Application of Game Theory Based Design to U.S. Space Systems

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DEVELOPING A METHODOLOGY FOR SIMULATION-BASED GAME THEORETIC SYSTEM DESIGN

THESIS

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Abstract

Modern defense systems are designed to static sets of requirements and specifications. This process has worked well in the past but fails to account for the strategic interdependence of design choices made prior to operating systems. However, as the world becomes increasingly competitive, and as the space domain becomes increasingly congested and contested, such interdependence of design can be described using Game Theory. Game Theory is the mathematical study of strategy, actions and payoffs between rational, self-interested actors. Limited extant research has applied Game Theory to system design but focused on a single system being designed cooperatively within a single company and incorporated regression derived payoffs. This research extended Game Theory Based Design to non-cooperative "red vs. blue" actors designing their own systems and incorporated physics-based simulation to determine payoffs. The hypothesized scenario has blue conducting Space Situational Awareness from a polar geosynchronous orbit, and red conducting geosynchronous proximity operations. The output from this AGI System Tool Kit (STK) simulation allows analysis of any Nash Equilibria of the satellite designs. Results indicate Game Theory can be applied to the design of U.S. Space systems to account for strategic interdependence between non-cooperative or hostile nations.

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Benjamin C. Donohoo

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I. Introduction

General Issue

In recent years, the way the space domain is viewed has shifted from a benign, uncontested environment to one that is becoming congested and contested. This shift is due to an ever-increasing number of satellite launches and continuing advancements in science and technology that one day, may allow for the realization of war in the space domain. This paradigm shift is made obvious by the heavy investment in anti-satellite (ASAT) technology by the United States, China, India, and Russia (Birkeland, 2020). The United States also recently established the United States Space Force (USSF) as an armed force within the Department of the Air Force. The USSF is tasked with protecting the interests of the United States in space and deterring aggression in, from, and to space. As a real-world example, in January of 2007 the People's Republic of China launched a direct assent ASAT and destroyed one of their own decommissioned weather satellites (Zissis, 2007). In addition to ASAT's there are many other technologies that could be utilized to conduct warfare in space such as: cyber-attacks, rendezvous/proximity operations, directed energy weapons, and jamming/spoofing. All of these emerging technologies leave the United States space community scrambling to adapt to designing, building, and operating satellites and constellations for and in a congested and contested environment.

Problem Statement

Based on this congested and contested space environment, it is crucial that the U.S. space community begin to better account for possible hostile actions when designing new space systems. Doing so will help ensure the systems designed are capable of surviving, mitigating, or negating possible future attacks. In order to address this concern, this research will examine how Game Theory can be applied to system design, when designing for a future confrontation against a hostile or antagonistic system. Some decision makers may unknowingly apply a concept of Game Theory when they "rack and stack" design choices prior to making a decision. However, a formalized approach to integrating Game Theory in a non-cooperative/hostile environment does not exist. This thesis examines this integration to better account for the strategic interdependence of design choices made prior to building systems that will operate against other systems.

Investigative Questions

This thesis will answer the following questions.

1. How can Game Theory be used to inform systems design?
2. How do the value/utility functions of a hypothesized "game" relate to system design parameters and overall mission effectiveness?
3. How can physics-based simulation be integrated to calculate game payoffs?

Research Focus

This research will first develop a methodology for applying Game Theory to the design of two systems that will operate in a "red vs. blue" scenario. This methodology will then be applied to a portion of a Space Situational Awareness (SSA) scenario posed in a 2018 AFIT thesis by Felten. In this scenario, a Polar GEO SSA satellite will be tasked with observing a GEO satellite. The GEO satellite is tasked with performing a proximity operations mission. Game Theory will be implemented to determine what aperture size yields the most value for the Polar GEO satellite based on the size and reflectivity of the GEO satellite. AGI System Tool Kit (STK) was utilized via Python scripting to model this scenario over a 6-month time frame. Information derived from this scenario is then used to calculate the game payoffs for possible combinations of the design variables mentioned earlier to determine the Nash Equilibria thus proving Game Theory can be applied to a "red vs. blue" two system design scenario.

Methodology

The steps enumerated below are required to apply a game theoretic approach to systems design in a "red vs. blue" scenario.

1. Identification of players and environment
2. Determination of the mission objective of the system being designed
3. Identification of design variables and the bounds of the variable that will be manipulated by both players
4. Generation of value/utility equations to capture system performance
5. Simulation to generate data for each combination of design variables
6. Determine Best Reply Correspondence for both players
7. Identify Nash Equilibria from the intersection of Best Reply Correspondence
8. Sensitivity analysis to ensure proper value weighting

Assumptions/Limitations

This thesis will focus on a select few design variables in the GEO SSA scenario. These variables are aperture size for the observing satellite and satellite radius and reflectivity of the satellite being observed. Additionally, the satellite being observed is assumed to be spherical and diffusely reflective (Cognion, 2013). The spherical assumption simplifies calculations, while the diffuse reflection accounts for rough or textured surfaces such as heat blankets and mylar film. This research was scoped to these design variables to show that Game Theory can be applied to the design of two systems in a "red vs. blue" scenario.

Preview

The structure of this thesis is as follows. Chapter II will provide a review of the topics that contributed to this research. Chapter II will also delve further into developing the problem space addressed in the later chapters. Chapter III will provide a roadmap of the methodology that can be used to apply Game Theory to system design in "red vs. blue" scenarios. Chapter IV contains an example of applying Game Theory to designing a GEO SSA system. Finally, Chapter V will draw conclusions and provide recommendations for future work.

II. Literature Review

Chapter Overview

The purpose of this chapter is to review the main topics that contributed to this thesis, define key terms, and develop the problem space. Game Theory, Game Based Design, and Gaming Space are explored to understand how a "game" can be utilized to determine design parameters of a system. Geosynchronous Earth Orbit (GEO) Space Situational Awareness (SSA), will serve as the example for applying Game Theory the design of a space system.

Game Theory

"At its core, Game Theory is the study of strategic interdependence - that is, situations where my actions affect both my welfare and your welfare and vice versa" (Spaniel, 2014). Essentially, Game Theory is the study of mathematical models that describe the strategic interaction among rational decision makers. One of the most critical aspects of the previous definition is the fact that the decision makers must be rational. If one or more of the decision makers are not rational, then it would be nearly impossible to generate a mathematical model that represents how they make decisions. Spaniel goes on to states that "Game Theory analyzes what should happen given what players desire" (Spaniel, 2014). A classic example of Game Theory is the prisoner's dilemma described below and captured in a standard Normal Form in Table 1.

> Two thieves plan to rob an electronics store. As they approach the backdoor, the police arrest them for trespassing. The cops suspect that the pair planned to break

in but lack the evidence to support such an accusation. They therefore require a confession to charge the suspects with the greater crime.

Having studied Game Theory in college, the interrogator throws them into the prisoner's dilemma. He individually sequesters both robbers and tells each of them the following:

We are currently charging you with trespassing, which implies a one-month jail sentence. I know you were planning on robbing the store, but right now I cannot prove it-I need your testimony. In exchange for your cooperation, I will dismiss your trespassing charge, and your partner will be charged to the fullest extent of the law: a twelve-month jail sentence.

I am offering your partner the same deal. If both of you confess, your individual testimony is no longer as valuable, and your jail sentence will be eight months each. (Spaniel, 2014)

Table 1 - Prisoner's Dilemma Payoffs

	Quiet	**Confess**
Quiet	-1, -1	-12, 0
Confess	0, -12	-8, -8

In Table 1, Player 1's actions or strategies are shown in the rows and Player 2's is in the columns. Each cell shows the payoffs (Player 1, Player 2) for a combination of their played strategies, with negative numbers reflecting a penalty for time to be served. After reviewing Table 1, if both prisoners remain quiet, they will receive the lightest sentence, however if either prisoner confesses the other prisoner will receive the maximum jail time. Since each prisoner is self-interested and only cares about their time in jail, the confess option seems to provide the lowest jail time, (0). However, if both prisoners confess, they will both receive 8 months in jail. Since the prisoners are not allowed to communicate and the other's silence is never guaranteed, the only sustainable option is to confess, thus ensuring either 0 months or 8 months in jail, both preferable to 12 months. So, although it is not the most desirable outcome, the prisoners should both confess and take the slightly reduced sentence. The option where both prisoners confess is called a Nash Equilibrium.

Key Terms

Nash Equilibrium – A set of strategies for both players, such that neither player is incentivized to change their strategy given what the other player is doing.

