EX has the ability to use many of the existing resources from the aircraft already in service including; ground equipment, hangars, simulators, and support gear, and it has the ability to be armed with a variable load of weapons [34]. The F-15EX is a good candidate for the ABAD mission, but currently there is not sufficient (unclassified, publicly available) information on the fighter specifications of the F-15EX. Since the F-15EX is in the F-15 series of fighters which have similar characteristics, the F-15E's specifications will be used to model the F-15EX.

An additional factor that must be considered in this application is the advent of a directed energy (DE) weapon system. The potential benefits and limitations of DE weapons were briefly discussed in Chapter I. The Air Force plans to integrate high energy LASER (HEL) pods onto fighter aircraft in the near future as shown in Figure 7, which is from a presentation given at Air Force Research Laboratory (AFRL) [1]. The DE pods can be mounted directly to the center of the fuselage of an F-15 with power greater than 10 kW. However, the Department of Defense has updated the power expectations due to the requirement to have a DE weapon capable of shooting down cruise missiles. The Pentagon is negotiating contracts with three different contractors for the development of 300-kilowatt DE weapons which will be powerful enough to destroy or disable cruise missiles [17]. Combining the ability of DE weapons to engage for the duration of combat air patrols with the existent abilities provided by AIM-9 Sidewinders and AIM-120 AMRAAMs increases the overall capability for fighters to provide anti-access area defense (A2/AD).

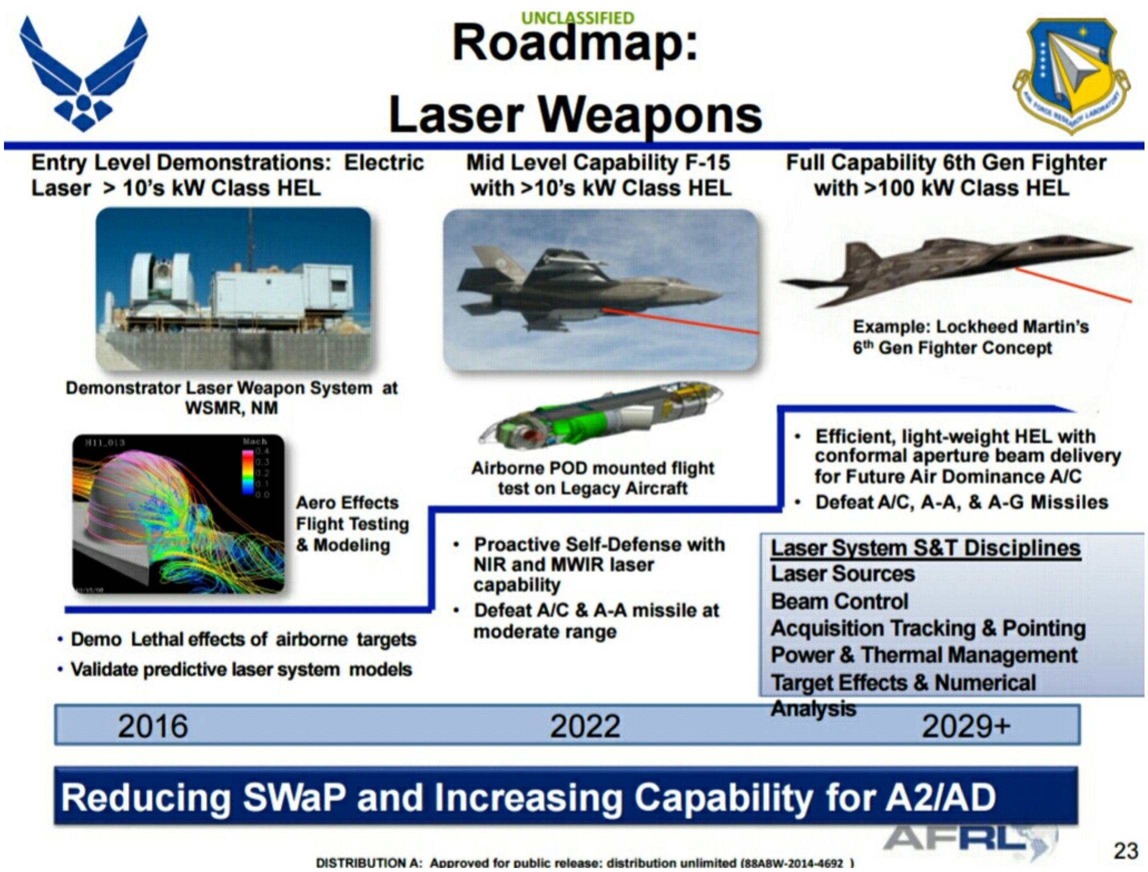

Figure 7. Air Force Research Laboratory potential road map to the directed energy capability demonstrating a high energy LASER pod mounted on fighter aircraft [1]

Given a known aircraft with known specifications the representative scenario can be approximated with the updated sets, and parameters.

4.2.1 Sets for the Representative Scenario

- T: The discrete time increments at which decisions are made, indexed by $t \in T = \{0, 1, ..., 100\}$, where each increment is a tenth of a minute (i.e. six seconds).

- N: The set of nodes within the network indexed on i or j, alternatively, and which represent the possible locations of fighter aircraft. The network in this scenario is composed of 100 total nodes along a corridor which is 25 nodes wide with a height of 4 nodes. The nodes are numbered in order from 0 to 99 with node 0 corresponding to the origin of the (x, y)-coordinate plane at $(0, 0)$, and node 99 corresponding to the point $(24, 3)$.

- A: The fighters can only travel along undirected arcs from adjacent nodes either vertically, horizontally, or diagonally.

- $G(N, A)$: The complete network composed of nodes N and arcs A. The network is 240 miles wide along the x-axis, and 30 miles long along the y-axis. The total area covered by the square tesselated network is a rectangular corridor that is 7200 miles^2. Figure 8 depicts the entire network as the shaded region of the flight corridor.

- F: The set of fighters traversing the network for this scenario consists of two F-15EX aircraft, indexed by $f \in F = \{1, 2\}$. The fighters are located at coordinates $(24, 0)$ and $(24, 2)$ as shown in Figure 8 by the blue dots labeled F1 and F2.

- M: The set of cruise missiles traversing the network for this scenario is 4, indexed by $m \in M = \{1, 2, 3, 4\}$. Missiles 1 through 4 are detected at coordinates $(1, 0)$, $(0, 3)$, $(4, 2)$, and $(10, 0)$, as represented by the red dots labeled M1 through M4 on Figure 8. Each missile is targeting a high value air base located at coordinates $(24, 3)$ shown as the green X labeled AB in Figure 8.

- W: The F-15 Eagle series fighters carry up to eight air-to-air missiles, either four AIM-9 Sidewinders and four AIM-120 AMRAAMs, or eight AIM-120 AMRAAMs as shown in Table 1 [9]. The set of weapons that can be used in an engagement of cruise missiles is composed of the AIM-9, AIM-120, and a DE weapon indexed by $w \in W = \{1, 2, 3\}$, respectively.

