Multi-Objective Evaluation of Target Sets In A Logistics Network

Paul D. Emslie

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MULTI-OBJECTIVE EVALUATION OF TARGET SETS FOR LOGISTICS NETWORKS

THESIS

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This thesis addresses the selection of target sets to disrupt an adversary's logistics network in a wartime environment. In the presence of many objectives—such as reducing maximum flow, lengthening routes, avoiding collateral damage, all at minimal risk to our pilots—the problem of determining the best target set is complex. Previous efforts have not adequately considered the value of the remaining network functionality after target destruction. In addition, current network targeting procedures optimize target sets with respect to only a single metric. This thesis uses a multi-objective decision analysis framework capturing actual targeting decision-maker values and preferences to evaluate and analyze 10 alternative target sets. Sensitivity analysis and persistency analysis on the results give insight as to how to select better target sets to meet stated strategic objectives.
The views expressed in this thesis are those of the author and do not reflect the official policy or position of the United States Air Force, Department of Defense, or the U. S. Government.
MULTI-OBJECTIVE EVALUATION OF TARGET SETS
FOR LOGISTICS NETWORKS

THESIS

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Degree of Master of Science in Operations Research

Paul D. Emslie, B.S.
2nd Lieutenant, USAF

March 2000

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With gratitude,

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Abstract

This thesis addresses the selection of target sets to disrupt an adversary's logistics network. In the presence of many objectives—such as reducing maximum flow, lengthening routes, avoiding collateral damage, all at minimal risk to our pilots—the problem of determining the best target set is complex. Previous efforts have not adequately considered the value of the remaining network functionality after target destruction. In addition, current network targeting procedures optimize a single metric. This thesis uses a multi-objective decision analysis framework capturing actual targeting decision-maker values and preferences to evaluate and analyze ten alternative target sets. Sensitivity analysis and persistency analysis on the results give insight as to how to select better target sets to meet stated strategic objectives.
Chapter 1. Introduction

1.1 Background

"Logistics provides the means to create and to support combat forces. Logistics is the bridge between the national economy and the operation of combat forces. Thus, in its economic sense it limits the combat forces which can be created; and in its operational sense it limits the forces which can be employed" [Eccles, 1959: 315].

This pronouncement by Navy Rear Admiral (Ret) Henry Eccles, in his book *Logistics in the National Defense*, states the extent of dependence a military necessarily has upon its supply chain. No military force can expect to fight effectively with a weak or crumbling logistical infrastructure. On the other hand, a force which can effectively disrupt the logistics of an opposing army will be in much better stead to overpower its enemies' fighting forces.

Joint doctrine acknowledges this goal as equal to the attack of fielded forces. Joint Publication 1-02 identifies the function of targeting as "*delaying, disrupting, disabling, or destroying enemy forces or resources critical to the enemy*" [1999: 447, italics added]. Accomplishing such a goal means interfering with the entire complex of interconnections in the enemy's logistics chain, termed a line-of-communications (LOC) network. An LOC is "a route, either land, water, and/or air, which connects an operating military force with a base of operations and along which supplies and military forces move" [JP-1-02, 1999: 262].

Following the goals of targeting, delay and disruption along LOCs are precisely the objectives of interdiction operations. They can enable friendly forces to strengthen their position prior to conflict, help to drive enemy forces into an inferior situation, thus
enabling their defeat in detail, or simply reduce the enemy's front-line forces to operational impotence by preventing timely arrival of supplies and reinforcements. JP 3-03 confirms “Attacks on enemy lateral LOCs can channel movement, impair reinforcement, reduce operational cohesion, and create conditions for defeating the enemy in detail” [1997: I-4]. Indeed the criticality of LOCs to success cannot be overstated: “The greatest secret of war and the masterpiece of a skillful general is to starve his enemy” [Frederick the Great, quoted in JP 3-03, 1997: I-3].

Naturally, the desire is to accomplish this objective with a minimum of losses and expense. Field Marshal Bernard L. Montgomery said “They forget that the whole art of war is to gain your objective with as little loss as possible” [JP 3-03, 1997: IV-1]. This is done both by proper targeting and good weaponeering. An example serves to illustrate the point.

“... from 1965 to 1972 during the Vietnam War, hundreds of sorties dropping thousands of tons of unguided ordnance failed to close the heavily defended Thanh Hoa and Paul Doumer bridges permanently. All supplies coming into Hanoi or moving southward by rail passed over these two key bridges. When precision-guided munitions became available, the first strike on each target with laser guided bombs resulted in two dropped bridges with no friendly losses” (JP 3-03, 1997: IV-3).

In modern times, it is even more critical not only to protect allied combat forces, but also to minimize the impact on non-combatants and civilian society, the environment, and cultural and humanitarian facilities. In a time when budgets are tight but weapons are smarter and more expensive, it is also highly desirable to preserve weapons stocks by using them efficiently and appropriately.

This thesis addresses the land transportation infrastructure in a theater by comparing target sets designed to interdict the flow of forces and resources along LOCs.
The target sets in the scenario used in this research are made up of stationary components of a land logistics network, specifically road and rail bridges. The particular scenario will be explained in detail in Chapter 4.

Figure 1. Overlay of Joint/AF/Navy with Army/Marine Corps Targeting

Doctrine from AFPAM 10-225 [1997: I-3]

AFPAM 10-225 describes the joint targeting process. Figure 1 overlays the traditional US Air Force and Navy view in the inside circle with the Army and Marine
Corps targeting representation on the outer circle, thus illustrating the fundamental unity and agreement among services for joint targeting. This cooperation between services is critical to inter-service trust and the success of joint operations, and strategic interdiction targeting is no exception.

Starting at the top on the inside circle, a commander’s objectives and guidance define the direction and purpose which guides the targeting process. They provide the basis for comparison of the desirability of any target and target set—the foundation for evaluating alternative targets or target sets, weapons assignments, attack plans, and appropriate damage assessments. Once objectives and purpose are established, targets are developed as alternative ways to meet those objectives. Portfolios of targets will satisfy those established objectives to varying degrees. It is the task of this research to provide the means for evaluating that level of satisfaction of commander’s objectives for candidate target sets. Following target set selection, weapons must be recommended appropriate to the goals and purposes for choosing the targets, with consideration for the current supplies of different weapons and the relative priority of targets. At the same time, the reverse arrows emphasize the fact that weapons constraints should influence the selection of targets, and operational constraints on targeting may influence the commander’s realistic goals.

The next step is Force Application, where the job of attacking each target is given to the unit for execution. Execution Planning follows, where the delivery vehicle and personnel are assigned and information such as angle and direction of attack and precise timing are worked out. Last in the process is combat assessment, which enables the
commander to note what have been the effects of the effort and to determine updated objectives. [AFPAM 10-225, 1997: I-4 – I-9].

The outside circle describes a similar process. In this case Army doctrine [FM 6-20-10] is used to explain the concept of Decide, Detect, Deliver and Assess (D3A), although Marine Corps doctrine could also be referenced (MCRP 3-1.6.14 TTP). The current situation is assessed as the framework for a decision of immediate and future action. This decision incorporates the elements of commander’s objectives and target selection through use of a High Payoff Target List (HPTL) and various intelligence products. The decision then leads to the detection and tracking of selected targets to be destroyed so weapons can be employed against them. The next step is the delivery, where the weapon is overseen from launch to destination. After the attacking action is complete, the new state of the system is assessed, forming the framework for a new iteration of the decide function [FM 6-20-10, 1996: Ch. 2].

It is observed that the Commander’s Objectives and Target Development steps on the inside circle correspond to the Decide step on the outer process diagram. The focus of this research is the quantification of the commander’s objectives in order to make the target-development process more consistent with those objectives.

The selection of target sets must be evaluated on their contribution to accomplishment of the commander’s objectives as well as for their system criticality and vulnerability [AFI 14-117: 4], implying concern for the costs associated with their destruction. Furthermore, in a world of limited resources, any decision will involve tradeoffs among competing objectives for resource allocation.
The targeting agency sponsoring this research frequently deals with networks with thousands or tens of thousands of connection points and lines of communications between them. Even considering that many of the possible targets may be screened out, picking the “best” set in a multi-objective setting grows very quickly beyond what a targeter can compare and exhaustively judge. The greater the number of objectives he or she must account for, the less realistic it is to expect to find the “best” set of targets, even by his or her own definition and preferences. Using extensive experience and judgement, the experts arrive at what is thought to be a good target set, but even one comparison between two alternative tasks can be a daunting challenge in a multi-objective setting.

The experienced targeter applies experience, analysis and judgement, coupled with a series of “rules of thumb.” With careful consideration, he or she can generate a target set to meet two or three objectives at once to varying levels. According to a chief analyst for the sponsoring agency, tradeoffs between various objectives are compared and contrasted mentally, but many different objectives will inevitably be reduced to a few key considerations, leaving the rest as constraint criteria for disqualifying individual targets. This research structures the varied goals and tradeoffs to incorporate the decision-maker’s actual preferences in a way that can be applied to any LOC network target set. The moderate level of up-front value elicitation effort can enable very fast comparison of many target sets on a composite merit of all of the measured considerations. This can reduce the array of alternatives to a few good alternatives, providing insight to the decision-maker both on the choice between the guaranteed good alternatives and in the generation of even better alternatives using insight gained from sensitivity and persistency analysis.
1.2 Problem Statement

The evident complexity of the decision calls for a methodology to help quantify the commander's objectives to provide insight and decision support that is both true to his preferences and theoretically valid. The methodology must account for the multiple objectives expressed with respect to an LOC network environment. As mentioned above, these objectives include, but are not limited to, delaying the enemy, disrupting (reducing) the flow and the paths through the network and funneling the enemy into a few locations for defeat in detail. In addition, costs should be modeled in a way that captures multiple aspects, including but not limited to risk to allied and civilian personnel, weapons costs, cultural and social collateral damage, and possible obstruction to allied troops once the territory has been liberated. These concerns of the targeting and weaponeering decision-makers are more fully developed in Chapter 3 and Appendix A. Finally, the objectives must be structured in such a way that diverse target set alternatives can be evaluated on a common scale.

A clarification is appropriate at this point. Several of the objectives mentioned thus far are described in terms of their results rather than in characteristics of individual targets. In other words, the desired characteristics for a good target sets in reality are characteristics of the LOC network that remains after the targets have been attacked. Objectives such as reducing the flow capacity or delaying and disrupting items traveling through the network are all really descriptions of what the commander would like to see in the degraded network in terms of reduced functionality. The new network characteristics such as flow or delay are generally not obtainable by simply adding attributes of the individual targets, but instead must be measured on the degraded network.
using network algorithms and accurate battle damage assessment. Chapter 2 demonstrates that this critical concept has not been adequately addressed in the open literature, motivating the need for a methodology to account for it.

1.3 Scope and Limitations

Judging where a system ends is no small task. MIT Transportation Professor Marvin L. Manheim observes that

"...the activity system of a metropolitan area or a metropolitan region or a developing country consists of many subsystems, overlapping and interrelated—social structures, political institutions, housing markets, and so on. Transportation is only one of these subsystems" [1979: 13].

Nevertheless, the focus of this research is restricted to theater logistics—specifically the land component, consisting of stationary, tangible, infrastructure targets. This restriction simplifies the decision by narrowing the focus of items under consideration. The typical components of a LOC network include bridges, highways, railroads, rail stations, intersections, and intermediate warehouses or storage locations. The supply depots might be factories, seaports, airports, or other warehousing and delivery facilities, while the receiving locations would typically be front-line military units. However, because of the relative ease of bypassing obstacles in a road on flat terrain, the components targeted in the scenario used here are exclusively bridges. The greater difficulty associated with repairing bridges as compared to circumventing a road crater makes bridges highly preferred, and highly effective targets.

The time context of this research is for the contingency and pre-hostility planning phases. The general applicability of the model to a variety of scenarios means that even with very short notice it can be easily modified to fit the situation. Because this research
provides the means to quantitatively evaluate existing alternatives in a timely manner, the
generation of target sets is not considered here.

1.4 Thesis Overview

This chapter has introduced the background and motivation for the research at hand. Chapter 2 will survey the relevant literature, including an overview of network and graph theory concepts, clustering approaches, and multi-attribute decision analysis. The specific methodology used in this research is selected and presented in Chapter 3, with a description of the scenario presented as a motivation and context for the methodology. The chapter details the model construction process and conclude with a discussion of a possible method of calculating the number of Strike Packages as one piece of the model. Chapter 4 presents an analysis of the 10 alternatives in this scenario, including rank ordering, sensitivity to certain parameters, cost-benefit analysis, and persistency. Finally, Chapter 5 provides insights particular to this problem—made possible by the results from Chapter 4—as well as insights general to the method and suggestions for further research in this area. A detailed description of the measures used is provided in Appendix A.
Chapter 2. Literature Review

2.1 Introduction

This chapter surveys the relevant literature, giving a theoretical baseline for further research. Because the problem is based on a complex network structure, a brief exploration of graph and network theory is provided to identify fundamental concepts, terms and capabilities. A short discussion of clustering methods follows, serving as background for one of the measured target set attributes. Since the sponsoring organization finds itself in a complicated, multi-objective decision situation, the next section addresses decision analysis, investigating its capabilities and possible value added for this thesis. Finally, a review of the most relevant recent studies on military networks, target set evaluation and similar topics establishes a background for the present effort.

2.2 Network and Graph Theory

This section introduces basic network and graph theory concepts to help with an understanding of the measures developed later in this chapter and described in detail in Appendix A.

2.2.1 Terms and Definitions

Recall from Chapter 1 that some metrics for the benefits of a target set may be network measures for the residual network. These network measure scores can be derived by running standard algorithms on the LOC network. A survey of relevant terms and concepts is desirable.
A graph is a collection of vertices or nodes together with adjoining links or edges. A node is a mathematical point and may be incident with any number of links. In Figure 2, the numbered circles are nodes. A link is a connection between two nodes. In Figure 2, the line between nodes 1 and 2 is a link, designated 1-2. Although the graph theory community usually speaks of vertices and edges, the network optimization community prefers the terms nodes and links. This thesis will conform to the latter convention.

A path is a sequence of nodes and links in which no node or link is repeated, except possibly at its extreme end nodes. Such an end node repetition delineates the path as a closed path. An example of a path would be 1-2-4-6-7, and a closed path might be 1-2-4-6-3-1. A network is connected if a path exists between each pair of nodes. The graph in Figure 2 is connected.

Direction may be assigned to links of a graph, as in Figure 2. Links 5-7 and 6-7 are directed, allowing flow into node 7 but not back to other nodes in the graph. All other links in the network are undirected and may be traversed in either direction. A graph is directed if all links have definite direction; it is undirected if all links may be traversed in either direction. The graph is mixed if there are some links of each type, as
there are in Figure 2. Two paths are node-disjoint if they share no nodes in common except the endpoints, necessarily implying that they also have no links in common. For example, path 1-2-5-7 is node-disjoint from 1-3-4-6-7. Paths 1-2-4-6-7 and 1-3-4-5-7 are not disjoint, since they share node 4.

A cutset is a set of nodes and/or links that breaks all paths between two nodes $s$ and $t$. An example of a cutset in Figure 1 might be links (4,6) and (3,6) together with node 5. Finally, a cutset may be termed minimal if it contains no proper subset that is also a cutset. In other words, if any element from a minimal cutset is removed, it will no longer disconnect the source and sink nodes [Patvardhan, Prasad, and Pyara, 1995: 347]. Two nodes are termed adjacent if they are directly connected by a link. A clique is set of nodes in which every pair of nodes is adjacent [West 1996: 3]. An independent set is a collection of nodes no two of which are adjacent [3].

The nodes of a graph may be colored by assigning labels (colors) to each node such that no two adjacent nodes are the same color. The chromatic number of a graph is the minimum colors required to color a graph. The chromatic number identifies a minimum number of independent sets in the graph [3]. The challenge of identifying the chromatic number in a general graph is an NP complete problem, which means no algorithm exists to find the correct answer in an amount of time polynomially-related to the size of the graph [Beineke and Wilson, 1997: 12]. In addition, the complement of a graph $g$ is the graph on the same set of nodes such that pairs of nodes in complement are adjacent if and only if they were not adjacent in $g$. The union of the edge sets of a graph and its complement, then, defines a clique on the node-set of the graph [West, 1996: 3].
Further background in graph theory may be found in West’s text, *Introduction to Graph Theory* [1996], especially sections 1.1 and 4.3.

### 2.2.2 Network Algorithms

The network optimization community often deals with characteristics such as the shortest path between two particular nodes, maximum flow between particular nodes in a capacitated graph, and node-clustering. The shortest path may be defined in terms of cost, distance, time, or virtually any single-dimensional measure. Dijkstra’s Algorithm yields an optimal solution in polynomial time [Evans and Minieka, 1992: 8]. Maximum flow refers to the greatest amount of material that may be pushed through a network per time period, and is found using a variety of approaches, from flow-augmenting path schemes to pre-flow push and numerous dynamic algorithms [Evans and Minieka, 1992: 184-227]. Some of the many sources for algorithms and problem-solving techniques are Evans and Minieka [1992], Bertsekas [1991], and Ahuja, Magnanti, and Orlin [1993].

Although this research is not primarily focused on the problem of finding all cutsets, or even the minimum cutset, by some single-attribute measure, several of the relevant military efforts in recent years have used the concept for generation of their alternatives. For the scenario developed in Chapter 3 the concept of a cutset is not practically relevant. There are generally far more paths of some sort between a set of rear and forward areas than it would be reasonable to spend munitions on to sever, so the cutset discussion is relevant primarily as background for the recent contributions. These contributions are not mentioned for their cutset theory, but rather their multi-attribute relevance. With that motivation, a short discussion of cutsets follows.
The literature reveals a wide variety of algorithms that can generate all minimal cutsets. Some of the algorithms identify link-cutsets in undirected graphs [Ghosh and Singh, 1993; Ahmad, 1990; Prasad, Sankar and Rao, 1992; Singh, 1995]. Some of these may also be applied to directed networks [Ghosh and Singh, 1993; Ahmad, 1990]. Node-cutsets may also be generated in directed networks [Shier and Whited, 1985], or undirected networks [Prasad, Sankar and Rao, 1992; Patvardhan, Prasad, and Pyara, 1995]. The last reference in particular will be significant as the algorithm used by Leinart in his Telecom disruption effort in section 2.5.2.