Game Based Design

Game Based Design is a term coined by Matthew C. Marston in his July 2000 Ph.D. dissertation presented at the Georgia Institute of Technology. "Game Based Design is the culmination of nearly 50 years of continuous research in decision theory, Game Theory, and design theory" (Marston, 2000). Marston goes on to define Game Based Design as "the set of mathematically complete principles of rational behavior for designers in any design scenario." Marston describes the process of applying Game Based Design to multiple scenarios of varying complexity, with and without collaboration, and in single and multi-designer situations. This process results in a Best Reply Correspondence (BRC) for each design variable and the intersection of these BRC is the Nash Equilibrium for the design.

Key Terms

Best Reply Correspondence (BRC) – The values of the system variables that minimize the deviation from the pre-established end goal.

Marston's dissertation covers in great depth scenarios involving multiple designers from a single company, or a coalition of companies responsible for design choices that will roll up to create a system. This application is the logical first step in applying Game Theory to the design of a system. In this body of research, adapting Marston's technique to treat two countries (Red and Blue) as the designers, and the space system or systems interacting in the space domain as the complete system will be examined. Marston's work will serve as a guide to show how Game Theory can be applied to the design of a complex system, rather than a simple problem such as the prisoner's dilemma discussed earlier.

Gaming Space

Gaming Space is the result of research conducted by the RAND Corporation that intended to develop a methodology of applying Game Theory to determining the deterrent value of space control options (Morgan et al, 2018). The Defensive Space Analysis Tool or DSPAT was also developed as part of this research. DSPAT allow a user to input a multitude of scenario variables to assess what could happen in any given space warfare scenario. For a scenario there are Offensive Space Capabilities (OSC), Defensive Space Capabilities (DSC), and Counter Defensive Space Capabilities (C-DSC) Each scenario is then scored in three areas; Mission Effectiveness, Escalation Risk, and

Political Cost. Mission Effectiveness is a measure of how well the space asset can perform it mission both before and after an attack. Escalation Risk shows what impact each attack and counter-attack has on the use of force continuum. Finally, Political Cost measures how an attack or counter-attack will be viewed politically. The DSPAT tool was developed to help space operators and developers understand how their systems would fare in space warfare and what impacts their actions would have.

Geosynchronous Earth Orbit Space Situational Awareness (GEO SSA)

Geosynchronous Earth Orbit (GEO) has a very unique quality that is of great interest to space enabled countries. A satellite placed in GEO will have an orbital period that exactly matches the rotation of the Earth. This feature enables continuous observation and high-speed data relays for both Department of Defense (DoD) and commercial missions. This fact makes GEO satellites and the protection of those assets vital to the U.S. and our allies. One possible method of disrupting these capabilities would be via rendezvous/proximity operations by a hostile nation. The ability to observe and predict non-friendly maneuvers in GEO is crucial, and highly dependent on mature GEO SSA capabilities (Felten, 2018). The first step in establishing SSA is to be able to accurately and reliably detect the objects in orbit that may cause harm to an asset.

Key Terms

SSA – Space Situational Awareness refers to finding, characterizing, tracking, and predicting the future location of objects in orbit.

Rendezvous/Proximity Operations (RPO) – Two or more satellites that have matching planes, altitude, and phasing. Two or more satellites that at some time during their orbits come within a few kilometers of each other.

GEO SSA is not of any importance when it comes to applying Game Theory to the design of space systems. However, in this thesis GEO SSA provides the example that the methodology of Game Theory Based Design will be applied to. Therefore, a basic understanding of the GEO orbit and the importance of SSA is required.

Ever since Sputnik, humans have been trying to gain the "upper-hand" in the space domain. The 1967 Outer Space Treaty proves that we have been concerned with the possibility of warfare in space for more than five decades. However, more recent events have accelerated us away from the point of viewing space as benign and uncontested. "Given that the number of states with space capabilities is growing with time, the likelihood of a war in space will correspondingly increase in the future. Further, since the U.S. is the predominant user of space, it is also the most vulnerable in that medium" (Kleinberg, 2007). Compounded with recent events, it is likely that space will be the newest warfighting domain. With that being said and GEO being of great interest to all space enabled countries, GEO SSA is of more importance than ever. The U.S. must ensure that the systems designed and launched for the mission of observing objects in GEO are prepared for whatever may happen. The application of Game Theory to the design of these systems could help to ensure that the designers consider all possibilities, for all actors involved.

Summary

With the introduction of space as a warfighting domain there is significant work to be done to fully understand what this paradigm shift means. There are new design choices, new benefits and/or consequences, and new tactics that will be discovered in the coming years. While Game Theory is typically applied to real world moves and countermoves of one opponent versus another, this research is focused on applying Game Theory in a different manner. Can Game Theory be used as a design tool, to influence system parameters and quantify the benefits and/or consequences of those design choices prior to building the system? Chapter III will utilize information from Game Theory and Game Based Design to develop a methodology for applying Game Theory to the design on systems that will operate in a “red vs. blue” scenario. The DSPAT tool will not be used as it was found to apply only to operational moves and counter moves, not engineering design choices. Additionally, physics-based simulation will be incorporated as a new method of determining the game payoffs of a space scenario.

III. Methodology

Chapter Overview

The purpose of this chapter is to outline the methodology that can be used to apply Game Theory Based Design to the design of a system in a “red vs. blue” scenario. This chapter will begin by extracting the necessary steps from Game Based Design as presented by Marston as it applies to a single system designed in a cooperative setting. Next, extensions will be developed, allowing a method to capture the design of two interacting confrontational systems. Next this new methodology will be applied to part of the idealized GEO SSA system presented by Felten. Felten identified a constellation of twelve Polar GEO satellites with 0.15-meter aperture telescopes as the most effective system for observing then 813 known GEO Resident Space Objects (RSO) (Felten, 2018). From this architecture, a single Polar GEO telescope is tasked with observing a GEO RSO as the example for applying Game Theory Based Design.

Methodology

A thorough examination of Game Theory and Game Based Design resulted in the Methodology Flow Chart shown in Figure 1. There are eight steps required to apply Game Theory to the design of two systems in a “red vs. blue” scenario. The following section will walk through each step of the flow chart, while providing examples.

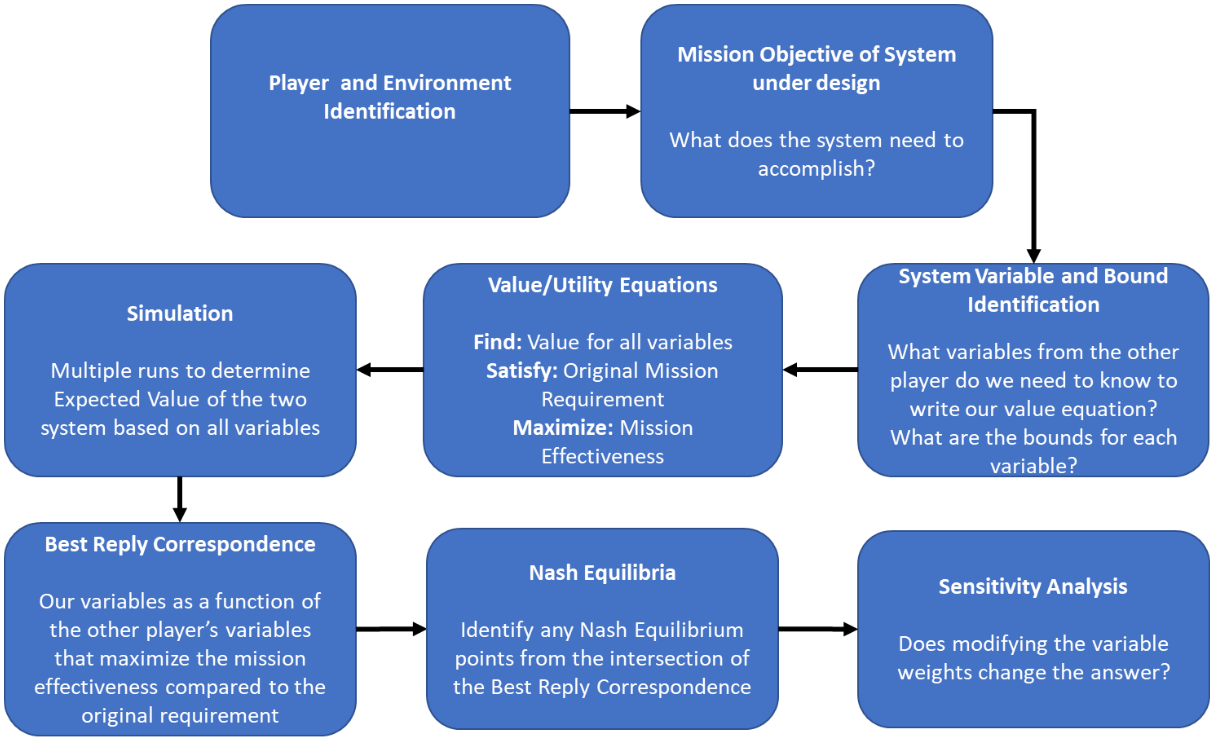

Figure 1 - Methodology Flow Chart

The first step in applying Game Theory to system design is to identify who the "players" are and the environment to which they are constrained. In Marston's work the "players" were shown as engineers working for different departments within a company or two companies collaborating on a project. The environment was most commonly collaborative either with or without communication between design engineers. This step ensures that all designers are accounted for and everyone is aware of their impact on the system.