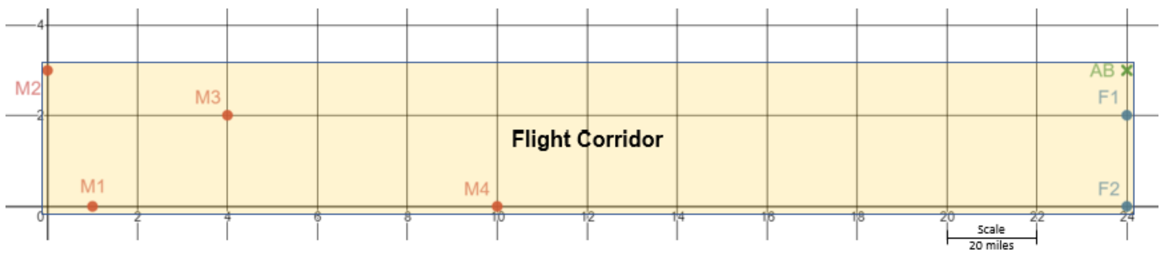

Figure 8. Model Representative Instance: Two fighters providing DCA to an air base with four inbound cruise missiles

Table 1. The fighter specific parameters for the representative scenario [15]

F-15E		
Speed (mph)	**Speed (mi/min)**	**KE Armament**
1875	31.25	four AIM-9s & four AIM-120s; or eight AIM-120s

4.2.2 Parameters

- d_{ij}: The distance between horizontally or vertically adjacent nodes is 10 miles, and the distance between diagonally adjacent nodes is $10 * \sqrt{2}$ miles (approximately 14.14 miles). Similarly, the distance between non-adjacent nodes can be determined by the Pythagorean Theorem.

- τ_{ij}: The time required for a fighter to traverse arc (i, j) is dependent on the speed of the fighter, which is shown in Table 1 for this scenario. Each fighter is assumed to maintain this speed throughout the engagement. A fighter can travel between adjacent nodes in 19.2 seconds either vertically or horizontally, and 27.15 seconds diagonally.

Table 2. Fighter weapon model parameters [3, 4, 33, 37]

Weapon	AIM-9	AIM-120	DE LASER
Range (miles)	6.2	31.1	10
Speed	Supersonic	Supersonic	Light

- a_{imtw}: A parameter indicating missile m is covered from node i at time t by a weapon w dependent on the weapon's range as indicated in Table 2. The missile's location is determined via (1) the location and time at which missile m is detected, (2) the target of missile m which is either known or inferred based on the direction of travel from point of detection, and (3) the speed, s_m, at which missile m is traveling, which is shown in Table 3. The Kh-55 Russian cruise missile is used to approximate the performance characteristics of the Chinese cruise missile Hong Niao series (HN-1/-2/-3) based on their similar shape and size [22].

Table 3. Cruise missile model parameters [37]

Cruise Missile	Speed (mph)	Speed (mi/min)	Range (miles)
Kh-55	614	10.23	1349.9

- b_{fit}: Fighters 1 and 2 enter the network at nodes 74, and 24, respectively, both at time period $t = 0$ which corresponds to $b_{1,74,0} = b_{2,24,0} = 1$.

- u_{fw}: The upper bound of the number of engagements each fighter f can conduct in this scenario is informed by the capabilities of the F-15E as shown in Table 1. Each fighter is assumed to carry a standard weapons loadout of $u_{f1} = 4$ AIM-9 missiles, and $u_{f2} = 4$ AIM-120 missiles, in addition to a directed energy capability indexed by $u_{f3} = 100$ LASER engagements. The magazine depth of a directed energy weapon is assumed to be dependent only on the amount of time it takes for the system to reach the appropriate power level. Therefore, in this scenario, the directed energy weapon will be limited by the time required between engagements (δ_3) and not by its limit for total number of engagements.

- υ_w: The upper bound on the number of engagements a weapon can conduct at a given time is assumed to be 1 for each weapon type in this scenario.
- δ_w: The amount of time required between engagements for weapon type w. The time between engagements for the KE weapons is assumed to be one time period, or $\delta_1 = \delta_2 = 1$. The time required between engagegements for the DE weapon is 20 time periods, $\delta_3 = 20$. The limitation on DE weapons is a conservative approximation that takes both the time required on target and the time to regenerate the power levels of the weapon system into account.

4.3 Pareto Frontier: Examination of the Representative Scenario

Problem formulation P2 in Section 3.3 introduced the ε-constraint Method of multi-objective optimization. The ε-constraint Method induces the entirety of the Pareto optimal frontier depending on the intervals of ε chosen. The representative scenario outlined in Section 4.2 is used to determine the efficient solutions. Formulation P2 is solved for each integer combination of $\varepsilon_2 \in \mathcal{E}_2 = \{0, 1, 2, ..., 400\}$ and $\varepsilon_3 \in \mathcal{E}_3 = \{0, 1, 2, 3, 4\}$. Each combination of ε-constraints is processed through the Network-Enabled Optimization System (NEOS) [12, 13, 20]. The total number of runs solved is $|\mathcal{E}_2| * |\mathcal{E}_3| = 2005$.

An important consideration for finding the optimal route for a set of fighters is computational time, because each fighter makes routing decisions prior to the arrival to the next node. Therefore, the time to traverse an arc between nodes is the upper bound on the time that should be allotted to determine a routing solution. In this instance the upper bound on solver time for routing decisions is

$$\textit{Decision Point Time Threshold} = \min\{\tau_{ij} \mid \forall\ (i,j) \in A\} = 19.2 \text{ seconds.}$$

To determine how efficient the model is, each iteration is allowed to exceed the decision point time threshold. However, due to the number of required computations using the ε-constraint method, the time resource limitation for the solver is set to 1000 seconds. Figure 9 shows the computational time for each run conducted using the ε-constraint method. When $\varepsilon_3 = 0$ the computational time is low since the optimal route for each fighter is to not route through the network at all. Similarly, when ε_2 is low the computational time is low because there are fewer routes that are possible for the fighters to traverse while still satisfying the $f_2(\boldsymbol{y}) \leq \varepsilon_2$ constraint. However, as ε_2 increases the number of possible routes increases drastically and a optimal solution takes much more time to compute. Evidence of the relationship between computational time and the available number of routes is depicted in Figure 9 with a steep increase in computational time from 0 up to 1000 seconds as ε_2 increases for each value of $\varepsilon_3 > 0$. Of the 1604 iterations with $\varepsilon_3 > 0$, 60% solved within 1000 seconds and 31% have a computational time that is less than or equal to 19.2 seconds.

In addition to computational time of the model, the solution type returned is also important. Figure 10 shows the model status returned for each iteration. For this model, there were only three values returned for the model status. A model status value of 1 indicates the solver was able to verify the solution as the global optimal solution. A model status value of 8 indicates an integer solution exists, although it may or may not be the optimal solution. A model status value of 10 indicates that there is no feasible integer solution. For $\varepsilon_3 = 0$ all solutions are optimal. For all $\varepsilon_3 > 0$, the solutions are infeasible if the time allotted to traverse the network given

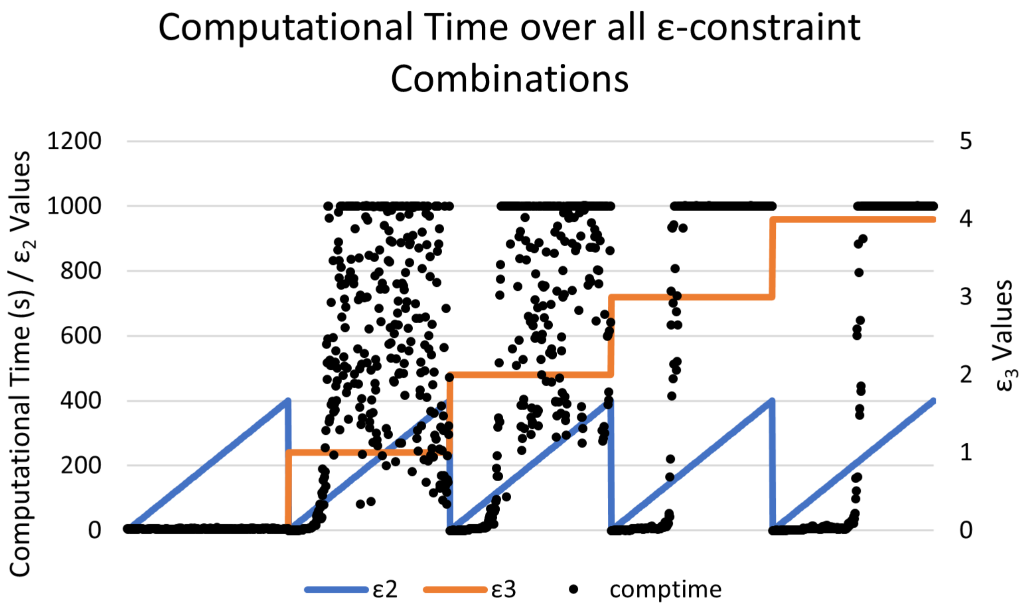

Figure 9. Computational time for each iteration with ε_2 ranging from 0 to 400, and ε_3 ranging from 0 to 4

by ε_2 is less than any possible path to engage at least ε_3 missiles. As ε_2 increases the model status at $\varepsilon_3 = 1$ and $\varepsilon_3 = 2$ vacillates between integer feasible and optimal solutions. There are still some global optimal solutions for $\varepsilon_3 > 2$ at ε_2 values that are slightly larger than the $f_2(\boldsymbol{y}) \leq \varepsilon_2$ constraint. However, for large values of ε_2 and $\varepsilon_3 > 2$ the solutions are only integer feasible.