Mixed cutsets may be easily generated via a standard node-cutset algorithm by conversion of links to nodes, illustrated in Figure 3. This conversion involves replacing a link with a node and two links, so that the node assumes the attributes of the original link, and the two new links connect the new node between the former two endpoint nodes [Frank and Frisch, 1971: 305].

![Figure 3. Converting Links to Nodes](image)

Cutting the link in the top figure and cutting the middle node in the bottom figure are equivalent.
Alternatively, mixed cutsets can be generated with an edge-cutset algorithm for directed graphs, using a technique called node-splitting, described in Ahuja, Magnanti, and Orlin [1993: 41] and demonstrated in Figure 4. This process yields a vastly more complex graph and is not preferred for finding mixed cutsets in an original undirected graph.

![Diagram of node-splitting](image)

**Figure 4. Converting Nodes to Links**

Recently, Whiteman [1998] has improved upon the traditional integer-programming dual of a maximum flow problem [Bazaraa, Jarvis, and Sherali, 1990: 566]. This method finds the optimal set of nodes to strike in a directed network by assigning weapons and calculating effectiveness distributions using Probability-of-Kill (PK) data.
Whiteman's research addresses the possibility of incomplete cuts by minimizing cost with a bound on remaining flow, obtaining fully optimal solutions. Along similar lines, Curet has addressed generation of multiple optimal solutions to the minimum link-cut problem using a method called \textit{netpdcut} [1999]. His algorithm demonstrates the feasibility of using a standard network or LP algorithm to find \textit{all} optimal solutions rather than just one. Both of these approaches, however, have the disadvantage of addressing only a single objective with assumed linear additive properties.

2.3 Clustering

Clustering is an approach to classifying data, for the purpose of identifying groups of a single type [Höffner, Klawonn, Kruse, and Rukler, 1999: 1]. Various measures of similarity or distance between data may be used, depending on the problem. Likewise, the selection of an appropriate approach and algorithm is also dependent on the scenario. According to Iyer and Aronson, "clustering techniques are broadly classified as hierarchical, optimization, density search, clumping and other techniques" [1999: 67].

Standard statistical packages like JMP [SAS,1999] have hierarchical agglomerative and disaggregating procedures where the number of clusters is found as a function of the magnitude of the distance measure. In JMP, any of five linkage methods generates a dendogram like the one in Figure 5. In hierarchical clustering, each point is initially assigned its own cluster, and clusters are joined successively as some measure of cluster distance (or dissimilarity) grows. The process stops when all points are in a single cluster, and the user may trace the clustering process back to any arbitrary threshold and discover the arrangement of data points in clusters at that step. In Figure 5, the
data points are listed on one axis and the distance of measure on the other. The user may simply move along the x-axis to the desired distance measure or the desired number of clusters. Tracking the cluster history of a data point is straightforward. For example, Figure 5 shows point 13 joined to 12 and then to 14 and 15, next to 10 and 11, then to 16 and 17, followed by points 5 through 9, points 18 through 26, and finally bundled into one giant cluster containing all the points. As an illustrative aid to reading the dendogram, the star indicates a point at which there are four clusters remaining.

![Sample Dendogram for Cluster Analysis](SAS, 1999)

Figure 5. Sample Dendogram for Cluster Analysis [SAS, 1999]
Methods also exist to improve a cluster partition, sometimes called a density search. One older source for such methods is chapter 7 of the monograph *Cluster Analysis for Applications* [Anderberg, 1973]. Such methods are relatively simple and quick but fail in the same situations as the hierarchical methods (demonstrated in Chapter 3). Optimization methods do have the capability to find the minimum number of partitions necessary to cover a data set given a fixed distance threshold, which is an NP-hard problem [Iyer and Aronson, 1999: 67]. A problem is termed NP-hard when there is no polynomial algorithm guaranteed to return an optimal solution in a period of time polynomially related to the number of original data points [Reeves, 1995: 8-9]. Optimization is a strong choice if it is important to know a solution is the best possible, but may exhaust computing resources for larger problems because of its combinatorial growth.

### 2.4 Decision Analysis and Value-Focused Thinking

Decision-making is a normal, ongoing activity practiced by any autonomous person. Many decisions are easy, routine, and relatively unimportant in terms of the magnitude of possible consequences. Sometimes, however, the complexity and significance of a decision is more critical. In such cases, a method to clarify and analyze the problem is desired.

Decision analysis is the science of modeling decisions quantitatively to provide understanding and insight to assist a decision-maker in choosing among available alternatives. A decision may be defined as an irrevocable allocation of resources. Sometimes the resources may be more or less continuous, while at other times they are
discrete and perhaps irregular. Sometimes the possible alternatives are relatively few, and the constraints are more implicit.

Clemen lists some factors that make decisions difficult. First, the decision may be complex, involving great diversity of the courses of action, time frames, and opinion among the involved parties. It may be composed of several decisions to be made in succession, with the substance of later decisions depending heavily on the earlier choices [Clemen, 1996: 2]. Second, the decision may involve considerable uncertainty in the eventual consequences, with an insufficient understanding of the dependence of probabilities and their distributions [2]. Third, there may be multiple objectives, but progress in one objective frequently impedes progress in another. Fourth, the framing of a problem may have a major impact on one's preferences. That is, slight changes in perspective between multiple people could yield significantly different preferred options [3].

It is precisely these difficulties of multiple objectives, significant uncertainty, and complexity in the decision structure and the form of alternatives that decision analysis can help. By focusing on values rather than alternatives, better alternatives may be found. Sensitivity analysis can quantify and analyze uncertainty, so discussion and effort may be devoted to the parameters that really matter to the final result.

2.4.1 Values

Having provided a motivation for a better decision-making process, the focus turns to values. Keeney [1992: 49] compares approaches to the decision-making process.
in Table 1, advocating value-focused thinking rather than the more commonly-practiced alternative-focused thinking.

Table 1. Comparison of Alternative- with Value-Focused Thinking

[Keeney, 1992: 49]

<table>
<thead>
<tr>
<th>Alternative-Focused Thinking for Decision Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Recognize a decision problem</td>
</tr>
<tr>
<td>2. Identify alternatives</td>
</tr>
<tr>
<td>3. Specify values</td>
</tr>
<tr>
<td>4. Evaluate alternatives</td>
</tr>
<tr>
<td>5. Select an alternative</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Value-Focused Thinking</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>For Decision Opportunities</strong></td>
</tr>
<tr>
<td><strong>Before Specifying Strategic Objectives</strong></td>
</tr>
<tr>
<td>1. Identify a decision opportunity</td>
</tr>
<tr>
<td>2. Specify values</td>
</tr>
<tr>
<td>3. Create alternatives</td>
</tr>
<tr>
<td>4. Evaluate alternatives</td>
</tr>
<tr>
<td>5. Select an alternative</td>
</tr>
<tr>
<td><strong>After Specifying Strategic Objectives</strong></td>
</tr>
<tr>
<td>1. Specify values</td>
</tr>
<tr>
<td>2. Create a decision opportunity</td>
</tr>
<tr>
<td>3. Create alternatives</td>
</tr>
<tr>
<td>4. Evaluate alternatives</td>
</tr>
<tr>
<td>5. Select an alternative</td>
</tr>
</tbody>
</table>

The difference may be seen in the order of value development and alternative generation, framing the problem differently to create alternatives more in line with the identified values. Whereas alternative-focused thinking may excessively constrain the decision context and lead to his picking from a group of uniformly poor alternatives, VFT encourages a broader perspective and enables better alternatives to be found [Keeney, 1992: 50-51].

Stated simply, a value is that which the decision-maker is interested in achieving through the decision at hand [Keeney, 1992: 33]. Values are interpreted by objectives,
which may then be broken into sub-objectives in a structured framework called a value hierarchy.

2.4.2 Value Hierarchies

A value hierarchy is used to categorize objectives important to the decision under study. One moves down in the hierarchy by asking the question "What do I mean by that?" or "How do I achieve that?" The hierarchy ends when every bottom consideration is measurable in the form of a single attribute. Properties of measures will be discussed in the following section. Going up in the hierarchy involves asking the question "Why is that important?" The top of the hierarchy is reached when no other reasons can be provided. "It's just important for its own sake, in and of itself."

A valid value model must pass several tests. It must be complete, non-redundant, operable and as small as possible [Kirkwood, 1997: 16]. The first two criteria may be achieved by examining each consideration together with its immediate descendents. Sub-objectives and measures must be mutually exclusive (complete) and collectively exhaustive (non-redundant) with respect to their parent objective. A set of considerations is collectively exhaustive with respect to the parent objective if there is nothing included in the parent concept which is not captured in one of the sub-considerations [17]. A set of considerations is mutually exclusive if there is no characteristic being measured or accounted for by more than one of the considerations in the set [17-18]. The hierarchy must be operable—that is, it must be understandable to the parties who must use it [18]. Finally, a small model is easier to explain and use, making it more valuable and efficient as a decision-making aid [18-19].
2.4.3 Metrics and Value Functions

The bottom-level considerations in the hierarchy are called measures, and they can be of various types. Kirkwood discusses two types of classification for measures: natural versus constructed, and direct versus proxy. He states "a natural scale is one that is in general use with a common interpretation by everyone," whereas "a constructed scale is one that is developed for a particular decision problem to measure the degree of attainment of an objective" [1997: 24]. To describe the second classifier, "A direct scale directly measures the degree of attainment of an objective, while a proxy scale reflects the degree of its associated objective, but does not directly measure this" [1997: 24]. For example, the mileage of a car is a natural, proxy measure for the amount of wear and tear on the car. The mileage does not directly measure the wear but it is accepted and understood by most people as being reasonably correlated to the actual objective. After deciding on the best metric for an objective, appropriate ranges are determined based on the highest and lowest possible levels of the attribute that could appear in an alternative.

Once the appropriate metrics and ranges have been determined, the next step is to assess value functions. A value function transforms a measure—a bottom-level consideration in the hierarchy—into a value. Value is usually put on a scale of 0 to 1, with 0 corresponding to the worst possible level of the attribute likely to be seen in any alternative, and 1 corresponding to the best level of the attribute. For the tradeoffs to be meaningful, the value function for each measure must be monotonic, that is, higher scores on a metric are either always more preferred or always less preferred [Kirkwood, 1997: 228].
2.4.4 Preferential Independence and Value-additive Models

An important assumption in the formation of hierarchies are that measures must be preferentially independent. This means that a change in the score on one measure does not change the preferences in other measures. Algebraically, if Y and Z are a partition of \( (X_1, X_2, \ldots X_n) \) where n is the number of measures, "then Y is preferentially independent of Z if the rank ordering of alternatives that have common levels for all attributes in Z does not depend on these common levels" [Kirkwood, 1997: 238]. This is not to say that a car buyer's disappointment with a low gas mileage cannot be lessened by the same car's high horsepower. Rather, preferential independence implies his preference for high gas mileage does not reverse itself depending on a corresponding level of horsepower. In other words, if he prefers high gas mileage on a 200-horsepower car but prefers low gas mileage on 100-horsepower cars then mileage and horsepower are not preferentially independent. If preferential independence holds between all such partitions of attributes, then we have mutual preferential independence and may use an additive value model [1997: 239].

To define an additive value function we need the concept of strategic equivalence. Two value models are strategically equivalent if both give the same preference ranking for any given set of alternatives [1997: 229]. A value function is additive, then, "if it is strategically equivalent to a value function of the form

\[ v(x) = \sum_{i=1}^{n} \lambda_i \cdot v_i(x_i) \]  

[1997, 230]. In this expression, x is an alternative with each \( x_i \) representing the \( i^{th} \) of \( n \) attributes, \( v_i(x_i) \) is the single-attribute value for measure \( i \), and \( \lambda_i \) is the weight given to single attribute value \( i \) [1997: 230].
Each alternative is assigned a particular score for each metric based on its attribute with regard to the corresponding value consideration. Each metric score can be converted to value by means of a value function, putting it in the common unit of value. Preferences or trade-off values determine the weights assigned to each measure, and the entire weighting structure is combined into a total value function. Once an alternative has been assigned the correct score on each measure, it is evaluated via the overall value function. This result is its total value, also denoted the composite or overall value. The specific calculations for this are demonstrated in Section 3.4, especially Table 6. For any parameter or score with significant uncertainty, sensitivity analysis may be performed to determine if reasonable variation in the parameter results in changes in the final ranking of alternatives, that is, if a change in the parameter results in a change in the recommended choice. For a more detailed introduction to Decision Analysis and Multi-Attribute Utility Theory the reader is referred to Keeney [1992] and Kirkwood [1997].

2.5 Previous Combinations of Network Evaluation and Decision Analysis

This thesis follows significant strides in the area of applied decision analysis in the realm of network theory. Three previous efforts in particular—Davis [1997], Leinart [1998] and Wallace [1999]—establish a chain of contributions to the operational military that will be briefly mentioned.

2.5.1 A Methodology for Evaluating and Enhancing C4I Networks [1997]

Davis developed methodology to evaluate competing, alternative upgrades to a communications network. Using a network-flow algorithm, she identified bottleneck links in a capacitated tactical communication network, identified possible upgrades,
evaluated them, and proposed an upgrade plan [Davis, 1997, and Davis, Deckro, and Jackson, 1999]. Evaluation of upgrade plans was accomplished using the value hierarchy shown in Figure 6, where the overall goal is to maximize the total expanded system effectiveness.

![Diagram of Davis' Network Upgrade Value Hierarchy](image)

**Figure 6. Davis' Network Upgrade Value Hierarchy [1997: B-1]**

Value functions translated scores on 21 individual measures to values. The scores assigned to a single upgrade plan were functions of the attributes of each piece of the upgrade, aggregated in most cases by simple averaging of the individual target scores on each measure. Davis' work was a ground-breaking effort to structure the network upgrade challenge in a VFT framework. She integrated a solid network analysis of bottlenecks and upgrades with a VFT approach, laying a baseline for future DA network application efforts. Although she attempted to address the value of the upgraded network in order to choose among upgrade alternatives, the measures were calculated by averaging attribute scores across all links rather than by a network algorithmic approach. Costs of destruction were not relevant to the problem and were not modeled.
2.5.2 A Network Disruption Modeling Tool [1998]

Leinart produced a tool for comparing target sets for an enemy communications network, a perspective that is opposite to Davis' work. The network was a composite of ground, cellular and radio telephone systems and other telecommunications systems.

After writing Visual Basic code for Patvardhan's cutset algorithm in an Excel environment, Leinart evaluated all 9,079 cutsets of both nodes and links according to the hierarchy shown in Figure 7. He ranked the target sets, and conducted sensitivity analysis on weights and persistency analysis on individual targets. Persistency measures the frequency of appearance of a particular node or link in the top target sets and is an indicator of its individual importance as a target.

Leinart's contributions are concentrated in the VB/Excel tool he built and in the idea of using DA as an approach to network interdiction. All individual value functions were assumed linear, and the aggregation of individual target attributes into a composite target set score for each measure was done by simple averaging of the component target

Figure 7. Leinart's Network Disruption Value Hierarchy [1998: 4-5]
attributes. Within each measure, the average of individual target scores was taken as the
target set score and transformed into a value for that measure. While this type of
aggregation has definite weaknesses, which are addressed in the research at hand, it
represents the commander's multiple objectives far better than the single-objective
models of Whiteman [1998] and Curet [1999], mentioned in the previous section. Costs
were addressed only in the low-resolution measure *Cardinality*.

2.5.3 *Multi-Discipline Network Vulnerability Assessment* [1999]

Wallace reverted to the protection goal of Davis' work [1997], seeking to identify
vulnerabilities in a notional US defense network. Building on Leinart's Visual Basic tool
[1998], Wallace used a cutset algorithm to generate her 34,285 alternatives. She selected
the already-mentioned directed networks algorithm by Shier and Whited [1985], given
the unidirectional character of much of her network topology. Her network incorporated
power and water sub-networks as support components to a space navigation and
communications network, and the tool assessed vulnerabilities in the component sub-
networks as well as in the top-level network. Her aggregation was again done by
averaging individual target scores to obtain a set score for each measure, and the value
functions were assumed linear.

Her work represents the first application of a multi-objective value model to a
multi-discipline network. Generation of all cutsets in the top-level network ensured the
highest vulnerabilities were noted. However, costs were not modeled and no alternatives
were considered other than complete cutsets.

Table 2 briefly summarizes the contributions mentioned to this point.
Table 2. Previous Network Improvement and Degradation Analysis Efforts

<table>
<thead>
<tr>
<th>Contributor</th>
<th>Year</th>
<th>Description</th>
<th>Multi-Attribute Modeled Costs</th>
<th>Addressed Value of Remaining Network</th>
<th>Addressed Partial Cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whiteman</td>
<td>1998</td>
<td>IP, Partial cuts with PKs</td>
<td>No Single-attribute</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Curet</td>
<td>1999</td>
<td>Find all optimal cuts by cost</td>
<td>No Single-attribute</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Davis</td>
<td>1997</td>
<td>C4I network upgrades</td>
<td>Yes</td>
<td>No Gross Approximation</td>
<td>No</td>
</tr>
<tr>
<td>Leinart</td>
<td>1998</td>
<td>Telecom network cutsets</td>
<td>Yes # Targets</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Wallace</td>
<td>1999</td>
<td>Multi-discipline network Vulnerability</td>
<td>Yes No</td>
<td>No No</td>
<td>No No</td>
</tr>
</tbody>
</table>

2.6 Logistics Doctrine

The task of logistics interdiction is to prevent those involved in the enemy logistics planning and execution process from accomplishing their mission. As a proxy for enemy logistics doctrine it is useful at to survey U.S. logistics doctrine. In that the Army is usually responsible for land logistics in a combat theater, Army logistics doctrine has the greatest application to the current problem.