Next, the mission objective of the system being designed must be identified along with any constraints that must be met. One example from Marston is the development of a family of electric motors designed by three different engineers with differing objectives. This family of electric motors has many objectives, some of which are; a power output of 300 Watts, an efficiency of 0.7, and a mass of 0.5 kg (Marston, 2000). This is one of the most crucial steps, as it establishes the requirements for the system that are non-negotiable.

The third step is to identify all of the design variables that are controlled by the different designers. Along with the design variables, the bounds of each variable also need to be established. This step must be completed by each designer, and needs to account for the variables controlled by the other designers as well as the variables under their control. Continuing with the family of electric motors, the power designer controls the current and power of the motor. The power designer also has to consider the number of turns and the thickness of the wire chosen by the configuration designer, and the diameter of the motor chosen by the platform designer. An example of the bounds that must be considered is that the platform designer may choose either a 5.0 cm or a 6.5 cm diameter motor (Marston, 2000). This step ends with an understanding of all variables controlled by all designers and the bounds of those variables that may be implemented in the design.

The next step is the generation of utility or value equations that are used to evaluate the system being designed. Marston implemented regression fitting to develop single attribute utility functions for each design variable in the family of electric motors. The stochastics captured the variation and uncertainty in the design. Figure 2 shows the results for the power designer. Another method of accomplishing this is the use of response variables that capture the impact on system performance from each player's design variables. Either way, these equations will be used in the next step to generate data for each design based on manipulation of the design variables.

```
       *** Linear Model ***

Call: lm(formula = V2 ~ sp + sp^2 + sp^3, data = new.data, na.action = na.omit)
Residuals:
       1       2       3       4      5       6       7       8        9
 -0.03741 0.02723 0.03577 -0.0191 -0.013 -0.0191 0.03577 0.02723 -0.03741

Coefficients:
                Value Std. Error  t value Pr(>|t|)
(Intercept)    1.0130   0.0263    38.5426   0.0000
         sp    0.0000   0.0591     0.0000   1.0000
    I(sp^2)   -0.9756   0.0391   -24.9619   0.0000
    I(sp^3)    0.0000   0.0728     0.0000   1.0000

Residual standard error: 0.03934 on 5 degrees of freedom
Multiple R-Squared: 0.992
F-statistic: 207.7 on 3 and 5 degrees of freedom, the p-value is 0.00001149
```

Very small error and good fits so this is an acceptable utility function for Power

$$u(power) = 1.013 - 0.9765\left(\frac{power - 300}{200}\right)^2$$

Figure 2 - Power Designer Utility Function (Marston, 2000)

The next step in the process of applying Game Theory to system design is to generate data that will be used to inform the final decision. There are many methods that can be applied to accomplish this task. Marston implemented regression modeling utilizing the utility functions generated earlier to generate a response surface. The method used for this step will depend on the scenario and system being analyzed, the players involved, and the overall fidelity of the model. Some other possible methods include: Monte Carlo Simulation, Physics-Based Simulation, Space Filling Design, etc. (Marston, 2000). The results of this step will be the expected value of each combination of possible design variable choices. These expected values are then entered into the Normal Form matrix that shows the values for each player as shown in Table 1. This matrix will be used in the next step, construction of the Best Reply Correspondence.

The construction of the Best Reply Correspondence is where Marston's Game Based Design meets a classic element of Game Theory, represented in Normal Form, also known as the strategic or matrix form. The Normal Form matrix is a representation of a "game" where the players make decisions simultaneously and the value of each move is determined by the combination of the moves made by all players. Recall that the matrix shown for the Prisoner's Dilemma in Table 1 showed the expected jail times for each suspect based on either their confession or silence. Figure 3, shows the transformation from jail time in months to an order of preference with the least preferred outcome marked with a 1 to the most preferred outcome marked with a 4.

	Quiet	Confess
Quiet	-1, -1	-12, 0
Confess	0, -12	-8, -8

	Quiet	Confess
Quiet	3, 3	1, 4
Confess	4, 1	2, 2

	Quiet	Confess
Quiet	3, 3	1, 4*
Confess	4*, 1	2*, 2*

Figure 3 - Transformation of Normal Form Matrix for The Prisoner's Dilemma

In the right most matrix in Figure 3, asterisks have been placed beside certain outcome payoffs. This is a method described in *Game Theory 101* to mark the Best Reply Correspondence for each player. The method works as follows. For Player 1 (Rows) it is first assumed that Player 2 will remain quiet, so in that case the best response for Player 1 is to confess. This results in Player 1 being set free and Player 2 serving the maximum jail time. This is done for the next option that Player 2 has which is to confess, and if Player 2 confesses then Player 1 should also confess to minimize his/her jail time. The same methodology is applied to Player 2's choices (Spaniel, 2014). It should be noted that one of the boxes has an asterisk for both players, this is the final step of the process, the Nash Equilibrium.

The Nash Equilibrium is the point where the Best Reply Correspondences for both players intersect. At this point, there is no incentive for either player to deviate to any other point given that the other play will not change their decision. Therefore, the idea that both prisoners should confess established in Chapter II holds true to this method, where the only point that has an asterisk for both players is where they both confess.

A step not covered in Marston's work is the addition of a sensitivity analysis conducted at the end of the process. Conducting a sensitivity analysis will determine what if any impact there is on the Nash Equilibria based on changes made to the model (Kirkwood, 1997). Most commonly, the weights assigned to each variable are manipulated and the simulation is repeated to see if the end result changes. This analysis can be incredibly useful when multiple solutions are close in overall value. When conducting a sensitivity analysis there are a few possible outcomes.

1) No change in the results no matter what.
2) A change in the results when the variable weight is near the original values (less than a 10% difference).
3) A change in the results when the variable weight is not near the original values (greater than a 10% difference).

In the first case, the value equations used should be reviewed to ensure they are not overly dependent on a shared variable, as this would cause the expected values to synchronously increase and decrease. In the case of the second outcome, the value equations should be re-evaluated to ensure they accurately represent the system value and stakeholder values. A change to the value equations is not necessary but may be beneficial. Finally, the third outcome shows that assigned variable weights result in an expected value that is not near a cross over point of the sensitivity analysis. This result is stable and the expected value of the system is not questionable as it relates to the value equations.

Application of Methodology to Space System Design

Now that the methodology for applying Game Theory to the design of systems in a “red vs. blue” scenario has been established, the next step is to apply it to a realistic example. This is where part of the optimized Polar GEO SSA system described by Felten will be evaluated. This optimized SSA system consisted of two orbital planes separated by 90 degrees, each with six satellites with 0.15-meter aperture telescopes designed to find and track over 800 GEO Resident Space Objects (RSO) (Felten, 2018).

In this scenario there are two players, in this case countries with Player 1 being blue and Player 2 being red. There is no communication between the players and they are both self-interested. The systems will operate in the following space environments; blue will be in Polar GEO conducting an SSA mission and red will be in GEO conducting a rendezvous/proximity operations (RPO) mission. Blue is tasked with observing red, while red is trying to avoid detection while conducting the RPO mission. The mission objective of the blue system is to observe a spherical RSO with a radius of 0.3 meters and a coefficient of reflectivity of 0.175 at least 30 percent of the time. From this one mission requirement it can be determined that the aperture size of the blue system is dependent on the radius and the coefficient of reflectivity of the red system. Blue now needs a range of values that red may implement for both radius and coefficient of reflectivity. A method used by Marston to capture all of the information required to begin applying Game Theory to the design of a system is shown in Table 2.