Given the full set of solutions using the ε-constraint method within the computational time restriction of 1000 seconds, the efficient solutions can be extracted. Table 4 shows the remaining seven solutions along the Pareto frontier. Each Pareto efficient solution is a global optimal integer solution given the parameters for the ε-constraints. Not only did each Pareto efficient solution solve within the computation resource limitation of 1000 seconds, but also within the decision point time threshold. Since each of these solutions is within the decision point time threshold, the fighters will be able to efficiently determine an optimal path to engage a set of cruise missiles.

The number of possible ε-constraint combinations is too large for effective pro-

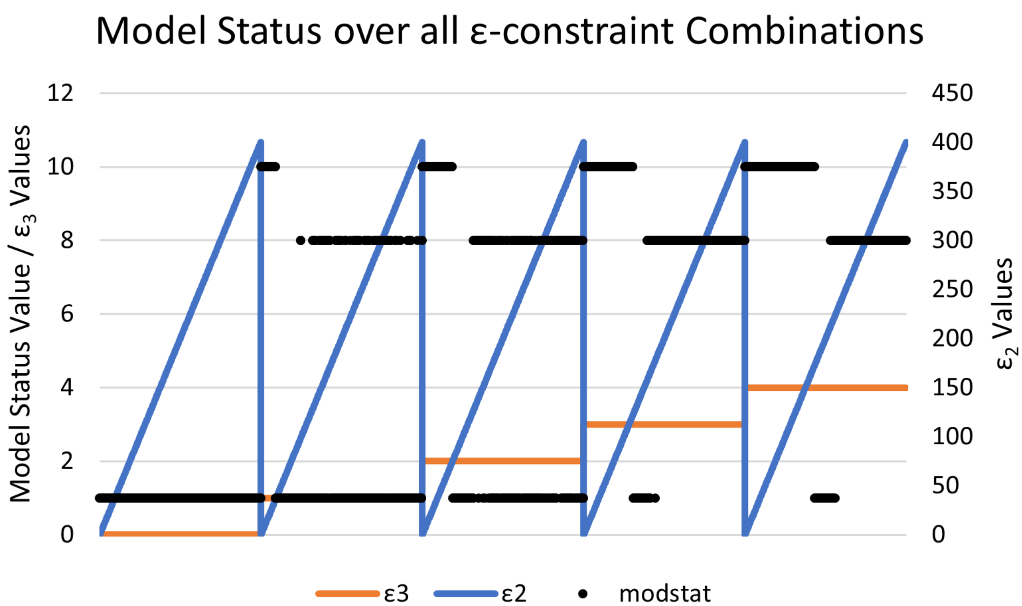

Figure 10. Model Status for each iteration with ε_2 ranging from 0 to 400, and ε_3 ranging from 0 to 4 where modstat = 1 is optimal, modstat = 8 is integer feasible, and modstat = 10 is integer infeasible

Table 4. The Pareto frontier of solutions using the ε-Constraint Method with associated computation times in seconds

Pareto Optimal Solutions

min $f_1(\boldsymbol{x})$	ε_2	ε_3	Computation Time (s)
0	0	0	4.54
12	36	1	5.32
13	75	2	7.74
15.828	124	3	7.56
16	123	3	12.46
17	174	4	8.39
32.414	173	4	6.48

cessing. In this instance, the total sum of computational time for the $|\mathcal{E}_2| * |\mathcal{E}_3| = 2005$ iterations is almost 245 hours. The pairwise scatterplot matrix in Figure 11 shows the relationship between objectives $f_1(\boldsymbol{x})$, $f_2(\boldsymbol{y})$, $f_3(\boldsymbol{z})$, and the computational time. Only feasible solutions are included in the scatterplot matrix to highlight the comparison between optimal and integer feasible solutions. The blue dots indicate a global optimal solution (modstat = 1), and the green dots indicate a suboptimal integer feasible solution (modstat = 8), which are also shown in Figure 10. Figure 11 shows a high correlation between $f_2(\boldsymbol{y})$ and $f_3(\boldsymbol{z})$ with $r = 0.9949$. The strong correlation between the time limit to engage missiles and the number of missiles to engage suggests that one of the objectives is sufficient for the model.

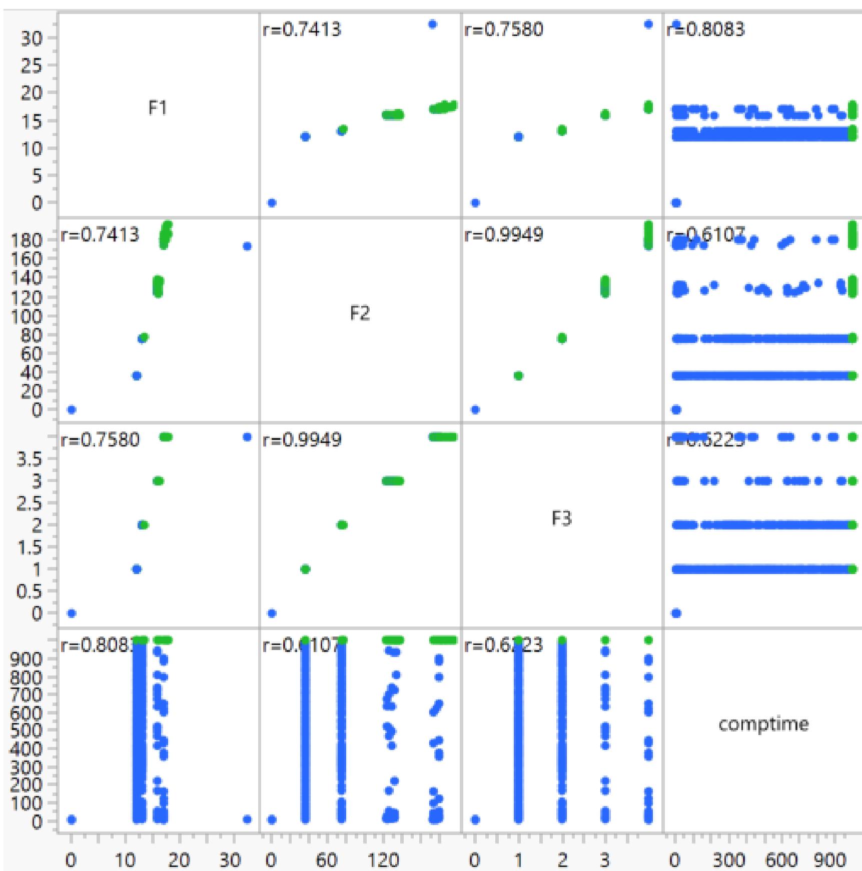

Figure 11. Pairwise scatterplot matrix comparing the three objectives and the computational time for all feasible solutions

Since $f_2(\boldsymbol{y})$ has such a wide range of possible ε_2 values, the objective $f_3(\boldsymbol{z})$ will be used as a comparison to the original model where ε_3 ranges from 0 to 4 missiles. Therefore, P2 is adjusted to remove constraint (17) for objective $f_2(\boldsymbol{y})$ and solved with the same resource limitation for computation time of 1000 seconds. The results of the ε-constraint method with only objective $f_3(\boldsymbol{z})$ are shown in Table 5. The computation times required for $\varepsilon_3 > 0$ are not within the decision point time threshold, and therefore are not effective in the real-world fighter application. Of note, the solutions in Table 5 are the same as the Pareto efficient values in Table 4 for the two objectives $f_1(\boldsymbol{x})$ and $f_3(\boldsymbol{z})$, which implies that all the solutions are indeed optimal but the latter three solutions in Table 5 are not proven optimal within the time resource limitation. The results from removing objective $f_2(\boldsymbol{y})$ imply that although the total number of iterations solved is substantially lower, the time to compute efficient solutions is outside of the decision point time threshold, and therefore objective $f_2(\boldsymbol{y})$ cannot be removed entirely despite its high correlation with objective $f_3(\boldsymbol{z})$.