First, a definition of the land transportation logistics system is important. FM 100-10-1 [1999] covers Theater Distribution and describes the elements under study:

The physical network of the distribution system consists of the quantity and capability of fixed structures and established facilities available to support distribution operations. It includes factories, airfields, seaports, roads, railroads, inland waterways (IWW), pipelines, terminals, road and railroad bridges/tunnels, and buildings. The resource network consists of the people, materiel, and machines operating within and over the physical network of the distribution system. It includes a mix of uniformed and civilian (US/HN government, military, and contractor) organizations and equipment. The combined physical and resource networks make up the infrastructure of the distribution system. Infrastructure capacity (net capability of the combined physical and resource networks) establishes the finite capacity of the distribution system. [Chap. 3, Sect. 1, Para. 3-4]
The Army gives nine principles of logistics in its FM 700-80 Chap1, Part 1, Section 3, Point 1-15 [1998]:

(1) **Logistics intelligence.** Commanders must have accurate and timely logistics information in order to provide effective logistics support.
(2) **Objective.** Logistics endeavors must be directed toward a clear and attainable objective.
(3) **Generative logistics.** The professional application of initiative, knowledge, and ingenuity, and the innovative exploration of technical and scientific advances are fundamental to the generation of logistics systems improvements.
(4) **Interdependence.** Logistics system efficiency requires effective interrelationships among all functional parts of the system.
(5) **Simplicity.** Simplicity is essential at all levels of the logistics system.
(6) **Timeliness.** Logistics support must be provided in the right quantity at the proper time and place for accomplishment of the mission.
(7) **Impetus.** The impetus of logistics support is forward to support the combat mission.
(8) **Cost-effectiveness.** Efficient management of logistics resources is essential to cost-effective logistics support.
(9) **Security.** Security of every facet of the logistics system must be maintained to preserve resources and insure sustained combat capability.

As these doctrinal citations highlight principles important to the logistician, the converse of those principles should be useful to the targeter seeking to disrupt enemy logistics. For instance, timeliness may be negated by an interdiction objective of maximizing time delay of materiel through the network. The principle of interdependence suggests an objective of breaking connections between various modes of the system, such as between road and rail, multiplying the effect of other targets within a mode of transportation. Another objective, Security, is defeated by gaining intelligence of the adversary's network; such defeat is assumed to be a fait accompli before this problem of LOC network interdiction is even begun. Traffic data and knowledge of convoy plans in advance would also help to defeat Security.
These resources as well as the Joint and Air Force interdiction doctrine surveyed in Chapter 1 lay the groundwork for the hierarchy in Chapter 3. The experts and decision-makers strengthened and supported the objectives taken from various doctrine sources in order to expand the hierarchy and make it relevant and applicable to the LOC scenario described in Chapter 3.

2.7 Summary

This chapter has reviewed key elements of the relevant literature for this project, progressing through network and graph theory, clustering, and value focused thinking as a background for the methodology. Since this effort is related to several previous theses, a summary of their focus and contributions gives a context for the present effort. Finally, a survey of US land logistics doctrine complements the joint interdiction doctrine and lays a baseline for organizing and implementing the values and objectives of actual decision-makers.
Chapter 3. Methodology

3.1 Introduction

This chapter develops a methodology for evaluating a target set against the commander's objectives. The problem is first structured and graphically depicted in a framework. A specific scenario motivates the discussion of important objectives and forms the backdrop for development of the hierarchy. The additive value model mentioned in Chapter 2 takes form in the way tradeoffs are elicited and used in the target set evaluation. Finally, although most calculations for the attribute scores of each target set are either well-established methods or proprietary knowledge, an alternative method of clustering targets into strike packages is presented to satisfy the particular framing of the Strike Packages measure.

3.2 Framework

An important part of the VFT process is the framework. The framework helps translate understanding of the problem into a graphical form, so that the function of particular inputs and outputs is made clear. For this problem, the final product is insight to help the decision-maker pick a good target set to interdict a LOC network. This insight will be drawn out from target set ranking, while sensitivity and persistency analysis will help identify the most robust target set and relative priority of individual targets. As described in Chapter 2, a multi-objective value model allows for quantification and sensitivity analysis of its results. This value model was built from the decision-makers in charge of the targeting effort and their representative experts/analysts.
Doctrine laid the groundwork for an initial straw-man model, which was then strengthened and refined through lengthy interaction and discussion with the decision-makers and experts. The alternatives—the target sets—must be evaluated based on target set attributes and the particular network topology.

The framework or process map used in this research is shown in Figure 8. As described above, the desired insight is derived by combining the initial products at the top and left of the figure. In Figure 8, initial sources of information—in the form of network data, constraints, and a list of alternative target sets—are combined to yield alternatives with input attributes. These attributes must match the metrics used in the value model, and the alternatives are evaluated from that data. These evaluated alternatives can be ranked and analyzed for sensitivity to virtually any parameter of the model or of an

![Figure 8. Project Framework for Research](image-url)
alternative. This information, through discussion and analysis, is turned into insight into improving the best target sets and restarting of the cycle if desired.

3.3 Value Model

As previously stated, the value hierarchy evolved out of doctrine and extensive discussion with experts and decision-makers with a wide variety of operational experience, including air interdiction, targeting, weaponeering, and logistics. Although the hierarchy is in fundamental agreement with the relevant doctrine mentioned in Chapters 1 and 2, it is tailored to the specific scenario with regard to quantification of the value model. When possible, the model was designed to be applicable to a broad spectrum of operational scenarios. The focus of the process is at the level of nominating options to the Joint Task Force Commander (JTFC) or theater commander.

3.3.1 Scenario Description

The scenario developed to exercise the value model methodology was obtained from the joint targeting agency sponsoring this research. The notional scenario is designed to be comparable to actual situations the sponsoring organization might deal with in the course of its duties. In this scenario, allied forces are in a prehostility environment in standoff with a rogue nation. Political solutions to the international situation have failed and the rogue nation appears to be preparing for armed conflict. Allied forces must be prepared to oppose and thwart the aggression of this nation by preparing specific plans should conflict become inevitable. Allied forces are concerned about the military and support infrastructure of this nation. There is concern both about the adversary's ability to support a war machine and about the potential effects on the
country's people who, for the most part, appear to be unwilling participants in the preparations. Allied forces would like to minimize those negative effects on the civilian population if the logistics structure is attacked.

Two enemy armor divisions are moving south to a designated area of assembly near the foe's southern border. The objective of the allied forces' counter-logistics campaign is to delay and disrupt the progress of opposition ground forces, through interdiction of the LOC network. A set of targets is being constructed to accomplish this goal.

While many facilities could be targeted, the specific focus directed by the commander of allied forces is to delay the land vehicles in their progress to the assembly area, so all targets actually considered are bridges. The list of possible targets contains 183 bridges, 72 of which are actually included in one or more of the ten alternative target sets. The approximate locations of these 72 targets are shown in Figure 9 along with the staging and assembly areas.

The underlying network is extremely dense and is not shown, but each displayed target is a bridge along a link in the network. These bridges will reduce the functionality of the network and have some measurable effects both in terms of risk to friendly forces, and in terms of civilians and society in the surrounding region of a target. These factors are expanded and defined in the hierarchy described in the next section. The target sets will be evaluated with respect to their specific effects in each of these measured areas.

Given this scenario, the evaluation methodology for each target set is designed to help the targeter choose among the target sets by identifying and balancing all the desired
The hierarchy is explained according a breadth-first search ordering, so more specialized objectives follow broader objectives.

The Value of Remaining Network objective describes the characteristics of the network left after the targets in the set have been attacked. The concept of the remaining network was introduced in Chapter 1, and is supported by the doctrinal objectives of disruption and delay [JP 3-03, 1997:1-2]. Disruption and delay imply effects, which in a network are not generally a linear combination of individual target attributes, so the object of concern must be the network left after the attack. Similarly, AFI 117-14 speaks of criticality [1997: 4], which necessarily implies the target is critical to something, so that the target’s destruction makes some measurable difference in the new functional topology of the network.

Additional Benefits denote those possible benefits of a target set for other reasons than those captured directly in the remaining LOC network. This concept follows the principle Economy of Force, in that multiple objectives are being addressed in a single strike [JP 3-0, 1995: A-1].

The Cost of Target Set groups together aspects of target destruction, such as risk to allied forces and affected non-combatants, political and environmental considerations, and obstacles to later use of targeted facilities after allied forces have possession of the surrounding territory. These are supported in the concept of vulnerability as expressed in AFI 14-117 [1997: 4] and also in the principles of Mass and Economy of Force [JP 3-0, 1995: A-1]. In Army terms, a target is only a high-payoff target if it has a high payoff in relation to its costs or disadvantages. The decision-makers at the joint targeting agency confirmed and supported these three top-level values, and verified that they cover
everything of interest to the commander in selecting the best target set. Furthermore, the experts confirmed that the concepts included in each of these top three values were distinct from one another, so no important value of a target set was covered under more than one of the top three values.

The measures under Value of Remaining Network are grouped into three categories: Quality of Best Remaining Routes, Remaining Functionality, and Recovery Time. Quality of Best Remaining Routes captures the opposition forces' capability to carry on their forward movement through the network to the assembly area. Remaining Functionality addresses the overall capacity of the remaining network for future convoys assuming complete knowledge of the new reduced network status and configuration. The third consideration, Recovery Time, indicates the time expected before reconstruction and repair crews would likely be able to restore the network to its original configuration, by restoring the destroyed targets to full operation. Within the Value of the Remaining Network, the panel of decision-makers confirmed that these three sub-values covered everything of interest in assessing the remaining network value, and that the three sub-values did not overlap in the concepts they measured.

Because this model will be used during times when data might not be abundant, Additional Benefits has only one measure. Future work will likely expand evaluation to multiple disciplines of infrastructure networks. The Additional Benefits value seeks to account for the target value in degrading infrastructures other than the LOC network. There is frequently operational emphasis on targets with "two-for-one" return, which accomplish multiple objectives with a single strike. Perhaps a bridge has a major communications link or power line running underneath it. Maybe dropping a bridge
would block a river, thus preventing barges from passing. This would disrupt river traffic as well as destroy the bridge. Of course, it is important to realize that targeting certain structures simply because of their appearance in two or more different infrastructure networks may in fact misstate their value to each network. Perhaps a target is relatively insignificant and redundant in each network, so its combined worth with respect to both networks is still small. If it is important enough to appear in the final approved target list for another infrastructure, however, it is useful to note that contribution when evaluating it an LOC context.

Finally, the objective Cost of a Target Set is divided into four categories:

Collateral Damage, Collateral Effects, Opportunity Cost, and Restoration Time.

Collateral Damage addresses the unintended damage associated with the destruction of a target. This damage might be to civilians, buildings, or sensitive areas of the environment in the immediate blast area. Collateral Effects differs from Collateral Damage in that it captures the combined effects of multiple targets on a single critical function of society, such as destroying all of several paths to a power or water supply. Opportunity Cost accounts for the total munitions expended as well as the risk to both delivery personnel and systems. Finally, when control of a target area is later in allied hands, it is desirable to minimize delay in restoring a capable logistics network as part of re-establishing an orderly society. Additionally, allied combat forces would prefer to be able to repair a needed bridge as easily as possible. The panel of decision-makers supported these sub-values as encompassing everything of interest in evaluating the cost of a target set. Furthermore, the panel confirmed the measures underneath each sub-value in Cost of Target Set were mutually exclusive. The four categories Collateral
Damage, Collateral Effects, Opportunity Cost and Restoration Time do not overlap in the concepts measured.

The two measures under Quality of Remaining Routes measure the delay of enemy forces and the target-richness of the environment after destruction of the targets. The panel of decision-makers agreed that these are fundamentally different concepts, since one is concerned with arrival time due to destruction of routes, while the other seeks to create bottlenecks for attacking forces themselves.

The four measures under Collateral Damage were also developed to cover all the relevant aspects of collateral damage valued by the commander. The Max Non-combatant Casualties at a single target is focused on the local impact of high casualties in a single area, while Total Casualties focuses on the value of civilian lives in the blast area of targets across the entire theater. These two are fundamentally different from each other and from the other two measures: Sensitive Structures and Environmental Impact. Sensitive Structures focuses on the damage to buildings with cultural or social significance, whereas Environmental Impact is a broad assessment of damage to water, soil, air, wildlife and vegetation, apart from their social or cultural impact.

The Opportunity Cost under Cost of Target Set is comprised of two measures, each measuring a different aspect of opportunity cost. The basic difference is between things designed to come back and weapons designed to be expended in the attack. Again, the panel of experts confirmed these measures covered the main areas of value to the commander, with no overlap between them.

The measures were constructed to be mutually exclusive and collectively exhaustive, as has just been demonstrated. The remaining requirement for an additive
model is that measures must be preferentially independent, as defined in Chapter 2. This property holds that the level of achievement in one attribute does not change the decision-makers' preference for achievement in the other attributes. In the elicitation process, care was taken to ensure this as well. Each value function was elicited without respect to the particular attribute levels of other metrics. Although correlation was expected between related attributes in the set of feasible alternatives, in no case does preference for an objective change direction or shape, so preferential independence was preserved.

The justification outlined above demonstrates that the model satisfies the requirements for an additive value model. Those requirements are that values and measures be mutually exclusive and collectively exhaustive within each level, and that mutual preferential independence exists among all measures. The reader is referred to Appendix A for a detailed explanation of metrics, their associated value functions, and
the rationale presented for their particular shape. To illustrate the process of constructing a measure, the measure Cumulative Delay is chosen for explanation.

3.3.3 Sample Measure: Cumulative Delay

Cumulative Delay measures the additional time expected before enough opposition forces assemble to be operationally effective. For this scenario, the experts agreed that operational effectiveness would be assumed to be 80% of opposition forces present at the assembly area. This question was especially interesting, since at least one person present was an expert on the likely nation’s combat force structure and communicated that they like to have two divisions participating in an attack and a third in reserve. If the third division were still on its way and expected to arrive reasonably soon, a commander might engage with only two of his divisions (66% of his force) actually on the scene. However, since in this scenario there are only two divisions in the convoy, the situation should be viewed at the division level rather than the theater commander level. Therefore in this case 80% of the division should be present before it is operationally capable.

The x-axis for this natural direct measure has a range of 0 to 50 hours. The upper bound of 50 is derived from the commander’s objective of a 72-hour time window before opposition force arrival, with a 22-hour baseline travel time under the original network configuration. The upper bound assigns no additional value or benefit to delaying forces beyond the requested 72-hour window. As with all the measures in this study, the panel was careful to make this measure flexible enough to model the commander’s preferences in almost any strategic situation. Construction of the value function was in almost all
cases a quicker and simpler process than deciding on the measure, so future situations and scenarios should have a greatly reduced development time for the model.

In an exponential value function such as the one used here, \( p \) is a parameter that determines the direction and rate of change in the slope [Kirkwood, 1997: 66]. Given a direction of preference, the decision-maker can specify the value of some quantity of the x-axis between the two extreme points to establish the curve. The parameter \( p \) is then determined by solving an equation using three points. In this case, the panel of experts determined 20 hours had zero value because if opposition forces are delayed less than 20 hours the allies are expected to lose the engagement. Since the commander wanted 72 hours delay and additional time had no marginal value, the score of 50 was assigned a value of 1. They concluded not every hour has the same associate marginal value however, and in particular the last 10 hours have the same marginal increase in value as the 20 previous hours. In other words, achieving 40 hours of delay is worth half as much as the full 50 hours of delay. Given these three points at (20,0), (40,0.5), and (50,1), the parameter \( p \) is determined from the equation:

\[
1 - \exp\left[\frac{40 - 20}{p}\right] = 0.5
\]

\[
1 - \exp\left[\frac{50 - 20}{p}\right] = 0.5
\]

The solution is to let \( p = 69.7 \). Further explanation of the exponential value function is found in Kirkwood [1997: 64-67]. Figure 11 shows the value function formula, followed by its graphical representation.
\[ V(x) = \begin{cases} 
0 & \text{when } 0 \leq x \leq 20 \\
\frac{1 - \exp\left[-\frac{(x - 20)}{69.7}\right]}{1 - \exp\left[-\frac{(50 - 20)}{69.7}\right]} & \text{when } 20 < x < 50 
\end{cases} \]

**Figure 11. Value Function of Cumulative Delay in Hours**

The entire set of value functions is summarized in Table 3, with minimum and maximum values and direction of single-dimensional value as the x-axis level increases. An increasing value function indicates more of the attribute is preferred, while a decreasing value function indicates less of the attribute is preferred. The reader will notice the first five measures (measuring benefits) generally have increasing functions, and last eight measures (measuring costs) all have decreasing functions. The only exception is the maximum flow, in which the measure is the percentage of maximum flow remaining. If the network does not have any remaining flow, the interdiction has been successful and total, which is highly valued. If all of the flow remains, the target set has not damaged this aspect of the network, which is an undesirable outcome.
Table 3. Measure Ranges and Direction of Change in Value

<table>
<thead>
<tr>
<th>Top Obj</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Measures</th>
<th>Units</th>
<th>Min Level</th>
<th>Max Level</th>
<th>Direction of Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Quality of Best Remaining Routes</td>
<td>Cumulative Delay</td>
<td>Hours</td>
<td>0 (20)</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Density of Choke Points</td>
<td>Choke Points</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Remaining Functionality</td>
<td>Max Flow Remaining</td>
<td>Percent</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Recovery Time</td>
<td>Span Length</td>
<td>Meters</td>
<td>0</td>
</tr>
<tr>
<td>Value of Remaining Network</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Additional Benefits</td>
<td></td>
<td></td>
<td>Targets in Another Infrastructure's Target List</td>
<td>Targets</td>
<td>Targets</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Cost of Target Set</td>
<td></td>
<td></td>
<td>Max Non-Combatant Casualties at Single Target</td>
<td>Casualties</td>
<td>Casualties</td>
<td>None</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Total Non-Combatant Casualties</td>
<td>Targets near Sensitive Structures</td>
<td>Targets</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Targets with Significant Environmental Impact</td>
<td>Targets with Significant Environmental Impact</td>
<td>Targets</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>Collateral Damage</td>
<td></td>
<td></td>
<td># of Sensitive Services Affected</td>
<td>Ordnance Expend</td>
<td>Services</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Collateral Effects</td>
<td></td>
<td></td>
<td></td>
<td>Expected # of Strike Packages</td>
<td>Munitions</td>
<td>30</td>
<td>180</td>
</tr>
<tr>
<td>Opportunity Cost</td>
<td></td>
<td></td>
<td></td>
<td>Packages</td>
<td>Packages</td>
<td>1</td>
<td>30</td>
</tr>
<tr>
<td>Restoration Time</td>
<td></td>
<td></td>
<td>Max Time to Rebuild</td>
<td>Months</td>
<td>Months</td>
<td>0</td>
<td>9</td>
</tr>
</tbody>
</table>
3.3.4 Weighting Method