Table 2 - GEO SSA Game Information

	BLUE	RED
GIVEN:	Mission: Observe the Red GEO RSO RSO Visibility = {A, f(R), f(CoRef)} Polar GEO SSA Architecture from Felten	Mission: Conduct GEO Prox-Ops Min. Payload Size = 1.5U LRD = {R, CoReff, f(A)}
FIND:	Aperture Size, A	Radius of RSO, R Coefficient of Reflectivity of RSO, CoReff
SATISFY:	Time RSO is visible >= 30%	Likelihood of RSO detection LRD <= 50%
BOUNDS:	0.1m < A < 0.5m	0.2m < R < 0.5m 0.155 < CoReff < 0.195
FUNCTION:	Maximize Expected Value Minimize Cost	Maximize Expected Value Minimize Cost

Once this table is filled out, the next step is to determine the value/utility equations that will be used to evaluate the system being designed. These equations must include all of the other players design variables that affect the system design and may include any other relevant variables. Equations 1 and 2 below show the linear expected value equations for Blue and Red.

$$EV_{Blue} = 0.75(Time\ Visible) + 0.25(Min\ Size\ Detectable) \quad (1)$$

$$EV_{Red} = 0.5(Aperture\ Size) + 0.25(Time\ Visible) + 0.25(Min\ Size\ Detectable) \quad (2)$$

At first glance, it appears that not all of the design variables are accounted for in these equations; however, that is not the case. Time Visible and Minimum Size Detectable are response variables dependent on the aperture size of blue and the radius and the coefficient of reflectivity of the red RSO. Chapter IV continues the process and will discuss these equations in depth.

Summary

The methodology discussed in Chapter III develops an extension from application of Game Theory to the design of a single system to applying Game Theory to the design of two systems in a “red vs. blue” scenario. This methodology was then applied to a GEO SSA problem based on previous research. Now that all of the information required to apply Game Theory to the GEO SSA problem has been established, Chapter IV will continue the process with simulation, results, and analysis.

IV. Analysis and Results

Chapter Overview

This chapter will begin at the Simulation block of the flow chart described in Chapter III for the application of Game Theory to a GEO SSA example. The simulation was completed by using Python scripting to generate an STK scenario. This scenario then generated the information required to apply Game Theory to the design two systems in a “red vs. blue” scenario. The output from this simulation is then compared to a system designed without Game Theory.

Simulation

For this example, it was determined that a physics-based simulation would be most appropriate. This simulation allows accurate modeling of space systems, their orbits, and their interactions. To accomplish this a Python script was written that controlled all of the variable that were then passed to STK where the scenario was run and data was collected for later analysis. First, the scenario timeline was set to run from 1 January 2020 to 30 June 2020. This timeline essentially allows for the scenario to model an entire year since the first six months are a mirror of the last six months of the year. Next, the two satellites were generated with blue having an altitude of 48,542 km and an inclination of 89 degrees, this creates the Polar GEO orbit described by Felten (Felten, 2018), see Figure 4. The red satellite also had an altitude of 48,542 km, but the inclination was set to 0 degrees to create a GEO orbit. A sensor to act as the telescope was then added to the Polar GEO satellite so that the access between this sensor and the GEO satellite could be calculated. From the access between the sensor and satellite, an

Azimuth, Elevation, and Range (AER) report was created. From this report only the range value was needed as it was used in the calculation of the visual magnitude of the GEO satellite.

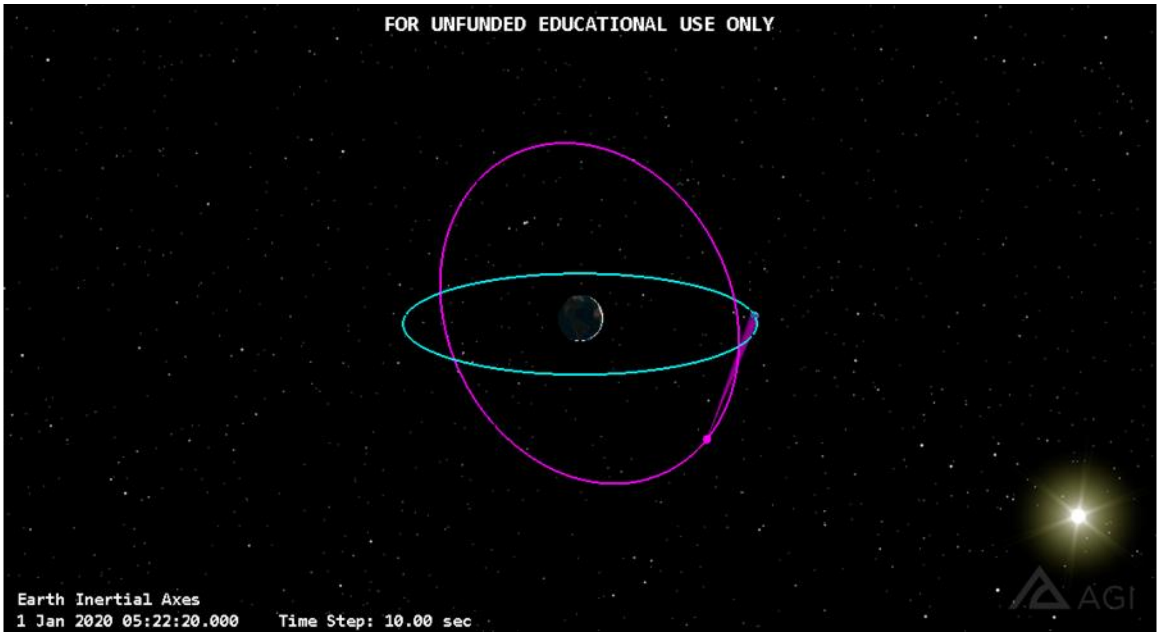

Figure 4 - AGI System Tool Kit Physics-Based Simulation

In calculating if a telescope can detect an object extant research has used Signal-to-Noise ratio. This research instead, used visual magnitude to determine if an object could be detected. Visual magnitude is dependent on: the distance from the object to the observer, the size of the object, the reflectivity of the object, and the angle created between the light source the object and the observer. This final piece is also known as the phase angle. Equation 3 shows how the phase angle (φ) is used to calculate the photometric signature (F_{diff}) of a satellite assumed to be a Lambertian (diffusely-reflecting) sphere (Cognion, 2013). See the section, STK Phase Angle Report in the Appendix for the Python code used.

$$F_{diff} = \frac{2}{3\pi} * Coreff * \frac{{r_{sat}}^2}{R^2} * (\sin(\varphi) + (\pi - \varphi)\cos(\varphi)) \tag{3}$$

where:

Coreff = the coefficient of reflection of the target, initially set to 0.175

r_{sat} = the radius of the target satellite in meters

R = the distance between the observer and the target in meters

φ = phase angle created between the sun, the object, and the observer

The photometric signature can then be entered into Equation 4 to determine the Apparent Magnitude (M) of the object. In this equation the *-26.7* is the visual magnitude of the Sun and is used to set a baseline for the object being compared. See the section, Apparent Magnitude Calculation in the Appendix for the Python code

$$M = -26.7 - (2.5 * log_{10}(F_{diff})) \tag{4}$$

These two equations determine the visual magnitude of an object at a single point in time. This is good information but not very useful when trying to analyze a system over a six-month timeframe. From the AER report, an hourly range report was generated for the entire six months. A similar phase angle report also had to be generated using Vector Geometry within STK. Figure 5 shows a visualization of the phase angle generated by STK.