Table 5. The Pareto frontier of solutions using the ε-Constraint Method for objectives $f_1(\boldsymbol{x})$ and $f_3(\boldsymbol{z})$

Pareto Optimal Solutions with ε_3-constraints			
min $f_1(\boldsymbol{x})$	ε_3	Computation Time (s)	Model Status
0	0	0.286	Optimal
12	1	479.3	Optimal
13	2	1000	Integer Feasible
15.828	3	1000	Integer Feasible
17	4	1000	Integer Feasible

4.4 Sensitivity Analyses

The ε-constraint method performs well to identify the full set of the Pareto efficient solutions and helps to develop insight into how to route combat air patrols for air base air defense. The computation times listed in Table 4 in this instance indicate that points on the Pareto frontier solved within the decision point time threshold. However, use of the ε-constraint method to solve for the multiple instances is not realistic for a fighter routing over a network. An *a priori* hierarchical multi-objective optimization approach can be used to determine a lexicographic optimal solution. Lexicographic optimization requires the objectives to be ranked in order of priority and then each objective solved in order. After an objective has been solved to optimality, the optimal solution is set as a constraint to be solved for objectives lower on the hierarchy [14].

Table 6. The lexicographic method of optimization for all permutations of objectives $f_1(\boldsymbol{x})$, $f_2(\boldsymbol{y})$, and $f_3(\boldsymbol{z})$

Lexicographic Optimization Permutations

First	Second	Third	Viability
min $f_1(\boldsymbol{x})$	min $f_2(\boldsymbol{y})$	max $f_3(\boldsymbol{z})$	Not Viable
min $f_1(\boldsymbol{x})$	max $f_3(\boldsymbol{z})$	min $f_2(\boldsymbol{y})$	Not Viable
min $f_2(\boldsymbol{y})$	min $f_1(\boldsymbol{x})$	max $f_3(\boldsymbol{z})$	Not Viable
min $f_2(\boldsymbol{y})$	max $f_3(\boldsymbol{z})$	min $f_1(\boldsymbol{x})$	Not Viable
max $f_3(\boldsymbol{z})$	min $f_1(\boldsymbol{x})$	min $f_2(\boldsymbol{y})$	Viable
max $f_3(\boldsymbol{z})$	min $f_2(\boldsymbol{y})$	min $f_1(\boldsymbol{x})$	Viable

Recall the three objectives (i.e., $f_1(\boldsymbol{x})$, $f_2(\boldsymbol{y})$, and $f_3(\boldsymbol{z})$) from P1 in Chapter III. The three objectives can be ranked in six possible distinct permutations as shown in Table 6. The first four lexicographic rankings are not viable candidates. If the first objective is to minimize $f_1(\boldsymbol{x})$, then the total distance traveled constraint will be 0 for the following second and third objectives. Likewise, if the first objective is to minimize $f_2(\boldsymbol{y})$, then the time constraint for the second and third objectives will be 0. Thus, the only viable candidates for lexicographic optimization are the combinations in which objective $f_3(\boldsymbol{z})$ is maximized first. Therefore, the remaining lexicographic optimization problems can be written as Problems **P3** and **P4**, as represented in formulas (19) and (20), respectively.

$$\mathbf{P3}: \; lex_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}}[\max f_3(\boldsymbol{z}),\; \min f_1(\boldsymbol{x}),\; \min f_2(\boldsymbol{y})] \tag{19}$$

$$\mathbf{P4}: \; lex_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}}[\max f_3(\boldsymbol{z}),\; \min f_2(\boldsymbol{y}),\; \min f_1(\boldsymbol{x})] \tag{20}$$

The problem formulations P3 and P4 optimize the objectives in the order as listed in (19), and (20), respectively. Table 7 shows the results from solving both of these problems. Problem P3 solved in 1024 seconds, and yielded an integer feasible solution. The solution for P3 is actually optimal as shown in Table 4, but the solver exceeded the maximum resource utilization when solving for objective $f_1(\boldsymbol{x})$. Problem P4 was solved five times with an average total solve time of 14.02 seconds, which is within the decision point time threshold. Of the five runs, all were within the decision point time threshold. The solution for problem P4 is the proven optimal solution. Both solutions for P3 and P4 are the same as the ε-constraint method generated Pareto optimal solutions, as indicated on Table 4 at $f_3(\boldsymbol{z}) = 4$.

Table 7. The Lexicographic Optimization method with the objectives $f_1(\boldsymbol{x})$, $f_2(\boldsymbol{y})$, and $f_3(\boldsymbol{z})$ in priority as listed in problem formulations P3 and P4

Lexicographic Optimization of P3 and P4

Problem	min $f_1(\boldsymbol{x})$	min $f_2(\boldsymbol{y})$	max $f_3(\boldsymbol{z})$	Total Time (s)
P3	17	174	4	1024
P4	32.41	173	4	14.02

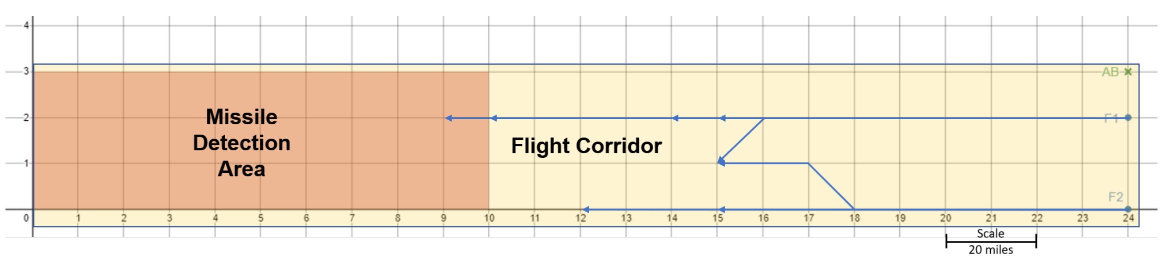

Figure 12. Model Representative Instance: Two fighters providing DCA to an air base with four inbound cruise missiles, each having randomly generated origins

Problem P4 for the single representative instance solved within the decision point time threshold; however, to determine how reliable the optimization is using the lexicographic method, Problem P4 was solved using randomly generated instances. Figure 12 shows the same network as described in Section 4.2, Figure 8. In Figure

12, the four missiles are detected within the area labeled ‘Missile Detection Area’. The origins of each missile are randomly generated using a uniform distribution with random seeds from 1 to 100. Each of the 100 different instances are solved using P4. Figure 12 shows the respective routes of each fighter as blue lines. The blue arrows depict the node at which routing through the network terminates. There are five unique routes from Fighter 1, and three unique routes for Fighter 2.