As mentioned earlier, weights represent the relative importance of the target set attributes with respect to the attribute ranges. All the weights must sum to 1, so relative importance is indicated by relative size of the weights. For example, a weight of 0.24 placed on the Cumulative Delay measure and a weight of 0.12 on the Strike Packages measure would together indicate that a change from worst to best in Cumulative Delay is twice as valuable to the decision maker as a change from worst to best in Strike Packages. Elicitation with experts in their respective areas of the hierarchy produced the equalities shown in Table 4. To produce the first comparison in Table 4, the expert judged that he would be indifferent between similar two alternatives, one of which delayed the enemy divisions for 30 hours but produced no chokepoints, while the other differed only by delaying the enemy divisions only 20 hours but producing 6 chokepoints per 100 km. Using the value functions available in Appendix A, weights are determined by solving systems of equations of the form

\[ \Delta V_1 \times W_1 = \Delta V_2 \times W_2 \]

together with the equation

\[ \sum_i W_i = 1 \]
Table 4. Comparisons of Value Increments for Weight Computation

<table>
<thead>
<tr>
<th>Benefits</th>
<th>Costs</th>
<th>Benefits v. Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The value increment associated with a change in the x-axis</strong></td>
<td><strong>The value increment associated with a change in the x-axis</strong></td>
<td><strong>The value increment associated with a change in the x-axis</strong></td>
</tr>
<tr>
<td>From</td>
<td>To</td>
<td>in Measure</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
<td>Cum Delay</td>
</tr>
<tr>
<td>20</td>
<td>35</td>
<td>Cum Delay</td>
</tr>
<tr>
<td>20</td>
<td>25</td>
<td>Cum Delay</td>
</tr>
<tr>
<td>22</td>
<td>150</td>
<td>Max Span</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>Sensitive Structures</td>
</tr>
<tr>
<td>180</td>
<td>30</td>
<td>Ordnance Expended</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
<td>Total Casualties</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>Sensitive Services</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>Sensitive Services</td>
</tr>
<tr>
<td>100</td>
<td>30</td>
<td>Total Casualties</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>Max Time to Rebuild</td>
</tr>
<tr>
<td>20</td>
<td>40</td>
<td>Cum Delay</td>
</tr>
</tbody>
</table>

After minimizing inconsistency in the equalities in Table 4 using the value functions from Appendix A, the weights were determined, as shown in Table 5. The *local* weight refers to the weight of a consideration relative to its fellow considerations under the same parent consideration (the consideration directly above it in the hierarchy). Reading from the left side in Table 5, more general values are broken out into more specific values. Bold lines help identify a consideration more clearly with its direct parent and sibling considerations. Each number other than the right-most column
<table>
<thead>
<tr>
<th>Target Set Value</th>
<th>Cost of Target Set</th>
<th>Value of Remaining Network</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Local Weights</th>
<th>Measures</th>
<th>Global Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.68</td>
<td></td>
<td>0.84</td>
<td>Cumulative Delay</td>
<td>0.246</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.16</td>
<td>Density of Choke Points</td>
<td>0.047</td>
</tr>
<tr>
<td></td>
<td>Additional Benefits</td>
<td></td>
<td>1.00</td>
<td></td>
<td>1.00</td>
<td>Max Flow Remaining</td>
<td>0.080</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td>Span Length</td>
<td>0.060</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Targets in Another Infrastructure's Target List</td>
<td></td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Max Non-Combatant Casualties at Single Target</td>
<td></td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Total Non-Combatant Casualties</td>
<td></td>
<td>0.118</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Targets near Sensitive Structures</td>
<td></td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Targets with Significant Environmental Impact</td>
<td></td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td># of Sensitive Services Affected</td>
<td></td>
<td>0.064</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Ordnance Expended</td>
<td></td>
<td>0.089</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Expected # of Strike Packages</td>
<td></td>
<td>0.123</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Max Time to Rebuild</td>
<td></td>
<td>0.077</td>
</tr>
</tbody>
</table>
represents the local weight of each value to the number's immediate right within the value to its immediate left. For example, the measures under Collateral Damage contribute 35 percent of the value of the Cost of Target Set.

The global weight of a measure is its quantitative priority relative to all the other measures in the hierarchy. A global weight may be computed for any consideration by taking the product of all the local weights between it and the top objective. In Table 5, then, global weights in the right hand column are equal to the product of the numbers to the left. For example, the global weight for Total Non-combatant Casualties (11.8%) is calculated as 55% × 35% × 61%. Since exact weights were calculated from the comparisons in Table 4, calculations based on the level of precision shown in Table 5 may not yield identical results.

Value increment comparisons were elicited from two sets of experts, one with logistics expertise for weighting the section of the hierarchy under Value of Remaining Network and Additional benefits, and the other with command weaponeering experience for the Cost of Target Set section of the hierarchy. The assignment of top-level weights between costs and benefits was done by comparison of the highest weighted measure from each sub-hierarchy.

A pie chart of the global weights is shown in Figure 12. The largest weights are apparent. Since the commander's stated goal was to delay the two approaching divisions by 50 hours, it is natural that the Cumulative Delay should predominate. The next most important measure, Strike Packages, accounts for the risk to allied pilots and the weapons platforms, and is described in detail in Appendix A. Its heavy weighting conforms intuitively to the decision-maker's strong desire to minimize the risk the lives of his or
her troops while meeting the mission objective. Third in weight is Total Non-combatant Casualties, again highlighting the decision-maker's high value for innocent human life in its range of 0 to 100 civilian casualties. The smallest weight is assigned to environment, flowing from the idea that a target with large environmental damage would be banned from the list of possible targets, so targets with some small environmental impact were not as heavily penalized. This low weighting is also explained by the fact that it is not yet rigorously measured.

Figure 12.  Global Apportionment of Weights to Measures
3.4 Evaluation Model

Appendix A contains the value function for each measure, with accompanying description and rationale for the unique shape of the value function.

The total value of a target set alternative may be found by the following equation, where \( x_i \) is the \( i^{\text{th}} \) attribute of the alternative, \( V_i(x_i) \) is the value associated with the alternative’s score in that measure (i.e. with the \( i^{\text{th}} \) attribute), and \( W_i \) is the global weight apportioned to the \( i^{\text{th}} \) measure in the value model.

\[
Value_{\text{Target Set}} = \sum_{i=1}^{13} V_i(x_i) \cdot W_i
\]

Table 6. Full Evaluation of One Target Set

<table>
<thead>
<tr>
<th>Top Obj</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Measures</th>
<th>Unit</th>
<th>Global Weight</th>
<th>Score</th>
<th>Value</th>
<th>Wtd Value</th>
<th>Wtd Value by Top Considerations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Value of Remaining Network</td>
<td>Quality of Best Remaining Routes</td>
<td>Cumulative Delay</td>
<td>Hours</td>
<td>0.246</td>
<td>33.32</td>
<td>0.279</td>
<td>0.068</td>
<td>Value of Remaining Network 0.200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Density of Choke Points</td>
<td>Choke Points</td>
<td>0.047</td>
<td>0.5</td>
<td>0.300</td>
<td>0.014</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Remaining Functionality</td>
<td>Max Flow Remaining</td>
<td>Percent</td>
<td>0.080</td>
<td>13.34</td>
<td>0.867</td>
<td>0.070</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recovery Time</td>
<td>Span Length</td>
<td>Meters</td>
<td>0.060</td>
<td>100</td>
<td>0.800</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Additional Benefits</td>
<td>Targets in Another Infrastructure's Target List</td>
<td>Targets</td>
<td>0.021</td>
<td>0</td>
<td>0.000</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recovery Time</td>
<td>Span Length</td>
<td>Meters</td>
<td>0.060</td>
<td>100</td>
<td>0.800</td>
<td>0.048</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Casualties at Single Target</td>
<td>Level</td>
<td>0.049</td>
<td>high</td>
<td>0.000</td>
<td>0.000</td>
<td>Value of Remaining Network 0.200</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Targets near Sensitive Structures</td>
<td>Targets</td>
<td>0.024</td>
<td>1</td>
<td>0.871</td>
<td>0.021</td>
<td>Additional Benefits 0.000</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Targets with Significant Environmental Impact</td>
<td>Targets</td>
<td>0.002</td>
<td>0</td>
<td>1.000</td>
<td>0.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Collateral Effects</td>
<td># of Sensitive Services Affected</td>
<td>Services</td>
<td>0.064</td>
<td>2</td>
<td>0.667</td>
<td>0.042</td>
<td>Cost of Target Set 0.301</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Opportunity Cost</td>
<td>Ordnance Expended</td>
<td>Munitions</td>
<td>0.089</td>
<td>80</td>
<td>0.833</td>
<td>0.074</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Restoration Time</td>
<td>Max Time to Rebuild</td>
<td>Months</td>
<td>0.077</td>
<td>4.5</td>
<td>0.251</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cost of Target Set</td>
<td>Collateral Services</td>
<td>Services</td>
<td>0.064</td>
<td>2</td>
<td>0.667</td>
<td>0.042</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Ordnance Expended</td>
<td>Munitions</td>
<td>0.089</td>
<td>80</td>
<td>0.833</td>
<td>0.074</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Expected # of Strike Packages</td>
<td>Packages</td>
<td>0.123</td>
<td>5</td>
<td>0.862</td>
<td>0.106</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Max Time to Rebuild</td>
<td>Months</td>
<td>0.077</td>
<td>4.5</td>
<td>0.251</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Value</td>
<td>1</td>
<td>0.501</td>
<td>0.501</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
In this scenario, attribute levels have been assumed deterministic, although uncertainty and risk aversion could certainly have been dealt with if desired. The methodology used here also is valid under uncertainty, but for now each target set alternative assumes a single point estimate for each attribute. For further detail in modeling preferences under uncertainty, see Section 9.3 of Kirkwood [1997: 245]. For an example of the calculation of a composite value, the attribute levels for Target Set 1, the corresponding single-dimensional values, and composite value for Target Set 1 are shown in Table 6.

3.5 Estimated Number of Strike Packages

The Estimated Number of Strike Packages is the 12th measure. The method of scoring was based on the premise that any group of targets within some specified proximity to each other could be organized into a single strike package. Within that proximity threshold, a single package of aircraft can attack up to 10 targets, thus reducing the risk to both platforms and personnel involved in the attack by eliminating excessive exposure to air defenses. The problem is to find the minimum number of clusters in a target distribution such that no cluster is more than 30 nautical miles (nm) in diameter (i.e. every pair of targets in a cluster is separated by no more than 30 nm), and no cluster contains more than 10 targets. In this case the measure of cluster distance is complete-linkage distance, where a target is part of a cluster only if is within 30 nm to every other target in the cluster [Godehardt, 1990: 53; and McQuitty, 1987: 19].

While the literature survey included methods for partitioning data into clusters by similarity, it was observed that hierarchical methods did not solve the problem just stated.
Figure 13 illustrates the difficulty. An agglomerative clustering procedure will join the middle two nodes first (d=10) and then join that cluster to one of the extreme nodes (d=35) and then join all four nodes (d=60) into a cluster. Disaggregating procedures will separate one of the two extreme nodes first (d=35), and then the other (d=10), before finally separating the middle two nodes (d=0). In neither case are the left two nodes ever clustered together, with the right two nodes put in another cluster, which is the minimum partitioning with d<30.

![Diagram of clustering problem with hierarchical procedures]

Figure 13. Clustering Problem with Hierarchical Procedures

Although optimization procedures could be adapted to solve the problem, computing time could quickly outgrow available computing resources. Conversely, improvement algorithms fail in the same situation as hierarchical procedures, as demonstrated above, so a better heuristic is needed. In the interest of contributing to the already-vast expanse of clustering literature, a graph theoretical complete-linkage algorithm is described in this section and an implementation is given in Appendix B. While it is still non-polynomial in computational time to solve for optimality, it uses two techniques to reduce the size of the problem, increasing the effectiveness of the heuristic solution.

If targets are given in Cartesian coordinates, a distance matrix may be easily calculated and links assigned to every pair of targets with mutual distance less than the threshold of 30 nautical miles (nm). The minimum number of clusters is then equal to 52.
the minimum number of disjoint cliques which covers all the nodes. This problem is seen
to be equivalent to finding the minimum number of independent sets in the graph
complement [West, 1996: 3]. As noted in Chapter 2, the problem of finding the
minimum number of independent sets in a graph is also known as the coloring problem
[West, 1996: 174]. Although the optimal solution cannot be assured by any polynomial-
time algorithm, several techniques may be applied to the original graph to make a
coloring heuristic more effective. That is, reduction of the original problem into smaller
sub-problems can enable a coloring heuristic to be applied to much simpler graphs and
thus to actually achieve optimality with greater frequency. Figure 14 illustrates this
process on a graph with one component. The colors given do not represent the only
possible coloring, but merely one possible coloring using the minimum of three colors.

Original Graph

![Original Graph](image)

Colored Complement

![Colored Complement](image)

Figure 14. Coloring the Complement of a Graph

The first of these techniques is to look at individual connected components as
candidates for strikes or groups of strikes before taking the complement. This may
reduce the size of the sub-problems significantly since the groups of targets in this scenario are likely to be separated by more than 30 nm width between groups. Each individual component may then be tested to see if the component forms a clique. If the component is not a clique, it may be further manipulated in a second technique.

This second technique is to identify nodes of degree 1 in the component. The degree of a node is the number of links incident to it. In real terms, this means finding the number of targets that are within 30 nm of only one other target. Such a target may be paired with its neighbor and considered a single strike package without increasing the minimum number of packages or clusters, as will be proved.

Theorem: There exists a minimum partition of a graph into cliques such that an arbitrary degree-1 node is in the same clique as its neighbor.

Proof: Let g be a graph whose nodes may be covered by a minimum of c disjoint cliques. Without loss of generality, let x be the 1-degree node with the lowest index and let its neighbor be y. Let C_y contain the nodes in the clique to which y belongs under the optimal graph partitioning into cliques. Clearly C_y must contain more than 1 node, since if it were a package by itself we could reduce the number of packages by combining it with x in a 2-clique. Then, the induced subgraph on C_y remains a clique if y itself is deleted [West, 1996: 3]. That is, the deletion of any node from a clique on n nodes must leave a clique on n-1 nodes, given n>1. Hence, deleting node y from clique C_y leaves a clique of at least cardinality one. Since y is x's only neighbor, the optimal clique partition must either pair x with y or designate x as a package by itself. If the former is true, we are done.
Otherwise, we now package $x$ with $y$ and leave the remainder of $C_y$ still a single package. The total number of packages, $c$, is unchanged.

The process of removing 1-degree nodes with their neighbors is continued on each component until no such nodes remain. At this point, the remainder of the former component may no longer be connected, and may not even exist. If a part of the component still remains and that remainder is connected, the complementation and coloring process begins. If it remains but is in multiple components, each piece may be complemented and colored separately. If no nodes remain, the algorithm proceeds to the next component. When the last component has been reduced by pairs, complemented and colored, the total number of strikes is reported. Illustrations of each of these techniques are shown in Appendix B.

Again, these two techniques are included to augment and speed up the heuristic, but are not necessary for the heuristic's theoretical validity. Furthermore, this algorithm is not the process used to score the 10 alternatives used in this scenario, but is one of the possible approaches to cluster computation in future applications of this research. The Mathematica code and brief re-explanation are included in Appendix B.

3.6 Summary

This chapter has outlined the methodology used for this thesis, beginning with the project framework to clarify the sources and use for various aspects of the model. Unveiling of the model progressed from a brief scenario description through the value hierarchy to elicitation of value functions and weights, and finally to the computation of
composite value for each alternative. Possible methods of clustering were discussed as a highlight of the score computation process.
Chapter 4. Scenario Results

For the scenario described in Chapter 3, the joint targeting agency generated 10 alternative target sets and provided the data and information needed to calculate scores for all the metrics in the hierarchy. The lists of targets in the sets and corresponding target set raw scores are given in Tables 7 and 8. The 72 targets covered in the 10 target sets were plotted in Figure 9 in Chapter 3. Table 7 shows the distribution of target set sizes, and is provided as a reference for later analysis. Intuitively, one might expect the smaller target sets like 4, 5, and 8 to be valued higher on costs while larger sets like 2 and 3 should probably achieve a higher level of benefits value.

Table 7. Ten Target Set Alternatives

<table>
<thead>
<tr>
<th>Number of Targets in the Set</th>
<th>Target Set Alternatives</th>
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<tbody>
<tr>
<td>16</td>
<td>1725</td>
</tr>
<tr>
<td>23</td>
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<td>14260</td>
<td>14260</td>
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</table>
Table 8. Target Set Data

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<tr>
<th>Top Obj</th>
<th>Value of Remaining Network</th>
<th>Target Set Value</th>
<th>Cost of Target Set</th>
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</thead>
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<tr>
<td>Level 2</td>
<td>Quality of Best Remaining</td>
<td>Collateral Damage</td>
<td>Collateral Effects</td>
</tr>
<tr>
<td>Level 3</td>
<td>Remaining Functionality</td>
<td>Non-Combatant</td>
<td>Opportunity Cost</td>
</tr>
<tr>
<td>Level 4</td>
<td>Recovery Time</td>
<td></td>
<td>Restoration Time</td>
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</table>

<table>
<thead>
<tr>
<th>Measures</th>
<th>Cumulative Delay</th>
<th>Density of Choke Points</th>
<th>Max Flow Remaining</th>
<th>Span Length</th>
<th>Targets in Another Infrastructure's Target List</th>
<th>Max Casualties at Single Target</th>
<th>Total Casualties</th>
<th>Targets near Significant Structures</th>
<th>Targets with Significant Environmental Impact</th>
<th># of Sensitive Services Affected</th>
<th>Ordnance Expended</th>
<th>Expected # of Strike Packages</th>
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</thead>
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<tr>
<td>Units</td>
<td>Choke Points</td>
<td>Percent</td>
<td>Meters</td>
<td>Targets</td>
<td>Level</td>
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<td>Targets</td>
<td>Targets</td>
<td>Targets</td>
<td>Services</td>
<td>Munitions</td>
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<td>Scores</td>
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<td></td>
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<td>1</td>
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<td>54</td>
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</table>

58
**Title and Subtitle**
Multi-objective Evaluation of Target Sets in a Logistics Network

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**Performing Organization Report Number**
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**Abstract**
This thesis addresses the selection of target sets to disrupt an adversary's logistics network in a wartime environment. In the presence of many objectives—such as reducing maximum flow, lengthening routes, avoiding collateral damage, all at minimal risk to our pilots—the problem of determining the best target set is complex. Previous efforts have not adequately considered the value of the remaining network functionality after target destruction. In addition, current network targeting procedures optimize target sets with respect to only a single metric. This thesis uses a multi-objective decision analysis framework capturing actual targeting decision-maker values and preferences to evaluate and analyze 10 alternative target sets. Sensitivity analysis and persistency analysis on the results give insight as to how to select better target sets to meet stated strategic objectives.