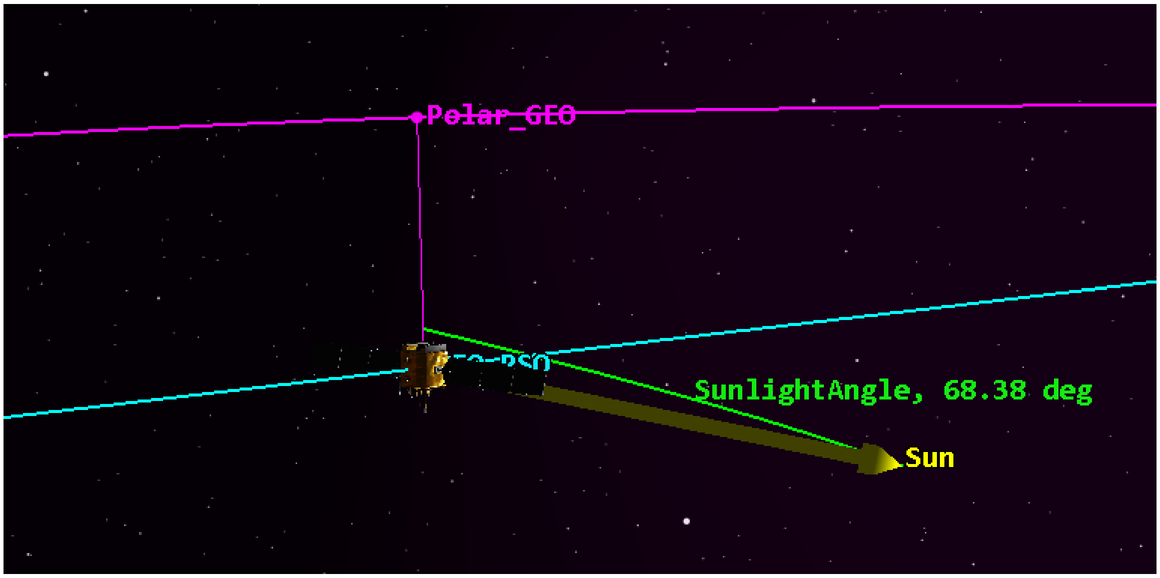

Figure 5 - STK Visualization of Phase Angle

Once an hourly phase angle report had been generated, the visual magnitude equations could be used to calculate the apparent magnitude of the GEO satellite hourly over the entire six-month scenario.

Now that the apparent brightness of the GEO satellite at all times is known, the limit of what can be seen by the telescope must be determined. To accomplish this an equation to calculate limiting magnitude (LM) of space-based telescopes based on aperture sized had to be developed. Equation 5 shows this calculation for ground-based telescopes (First Light Optics Ltd., 2021).

$$LM = 7.5 + (5 * log_{10}(Aper)) \tag{5}$$

where:

7.5 is used to account for: extinction, scattering, and absorption as light passes through the Earth's atmosphere (Flanders & Creed, 2008).

Aper = the diameter of the telescope's aperture in centimeters.

Equation 6 is the approximated equation used to determine the limiting magnitude of space-based telescopes. See the section, Limiting Magnitude Based on Aperture Size in the Appendix for the code used.

$$LM = 9.6 + (5 * log_{10}(Aper)) \tag{6}$$

The value used to account for atmospheric effects has been increased to *9.6* to increase the limiting magnitude since the light no longer passes through the Earth's atmosphere. This modified value is based on published limiting magnitudes of space-based telescopes. Figure 6 shows a comparison of Equation 6 against these published data points.

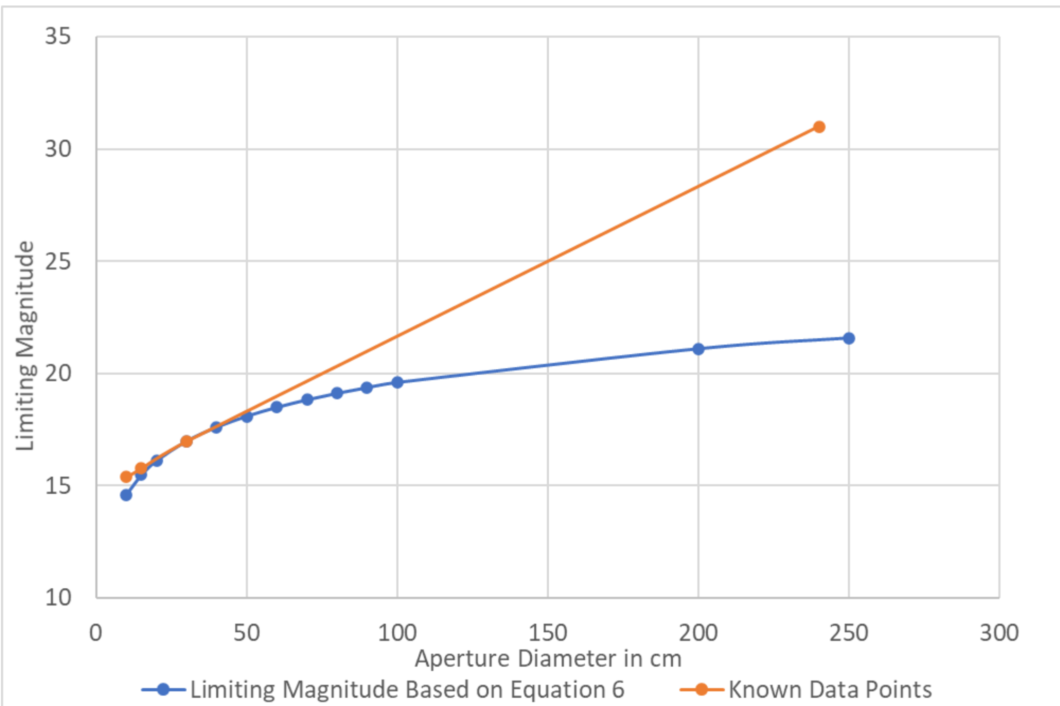

Figure 6 - Limiting Magnitude Comparison

The three known data points in the lower left of Figure 5 are satellites tasked with observing and tracking objects in space similar to the mission of observing a GEO satellite from a Polar GEO orbit. From left to right these satellites are; Geometry

Optimized Space Telescope (GeOST), Sapphire (Canadian satellite), and Space-Based Space Surveillance (SBSS) (Ackermann et al, 2015). It can be seen that these data points fit quite closely to the approximation calculated by Equation 6. The one data point that does not fit the curve is the Hubble Space Telescope (HST), this is due to the fact that Hubble can “stare” at a single point in space for hours or days. This continuous collection of light significantly increases the limiting magnitude of Hubble.

With the apparent magnitude of the GEO satellite now known hourly over the six-month scenario and an accurate method of determining the limiting magnitude of space-based telescopes, Equations 1 and 2 can be revisited. See the section, Expected Value Calculation in the Appendix for the code used.

$$EV_{Blue} = 0.75(Time\ Visible) + 0.25(Min\ Size\ Detectable) \tag{1}$$

$$EV_{Red} = 0.5(Aperture\ Size) + 0.25(Time\ Visible) + 0.25(Min\ Size\ Detectable) \tag{2}$$

Time Visible is calculated by determining how often the apparent magnitude of the GEO satellite is less than or equal to the limiting magnitude of the telescope. Minimum Size Detectable is calculated by taking the average distance and phase angle from the scenario and a starting with a GEO satellite radius of 1 meter. The radius is then iteratively reduced by 0.01 meters until the apparent magnitude of the GEO satellite is equal to the limiting magnitude of the telescope. At this point, the GEO satellite is barely observable under averaged conditions. The calculated values are then multiplied by the appropriate normalized weighting factor as determined by the single value attribute functions (Buede

& Miller, 2016). This allows Equations 1 and 2 to be solved for any combination of values. Figures 7 and 8 show visualizations of the single value attribute functions used.