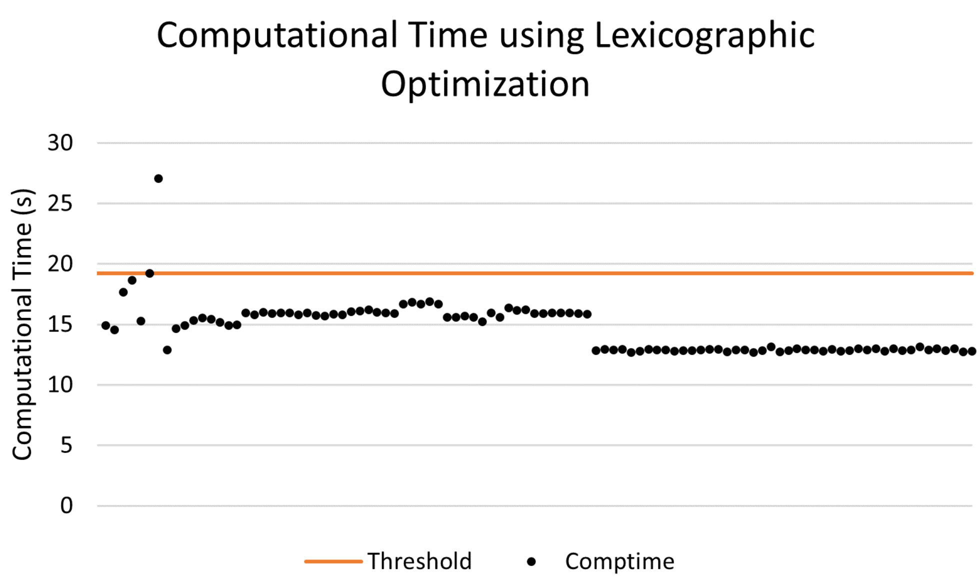

Figure 13. Computational time from lexicographic optimization with problem formulation P4 using a random uniform distribution to generate missile origins

Figure 13 shows the computational time required for each of the 100 iterations. The decision point time threshold is indicated by the orange line at 19.2 seconds. Of the 100 iterations, 98% solved within the threshold. The two that exceeded the threshold were 19.235 and 27.078 seconds, respectively. At each iteration, the response parameters are set to zero to ensure a cold start. However, the computational times for the 4 missiles seem to level off toward the latter half of the iterations. This could indicate a warm-up period for the solver. Throughout the iterations, all four missiles were engaged, totaling 400 missile engagements. Of the 400 missile engagements, 324 were conducted with w_2, the AIM-120 AMRAAM, and 76 were conducted with w_3, the DE weapon.

The lexicographic method appears to perform well the majority of the time in the representative instance over randomly generated missile origins. To determine the limitations of the method, a single fighter is considered for routing through the network. Fighter 1 is retained in the network with initial location at node 74 corresponding to coordinates $(24, 2)$. The number of missiles detected in the scenario is incrementally increased, starting at one incoming cruise missile. The origins for the cruise missiles are within the missile detection area highlighted in Figure 12. For each increment of missiles, the origins are randomly generated with a uniform distribution using random seeds from 1 to 30. Figure 14 depicts the single fighter instance with increasing number of incoming missiles detected. For a single fighter, the computational time to optimally route to destroy up to four incoming cruise missiles is within the decision point time threshold as shown by the points that fall below the orange line. At five incoming cruise missiles, the computational time starts to exceed the decision point time threshold with 14 of the 30 iterations above 19.2 seconds.

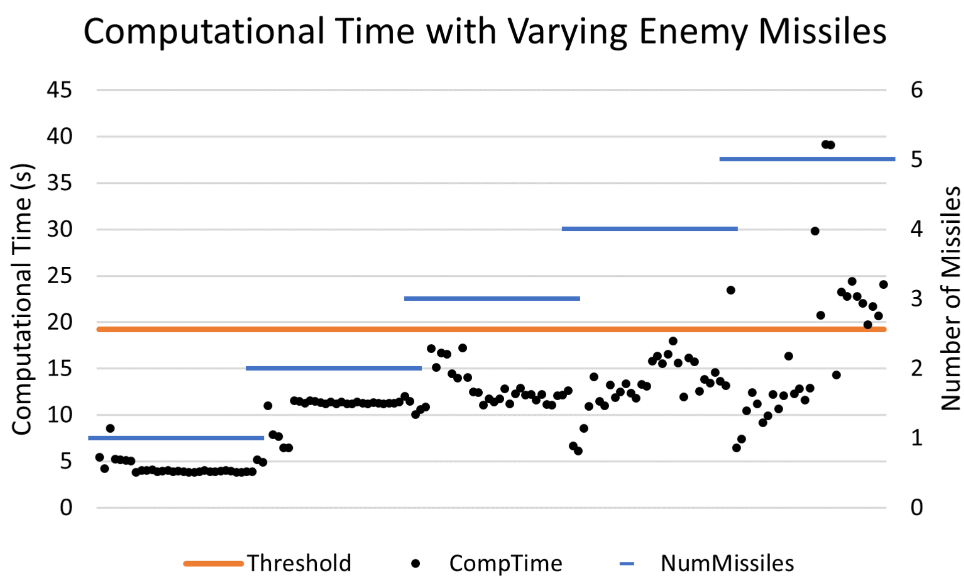

Figure 14. Computational time for varying missiles with 30 random iterations each using lexicographic optimization from problem formulation P4

V. Conclusion

This chapter summarizes the contributions of this research and identifies extensions to improve its examination of the defensive counterair for air base air defense problem. Section 5.1 summarizes the research and provides key insights. Section 5.2 provides recommendations for the solution methodology as well as potential the problem statement.

5.1 Conclusions

The protection of air bases is an important factor in maintaining a strategic advantage. Air bases are extremely important in the U.S. military's ability to rapidly project forces globally, and deter adversarial aggression. Due to their strategic importance, air bases in contested regions are subject to constant threats from adversaries such as China and Russia who attempt to erode American influence and security. Adversaries are focusing on efforts to expand the range of their missile strike capabilities that could hinder the force projection ability of the U.S. military. The *Joint Operating Environment 2035* [31] indicates that rival states may implement advanced multi-layered attacks with long-range cruise and ballistic missiles to maintain standoff from the U.S. and allied forces. The strategies that adversaries may employ include anti-access/area denial with coordinated attacks from aircraft and missiles [32]. The *National Defense Strategy* [25] key modernization capabilities include missile defense. The investment focus is on "layered missile defenses and disruptive capabilities for both theater missile threats and North Korean Ballistic missile threats" [25].

A layered missile defense system for an air base is comprised of both ground defense systems as well as combat air patrols providing defensive counterair protection. This research focuses on air base air defense from a combat air patrol. Fighter aircraft are used for defensive counterair operations to protect high value assets, such as an air base. Two to four fighters are placed in a combat air patrol to form a protective barrier for the air base. Fighters providing air base air defense are usually armed with air-to-air weapons capable of line of sight, or even beyond line of sight engagements using radars. The Air Force primarily uses two types of air intercept missiles (AIMs): the AIM-9 Sidewinder which is a short range, supersonic, heat-seeking missile, and the AIM-120 advanced medium-range air-to-air missile with beyond visual range capability. In addition to the kinetic energy weapons fighters use for air to air engagements, the Air Force Research Laboratory has established a road map to employ directed energy weapons on fighters [1]. Directed energy weapons cost less per engagement, but they must maintain precise contact with a target for a sustained period of time to be effective. A 300-kilowatt laser may be powerful enough to destroy an incoming cruise missile [17]. Directed energy weapons are not a replacement to the kinetic energy weapons, but they will augment a fighter's ability to engage air threats.