**Subject Terms**
decision analysis, multi-objective, targeting, logistics interdiction, clustering, partitioning, sensitivity analysis, persistency analysis, cost/benefits analysis, Value-Focused Thinking, network disruption

**Security Classification of:**
Unclassified

**Limitation of Abstract**
UL

**Number of Pages**
154
Table 8 contains a spreadsheet representation of the hierarchy, and shows the scores of each target set in all 13 measures of the hierarchy. It will be useful as a reference for target set ranking and other analysis. The ranges can be quickly compared to the possible ranges of levels provided in Table 3. Note the relative invariance of Span Length, Sensitive Structures, Environmental Impact and Strike Packages compared to the range of possible levels on each measure. The value function for each measure is shown in Appendix A. Because of the particular shapes of each value function, there will appear less variation in value for Span Length and Strike Packages among the 10 alternatives. In contrast, Sensitive Structures and Environmental Impact both concentrate their biggest changes in value near the low end of the attribute range, so more variation in value will be found among the target set alternatives in these two measures.

Calculation effort for each of these attributes also varies significantly. Cumulative Delay, Chokepoint Density and Maximum Flow would be calculated using traffic and road capacity data, and Chokepoint Density may also involve either simulation or some queuing theory. The two casualties measures are difficult to estimate in advance, although such estimations are made. Sensitive Structures and Environmental Impact requires some preliminary weaponeering to estimate the effective range of blast effects, while Collateral Effects calls for significant knowledge of a variety of military and civilian infrastructure and facilities in the region. The Strike Packages can be calculated using target coordinates. Ordnance Expended requires preliminary weaponeering. Both Time to Rebuild and Recovery Time might use the same set of raw data, although they would be based on two different sets of national resources and technology to judge the
time to repair bridges to some required use level. Leaving the raw scores and discussion of required data types, the analysis begins with the overall ranking of the 10 target sets.

4.1 Ranking

Figure 15 shows all 10 alternatives in addition to a perfect baseline alternative for comparison. The chart incorporates the cost-benefits weighting derived in the overall hierarchy. In this analysis, “benefits” of a target set are considered to be the two sub-hierarchies under Value of Remaining Network and Additional Benefits. The weight

![Total Value Chart](image)

**Figure 15. Target Set Value by Top Considerations**
distribution between costs and benefits is 45% for benefits and 55% for costs. Clearly
Target Set 9 is the highest ranked choice, with a composite value of 0.583, while the
Target Set 3 is the lowest ranked with a composite value of 0.388.

Figure 16 shows a similar graph, broken out by subcategories from the benefits
side of the hierarchy. It is evident that none of the alternatives does very well on the
highest-weighted category: Quality of Best Remaining Routes. That does not drive the
ranking however; it only assures the absence of any really high-value alternatives since
the model is additive. The worst target set is especially notable for its high Cumulative

<table>
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<tr>
<th>Benefits Value</th>
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<tr>
<td>0.00</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>8</td>
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<td>7</td>
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<td>5</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>Ideal Target Set</td>
</tr>
</tbody>
</table>

**Figure 16. Weighted Value for Benefits of Target Sets**

Delay relative to the other sets. However, since no set does better than 40.4 hours on the
50-hour delay objective, and since half the value in the delay measure is located in that
last 10 hours, the entire range of sets is severely limited on its total value. A universally poor performance on the highest-weighted measure means all the target sets can do no better than .87 of the full value, before even considering their performance on the other measures. Target Set 3's 28 hard targets bring down the Ordnance Expended value (from Table 8). Moreover, Set 3's general poor performance from all the variable cost measures is lowered even further by its failure to noticeably reduce even the Maximum Flow through the remaining network (66.7% max flow remaining).

Figure 17 shows a complementary bar chart to Figure 16, by displaying the main categories of cost measures. Alternative 9 does very well in the Ordnance Expended and Expected Number of Strike Packages, which is reasonable since it includes just 14 targets, all with relatively little hardening. Although it is not the best on Total Non-Combatant Casualties, it does avoid a "High" attribute level on Max Non-Combatant Casualties at a Single Target. It is observed that Target Sets 9, 10, and 8 are generally strong, without significant deficiencies in any of the cost-side considerations. Target Sets 4 and 5 are to be especially noted since they score very well under costs except in collateral effects. However, this illustrates the heavy value loss attached to the fourth Sensitive Service Affected by the attack, which occurs only in Target Sets 4 and 5. (See Section A.2.10 for the Sensitive Services value function.) In other words, the marginal value lost by the fourth sensitive service is more severe than earlier sensitive services because of the exponential value function. Target Set 3 does poorly throughout the Cost sub-hierarchy, keeping some value in Opportunity Cost only because the range is so large for Strike Packages and even Target Set 3 only requires 9 packages. The best target set
on that measure requires 3 packages, while the allowable range goes up to 30. (See Section A.2.12 for Strike Packages value function.)

Figure 17. Weighted Value for Costs of Target Sets

4.2 Cost v. Benefits

A Cost v. Benefits plot is a quick way to identify dominance of certain alternatives and identify alternatives on the frontier. Table 9 gives the localized values of each target set under the benefits and costs sides of the hierarchy, respectively. Based on those coordinates, the plot in Figure 18 shows the scattering of target sets by cost value and benefits value. The benefits value is given on a scale from 0 (the worst) to 1 (the
best), and the cost value is also on a scale from 0 (the worst) to 1 (the best).

Computationally, this is the overall benefits divided by the benefits weight to obtain the normalized benefits value, or the overall cost value divided by the cost weight to obtain the normalized cost value. In the following formulas, V and W are as described in Section 3.4, and i is the measure index, going from 1 (Cum Delay) to 13 (Restoration Time).

\[
Value_{\text{Target Set, Benefits}} = \frac{\sum_{i=1}^{5} V_i(x_i) \cdot W_i}{\sum_{i=1}^{5} W_i}
\]

\[
Value_{\text{Target Set, Costs}} = \frac{\sum_{i=6}^{13} V_i(x_i) \cdot W_i}{\sum_{i=6}^{13} W_i}
\]

In other words, Target Set 1 has a benefits value of 0.20 / 0.45 and a cost value of 0.30 / 0.55, drawing values from Table 5. The cost value axis is oriented this way because the fact that more is worse has already been accounted for in the individual measures. Therefore, those target sets with low actual costs have high cost value, and target sets with high actual costs have low cost value.
Table 9. Target Set Local Values in Benefits and Cost Sub-hierarchies

<table>
<thead>
<tr>
<th>Target Set</th>
<th>Benefits</th>
<th>Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.441</td>
<td>0.551</td>
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<tr>
<td>2</td>
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<td>0.452</td>
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<td>10</td>
<td>0.261</td>
<td>0.742</td>
</tr>
</tbody>
</table>

Given the entire value model, it is possible to think of the empty target set as representing the option to sponsor no attack at all. This could be thought of as the base-case or the status quo alternative, where the original network is left fully intact. This alternative would capture the entire value under costs, in that it would involve no risk, no collateral damage, no time to rebuild, and so forth. For the purpose of establishing a bound, the null target set can be assumed to have no benefits, although it is acknowledged there still might be some non-zero density of choke points. Thus the empty set is guaranteed all the value under the cost measures, and it can do no worse than none of the value under the benefits measures. The empty set may indeed have more than zero value under benefits in reality, but for this a fortiori argument it is sufficient to let it be bounded below. It then takes on a value equal the weight for Costs, which is 1 minus that of Benefits.
A scatterplot of the weighted benefits on the vertical axis and weighted cost value on the horizontal axis shows the distribution within the top-level values of the hierarchy. As before, the four measures under Value of Remaining Network and one under Additional Benefits are subsumed under the heading Benefits, while the remaining eight from the Cost of Target Set are taken to contribute to the Cost Value.

In the following discussion, the terminology "A dominates B" signifies that A is at least as good as B on both the cost value scale and the benefits value scale and better on one of the scales. It does not necessarily mean A is better on every measure within the sub-hierarchies. Figure 16, then, shows a clear dominance of certain alternatives.
regardless of the relative importance assigned to the Benefits Value. Both values have been normalized, so that the slope of an iso-value curve if equal weight were given to Benefits and Costs would be defined by a line through the points (0,1) and (1,0). It is immediately clear from the plot that alternatives 4 and 5, 8, and 10 are dominated by 9, and 2 and 7 are dominated by alternative 1, assuming fully deterministic scores for the named alternatives.

The weight assigned to Benefits that would make alternative 3 and 9 equal in value is found by swinging the weight of Benefits from its present value (0.45) to 1, with cost parameterized as 1 minus benefits weight, and noting where Target Set 3 is preferred to Set 9. The appropriate breakpoint on Benefits weight is found to be 0.75, while it is currently 0.45, inclining the line more clockwise so Set 9 is preferred. If the Benefits weight is 0.75, Target Sets 1, 3, 6, 7 and 9 are all essentially the same composite value. Along that line of equal preference, a higher level of benefits can be traded for higher cost value. Again referring to Table 7, Target Set 1 loses 7 hours of delay compared to Set 3, but has the advantage of reducing flow by almost a factor of 8, as opposed to Set 1’s reduction by only one-third from the original network flow capacity. In addition, Target Set 1’s biggest gains come in keeping the collateral effects modest compared to Set 3 and in saving over 50% of the munitions required for Target Set 3.

Another key observation follows from the discussion of the empty target set and the Cost versus Benefits chart in Figure 18. This observation is that weighting benefits at less than 0.39 causes the most preferred target set to be the empty set, amounting to no attack at all. Since the empty target set was set as a lower bound to the value of doing nothing, and it currently has a value of 0.55, equal to the weight on costs, it is clearly
superior to at least 9 of the target set alternatives. For the sake of the remaining analysis demonstrating the methodology, it will be assumed that the empty target set is not an option. The point is raised again only in Chapter 5.

4.3 Sensitivity Analysis of Key Uncertainties

The joint target agency has indicated that the data for two measures are particularly uncertain. The data used in these measure calculations, while realistic, is highly speculative for the particular target sets. These two measures are Total Non-combatant Casualties and Sensitive Services Affected, and sensitivity analysis on these two scores should be particularly helpful. As noted in Chapter 3, predicting the number of people likely to be within the blast area of a target far in advance of actual munitions delivery is not an easy task. Furthermore, the sponsoring organization confirmed that the Sensitive Services Affected by a target set might potentially involve significant guesswork and inference from data on the operations of normal civil society.

To demonstrate a methodology, a small change in the scenario is postulated. Suppose new intelligence indicates a likely shift in the refugee population in the areas surrounding targets 1886, 4459, 4460, 5264, 6801, and 9830. Reports indicate this shift will not cause the casualties at any one target to increase to a high level, but might increase the total for these targets by 40 or more. A net decrease is also possible due to the refugee movements out of those areas. In addition, intelligence also reveals some uncertainty as to whether target 1886 might be along the only remaining coal-supply route to a power plant. This power facility provides energy to several emergency facilities, but sources are not clear as to whether some of these facilities may have been
converted to enemy bases of operation. Given these circumstances it is possible that the sensitive services affected by Target Set 9 could move down to 0 or up to 3 or 4. Sensitivity analysis can reveal the degree of these changes that would make Target Set 9 no longer the most preferred target set. Since these particular targets in Set 9 are not in common with the second-ranked target set (Set 10), nor with the target set with the best performance in benefits (Set 3), it is possible to vary these scores only for Target Set 9. If the targets were in common, these three target sets would need to be reevaluated for new scores.

Sensitivity analysis reveals the changes in a given output value due to changes in particular input parameters, in this case the attribute levels of Total Casualties and Sensitive Services. Since the objective of concern for this study is the recommended target set, sensitivity analysis is performed only by varying the attribute levels of Target Set 9. However, since an additive model has been established, sensitivity analysis can be applied to any combination of input parameters on any alternatives, including the attribute levels of particular parameters or local or global weighting in the model. In the current weighting scheme, Target Set 9 is the top value by a significant margin. For this reason, sensitivity analysis is performed on both measures separately and then a two-way sensitivity analysis is performed with the two parameters varied simultaneously. The first two plots include the top two alternatives: Set 9 and Set 10.

Figure 19 shows sensitivity of total value to changes in Total Casualties of the Target Set 9, initially the top ranked target set. Target Set 9 currently reports 21 casualties, but drops to second place if that number is increased to 62 casualties.
Figure 19. Sensitivity Analysis on Total Casualties Caused by Target Set 9

The current Sensitive Services Affected score of Target Set 9 is 1 service affected. As the Sensitive Services Affected by Target Set 9 increases to 2, 3, and then 4, there is no change in rank, although at 4 services it is nearly equally preferred with Target Set 10, as Figure 20 demonstrates. The reader may note in both graphs that the path traced by the total value of Target Set 9 is, as expected, a scaled copy of the value function for the metric being considered. The single-dimensional value functions are in Figures 34 and 37 respectively in Appendix A.

The sensitivity analysis revealed that a sufficiently large change in Total Casualties of Target Set 9 results in a change in the top-ranked target set and a change in its Sensitive Services affected makes Set 9 nearly equal with Set 10. It is also possible to identify the two-dimensional region where Target Set 9 is the best alternative as well as
the region where another target set becomes best. Figure 21 shows those two regions. Since the second-ranked target set, Set 10, has a total target set value of 0.53, the decision-maker may be particularly concerned with the region in which Target Set 9 drops below that 0.53 value. The key region is in the upper right hand corner, and indicates that simultaneous changes in the two variables in a positive direction induce a change in the preferred alternative. For instance, from the current levels of 21 casualties and 1 sensitive service affected by Target Set 9, an increase of 25 total casualties and 2 sensitive services would be sufficient to change the preferred target set. Therefore, since this analysis shows the top ranking is sensitive to moderate changes in these two scores, it is recommended that further intelligence be gathered on the refugee movements and the facilities fueled by the power plant.
Figure 21. Two-Way Sensitivity Analysis (Strategy Region Graph) of Target Set 9 Composite Value

Figure 22 shows a one-way sensitivity analysis of the weight given to the benefits side of the hierarchy. Again, this side of the hierarchy includes both the Value of Remaining Network and Additional Benefits objectives. The plot was generated by parameterizing all the global weights in terms of the benefits weight and swinging the benefits weight from 0.2 to 1.0. This fully encompasses the reasonable range, since it was already observed the empty set is best for any weight below 0.39.

Figure 22 reveals a relatively low sensitivity of the top alternative to the weight on benefits, since it can increase past a 3:1 ratio to cost weight without changing the top rank. If a decision-maker values the benefits higher than that, however, Target Set 3 becomes the top-ranked target set. Target Set 3 has the greatest Cumulative Delay but also highest costs, and is the least preferred target set in the 45:55 weight distribution.
currently used. The correspondence of the top line frontier to the slope of the frontier in
the cost versus benefit plot in Figure 18 is apparent.

Figure 22. Weight Sensitivity for Benefits

Cumulative Delay is the only value used explicitly in the current target selection
process. With the addition of several aspects of cost, the current decision-maker input
was to weight costs at 55% of the value of the entire target set. However, a different
decision-maker might would value benefits differently than they are weighted in this
model. Therefore, sensitivity analysis is done on the benefits weight with the Total
Casualties for Target Set 9. Figure 23 shows various strategy regions as the benefits
weight and total casualties of Target Set 9 vary simultaneously. The curve indicates
changes in the top rank.
Similarly, since the number of Sensitive Services Affected by Target Set 9 appeared to be influential, one might be concerned about the possibility of a rank change if that score and the benefits weight vary simultaneously. In Figure 24, a similar format is shown with the vertical axis now representing Sensitive Services, again noting the various strategy regions on the graph. If benefits were weighted at only one-quarter of costs (benefits weight = 0.2) and Target Set 9 actually affected 3 services instead of 1, then 10 would become the preferred alternative, although only by a small margin. It
should be noted that between 0.65 and 0.75 for the weight of Benefits, Target sets 3, 9, and 10 are very close and should probably be considered equivalent.

Figure 24. Two-Way Sensitivity Analysis on Benefits Weight and Sensitive Services Affected by Target Set 9

Sensitivity analysis was also performed on each measure weight, where the other weights remained in proportion to each other. In Figure 25, the bars for each measure indicate the range over which the weight may vary with no change in the top-ranked target set. The stripe on each attribute's bar indicates its present weighting. It is evident that the top alternative is rather insensitive to changes in individual weights. The smallest range is in the Additional Benefits (Targets Duplicated) measure, where the current weight is 0.02. Weighting by a new decision-maker which gave more than .21 of the weight to Additional Benefits would put alternative 10 in the top position, assuming other weights were kept in proportion. For each attribute the weight was swung from zero to the point at which a change in the preferred target set appeared. The weights of all other measures stayed in proportion to each other, and together assumed the weight.
remaining after the attribute weight of interest was fixed. The six attributes for which the bar extends to a weight of 1.0 indicates the superiority of Target Set 9 in a broad range of areas. Setting a measure weight to 1.0 makes it the only attribute considered, so no change in rank indicates Set 9 is the best of all the target sets on that measure considered alone. Set 9 may be tied for the top rank in some individual measures.