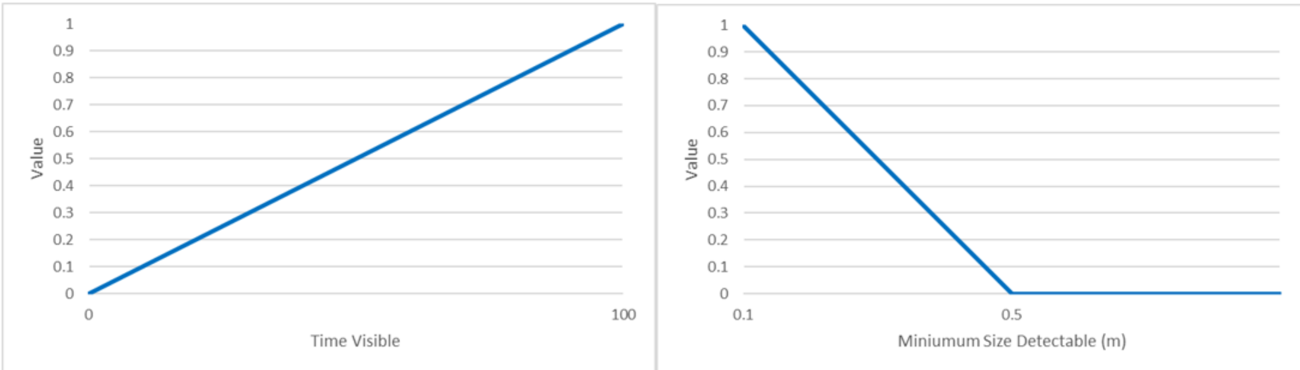

Figure 7 -Visualization of Blue Single Value Attribute Functions

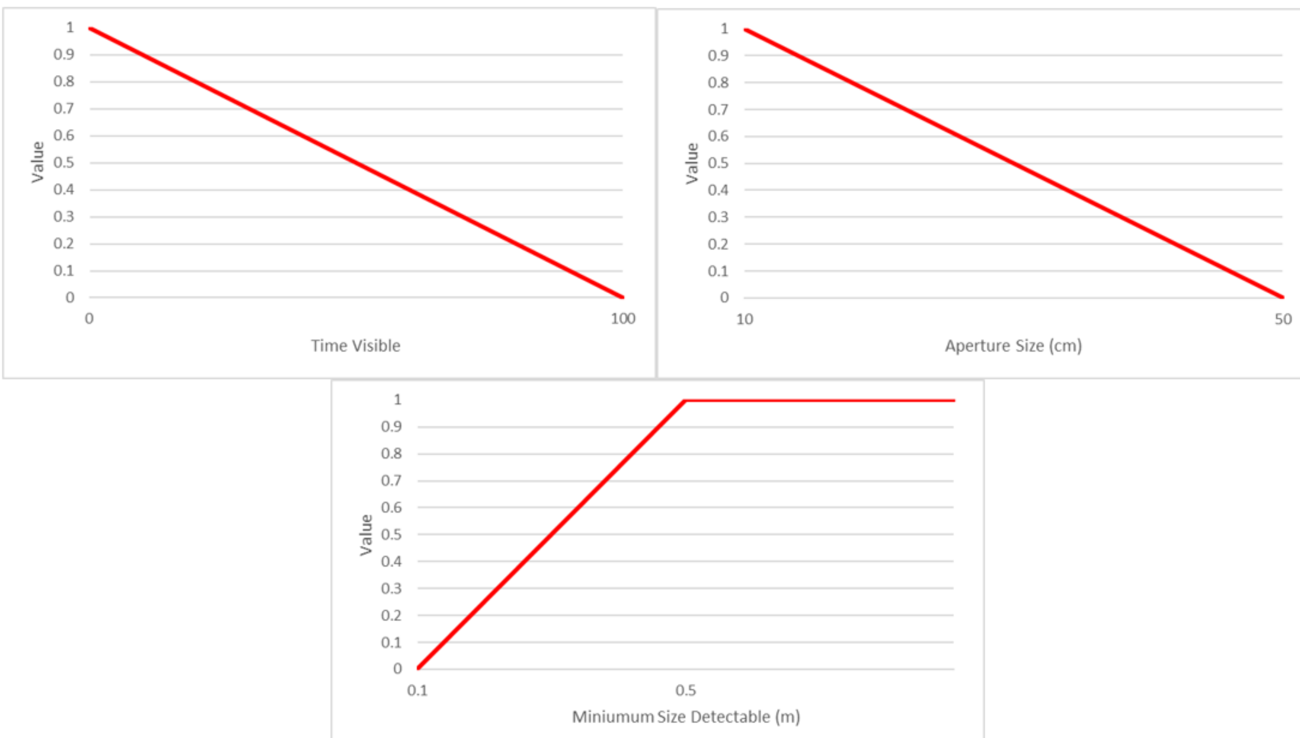

Figure 8 - Visualization of Red Single Value Attribute Functions

Results

The expected value for both the blue and red systems can now be calculated by entering combinations of the variables into the Python script that controls the STK Scenario. This will generate a list that can then be put into the normal form matrix shown in Table 3.

Table 3 - Expected Value of Blue and Red Systems

RSO Size(m) & Reflectivity / Aperture in m	.2, .155	.2, .175	.2, .195	.35, .155	.35, .175	.35, .195	.50, .155	.50, .175	.50, .195
.2m (-1 step)	0.09184 0.84438	0.09819 0.84226	0.10368 0.84043	0.16857 0.81880	0.18127 0.81457	0.19157 0.81114	0.24341 0.79386	0.25835 0.78888	0.27346 0.78384
.3m (meets Req)	0.19948 0.64596	0.20720 0.64338	0.21681 0.64018	0.31106 0.60877	0.32822 0.60304	0.34384 0.59784	0.45131 0.56202	0.49405 0.54777	0.54503 0.53077
.4m (+1 step)	0.32063 0.43437	0.33316 0.43019	0.34535 0.42613	0.48251 0.38041	0.51667 0.36902	0.55083 0.35763	0.72267 0.30035	0.75048 0.29108	0.77657 0.28239

Table 3 shows a significant amount of information but is difficult to decipher. Table 4 converts the expected value to an ordered preference with 1 being least preferred and 27 being most preferred. In addition, the asterisks method of finding the Best Reply Correspondence is applied.

Table 4 - Ordered Preference for Blue and Red Systems

RSO Size (m) & Reflectivity / Aperture in m	.2, .155	.2, .175	.2, .195	.35, .155	.35, .175	.35, .195	.50, .155	.50, .175	.50, .195
.2m (-1 step)	1 27*	2 26	3 25	4 24	5 23	6 22	10 21	11 20	12 19
.3m (meets Req)	7 18*	8 17	9 16	13 15	15 14	17 13	19 12	21 11	23 10
.4m (+1 step)	14* 9*	16* 8	18* 7	20* 6	22* 5	24* 4	25* 3	26* 2	27* 1

In Table 4 it can be seen that the order of preference for red starts with the smallest aperture and the smallest and least reflective GEO satellite and progresses linearly. Blue however has a few points where a smaller aperture with a larger and more reflective GEO satellite is preferred over a larger aperture. Exploring an area of Table 4 that includes this preference is shown in Figure 9. Figure 9 shows the conversion of the ordered preference and the implementation of the asterisks method to mark the Best Reply Correspondence.

RSO Size & Reflectivity **Aperture in m**	**.2, .155**	**.50, .195**
.2m (-1 step)	1 27	12 19
.3m (meets Req)	7 18	23 10

RSO Size & Reflectivity **Aperture in m**	**.2, .155**	**.50, .195**
.2m (-1 step)	1 4*	3 3
.3m (meets Req)	2* 2*	4* 1

Figure 9 - Close Look at Red and Blue

This matrix now looks very similar to the matrix from the Prisoner's Dilemma but with a rotation of the payoffs. This rotation does not appear to be important and is only due to the way the matrix is constructed. The Nash Equilibrium of this example is when blue implements the largest possible aperture and red has the smallest and least reflective satellite. An online Nash Equilibria solver was used in conjunction with the asterisks method to ensure the Nash Equilibria was determined correctly (Avis et al, 2010).

Finally, a sensitivity analysis can be conducted. Using a Python script, the weight assigned to each single value attribute function was varied. For blue the weight (w) applied to Time Visible was varied from 0 to 1.0 in increments of 0.1 with the remaining weight (1-w) applied to Minimum Size Detectable. The weighting applied to the red system remained the same for the first sensitivity analysis. Next, for red the weight (w) applied to Aperture Size was varied between 0 and 1.0 in increments of 0.1. The weighting applied to Time Visible and Minimum Size Detectable was split evenly from the remaining weight by ((1-w)/2). In this scenario, the blue weighting was the same as in the original problem. The expected value based on the new weights was then calculated for each combination of design variables. The Nash Equilibrium was then found using the online solver developed by Avis et al. One of the outputs from this solver can be found in the Appendix. It was found that varying the weights for 1 player while the other player's weight remained constant had no effect on the Nash Equilibrium. In each case the Nash Equilibrium was found when blue implemented the largest possible aperture and red implemented the smallest and least reflective RSO.

When conducting this sensitivity analysis, it was expected that the Nash Equilibrium would change at one point or another. When this did not happen, the value equation where re-examined. It was found that red's value equation (Equation 2) was overly dependent on blue's aperture size.

$$EV_{Red} = 0.5(Aperture\ Size) + 0.25(Time\ Visible) + 0.25(Min\ Size\ Detectable) \quad (2)$$

Both Time Visible and Minimum Size Detectable are response variables that account for the aperture size implemented by blue. To reduce this over dependence, a new value equation for red was developed, Equation 3.

$$EV_{Red} = 0.5(Time\ Visible) + 0.50(RSO\ Radius) \quad (3)$$

This new equation now values less time being seen by blue and a larger RSO Radius. With a larger RSO Radius, red would be capable of carrying more fuel and larger payloads which increases their mission capabilities. The single value attribute functions can be seen in Figure 10.