Given the motivation for air base air defense, this research seeks to determine the optimal positioning and maneuvering of a fixed number of fighter aircraft in a combat air patrol for defensive counterair to maximally and effectively attrit the inbound cruise missile threat to a stationary high value asset. The problem formulation identifies multiple objectives for the model: minimize the distance traveled for the fighters, minimize the time utilized to engage the incoming cruise missiles, and maximize the number of cruise missiles destroyed. The model is a mobile routing problem with a mobile demand. The network used for the model consists of nodes overlayed on a Cartesian plane for a specified region. Each combination of integer points within the region correspond to a node. The fighters can traverse the network via arcs connecting the nodes either horizontally, vertically, or diagonally adjacent. For a solution to be useful for real world applications, the routing decision for the aircraft would need to occur within a specific time period that is determined by the minimum amount of time that is required to traverse from one node in the network to the next adjacent node, which is referred to as the *Decision Point Time Threshold*.

This research used the ε-constraint method as the multi-objective optimization technique to determine the full set of Pareto efficient solutions. The objective functions respectively related to the time allotted for missile engagements and the total number of cruise missiles destroyed inform the ε-constraints because these objective functions are integer-valued. Both ε-constraint bounds are varied over all possible values, and the problem is solved iteratively to minimize the distance traveled by each fighter. The ε-constraint method yields the entire Pareto frontier of solutions for the representative scenario. Most notably, each Pareto efficient solution from the ε-constraint method solved within the decision point time threshold.

The ε-constraint method determined efficient solutions, but it is not a viable method for an actively mobile facility to implement within the specified decision point time threshold. Therefore, another multi-objective optimization technique is required that can efficiently be solved within the time threshold. An *a priori* hierarchical multi-objective method called lexicographic optimization is used to determine if there is a ranking of objectives that solved more efficiently. Lexicographic optimization will solve the model iteratively over each objective. The optimal objective value after each iteration is set as a constraint for subsequently solved objectives which are lower on the hierarchy. Lexicographic optimization that maximizes the number of cruise missiles destroyed, minimizes the time allotted for missile engagements, and finally minimizes the fighter distance traveled solves within the decision point time threshold 98% of the time in the representative scenario. The lexicographic method is also evaluated using a single fighter routing over the network and incrementally varying the number of incoming cruise missiles. A single fighter is able to engage up to four incoming cruise missiles while still making the routing decisions within the decision point time threshold. The lexicographic method for the defensive counterair problem is promising to determine which cruise missiles a fighter engages, what time period to conduct the engagements, and which weapon to use for a given engagement. The AIM-120 is the primary weapon utilized in all the scenarios, due mostly to its range. However, with 30 random instances of four incoming cruise missiles, the directed energy weapon is used 23 instances, and the AIM-9 is used 7 instances.

The first research objective in this research addressed whether a fighter aircraft can assume an optimal route traversing the network such that the weapons can most effectively engage the cruise missiles. This objective was achieved by using lexicographic optimization with the hierarchy of objectives that can be solved efficiently is: maximize the cruise missiles destroyed, minimize the time allotted for missile engagements, and minimize the distance a fighter travels. A fighter can optimally route to destroy up to four incoming cruise missiles within a reasonable computation time as determined by the speed of the fighter and the distance between adjacent nodes.

The second research objective analyzes the tradeoffs between effectively engaging the incoming cruise missiles and minimizing the total cost of all the engagements. The lexicographic method assures efficiency in cruise missile engagement by optimizing the maximum number of cruise missiles engaged as the first priority. The distance traveled is the final in priority of the lexicographic method based on the limitations imposed by the empirically observed computation times, which accounts only for fuel costs. The cost of each engagement from an air intercept missile or a discharge from a high energy laser are not explicitly modeled, although the model identifies which weapon type a fighter will use to destroy incoming cruise missiles. Determining the efficient frontier to include the minimum cost over the total engagements is possible, as demonstrated via the ε-constraint method, albeit with computational challenges that render it an impractical method for use with current computing and software advancements.

5.2 Directions for Future Research

The research presented herein was effective in determining an optimal route for a fighter aircraft to destroy incoming enemy cruise missiles using the lexicographic method of optimization. The model was useful at determining whether a cruise missile was within range of a particular weapon at a given time; however, for more accurate results, the model could implement the probability of a successful engagement instead of the current binary variable for whether a missile is engaged or not. The model can then be adjusted to maximize the expected number of missiles destroyed based on the fighter weapons' known effectiveness rates. Additionally, the directed energy weapon's full limitations are not explicitly modeled. The time required between directed energy weapon engagements is assumed to be two minutes, however, the time that the laser must focus on a specific point of a target is not considered, nor is the angle at which the laser is oriented to the incoming cruise missile. Both the range and time on target requirement for a laser are functions of the power of the laser, thus the expected rate of successful missile engagements using a directed energy weapon should be analyzed at varying power levels. With the additional considerations of weapon effectiveness rates, and directed energy weapon limitations, the validity of the results would increase.

The granularity used in the representative scenario is a network consisting of 100 nodes overlayed on a two-dimensional Cartesian plane. The vertically and horizontally adjacent nodes are 10 miles apart, a distance which corresponds to a minimum 19.2 seconds to traverse adjacent nodes when the fighter is traveling at 1,875 mph.

Increasing the granularity of the network would increase the number of nodes and their proximity to one another, which in turn decreases the decision point time threshold. Also, a more accurate representation of the network would be three-dimensional to account for altitude differences of the fighters in relation to the incoming cruise missiles. An increase in the number of nodes in the network increases the number of possible routes for a fighter. The increase in the possible combinations for routing drastically increases the computation time to determine optimal routes for cruise missile engagements. These issues in computation time for an active mobile threat to an air base are time sensitive, and thus merit the use of a heuristic to speed up computation. A preliminary instance could establish the resource limitation of the CPLEX solver to not exceed the decision point time threshold. The solution that the solver yields may not be a proven optimal solution, but a viable means to engage a sufficient number of cruise missiles.

The representative scenario used for testing was a network of 100 nodes arrayed in four rows of 25 nodes that limited the fighter maneuvering to a narrow corridor. The dimensions of the network limit the ability to generalize the results to a realistic scenario. Perhaps to determine the generalizability of the model, a series of multiple scenario types can be generated to validate the scalability for larger engagement zones. Furthermore, given a specific scenario in a geographic region, an intelligence estimate could determine which regions pose a cruise missile threat to an air base with some uncertainty. Given a known location for a high value asset (e.g., an air base), for which a fighter is providing defensive counterair support, the patrol space can be more refined, and thus can more accurately represent the maneuver and engagement area.