Figure 25. Allowable Variation of Global Weights in which Target Set 9 Remains the Preferred Set

4.4 Persistence

As a further source of insight, an illustration of persistency analysis is included. The reader is advised that results should be taken as a demonstration of what could be done on a larger collection of target sets. Persistency is meaningless for a small number
of target sets. Nevertheless, it is perhaps instructive to consider a few ways persistency could be measured in order to suggest possible additions or omissions of individual targets that could further improve the value of the best target set.

The objective of persistency analysis is to discover the most important individual targets in meeting the commander’s objectives. Leinart [1998: 4-22] observed the recurrence of certain node and links in the top 20 target sets. He reasoned that the true relative importance or at least ranking of the individual targets should be correlated to their frequency of occurrence in the best target sets. Ideally, a target should be rewarded with a high persistency score if it is in a good target set, and, if applicable, might be penalized with a lower persistency score for being in a bad target set. This is different from previous efforts of persistency analysis in that Leinart looked for persistency among the top alternatives from over 9000 total alternatives. With only 10 alternatives, it would be meaningless to draw conclusions by looking only at the top 2 or 3 target sets, so the following calculations incorporate all 10 alternatives. Rather than counting the recurrence of targets in the better target sets, this work will use frequency of occurrence in both the better and the poorer target sets. The use of the terms “good target set” and “bad target set” are to be understood only in relative terms. The set of 10 alternatives is not necessarily a flawless representation of the population of target sets, and the worst of the 10 target sets may still in fact be a good target set. Conversely, the best target sets may actually be bad since they do not fully meet the commander’s objectives.
Table 10. Categorical Persistency by Composite Score Ranking

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<th>Middle 4</th>
<th>Last 3</th>
<th>Overall</th>
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A rough approximation to persistency is with a composite score that accounts for membership in good and bad target sets. Since membership in the middle four sets does not really declare a target to be either desirable or undesirable, a simple metric would entail subtracting its repeated membership in the bottom three sets from its repeated
membership in the top three sets. In other words, a target set in two of the top three sets and in one of the bottom three would be assigned a persistency of 1, regardless of its membership in the middle four sets. By this method, targets 1748, 4459, 5982, 6015, 6801, and 9023 rate the best, each with membership in two more of the best target sets than of the worst sets. On the other side, targets 1293, 2357, 2429, 2434, 2548, 2635, 2645, 2748, 2752, 2836, 5266, and 5348 rate the worst, each with membership in two more of the worst sets than of the best sets, as shown in Table 10. Shaded boxes indicate “bad” targets, while bordered boxes indicate “good” targets.

Upon further consideration, this seems an unnecessarily gross method of target rating, since the full target set ranking is readily available, not just three categories. Furthermore, since each target set has a total value on an interval scale, where the value difference between the top two sets (9 and 10) is far greater than between the next two (8 and 1), persistency could be assessed in terms of the target set values for a more accurate measure.

At this point two approaches present themselves. The first is to let a target set contribute to persistency based on its rank, and the other is to let it contribute based on its calculated value from the value model. For each of these options, it seems more insightful to separate benefits and costs once again, generating four values for each target.

<table>
<thead>
<tr>
<th>Table 11. Persistency Metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rank-based Benefits Score</td>
</tr>
</tbody>
</table>

79
The first two measures will take the form below, where \( i = 1 \ldots 72 \) is the target index, \( j = 1 \ldots 10 \) is the target set index, and \( R_j \) is the scaled rank of the target set. \( I_{ij} \) is an indicator variable set to 1 if target \( i \) is in target set \( j \), and 0 otherwise. Defining \( R_j = -2/9 \times j + 11/9 \) gives top target set a value of 1 and the bottom (the 10th) target set a value of -1. The other sets are evenly spaced between those values, based on their rank. For the generalized case of \( N \) target sets, the transform function is as follows. Again, \( j \) is the index of the target set.

\[
R_j = \frac{-2}{n-1} \cdot j + 1 + \frac{2}{n-1}
\]

For the first score, ranking is determined solely within the benefits section of the model, and for the second it is determined within the costs section of the model, but again in terms of cost value (more is better).

\[
\sum_j I_{i,j} \times R_j
\]

The last two scores in Table 11 use the actual scores of the target sets in their respective parts of the value model. The target set values are normalized so that the best target set is assigned a 1 and the worst is assigned a -1. The difference from the first two scores is that mediocre target sets need not be at regular increments within that range. In these
Table 12. Local Values Rescaled for Target Persistency Calculations

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methods 3 and 4 from Table 11, the interval scale of value is preserved, whereas methods 1 and 2 reduce it to an ordinal scale. Using the target set local values already presented in Table 9, the distributions are each scaled to better account for the undesirability of a target from its presence in a poorer target set. Rescaling the benefits values and the costs values so that the spread of benefits and costs are from -1 to 1, the scores for Table 12 are obtained.

For each target, then, a similar formula assigns benefit and cost scores to each target. In the formula below, \( i = 1 \ldots 72 \) is the target index, \( j = 1 \ldots 10 \) is the target set index, and \( V_j \) is the scaled value of the target set, once for benefits and once for costs.

\[
\sum_j I_{i,j} * V_j
\]

Each target now has a score from each of the four measures presented in Table 11, and targets are ranked within each measure, yielding different ranking structures. Table 13 shows the four scores for each target, arranged in numerical order of the target index.
Figure 26 details the distribution of targets under the first two methods (using the second and third columns of Table 13), based on target set ranking (ordinal scale), while Figure 27 shows their distribution under the last two measures (using the fourth and fifth columns of Table 13), based on target set value (interval scale).
Figure 26. Rank-based Target Persistency: Cost v. Benefits

Figure 27. Value-based Target Persistency: Cost v. Benefits
The targets establish a frontier, with the best targets appearing toward the upper right of the graphs while the worst targets appear toward the lower left of the graphs. Targets 4459, 1886, 4460, 5262 and 9918 stand out as particularly strong targets, with high benefits and low costs (corresponding to high cost value in their target sets and high cost persistency of the individual targets). The poorest targets are those five unique to Set 2, with relatively low benefits and high costs.

The reader will note that while in the rank-based plot the points appear evenly spaced around the line \(y = -x\), the value-based plot appears biased in the positive direction on both axes. This occurs because the distribution of target set scores under cost value and also under benefits value are skewed heavily in the positive direction. Furthermore the target sets given tend to complement high benefits with high costs (low cost value), so the targets in the "bad" target sets overall still tend to score well in benefits. This causes them to be pushed into the second quadrant. This follows well from the cost-benefit plot in Figure 18. If one draws a rectangle aligned to contain all the points but no more space than is needed, it is obvious that only Target Set 2 is in the third quadrant of the rectangle, so only Set 2 can subtract from a point's score in both the benefit and cost directions. Therefore, Target Set 2 must contain any targets in the third quadrant for two-dimensional persistency, and as it happens, only those points unique to Set 2 are found in the third quadrant. This is an insight not given by the rank-based persistency measures.

Since Target Set 8 does very poorly in benefits, it might be improved by adding some targets from the highest scores in benefits persistency. It is one of the smallest target sets now, which allows a rather impressive value for costs but also gives it a poor benefits value. It might be improved substantially by adding one or more of targets 1725,
5262 or 9918. Conversely, substituting target 1859 in high-benefits, high-cost Target Set 1 for its lower-persistency targets would be advisable. Deleting a mediocre target like 13133 from an already strong Target Set 10 would possibly improve its cost score without hurting benefits significantly.

While ranking revealed the best target set to be Set 9, persistency analysis picked out the most important and critical targets within that good target set. The key targets in the set are those along the frontier of Figure 25. These are targets 4459, 4460, 5262, 5264, and 9918. If the mission needs to be a one-strike operation, raising the probability of kill should be especially focused on these targets.

The caution is emphasized that this is not to be used as a substitute for full evaluation of a subset of a target set when the commander does not intend to hit the lower priority targets. As an example, if a new target is added to a target set along a path which was already interdicted by an original member of the target set, little improvement would be expected. A modified set must still be evaluated in the overall value model to see the marginal improvement, if any, but considering persistency helps to direct that search more intelligently.

4.5 Summary

This chapter has documented analysis of 10 alternatives. Cost-benefit plots yielded a clear picture of the tradeoffs inherent in this decision, complemented by a ranking and evaluation of target sets on an interval scale. Sensitivity analysis on individual weights, on benefits-cost weights, and on two uncertain attribute levels of the top alternative all reveal what level of parameter certainty must be attained in order to be
confident of correctly identifying the best target set. Finally, for further insight on constructing better alternatives, persistency analysis of individual targets enables a rough-cut ranking to be assigned to targets within a target set, based on their presence or absence in the higher- and lower-value target sets. Again, persistency is not presented for the purpose of drawing real conclusions for this scenario, but rather to suggest a methodology for use in larger problems with more target set alternatives.
Chapter 5. Further Research and Conclusion

5.1 Insights

This research has concentrated on improving the evaluation methods for target set selection in an enemy lines-of-communication network. Previous research has focused on the generation of targets, but has had trouble with proper aggregation of target characteristics into target set measures. The use of a realistic network from a high-level joint targeting agency gives the problem of this research greater operational relevance. In addition, the input from decision-makers and experts whose job it is to pick targets in LOC networks has been the basis for a multi-objective value model with wide applicability.

5.2 Insight for Example Scenario

Since care was taken to make the model true to axioms of decision theory, the final score is on an interval scale. This means that the value differences between total values of different alternatives are mathematically meaningful. It is observed from this that Target Set 9 is the highest value set by a large margin, while Target Set 3 is has a much lower value than the next poorest alternative. However, in the center of the value distribution, 1, 6, 7 and 8 are essentially equivalent in total value.

One of the most significant observations, however, is that the current weight assignment between benefits and costs makes the empty target set preferred to everything except Target Set 9. As Chapter 4 illustrated, the value of the empty set is assigned as a lower bound on its true value, so its high performance relative to the actual target sets
provided suggests the need for better target set generation techniques. The current method is not sufficient for generating "good" target sets relative to no interdiction attack at all. The author acknowledges that this empty set alternative is probably not feasible, and for this reason is not recommending it as a good alternative, but rather using it to motivate the search for feasible target sets that will achieve higher value in the model.

Another significant fact the decision-maker should know is that the primary objective of 50 hours cumulative delay was not met by any target set. Not only was it not met, however, it is not even approached by the top-ranked target set. Target Set 9 has a cumulative delay of 23.7 hours, although it is the second best at reducing the maximum flow, reducing it by a factor of 5.

Although the model is relatively insensitive to individual weights and even to the top level weighting of benefits v. costs, it is moderately sensitive to changes in the Total Casualties and the Sensitive Services Affected by Target Set 9, especially when they vary together. Under the special scenario developments of refugee migration and unclear intelligence reports on hospitals, the Target Set 9 could become less preferred. Sensitivity analysis on the two measures with the top-level weighting between benefits and costs revealed specific regions where Set 10 or Set 3 becomes the most preferred target set. If further intelligence reduces the uncertainty in one or more of these parameters, the new preferred target set can be read easily from the charts. By adding these possible real-world developments, the research illustrated how to address the new questions by testing the robustness of the top alternative. Lower-value target sets containing the same targets in question would also change value, but the direction of change would be the same. This would make a rank change with the top set unlikely,
however, so the analysis remains valid. The sensitivity analysis conducted for changes in the top ranked target set due to small changes in parameters demonstrates the power of the model for testing robustness of a target set.

Regarding the relatively small area of cost and benefits scores covered by the 10 target set alternatives, it should be noted that this smaller variability among the example target sets may serve to emphasize the versatility of the model. It should come as no surprise that several measures experienced rather little variability among the feasible target sets. Rather, the ranges were set in several cases purposely wider than will be ordinarily seen in order to assess the value for more extreme cases without changing the model. This advantage of application cannot be fully appreciated until it is applied beyond this demonstration. Furthermore, the flexibility of the model extends to a wide range of scenarios. The “maximum probable size of a target set” from several of the cost measures in Appendix B is a parameter that greatly increases the immediate adaptability to a different scenario with almost no revision effort needed for those measures.

The assumption of deterministic scores gives some clear dominance relationships with respect to benefits v. costs. Specifically, Target Sets 2, 4, 5, 8, and 10 should never be preferred since they are dominated by sets 1, 6, 7, and 9. If the decision-maker is indifferent between Sets 3 and 9 then he or she ought to consider 1, 6, and 7 as well, since they fall along the same frontier. If he or she would weight benefits any less than 3 times as important as costs then 9 is the clear best choice of the feasible alternatives. If benefits are weighted significantly more than 3 times the weight of costs then 3 is the clear winner of the 10 alternatives listed. However, since Set 3 is a distinct outlier both for its high benefits and its high costs, more high-benefit but lower cost solutions should be sought.
around the target set, since there may be better solutions around it that have not been sufficiently explored.

Persistency Analysis provided some illustrative insights by identifying targets particularly strong in both benefit and cost rankings of the target sets and suggesting them for inclusion in already-high-value target sets. Value-based persistency methods are seen to preserve an interval scale and yield superior insights regarding target distribution and the persistency frontier. Targets near the center of the frontier are obviously good choices but targets out in the wings of the frontier should also be considered. These might even be in the second and fourth quadrant to complement a target set with most of its targets in the opposite section of the distribution. In addition, certain targets along that frontier should give a good starting point for more intense searching within the target set space. This is because persistency has been stipulated as a reasonable proxy for a target's contribution to target set value. These best targets are 4459, 4460, 5262, 5264, and 9918, based on the interval-scale persistency measures suggested in Section 4.4. If Target Set 9 is selected for execution, greater priority should be assigned to these five critical targets for probability of kill planning. All this is said under the caveat that network effects are still non-linear relationships, and frequent occurrence in good target sets does not mean they will be good on their own. It is precisely the combined effects of targets in a network that prompted this research, and persistency is not designed to circumvent that. Rather, it is presented as a heuristic to give suggestions for further exploration of the solution space, and a way to prioritize targets within a target set.

In the area of organizing the value hierarchy, it is recommended that more effort be spent toward data-gathering on the environment measure so that it may begin to
influence the consideration of target sets in a real way. It is currently a rough, initial cut, but should be considered more seriously in future scenarios than current data allows.

5.3 Contribution of This Thesis to the Logistics Network Interdiction Subject Area

Previous recent efforts have applied the strengths of VFT to network theory, enabling a wide variety of network upgrade or disruption alternatives to be evaluated in a multi-objective framework. This research has improved upon those contributions:

- The research opens the door for improved target sets that may save lives, cost less, and accomplish more.
- It provides a basis for comparison in order to direct the search for alternatives in a smarter and more insightful manner.
- By addressing the value of the remaining network, this thesis accomplishes a more valid aggregation of the attributes into a network-oriented value. This value has been missed in previous work by concentrating on attributes of the targets rather than the desired outcomes in terms of the network configuration.
- The multi-attribute approach to the value of the remaining network allows several measures of quality and value to be quantified and included, whereas in the past they may have been inadequately addressed. Traditional optimization techniques such as network flow, shortest path, and cluster analysis are put to a new purpose, resulting in a value model that more accurately reflects the actual decision-makers’ objectives. The model integrates mathematically rigorous algorithms and other accepted tools into a single measure of value, thereby enabling application of the methodology to more complex problems. Just as importantly, by incorporating the decision-maker’s
trade-off values among all the concepts he or she considers relevant, the model engenders greater confidence in the results.

- The multiple types of cost considerations are made more explicit than in previous work, leading to more realistic trade-off values and high-level consideration of weapons effects. Furthermore, the measures used have been designed to transfer well to a variety of scenarios to further reduce the effort to apply this methodology to real situations. The fact that costs are now explicit also facilitates communication regarding the most important costs and how they compare to one another. This communication should reduce redundancy and counter-productivity that might result from unvoiced disagreement about the relative priority of these considerations.

- The cost measures also apply well to other types of infrastructure networks, being more generalized than some of the benefits measures. This should significantly reduce the workload of future studies into multi-discipline network value models and helping to keep the expanded model as small as possible and understandable to the decision-makers.

- Non-linear value functions accurately reflect the actual marginal changes in decision-maker preferences. This stimulates the search for more innovative and realistic aggregation methods, in contrast to the simple averaging prevalent in previous efforts. In other words, since the panel of decision-makers focused on what they would like to know rather than precisely how they might be able to get it, the model was value-focused and operator-oriented from start to finish. The need to obtain the sorts of information requested by the panel can motivate the entire organization of analysts and experts to pool their data and expertise.
• Finally, the multi-attribute model allows the decision-maker to test the robustness of alternatives by doing sensitivity and trade-off analysis. These techniques enable the decision-maker to concentrate effort on the parts of the model that have the greatest influence on the preferred alternatives.

These contributions will help the war-fighting community make more rapid and more precise targeting decisions. They also providing justification for those target set selections using the commander’s own expressed preferences and the situation-specific constraints and objectives.

5.4 Areas for Further Research

Since the contribution of this research has been in better aggregation techniques and incorporation of operational decision-maker preferences, the value model process has not been fully automated yet. In addition, several of the measures relied on calculation by the sponsoring organization, using equipment and processes with which the organization is already familiar. Future research could integrate the methodology presented here with the tools already present at the agency and make the input process more user-friendly.

In addition, the sponsoring organization has expressed significant interest in expanding this value model to include multiple disciplines of networks. While search time for good alternatives might expand faster than actual problem size, the evaluation process is expected to expand much slower than the problem size, since many measures are not specific to LOC networks. The basic structure of the hierarchy would require few
alterations, while new measures could be adapted to new network types to increase the power of the model over a range of infrastructure types.

One drawback of using network measures instead of exclusively attributes of individual targets is that finding good target sets is more difficult. Just as it is hard to account for the interactions and combined effects of target sets without a strong methodology like the one presented here, it is also difficult to select those target sets. Persistency and sensitivity analysis can focus and direct the search as this research demonstrates, but it would be even better to have a more systematic selection technique than is presently available. Sufficient methods for full cutset generation were surveyed in Chapter 2, but it remains to further research to establish a systematic method to select good sets which only degrade a network in a multi-attribute setting.