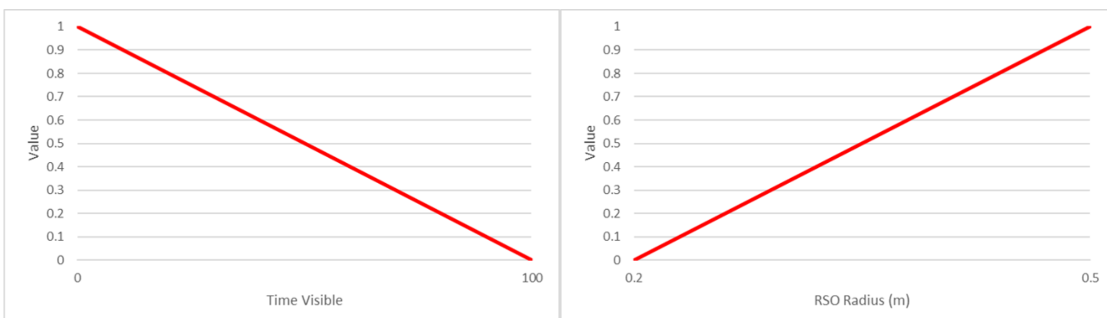

Figure 10 - Visualization of New Red Single Value Attribute Functions

Running the simulation again with red's new Expected Value equation and single value attribute functions results in the expected values for both players shown in Table 5.

Table 6 - New Expected Value of Red and Blue Systems

RSO Size(m) & Reflectivity Aperture in m	.2, .155	.2, .175	.2, .195	.35, .155	.35, .175	.35, .195	.50, .155	.50, .175	.50, .195
.2m (-1 step)	0.09184 0.43877	0.09819 0.43454	0.10369 0.43088	0.16857 0.63762	0.18128 0.62915	0.19158 0.62228	0.24342 0.83772	0.25835 0.82776	0.27346 0.81769
.3m (meets Req)	0.19948 0.40455	0.20721 0.39940	0.21682 0.39300	0.31106 0.58017	0.32823 0.56872	0.34385 0.55831	0.45131 0.73667	0.49406 0.70817	0.54504 0.67418
.4m (+1 step)	0.32064 0.36999	0.33317 0.36164	0.34535 0.35351	0.48251 0.51207	0.51668 0.48930	0.55084 0.46653	0.72267 0.60197	0.75048 0.58343	0.77657 0.56603

The conversion to an ordered preference is shown in Table 6. Table 6 also implements the asterisks method to mark the Best Reply Correspondences for both players.

Table 5 - Ordered Preference of New Expected Values

RSO Size(m) & Reflectivity Aperture in m	.2, .155	.2, .175	.2, .195	.35, .155	.35, .175	.35, .195	.50, .155	.50, .175	.50, .195
.2m (-1 step)	1 9	2 8	3 7	4 21	5 20	6 19	10 27*	11 26	12 25
.3m (meets Req)	7 6	8 5	9 4	13 16	15 15	17 13	19 24*	21 23	23 22
.4m (+1 step)	14* 3	16* 2	18* 1	20* 12	22* 11	24* 10	25* 18*	26* 17	27* 14

The ordered preference for red looks much different than before based on the new value equation. Red now prefers the largest and least reflective RSO while blue still prefers the largest aperture. The change in red's preference is due to red now preferring not to be seen but also preferring a larger radius. The Nash Equilibrium is now when blue implements the largest aperture and red implements the largest and least reflective RSO.

A sensitivity analysis can now be conducted based on this new information. The weights are varied for blue and red independently just like before. Since no changes were made to the blue value equation there is no new information to discuss. However, red's sensitivity analysis does have some interesting results. When the weight applied to RSO Radius is between 0% and 30% the Nash Equilibria returns to the original point where blue implements the largest aperture and red implements the smallest and least reflective RSO. From 40% to 90% weight on RSO Radius, the Nash Equilibria is at the point where blue implements the largest aperture and red implements the largest and least reflective RSO. At 100% weight applied to RSO Radius there are multiple Nash Equilibria at any instance of maximum RSO Radius. This analysis shows that the expected value of the red system is stable as it directly relates to the value equations.

Analysis

With the Nash Equilibria of these example now known, how do these systems compare with systems designed without using Game Theory? The original mission objective for the blue system was to observe a spherical GEO RSO with a radius of 0.3 meters and a coefficient of reflectivity of 0.175 at least 30 percent of the time. Using these values, the STK simulation shows that a 0.3-meter aperture would meet these requirements. At this point, without Game Theory the design process would be complete and the blue system would be built and launched.

The original results shown in Table 4 show that the Nash Equilibrium is when blue implements the largest possible aperture and red implements the smallest and least reflective RSO. Using Game Theory shows that a change to either RSO size or reflectivity to a smaller value would decrease the expected value of the blue system below what is acceptable. If red were aware of this and blue did not use Game Theory to inform the design, red could easily change a design parameter to negate the blue system, rendering it useless on orbit.

The new results shown in Table 6 show that the Nash Equilibrium is when blue implements the largest aperture and red implements the largest and least reflective RSO. The sensitivity analysis showed that the Nash Equilibria point varies when the weighting is varied. This is a simplified case as only performance of the systems was considered. There are many modifications that could be made to this simulation, some of which will be discussed in Chapter V.

Summary

Incorporating physics-based simulation with the methodology for applying Game Theory to the design of two systems develops the next step in system design via Game Theory. The GEO SSA example proves that a physics-based simulation can be used to accurately model the interaction between two systems and generate the expected value of both systems based on design variables. The result from the GEO SSA example fits what would be expected when two self-interested designers compete with only performance as a bounding condition. Chapter V will summarize this body of research and make recommendations for future research that may change the outcome if this simulation is run again.

V. Conclusions and Recommendations

Chapter Overview

This chapter summarizes the entire scholarly thesis. A summary of the research conducted will be provided while answering the Research Questions from Chapter I. Recommendations for future work and the significance of the research will also be discussed.

Research Conclusion

This research intended to advance the application of Game Theory to system design by answering the following research questions.

1. How can Game Theory be used to inform systems design?
2. How do the value/utility functions of a hypothesized "game" relate to system design parameters and overall mission effectiveness?
3. How can physics-based simulation be integrated to calculate game payoffs?

As shown in Chapters III and IV, the design of interacting systems can be informed by Game Theory. The mission objective of the end systems along with system performance requirements are used as a starting point. From here selected design variables are manipulated based on established bounds. The value/utility equations are solved for all combinations of design variables and the Best Reply Correspondences are found. In this case the BRC are based on the overall mission effectiveness determined by the combination of the value/utility functions. From these Best Reply Correspondences, any intersection is a Nash Equilibria of the two designs as they interact in contention.

The value/utility functions of a "game" are arguably the most important piece in the application of Game Theory to system design. These equations are responsible for capturing what is important to each "player". Great care should be taken when developing the utility functions to ensure they accurately represent the stakeholder's desires and the mission objective of the system. The value equations found in this research are simple linear equations designed solely for the demonstration of applying Game Theory to system design. An entire thesis could be written about the generation of these value/utility equations and the importance they play in determining the Nash Equilibria of any scenario.

Finally, Chapter IV showed how physics-based simulation can be utilized to calculate game payoffs. Through a Python scripted STK scenario the interaction between two satellites with varying aperture size, satellite size, and reflectivity was simulated. This six-month simulation allowed for actual data to be used to solve the value/utility equations. This integration greatly increases the fidelity of the system model by accounting for real-world effects on the space systems.

Recommendations for Future Research

This section discusses possible future research that would further advance the application of Game Theory to system design.

The first area of focus should be the generation of the utility/value equations. Does the output of the simulation change if one or more of the single value attribute functions are made non-linear? As an example, the value function for Percent Time Visible would likely be better represented as a decreasing return to scale (Buede &

Miller, 2016). As the percentage of time increases from zero there is likely a significant increase in value. However, as the percentage of time gets closer to 100% the added value is not as significant as it was in the beginning. This is most likely a better representation of the value that should be assigned to this single value attribute function. Additionally, investigate the addition of a cost model into the simulation to represent the cost associated with changing design parameters. If these changes are made, the sensitivity analysis should be conducted again. This analysis would determine if the non-linear weighting or the cost model affects the outcome of the game. It is expected that these changes would drastically change the output of the simulation and therefore the Nash Equilibria.