Appendix A. Executive Summary

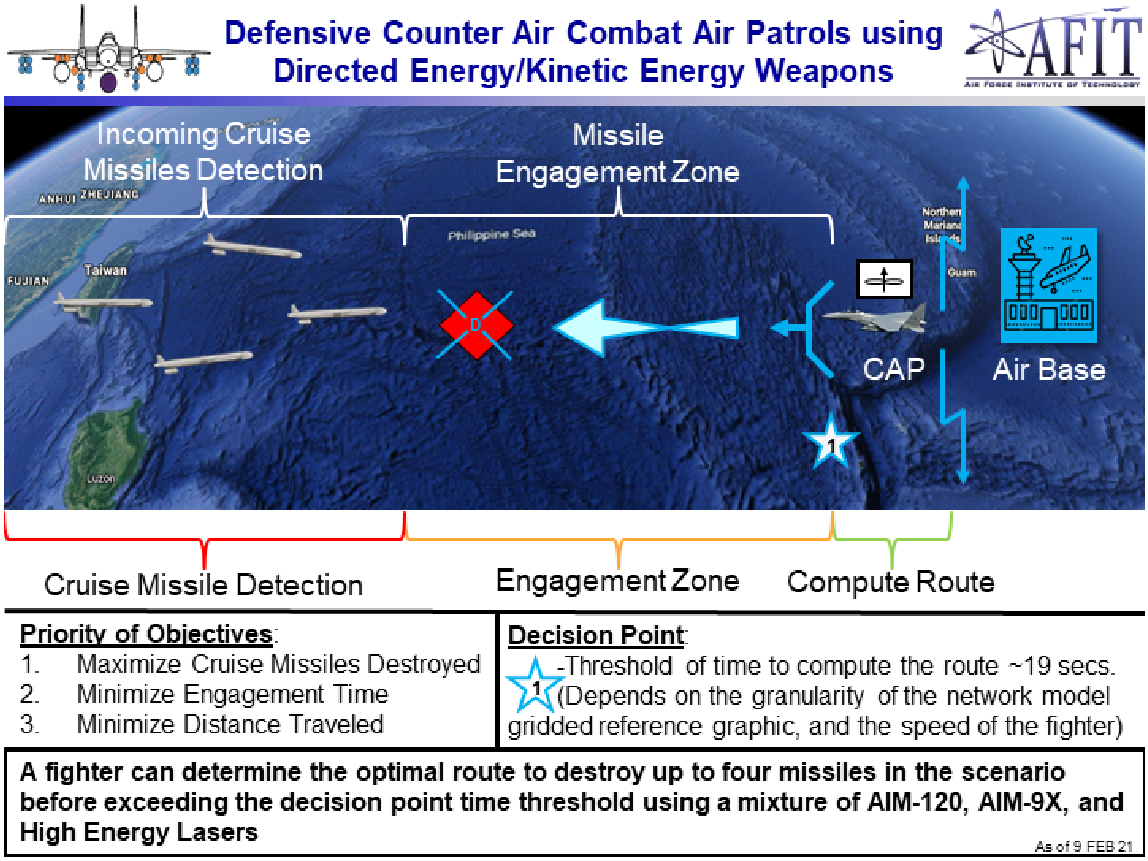

Figure 15. Overview of the model for routing using the lexicographic method of optimization