Finally, expansion on the persistency methods used here in further studies could enable smarter construction of the target sets themselves and prioritization of targets within each target set. While it is admitted that the sample size for the present analysis was insufficient to draw meaningful conclusions, the methodology is sufficiently powerful to yield valuable insights in future work.

5.5 Conclusions

This research improved the methodology developed by previous efforts, applying proven DA techniques to network optimization in a multi-objective framework. A realistic, notional problem was the context for this model, yielding insights and conclusions with immediate application and relevance. The solid basis of this research
should have direct benefits to the strategic targeting community and lead to greater capability and effectiveness in future targeting operations.
Appendix A. The Value Hierarchy

A.1 Explanation of the Value Hierarchy

As related in Chapter 3, the value hierarchy evolved out of doctrine and extensive discussion with experts and decision-makers who have a wide variety of operational experience, including air interdiction, targeting, weaponeering, and logistics. Although the hierarchy is in fundamental agreement with the relevant doctrine mentioned in Chapters 1 and 2, it is tailored to the specific scenario with regard to quantification of the value model. When possible, the model was designed to be generally correct for a broad spectrum of operational scenarios. The focus of the process is at the level of nominating options to the Joint Task Force Commander (JTF) or theater commander.
Figure 28. Full Value Hierarchy
The Hierarchy in Figure 28 shows the value structure agreed upon by the panel of experts involved in the study. The highest goal, to select the target with the highest overall value, is more explicitly given in the sub-objectives Value of Remaining Network, Additional Benefits, and Cost of Target Set. The Value of Remaining Network objective describes the characteristics of the network left after the targets in the set have been attacked. The concept of the remaining network was introduced in Chapter 1, and is supported by the doctrinal objectives of disruption and delay [JP 3-03, 1997: I-2]. Again, these imply effects, which in a network are not generally a linear combination of individual target attributes, so the object of concern must be the network left after the attack. Similarly, AFI 117-14 speaks of criticality [1997: 4], which necessarily implies the target is critical to something, so that the target’s destruction makes some measurable difference in the new status of the network.

Additional Benefits denote those possible benefits of a target set for other reasons than those captured in the remaining LOC network. This follows the principles of Mass and Economy of Force, in that multiple objectives are being addressed in a single strike [JP 3-0, 1995: A-1]. The Cost of Target Set groups together aspects of target destruction, such as risk to allied forces and affected non-combatants, political and environmental considerations, and obstacles to later use of targeted facilities after allied forces have possession of the surrounding territory. These are supported in the concept of vulnerability as expressed in AFI 14-117 [1997: 4] as well as the principle of Economy of Force [JP 3-0, 1995: A-1]. In Army terms, a target is only a high-payoff target if it has a high payoff in relation to its costs or disadvantages. In addition, each of these top-level values was confirmed and supported by the decision-makers at the joint targeting agency.
The measures under Value of Remaining Network are grouped into three categories: Quality of Best Remaining Routes, Remaining Functionality, and Recovery Time. Quality of Best Remaining Routes captures the opposition forces' capability to carry on their forward movement through the network to the assembly area. Remaining Functionality addresses the overall capacity of the remaining network for future convoys with complete knowledge of the new reduced network status and configuration. The third and last consideration, Recovery Time, indicates the time expected before reconstruction and repair crews would likely be able to restore the network to its original configuration, by restoring the destroyed targets to full operation.

For the sake of simplicity of use and because this model will be used during times when data might not be abundant, Additional Benefits has only one measure. Future work will likely expand evaluation to multiple disciplines of infrastructure networks, but a measure is needed to address the consideration in the meantime. The Additional Benefits value seeks to account target value in degrading other infrastructures than the LOC network. There is frequently operational emphasis on targets with "two-for-one" return, which accomplish multiple objectives with a single strike. Perhaps a bridge has a major communications link or power line running underneath it. Maybe dropping a bridge would block a river, thus preventing barges from passing. This would disrupt river traffic as destroying the bridge. Of course, it is important to realize that targeting certain structures simply because of their appearance in two or more different infrastructure networks may in fact misstate their value to each network. Perhaps a target is relatively insignificant and redundant in each network, so its combined worth with respect to both networks is still small. If it is important enough to appear in the final
approved target list for another infrastructure, however, it is useful to note that
contribution when evaluating it in an LOC context.

Lastly, the objective Cost of a Target Set is divided into four categories:
Collateral Damage, Collateral Effects, Opportunity Cost, and Restoration Time.
Collateral Damage addresses the unintended damage associated with the destruction of a
target, such as civilians, buildings, or sensitive areas of the environment in the immediate
blast area. Collateral Effects differs from Collateral Damage in that it captures the
combined effects of multiple targets on a single critical function of society, such as
destroying all of several paths to a power or water supply. Opportunity Cost accounts for
the total munitions expended as well as the risk to both delivery personnel and systems.
Finally, when control of a target area is later in allied hands, it is desirable to minimize
delay in restoring a capable logistics network as part of re-establishing an orderly society.
Additionally, allied combat forces would prefer to be able to repair a needed bridge as
easily as possible.
A.2 Measure Descriptions and Value Functions

Measures are organized in the order shown in the hierarchy, progressing downward and then to the right.

A.2.1 Cumulative Delay

Cumulative Delay measures the additional time expected before enough opposition forces assemble to be operationally effective. For this scenario, the experts agreed that operational effectiveness would be assumed to be 80% of opposition forces present at the assembly area. This question was especially interesting, since at least one person present was an expert on the likely nation’s combat force structure and communicated that they like to have two divisions present and a third in reserve. If the third division were still on its way and expected to arrive reasonably soon, a commander might engage with only two of his divisions (66% of his force) actually on the scene. However, since in this scenario there are only two divisions in the convoy, the situation should be viewed at the division level rather than the theater commander level. Therefore in this case 80% of the division should be present before it is operationally capable.

This natural direct metric is measured in hours with a range of 0 to 50 hours. The upper bound of 50 is derived from the commander’s objective of a 72-hour time window before opposition force arrival, with a 22-hour baseline travel time under the original network configuration. The upper bound assigns no additional value or benefit to delaying forces beyond the requested 72-hour window. As with all the measures in this study, the panel was careful to make this measure flexible enough to model the
commander's preferences in almost any strategic situation. Construction of the value function was in almost all cases a quicker and simpler process than deciding on the measure, so future situations and scenarios should have a greatly reduced development time for the model.

In an exponential value function such as the one used here, \( \rho \) is a parameter that determines the direction and rate of change in the slope [Kirkwood, 1997: 66]. Given a direction of preference, the decision-maker can specify the value of some quantity of the x-axis between the two extreme points to establish the curve. The parameter \( \rho \) is then determined by solving an equation based on three points. In this case, the panel of experts determined 20 hours had zero value because if opposition forces are delayed less than 20 hours the allies are expected to lose the engagement. Since the commander wanted 72 hours delay and additional time had no marginal value, the score of 50 was assigned a value of 1. They concluded not every hour has the same associate marginal value however, and in particular the last 10 hours have the same marginal increase in value as the 20 previous hours. In other words, achieving 40 hours of delay is worth half as much as the full 50 hours of delay. Given these three points (20,0), (40,0.5), and (50,1), the parameter \( \rho \) is determined from the equation:

\[
1 - \exp \left( -\frac{40 - 20}{\rho} \right) = 0.5
\]

\[
1 - \exp \left( -\frac{50 - 20}{\rho} \right)
\]
The solution is to let $p = 69.7$. Further explanation of the exponential value function is found in Kirkwood [1997: 64-67]. Figure 29 shows the value function formula, followed by its graphical representation.

$$V(x) = \begin{cases} 
0 & \text{when } 0 \leq x \leq 20 \\
1 - \exp \left[-\left(\frac{x - 20}{69.7}\right)\right] & \text{when } 20 < x < 50 \\
1 - \exp \left[-\left(\frac{50 - 20}{69.7}\right)\right] & \text{when } x \geq 50
\end{cases}$$

Figure 29. Value Function of Cumulative Delay in Hours
A.2.2 Density of Choke Points

Density of Choke Points assesses the susceptibility to attack. Susceptibility is the degree of danger opposition forces or convoys face from allied attack while traveling on the residual network after target destruction. The number of bottlenecks in a given length of route, then, is a measurable proxy for the susceptibility since a bottleneck creates a target-rich environment for the attack aircraft following the road. The natural proxy measure Density of Choke Points assesses the number of bottleneck locations per 100 km on all links used in a new maximum flow. A bottleneck is defined to be any point which experiences an increase in traffic by a factor of two or more with respect to the original network. The x-axis has a range of 0 to 6 choke points per 100 km. Discussion also included finding a percentage of routes with no vegetation and a 10% grade of incline or less, since these characteristics would be associated with easy approach and attack from the air. Such a measure could account for the increased difficulty of delivering munitions on target on winding mountain roads or on roads buried under thick jungle canopy. It was decided that implementing such an assessment would be left to future research, however.

The value function is shown in Figure 30. Here \( \rho = 1.42 \). The exponential value function reflects decreasing marginal value to higher densities. It is a more significant improvement to move from no choke points to one every 100 km than to move from five to six.
Figure 30. Value Function for Density of Choke Points by Choke Points per 100 km

\[ V(x) = \frac{1 - \exp\left(-\frac{x-0}{1.42}\right)}{1 - \exp\left(-\frac{6-0}{1.42}\right)} \]
A.2.3 Maximum Flow

Maximum Flow measures the greatest possible rate of transmission of forces or supplies through the network from the supply point to the receiving location. Maximum flow may use all available paths, and is constrained by the individual link capacities. This natural direct measure ranges from zero to the flow through the original network since the target set should only decrease flow, and uses only the link capacity attribute.

The value function is shown in Figure 31. The linearity of the value function indicates every additional unit flow per time is equal in value.

\[ V(x) = 1 - \frac{x}{100} \]

Figure 31. Value Function for Maximum Flow by Percent of Max Flow Remaining
A.2.4 Span Length

This metric is a natural proxy for the recovery time for the opposition forces to mend a bridge. A span length is the distance between pillar supports of a bridge, and is correlated to the extent of effort and resources required to bridge a given distance. The x-axis has a range of 0 to 150 meters and distinct changes in the shape of the graph reflect differing types of bridging techniques known to be used by the opposition forces in this scenario. Discussion included the observation that any recovery time greater than the commander’s desired delay would have a value of 1. Furthermore, evaluation of recovery time less than that level would require consideration of bridge capacity, bridge type, span (distance between supports), and gap (entire bridge length between banks). Although data was currently not readily available, the group agreed a metric like the one shown below could yield significant insight. The formulation is a time-weighted product that accounts for relative repaired value of damaged bridges, where $i$ is the index for bridges in the target set. The Time to Repair shows the likely time opposition would require to repair the bridge to some functional level. Percent Repaired is the percentage of capacity the partially or fully repaired bridge has with respect to its intact capacity. Capacity after Repair accounts for the actual capacity, keeping the score responsive to the actual new flow capacity.

$$X = \sum_i \text{Time to Repair}_i \cdot \text{Percent repaired}_i \cdot \text{Original Capacity}_i \div \sum_i \text{Time to Repair}_i$$

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Although such a measure was not used for this research, it indicates a desire to improve the model by directing effort toward particularly valuable types of data.

The value function for the chosen metric is shown in Figure 32. In this equation ρ=69.7. The discontinuity at 22 meters is due to the fact that the opposition forces in question have the equipment to bridge up to 22 meters of span almost immediately, whereas a longer distance requires different techniques and equipment. Subsequent decreasing slope reflects an operational understanding that an extra meter beyond 23 meters means more in terms of additional time than an extra meter at 100 meters. At 100 meters the function becomes linear, so every additional meter is the same impact. 150 meters is the breakpoint above which the structure is most likely a suspension bridge, irreparable in any practical amount of time relative to the desired delay time. At such a point, the bridge is considered to be permanently destroyed, although allied forces may be able to reconstruct it later.

\[
V(x) = \begin{cases} 
0 & \text{when } 0 \leq x \leq 22 \\
1 - \exp\left[\frac{-\left(x - 22\right)}{69.7}\right] & \text{when } 22 < x \leq 100 \\
1 - \exp\left[\frac{-\left(100 - 22\right)}{69.7}\right] & \text{when } 100 < x \leq 150 \\
0.004x + 0.4 & \text{when } 100 < x \leq 150
\end{cases}
\]
Figure 32. Value Function for Recovery Time by Meters of Span Length
A.2.5 Number of Targets in Target List for Another Infrastructure

This natural proxy measure is the only measure under the top-level objective Additional Benefits. It assesses the positive effects gained by the target set in the other three critical infrastructures: Energy/Power, Telecommunications, and POL (petroleum, oil and lubricants). Consideration of negative effects for the allies was discussed, such as hitting a bridge with a sewage line under it which would pollute the river below or which would degrade a power or communications network blue side wants to be able to use. In addition, the analysis of other infrastructure networks is a very important consideration but is beyond the scope of this initial effort. It was decided such complexity of network overlay analysis could be avoided by noting the number of targets already appearing in the target sets for other infrastructures, since they would already have accounted for any unique concerns and the LOC group need not duplicate their work. Therefore, the x-axis is the number of targets in common with other already-approved target lists, with a range of 0 to 4. The panel of experts felt four duplicated targets was an operational maximum for this scenario, and no additional benefit would be gained by additional overlap. This metric assumes target sets for other infrastructures have been developed before we assess it.

The value function is shown in Figure 33. The shape indicates every additional target duplicated by another network targeting group is equal value.

\[ V(x) = .25x \]
Figure 33. Value Function for Additional Benefits by Number of Targets Listed in Target List for Another Infrastructure
The Collateral Damage portion seeks to support US forces objectives to eliminate or at least minimize collateral damage to non-combatants and sensitive cultural and environmental sites.

A.2.6 Maximum Level of Non-combatant Casualties at a Single Target

This next measure is the first under the top objective Cost of Target Set, and is under the Collateral Damage sub-objective. It considers non-combatant casualties in a single location or target area. The x-axis uses the maximum level of casualties across all targets in the target set, where the number of expected casualties at a single target falls into one of three categories: None, Low (between 0 and 10, exclusive) and High (10 or greater). A casualty is either a death or an injury resulting directly from the target blast.

The value function is shown in Figure 34. The shape reflects that two or three casualties at a target, while unfortunate, has more limited local impact there, whereas 10 people in one local area will be much more devastating at a specific site.

\[
V(x) = \begin{cases} 
1 & \text{when } x = \text{None} \\
0.8 & \text{when } x = \text{Low} \\
0 & \text{when } x = \text{High}
\end{cases}
\]
Figure 34. Value Function for Maximum Non-combatant Casualties at a Single Target
A.2.7 Expected Total Non-combatant Casualties

Expected Total Non-combatant Casualties is the sum of expected non-combatant casualties from blast effects over an entire target set. For this natural proxy measure, the x-axis range is 0 to 100 total casualties. The upper bound of 100 assumes an operational reality that allies would not consider an LOC network target set with more than 100 expected non-combatant casualties.

This is a different concept from the previous measure, in that this measure attempts to put a penalty on total civilian casualties, while the max casualties at a single target measure assesses the localized effect. An increase or decrease in one of these measures does not change the preference for the other, so they maintain preferential independence. Furthermore, although they use the same attribute, the concepts and information gained are different, satisfying mutual exclusivity.

The value function is shown in Figure 35. For this function $p=-54.2$. The decreasing slope reflects a realistic perspective on the high uncertainty of casualties at a target. Additional casualties at the high end are likely accompanied by probability distribution with larger tails, so marginal value assigned to higher casualties are less than that assigned to the first casualties. It should be noted that all human life is valued, with the highest score being given to no loss of life.

$$V(x) = \frac{1 - \exp\left(-\frac{100 - x}{-54.2}\right)}{1 - \exp\left(-\frac{100}{-54.2}\right)}$$
Figure 35. Value Function for Total Expected Non-combatant Casualties
A.2.8 Number of Targets in Proximity to Sensitive Structures

This natural direct metric assesses the level of special weaponeering required to strike targets close to humanitarian, cultural or historic facilities. Before such a target may be selected, the recommending organization must demonstrate that a particular weapon used will not damage sensitive structures. Sensitive structures include schools, churches, mosques, temples, hospitals, and other buildings of appreciable cultural or historical significance. The x-axis has a range of 0 to M such targets that must be specially weaponeered. The x-axis depends on the scenario, using a parameter M to represent the maximum size of a target set. This is done to make the value function more operationally understandable. For this scenario, the maximum size of a target set may be estimated at 30 targets, so the parameter M = 30.

The value function is shown in Figure 36. In this function ρ=-7.40. The shape of the function again reflects a higher marginal value for re-weaponeering the first targets than the last ones. If certain targets are so critical that a few sensitive structures are put at risk, there is a decrease in value, suggesting the penalty for potentially damaging even the first sensitive target is extreme.

\[
V(x) = \frac{1 - \exp\left(-\frac{30 - x}{-7.40}\right)}{1 - \exp\left(-\frac{30}{-7.40}\right)}
\]
Figure 36. Value Function for Targets in Proximity to Sensitive Structures
A.2.9 Number of Targets Involving Significant Environmental Impact

The number of targets with environmental impact was viewed by the experts in the same way as the sensitive structures, not necessarily in the priority and trade-off value, but in the same terms of shape for the value function. For this natural direct metric, the x-axis is the same 0 to 30 targets having significant impact on the environment, where the upper bound is chosen as the largest probable number of targets in a target set. The value function is identical to that for Sensitive Structures in the previous measure, and is shown again in Figure 37. Again, there is assumed to be a high penalty or loss of value for even one site damaged.

$$V(x) = \frac{1 - \text{Exp} \left( -\frac{30 - x}{-7.40} \right)}{1 - \text{Exp} \left( -\frac{30}{-7.40} \right)}$$

Figure 37. Value Function for Targets with Significant Environmental Impact
A.2.10 Number of Sensitive Services Affected

The Number of Sensitive Services Affected is the only measure under the Collateral Effects heading, and refers to strategic effects one step removed from the blast. For example, in the case of a power loss, this means the disruption of major services without backup generators. Loss of power to a hospital would interfere with, life support systems, heating, ongoing medical procedures, or medicine refrigeration, while loss of power to emergency facilities like police or fire departments or to a water treatment facility would have possibly widespread serious consequences for the civilian population. While such collateral effects may be unavoidable in specific situations, the model penalizes such actions with decreased target set value.