Another area that could be investigated is the equation utilized to determine the limiting magnitude of a space-based telescope, Equation 6. No research was found that directly discussed how to calculate limiting magnitude based on aperture size. The equation used in this thesis is a solid starting point, but it is likely that the conversion from the ground-based telescope equation does not fully capture the differences. Further developing this method would be beneficial for future research as visual and limiting magnitude calculations would provide an additional method of determine if a space object is observable besides Signal-to-Noise ratios.

Significance of Research

The significance of the research conducted can be separated into three areas. First, a comprehensive methodology for applying Game Theory to the design of two systems that will operate in contention was developed. This methodology was developed based on extant research on the application of Game Theory to system design. This methodology allows design engineers and decision makers to better account for the strategic interdependence of design choices made by the U.S. and non-cooperative or hostile nations. Implementation of this methodology will help ensure all possible design choices and their benefits/consequences are considered.

The second area of significance is the integration of physics-based simulation into Game Theory Based Design. This integration allowed a Python scripted STK scenario to determine the payoffs for each player in the "game". This accomplishment enables actual data to be utilized in the value equations to determine the expected utility of the end system.

Finally, it was shown that conducting a sensitivity analysis at the end of the process adds value to the methodology. The sensitivity analysis showed that the original equation used to determine red's value was overly dependent on blue's design variable. With this information, the value equation was re-written, the simulation re-run, and the new results showed a significant change in the Nash Equilibria.

The methodology developed and demonstrated in this thesis provides a new method of designing systems for a congested and contested environment. Additionally, physics-based simulation allows for the realization of accurate models that can be used to determine the game payoffs for multiple scenarios involving multiple actors.

Appendix

STK Phase Angle Report

```
s.send(('angleDP = sat.DataProviders.GetDataPrvTimeVarFromPath("Angles/SunlightAngle").Exec(Donohoo_Thesis.StartTime, Donohoo_Thesis.StopTime, 60 \n').encode())

s.send(('time = angleDP.DataSets.GetDataSetByName("Time").GetValues() \n').encode())

s.send(('angles = angleDP.DataSets.GetDataSetByName("Angle").GetValues() \n').encode())

‘’’An error will occur the first time this script is run, here is how to fix it.

With the scenario running, right click GEO_RSO in the object browser,

select Report & Graph Manager, add a new report style named "SunlightAngle",

in the properties for that new style expand the angles data provider,

then expand SunlightAngle, add Time then Angle to the Report Contents and save,

restart STK and run the script again.’’’

s.send(('ReportCreate */Satellite/GEO_RSO Type Export Style "SunlightAngle" File "C:\\All Files\\AFIT Courses\\Thesis\\ReflectionData.txt" TimeStep 3600 \n').encode())
```

Apparent Magnitude Calculation

```
Reflectivity = Ref = 0.175    # Reflectivity of the Satellite

Radius = r = .30      # Radius of Satellite in m

Mag = []

with open('ApparentMagnitude.txt', 'w') as output:

   for pa, distance in zip(PA_Rads_Array, Meters_Array):

      P = (2/(3 * pi)) * (np.sin(pa) + ((pi - pa) * np.cos(pa)))

      RAP = Ref * (r**2) * P

      RAPd = RAP/(distance**2)

      M = -26.7 - (2.5 * np.log10(RAPd))

      list.append(Mag, M)

      output.write(str(M) + "\n")
```

Limiting Magnitude Based on Aperture Size Calculation

```
Aper = 10.00   # Aperature size in cm

LM = 9.6 + (5 * np.log10(Aper))

count = len([i for i in Mag if i <= (LM)])

print ('Limiting Magnitued =')

print (LM)

print (str(count))

print ('Time object is visible to .1m Aperture')

print ((count/len(Mag))*100)
```

Expected Value Calculation

```
EUblue1 = (0.75 * ((time1 - mintv)/(maxtv - mintv))) + (0.25 * (maxrad -
minsdla)/(maxrad - minrad))

EUred1 = (0.5* ((maxap - AperL)/(maxap - minap))) + (0.25 * ((maxtv -
time1)/(maxtv - mintv))) + (0.25 * ((minsdla - minrad)/(maxrad - minrad)))

print ('Expected Utility Scenario 1 =')

print(EUblue1)

print(EUred1)
```

Sensitivity Analysis

```
NO_SENS=1

if (NO_SENS):

   ##sensitivity analysis

   # adjust Blue Weights 0->1

   # using the range of 0, 10 but dividing by 10 (0 .1 .2 ...). Alternative is using
np.array(0,1,0.1)

   print('---begin Blue sensitivity---------------')

   workbook = xlsxwriter.Workbook('BlueSensitivity.xlsx')

   for w in range (0,11):  #step of .1 for weightB[0], 1-weightB[0]

      weightBtemp = (w/10, (1 - (w/10)))
```

```
        print('BlueWeight = ', w/10)
        s = 'BWgt=' + str(w / 10)
        print(s)
        worksheet = workbook.add_worksheet(s)
        for row in range(3):
            for col in range(9):
                NFMatrixRaw[row][col][0]=linear_UtilityB(TimeMatrix[row][col],
weightBtemp, MSDAperArray[row])
                #REd
                NFMatrixRaw[row][col][1]=linear_UtilityR(TimeMatrix[row][col],
weightR, AperArray[row], MSDAperArray[row])
                #print(row, ', ',col, ', ',TimeMatrix[row][col],',
',NFMatrixRaw[row][col][0],', ',NFMatrixRaw[row][col][1])
                s = "{:.5f}".format(NFMatrixRaw[row][col][0])
                s=s+'\n'
                s=s+"{:.5f}".format(NFMatrixRaw[row][col][1])
                #s=str(NFMatrixRaw[row][col][0]) +'\n' +str(NFMatrixRaw[row][col][1])
                worksheet.write(row, col, s)
    workbook.close()
    print('---done Blue sensitivity---------------')
```

Nash Equilibrium Solver Output

Solution Page

Please check that the matrices displayed below are as you intended. If not please go back and re-enter the game.

```
3 x 9 Payoff matrix A:

        0         0         0         0         0         0         0         0         0
 901/4000  901/4000  901/4000  901/4000  901/4000  901/4000  901/4000  901/4000  901/4000
  201/400   201/400   201/400   201/400   201/400   201/400   201/400   201/400   201/400


3 x 9 Payoff matrix B:

 84439/100000  84227/100000   21011/25000  81881/100000  81457/100000   40557/50000   39693/50000    9861/12500   15677/20000
  16149/25000             0  64019/100000  60877/100000   12061/20000    7473/12500   28101/50000  54777/100000   26539/50000
 43437/100000  43019/100000  42613/100000  38041/100000   18451/50000    8941/25000    7509/25000  29109/100000  28239/100000

EE = Extreme Equilibrium, EP = Expected Payoff

Decimal Output

 EE  1  P1:  (1)  0.000000  0.000000  1.000000  EP=  0.5025  P2:  (1)  1.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  0.000000  EP=  0.43437

Rational Output

 EE  1  P1:  (1)  0  0  1  EP=  201/400  P2:  (1)  1  0  0  0  0  0  0  0  0  EP=  43437/100000

Connected component 1:
{1}  x  {1}
```

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14. ABSTRACT
Modern defense systems are designed to static sets of requirements and specifications. This process has worked well in the past but fails to account for the strategic interdependence of design choices made prior to operating systems. However, as the world becomes increasingly competitive, and as the space domain becomes increasingly congested and contested, such interdependence of design can be described using Game Theory. Game Theory is the mathematical study of strategy, actions and payoffs between rational, self-interested actors. Limited extant research has applied Game Theory to system design but focused on a single system being designed cooperatively within a single company and incorporated regression derived payoffs. This research extended Game Theory Based Design to non-cooperative "red vs. blue" actors designing their own systems and incorporated physics-based simulation to determine payoffs. The hypothesized scenario has blue conducting Space Situational Awareness from a polar geosynchronous orbit, and red conducting geosynchronous proximity operations. The output from this AGI System Tool Kit (STK) simulation allows analysis of any Nash Equilibria of the satellite designs. Results indicate Game Theory can be applied to the design of U.S. Space systems to account for strategic interdependence between non-cooperative or hostile nations.

15. SUBJECT TERMS
Game Theory, GEO SSA, Game Based Design, System Design

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