Appendix B. Lexicographic Optimization Code

```
* Coded by Andrew S. Wilson, AFIT/ENS
* In support of Thesis Research
* Defensive Counter Air Combat Air Patrols for Airbase Air Defense
* Reduce output in lst file

$offlisting
$offsymxref offsymlist
option
    limrow = 0,
    limcol = 0,
    solprint = off,
    sysout = off;

Set
  i  nodes /i0*i99/
  f  fighters /f1/
  m  missiles /m1/
  t  time /t0*t100/
  w  weapons /w1*w3/;

Alias (i,j);
Alias (t,tHat);

Parameters
  xcoordi(i)  x-coordinate for node i
  ycoordi(i)  y-coordinate for node i;

scalar
  xwidth    'number of nodes in a row' /25/;
*  ywidth    'number of nodes in a column' /10/;
*Make sure xwidth*ywidth=card(i) ...i.e. 5x5 = 25 nodes

  xcoordi(i)=mod(ord(i)-1,xwidth);
  ycoordi(i)=(((ord(i)-1)-xcoordi(i))/xwidth);

*Use the Euclidean distance formula to calculate the distances
*   between points (x1,y1) and (x2,y2)
*Parameters dist(i,j) 'distance between nodes in the A/C
*   maneuver network G(N,A)';
```

```
dist(i,j)=sqrt(power(xcoordi(i)-xcoordi(j),2)+power(ycoordi(i)-
    ycoordi(j),2));

*******************************************************************
**************End Network Coordinate System Data******************
*******************************************************************

$ontext
Missile Parameters:  The origin (xo,yo) and destination (xd,yd) of each
missile is known.  Therefore, assuming that each missile travels in a
straight line along a plane, the location of a missile is known as a
function of its speed.  The distribution of missiles can be defined as
a normal  or uniform over some predetermined area.  Calculations follow
initialization of the origin and destination in the instance specific data.
$offtext

Parameters
  xo(m)    'x-coordinate origin for a missile'
  xd(m)    'x-coordinate destination for a missile'
  xm(m,t)  'x-coordinate for a missile at a given time'
  yo(m)    'y-coordinate origin for a missile'
  yd(m)    'y-coordinate destination for a missile'
  ym(m,t)  'y-coordinate for a missile at a given time'
  theta(m) 'Angle at which missile m is traveling along the x-y plane';


*******************************************************************
******************Begin Instance Specific Data*********************
*******************************************************************

************************Fighter Parameters*************************
Parameter
  b(f,i,t)    'supply at node i';

*set all supplies to 0
  b(f,i,t)=0;

*set starting positions for the fighters
  b('f1','i74','t0')=1;
*  b('f2','i24','t0')=1;

*Fighter Speed
Scalar
```

```
   FighterSpeed  'maximum speed for coverage' /0.3125/;
* 0.3*100 mi/min = 1800 mph;  so every 10 t's the fighter can travel
* 3 nodes if traveling vertically or horizontally

************************Weapon Parameters************************

*Fighter Weapon Range and upper bounds
Parameters
   r(w)  'maximum d(i,j) distance for coverage of weapon w'
   u(f,w)    'Weapon Capacity of weapon w on fighter f'
   upsilon(w)   'Number of engagements a weapon can conduct at a time'
   delta(w)     'Time required to charge a weapon for next engagement';

**Set Weapon specific range
    r(w)=1;
    r('w1')=0.62;
    r('w2')=3.11;
*   Sets a standard line of site distance "r(w)=LOS" for say AIM-9 and DE
*   and a AMRAAM, say AIM-120 at a different range

*Set Capacity of each weapon on board the fighter
    u(f,'w1') = 4;
    u(f,'w2') = 4;
    u(f,'w3') = 100;
*   We are giving each fighter a Standard load of the AIM-9 and AIM 120
*   (4 each). Weapon 3 is DE and is assumed to have a much higher capacity

*Set upper limit of engagement at a time
    upsilon(w) = 2;
    upsilon('w3')=1;
*    upsilon('w3') = 1;
*   The upsilon(w)=2 means a fighter can engage with 2 missiles at a time
*   upsilon('w3')=1 implies a fighter can only engage 1 of weapon type 3
*   at a given time
*   Make this parameter a time base one and set sum(w, upsilon(t,w) = 1
*   for all t in T

*Set change in time required between engagements delta(w)
    delta(w) = 1;
    delta('w3') = 20;
*   These are definitely assumptions on the characteristics of how quickly
*   a fighter can acquire a target after an engagement, or how long it takes
*   for a DE weapon to charge (whichever is the most restrictive)
```

```
************************Missile Parameters***********************

***Set missile speed
Scalar
  MissileSpeed 'missile speed'  /0.1023/;

*******************************************************************
*****************End Instance Specific Data**********************
*******************************************************************

*tau(i,j) is the distance that a fighter can travel at a given scalar
*   speed s parameter tau(i,j);

    tau(i,j)=dist(i,j)/FighterSpeed;

Parameter

  a(i,m,t,w)    'Coverage radius of weapon';
  a(i,m,t,w)=0;

*Given the known location of missile m at time t, the node i can cover it
*   if the missile falls within the coverage radius of the fighter's
*   weapon system.  The formula is:

*    a(i,m,t,w)$(sqrt(power(xcoordi(i)-xm(m,t),2)+power(ycoordi(i)-ym(m,t),2))_
        <= r(w))=1;

***We must create the arcs
Set
  arcs(i,j)  set of directed arcs between nodes i and j;

parameter
  d(i,j)    'distance between adjacent nodes if arcs(i,j) exists';

* Create an arc between every pair of (horizontally or diagonally) adjacent
*   nodes & calculate the distance between adjacent arcs.  Set the arc to the
*   cost (or distance) of traversal across the arc which is referenced in
*   d(i,j) in the objective function
```

```
loop(i,
     loop(j$(ord(j)>ord(i) and ord(j)<=card(j)),
          if(abs(xcoordi(i)-xcoordi(j))<=1 and
            abs(ycoordi(i)-ycoordi(j))<=1,
             arcs(i,j)=yes;
             arcs(j,i)=yes;
             d(i,j)=sqrt(power(xcoordi(i)-xcoordi(j),2)+power(ycoordi(i)
             -ycoordi(j),2));
             d(j,i)=d(i,j);
             );
        );
    );

*Display xcoordi, ycoordi, dist, b, xm, ym, a, arcs, d, tau;

*******************************************************************
******************Begin Epsilon-Constraint Method******************
*******************************************************************
* Create Epsilon constraints for f_2(y) and f_3(z) based on the multi-
* objective optimization technique.
scalars
  epsilon1        'upper bound on f1' /1000000/
  epsilon2        'upper bound on f2 for loop' /400/
  epsilon3        'Lower bound on f3' /0/;

*******************************************************************
******************End Epsilon-Constraint Method*******************
*******************************************************************

Binary Variable
  x(f,i,j,t)    'fighter flow on arc (i,j) at time t'
  beta(f,i,t)   'Indicator variable giving position of fighter at time t'
  y(f,m,t,w)      'variable for whether a fighter engages missile m at t'
  z(m)          'Indicator variable for missile m engagement';

Variable
  f1               'objective function value f1'
  f2               'objective function value f2'
  f3               'objective function value f3'  ;

Equations
  objfn1                'objective function for f1'
  objfn2                'objective function for f1'
```

```
  objfn3                'objective function for f1'
  f1x                   'epsilon2 constraint on obj 1'
  f2y                   'epsilon2 constraint on obj 2'
  f3z                   'epsilon3 constraint on obj 3'
  consflow(f,i,t)       'conservation of flow'
  fighterLoc(f,i,t)     'Location of fighter, f, at time t'
  cover(f,m,t,w)        'Missile coverage of demand at node i'
  weaponUB(f,w)         'Capacitated weapon w on fighter f'
  weaponTimeBound(f,t,w)'Capacitated weapon w on fighter f'
  weaponTypePerEngagement(f,t)  'Limits types of weapons that are used
  * per engagement'
  engaged(m)            'missile m engaged';

  objfn1..                  f1 =E= sum((f,arcs(i,j),t),d(i,j)*x(f,i,j,t));
  objfn2..                  f2 =e= sum((f,m,t,w), (ord(t)-1)*y(f,m,t,w));
  objfn3..                  f3 =e= sum(m, z(m));
  f1x..                     f1 =l= epsilon1;
  f2y..                     f2 =L= epsilon2;
  f3z..                     f3 =G= epsilon3;
  consflow(f,i,t)..         sum(j$arcs(i,j),x(f,i,j,t))-sum(j$arcs(j,i),
                                x(f,j,i,t-tau(j,i))) =L= b(f,i,t);
  fighterLoc(f,i,t)..       beta(f,i,t) =E= sum(j$arcs(j,i), x(f,j,i,
                                t-tau(j,i)));
  cover(f,m,t,w)..          y(f,m,t,w) =L= sum(i, a(i,m,t,w)*beta(f,i,t));
  weaponUB(f,w)..           sum((m,t),y(f,m,t,w)) =L= u(f,w);
  weaponTimeBound(f,t,w)..  sum(m,sum(tHat$(ord(tHat)>=ord(t) and ord(tHat)<=
                                ord(t)+ delta(w)+1),y(f,m,tHat,w))) =L=
                                upsilon(w);
  weaponTypePerEngagement(f,t)..    sum((m,w),y(f,m,t,w)) =L= 1;
  engaged(m)..              z(m) =E= sum((f,t,w), y(f,m,t,w));

Model LexLoop /all/;
LexLoop.optcr = 0;
LexLoop.reslim = 500;
*change reslim when conducting full test

* PRE-EMPTIVE OPTIMIZATION
epsilon1=500000;
epsilon2=500000;
epsilon3=0;
```

```
*******************

*Set the number of random seeds to start from for the loop
Sets
  RandomSeed /exec1*exec30/;

Parameters
  f1Lex(RandomSeed)
  f2Lex(RandomSeed)
  f3Lex(RandomSeed)
  xLex(RandomSeed,f,i,j,t)
  ylex(RandomSeed,f,m,t,w)
  betaLex(RandomSeed,f,i,t)
  comptime(RandomSeed);


loop(RandomSeed,
    execseed = ord(RandomSeed);

*Missile origins are randomly generated
    xo(m) = round(uniform(0,10));
    yo(m) = round(uniform(0,3));

*Set destination coordinates for all missiles
        xd(m) = 24;
        yd(m) = 3;

*Find the unit vector for the missile path to calculate the distance along
*   each axis travelled
*First conduct angle calculation

    theta(m) = arccos(sqrt(power(xd(m)-xo(m),2))/sqrt(power(xd(m)-xo(m),
        2)+power(yd(m)-yo(m),2)));

    xm(m,t) = xo(m) + MissileSpeed*(ord(t)-1)*cos(theta(m));
    ym(m,t) = yo(m) + MissileSpeed*(ord(t)-1)*sin(theta(m));

    a(i,m,t,w)$(sqrt(power(xcoordi(i)-xm(m,t),2)+power(ycoordi(i)-
        ym(m,t),2))<=r(w))=1;

*Ensure a Cold Start
    f1.L = 0;
```

```
    f2.L = 0;
    f2.L = 0;
    x.L(f,i,j,t) = 0;
    y.L(f,m,t,w) = 0;
    beta.L(f,i,t) = 0;
    comptime(RandomSeed) = 0;

    Solve LexLoop maximizing f3 using mip;
*    Display f1.l, f2.l, f3.l, LexLoop.resusd, LexLoop.optca, LexLoop.optcr,
*   LexLoop.modelStat, LexLoop.solveStat;

    epsilon3=f3.l;
    comptime(RandomSeed) = LexLoop.resusd;

    Solve LexLoop minimizing f2 using mip;
*    Display f1.l, f2.l, f3.l, LexLoop.resusd, LexLoop.optca, LexLoop.optcr,
*   LexLoop.modelStat, LexLoop.solveStat;

    epsilon2=f2.l;
    comptime(RandomSeed) = comptime(RandomSeed) + LexLoop.resusd;

    Solve LexLoop minimizing f1 using mip;
*    Display x.L, y.L, beta.L, z.L f1.l, f2.l, f3.l, LexLoop.resusd,
*   LexLoop.optca, LexLoop.optcr, LexLoop.modelStat, LexLoop.solveStat;

    f1Lex(RandomSeed) = f1.L;
    f2Lex(RandomSeed) = f2.L;
    f3Lex(RandomSeed) = f3.L;
    comptime(RandomSeed) = comptime(RandomSeed) + LexLoop.resusd;
    xLex(RandomSeed,f,i,j,t) = x.L(f,i,j,t);
    ylex(RandomSeed,f,m,t,w) = y.L(f,m,t,w);
    betaLex(RandomSeed,f,i,t) = beta.L(f,i,t);
);

********************

*Make a noise when done
execute 'powershell -c
(New-Object Media.SoundPlayer "C:\Windows\Media\Ring01.wav").PlaySync();'
```

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14. ABSTRACT
Air bases in key locations provide a strategic advantage to the countries that access them, and are at constant threat from adversaries. This research seeks to determine the optimal maneuvering of a fixed number of fighter aircraft in a combat air patrol for defensive counterair to maximally and effectively attrit the inbound cruise missile threat to a stationary high value asset such as an air base. The problem formulation identifies multiple objectives for the model: minimize the distance traveled for the fighters, minimize the time utilized to engage the incoming cruise missiles, and maximize the number of cruise missiles destroyed. The model is a mobile routing problem with a mobile demand, where the demand is the set of incoming cruise missiles that the fighter seeks to engage. The ε-constraint method is used as the multi-objective optimization technique to determine the full set of Pareto efficient solutions. An *a priori* hierarchical multi-objective method called lexicographic optimization is used to determine if there is a ranking of objectives that solved more efficiently. Lexicographic optimization is used to solve the model for each objective sequentially.

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