The x-axis is simply a count of sensitive services possibly affected, where a hospital is considered one service, emergency services count as one service, and a water treatment or sanitation facility each count as one service. The range is 0 to 4 services, valued so that the value increment associated with the jump from 2 to 4 such services is twice the value increment associated with the range 0 to 2 services.

The value function is shown in Figure 38. In this function $p=2.89$. 

119
\[ V(x) = \frac{1 - \text{Exp}\left[-\left(\frac{4-x}{2.89}\right)\right]}{1 - \text{Exp}\left[-\left(\frac{4}{2.89}\right)\right]} \]

Figure 38. Value Function for Sensitive Services Affected
A.2.11 Ordnance Expended

Ordnance Expended measures the Opportunity Cost of weapons used in the target set, under the top consideration Cost of Target Set. Ordnance Expended assesses the number of weapons expended to destroy all targets in the set. This is a gross measure intended not to prescribe the type of weapon for the weaponeer in the execution phase, but rather to account for the number of hard targets which require greater effort to destroy. The x-axis depends on the scenario, and uses a parameter M to represent the maximum size of a target set. This is done to make the value function more operationally understandable. For this scenario, the maximum size of a target set may be estimated at 30 targets, so the parameter M=30. For the elicitation and weighting, the expert panel was considering PGMs, both standoff and short-range, since rarely do allied forces use unguided weapons on targets. Furthermore, since this is an opportunity cost, the weighting will reflect the importance of using these munitions traded off against the ordnance available. If supplies are limited, the weight may increase dramatically.

This measure and the Strike Packages were both difficult to arrive at, since each of them intuitively suggested much higher-resolution assessments. As long as all production lines are open, or supply is abundant, weapon expenditure might be best measured as dollar cost of production, since such data per weapon are typically available from the production contract. However, it was determined that the actual assignment of weapons is a function a very large number of unpredictable operational considerations at the time of execution, and any attempt to predict them would be highly speculative.
reason, simply using a "typical" weapon and assigning quantities to targets based on target hardness was the most reasonable method at this stage.

The value function is shown in Figure 39. The value function evidences a threshold of value in the midrange. A very high score on the x-axis indicates the targets are hard targets, which is where the most value is lost. It is expected that any target set will use at least a small number of munitions, so value does not decrease substantially until 3 or 4 bombs are being expended per target in the largest probable target set.

\[ V(x) = \begin{cases} 
1 & \text{when } M \leq x \leq 2M \\
-\frac{1}{4M}x + \frac{3}{2} & \text{when } 2M < x \leq 3M \\
-\frac{7}{20M}x + \frac{9}{5} & \text{when } 3M < x \leq 4M \\
-\frac{3}{10M}x + \frac{8}{5} & \text{when } 4M < x \leq 5M \\
-\frac{1}{10M}x + \frac{3}{5} & \text{when } 5M < x \leq 6M 
\end{cases} \]

or in this specific case

\[ V(x) = \begin{cases} 
1 & \text{when } 30 \leq x \leq 60 \\
-\frac{1}{120}x + \frac{3}{2} & \text{when } 60 < x \leq 90 \\
-\frac{7}{600}x + \frac{9}{5} & \text{when } 90 < x \leq 120 \\
-\frac{1}{100}x + \frac{8}{5} & \text{when } 120 < x \leq 150 \\
-\frac{1}{300}x + \frac{3}{5} & \text{when } 150 < x \leq 180 
\end{cases} \]
Figure 39. Value Function for Ordnance Expended by Total Munitions
A.2.12 Expected Strike Packages

The Expected Strike Packages seeks to measure the operational risk to personnel and platforms based on the number of packages sent. The number of breaks allied forces must create in the air defense network is of primary concern here, and if several targets are clustered close to each other only one occasion of SEAD (Suppression of Enemy Air Defenses) need occur. As a proxy for this measure, a distance criterion was established, in which a cluster of targets may be assigned to a single package if they are all within a 30nm diameter. There is certainly some operational limit to the number of targets which may be attacked with only one package, so a target cluster is limited to 10 targets per package. With these constraints, the number of acceptable clusters is used as the x-axis, with a range from 0 to M, where M is the maximum probable number of targets in a target set, and is set for each scenario. Again, for this scenario M is set at 30.

The value function is shown in Figure 40. Here, each additional strike package is additional risk to both personnel and systems, and is equivalently valued. Reducing the number of strike packages from 30 to 29 is equally preferred to reducing it from 2 to 1.

\[
V(x) = -\frac{1}{M-1} x + 1 + \frac{1}{M-1}
\]

or in this scenario

\[
V(x) = -\frac{1}{29} x + 1 + \frac{1}{29}
\]
A possible improvement to the hierarchy with respect to this measure deserves mention at this point. During the development of the value function and the comparisons which produced the weights for the hierarchy, the experts involved consistently thought of this measure in terms of the risk to allied combat pilots who would be tasked to fly the missions. For future use of this hierarchy, it is suggested that a separate evaluation consideration be included under Cost of Target Set, perhaps called Operational Risk. The consideration of value of strike packages was not so much that pilots and aircraft would not be available to fly missions the next day against other targets, but rather that a comrade might die. For this reason, a separate box is possibly warranted in future use of the hierarchy.

Figure 40. Value Function for Number of Strike Packages by Clusters
A.2.13 Restoration Time

Restoration Time under Target Set

Cost is similar to Recovery Time under the Value of the Remaining Network, but assumes blue forces must repair instead of opposition forces. This factor is illustrated by situations in which Allied force members invest resources after conflict is resolved to restore certain functions in the defeated territory. In this case the blue side prefers targets which may be quickly rebuilt by the victorious blue side after termination of hostilities. This x-axis has a range from 0 to 9 months, and is measured on the maximum estimated restoration time of a single target over all the targets in the set. This assumes all can worked on at the same time.

The value function is shown in Figure 41. In this function $p = -1.91$. The first month is accepted to be chaotic even in the best situation, and normal society will rely on allied help regardless of the speed of progress in the restoration of previously-targeted facilities. After that point, further delay to restoring orderly society will take a toll on allied resources, so some value is lost. Past two months, the project has become a significant obstacle in the restoration process; significant value is lost, but it tapers off to no value as the required repair time approaches nine months and beyond.
\[ V(x) = \begin{cases} 
1 & \text{when } 0 \leq x \leq 1 \\
-0.3x + 1.3 & \text{when } 1 < x \leq 2 \\
\frac{1 - \exp\left(-\left(\frac{9-x}{-1.91}\right)\right)}{1 - \exp\left(-\left(\frac{9-2}{-1.91}\right)\right)} & \text{when } 2 < x \leq 9
\end{cases} \]

Figure 41. Value Function for Restoration Time by Months
Appendix B. Clustering Algorithm via Coloring

The following code in Mathematica [Wolfram Research, 1999] accepts a list of Cartesian coordinates of all targets and a specification of which targets are in the target set to be considered. The algorithm produces a distance matrix and then a graphical representation of these targets in the coordinate plane, which are then used to designate the number of clusters according to two constraints:

- A cluster consists of a number of targets every pair of which are closer than 30 miles to each other.
- Each cluster consists of at most 10 targets

The output produced is a graphical representation of the graph followed by the smallest number of packages necessary to cover the targets. This algorithm uses a number of built-in functions from the Mathematica package Combinatorica, one of which is the VertexColoring function. Also available is a function called ChromaticNumber which guarantees an optimal solution, but processing time is considerably longer. It is proposed that by dividing the packaging problem into smaller sub-problems, this heuristic can avoid some pitfalls not avoided in standard hierarchical or k-means clustering algorithms.

It is advisable to employ a standard package as well as the proposed approach and accept the minimum report of the two as a very good solution. If computation resources are abundant, using the ChromaticNumber function will find an optimal and the standard package is unnecessary.
Counting Clusters by Coloring the Complement

<<DiscreteMath`Combinatorica`

# Convert Coordinates to a Distance Matrix

```
Coordinates = Table[{150 * Random[], 150 * Random[]}, {i, 40}];
TableForm[Coordinates]
```

```
0.371383  59.574
26.8051   136.925
34.7844   73.8918
70.9771   2.33674
103.131   83.1527
23.3964   15.5676
57.5037   146.242
72.7424   6.50547
85.5972   1.08832
2.56688   133.323
25.5701   89.3651
44.6554   83.0794
25.1988   29.7911
17.8503   96.1545
140.414   105.899
96.8732   93.8178
37.2831   22.7466
73.4768   78.2502
129.779   26.505
0.734368  71.7447
44.1822   25.4167
44.6554   88.4222
18.6121   86.0516
103.512   5.34273
143.413   56.2605
85.6619   59.1882
2.99898   100.361
138.789   115.37
115.716   77.6146
65.3119   37.1202
135.936   51.1095
64.5775   115.376
91.7542   25.6928
66.41     26.9534
73.1421   89.6412
112.898   21.6107
79.7287   33.3808
27.2361   147.052
76.7297   83.0196
38.4474   147.052
```

```
Targets = Range[20];
```
Define Distance function

This function calculates the 2-norm between all pairs in the target set, generated from the coordinates of the target indices given in Targets. The distance matrix is stored as lower triangular, with the assumption that distance is independent of direction of travel.

```math
Distance[Targets_] := 
Table[Sqrt[(Coordinates[[Targets[[i]], 1]] - Coordinates[[Targets[[j]], 1]])^2 + 
          (Coordinates[[Targets[[i]], 2]] - Coordinates[[Targets[[j]], 2]])^2], 
          {i, Length[Targets]}, {j, i}]
TableForm[Distance[Targets]]
```

<table>
<thead>
<tr>
<th>0.</th>
<th>81.7429</th>
<th>0.</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.2727</td>
<td>63.5362</td>
<td>0.</td>
</tr>
<tr>
<td>90.8915</td>
<td>141.651</td>
<td>80.1875</td>
</tr>
<tr>
<td>105.43</td>
<td>93.3655</td>
<td>68.9714</td>
</tr>
<tr>
<td>49.666</td>
<td>121.405</td>
<td>59.4255</td>
</tr>
<tr>
<td>103.804</td>
<td>32.0812</td>
<td>75.8331</td>
</tr>
<tr>
<td>89.7432</td>
<td>138.273</td>
<td>77.3416</td>
</tr>
<tr>
<td>103.364</td>
<td>148.014</td>
<td>88.7822</td>
</tr>
<tr>
<td>73.7812</td>
<td>24.5045</td>
<td>67.6017</td>
</tr>
<tr>
<td>39.0191</td>
<td>47.5758</td>
<td>18.0091</td>
</tr>
<tr>
<td>50.1356</td>
<td>56.7271</td>
<td>13.4851</td>
</tr>
<tr>
<td>38.774</td>
<td>107.146</td>
<td>45.1304</td>
</tr>
<tr>
<td>40.5419</td>
<td>41.7422</td>
<td>27.9713</td>
</tr>
<tr>
<td>147.506</td>
<td>117.769</td>
<td>110.373</td>
</tr>
<tr>
<td>102.397</td>
<td>82.2664</td>
<td>65.2078</td>
</tr>
<tr>
<td>52.1415</td>
<td>114.658</td>
<td>51.2062</td>
</tr>
<tr>
<td>75.4533</td>
<td>74.9731</td>
<td>38.937</td>
</tr>
<tr>
<td>133.566</td>
<td>150.984</td>
<td>106.158</td>
</tr>
<tr>
<td>12.1761</td>
<td>70.2007</td>
<td>34.1177</td>
</tr>
</tbody>
</table>

Convert Distance Matrix to Adjacency matrix (1 for d<30, 0 for d>30)

Define ViewTargets function

This function takes a lower triangular distance matrix and puts links between all pairs less distant than 30 units from each other. The resulting adjacency matrix is coupled with the correct sets of coordinates in the format used by the Mathematica standard discrete math package "Combinatorica"
DefineGraph[Dist_] := 
(* Initialize adjacency list of zeros *)
Adjacency = Table[0, {i, Length[Dist]}, {j, Length[Dist]}];
(* Fill in links for targets pairs closer than 30 nm *)
Do[
  Do[
    If[Dist[[i, j]] < 30, Adjacency[[i, j]] = 1; Adjacency[[j, i]] = 1],
    {j, i - 1}
  ], {i, Length[Dist]}
];
(* Clear the diagonal *)
Do[Adjacency[[i, i]] = 0, {i, Length[Dist]}];
(* Make graph *)
Graph[Adjacency, Table[Coordinates[[Targets[[i]]]], {i, Length[Targets]}]]

View targets

ShowLabeledGraph[DefineGraph[Distance[Targets]]]

Count Strikes

Built-in function descriptions

?DeleteVertex

DeleteVertex[g, v] deletes vertex v from graph g.
GraphComplement

GraphComplement[g] gives the complement of graph g.

GraphIntersection

GraphIntersection[g, h] gives the graph defined by the edges which are in both graph g and graph h.

InduceSubgraph

InduceSubgraph[g, s] gives the subgraph of graph g induced by the list of vertices s.

CliqueQ

CliqueQ[g, c] yields True if the list of vertices c defines a clique in graph g.

Vertices

Vertices[g] gives the embedding of graph g.

VertexColoring

VertexColoring[g] uses Brelaz's heuristic to find a good, but not necessarily minimal, vertex coloring of graph g.

ChromaticNumber

ChromaticNumber[g] gives the chromatic number of the graph, the fewest number of colors necessary to color the graph.

ConnectedComponents

ConnectedComponents[g] gives the vertices of graph g partitioned into connected components.

DeleteVertices function definition

DeleteVertices is a trivial expansion on the built-in function DeleteVertex. It enables the programmer to delete multiple vertices at once. The graphs shown demonstrate that the function deletes the correct vertices regardless of the order in which they are listed.

DeleteVertices[g1_, dl_] := (
(* This function orders the deletion list dl and deletes those vertices from graph g1, going from highest index to lowest. *)
  dlNew = Sort[dl];
  ReducedGraph = g1;
  Do[
    ReducedGraph = DeleteVertex[ReducedGraph, dlNew[[i]]]
    , {i, Length[dlNew]}
  ];
  ReducedGraph
)
ShowGraph[Path[8]]
ShowGraph[DeleteVertices[Path[8], {2, 5, 3}]]

- Graphics -

- Graphics -

- Graphics -

- DeleteEdges function definition

DeleteEdges is an even more trivial expansion on the built-in function DeleteEdge. It enables the programmer to delete multiple edges at once, but it is not complicated by the re-numbering of vertices as in the function DeleteVertices.
DeleteEdges[gl_, dl_] :=
If[dl == {}, gl, DeleteEdges[DeleteEdge[gl, First[dl]], Rest[dl]]]

- DeleteDegreeOneVertices function definition

This function reduces the size of a connected component of a graph by pairing vertices of order 1 with their neighbor as a clique of order 2 and deleting them from further consideration. The routine ends when all remaining vertices are of degree 2 or greater. The remaining graph may no longer be connected or may be the null graph.

This action is guaranteed not to increase the minimum number of strikes necessary to cover the targets:
Proof: Let g be a graph whose vertices may be covered by a minimum of c disjoint cliques. Without loss of generality, let x be the 1-degree vertex with the lowest index and let its neighbor be y. Let Cy be the vertices in the clique to which y belongs under the optimal graph partitioning into cliques. Clearly Cy must contain more than 1 vertex, since if it were a package by itself we could reduce the number of packages by combining it with x in a 2-clique. Then, the induced subgraph on Cy remains a clique if y itself is deleted. That is, the deletion of any vertex from a clique on n vertices must leave a clique on n-1 vertices. This is true by definition. Hence, deleting vertex y from clique Cy leaves a clique of at least size one. Since y is x's sole neighbor, the optimal clique partition must either pair x with y or designate it as a package by itself. If the former is true, we are done. Otherwise, we now package x with y and leave the remainder of Cy still a single package. The total number of packages is unchanged. QED

This function is unnecessary to the clique-partitioning algorithm below, but is intended to reduce further calculation by reducing the size of the component to be further investigated. The vertex numbering of the remaining graph may be different, but will not impact the minimal number of remaining cliques or the assessment the number of necessary strikes.

The example problem shows how strikes are incremented as 1-degree vertices are deleted.
DeleteDegreeOneVertices[g2_] :=

NewGraph = g2;
Label[1];
Again = False;
Do[
  (* If vertex i is a leaf, locate its neighbor and delete them both, sending them together as a package *)
  If[
    \[Sum\]_{j = 1}^{Length[NewGraph[[1]]]} NewGraph[[1, i, j]] == 1,
    (* Look for the neighbor. When found, delete it and the leaf and break out of the loop (Scan again for degree 1 vertices). *)
    j = 1;
    While[
      NewGraph[[1, i, j]] = 0, j++];
    NewGraph = DeleteVertices[NewGraph, {i, j}];
    Strikes++;
    Again = True;
    Break[];
  ],
  , {i, Length[NewGraph[[1]]]}]
]
If[Again \&\& (Length[NewGraph[[1]]] > 2), Goto[1]]; (* If there are only 1 or 2 vertices left, send them as a package and return the null graph. *)
If[Length[NewGraph[[1]]] \leq 2,
  NewGraph = Graph[{}, {}];
  Strikes++;
  Print["Strikes = ", Strikes];
NewGraph
]
Strikes = 0
a = DeleteEdges[Wheel[7], {{2, 3}, {2, 7}, {3, 4}}];
ShowLabeledGraph[a]
If[V[DeleteDegreeOneVertices[a]] ≥ 1,
   ShowLabeledGraph[DeleteDegreeOneVertices[a]], Print["No vertices remain."]]
Strikes
0

- Graphics -
Strikes = 4
No vertices remain.
4

- CountStrikes function definition

Shirley is Bob's complement graph.
CountStrikes[g_] :=

Clear[Components];
Components[g] = ConnectedComponents[g];

While[Components[g] # {},
  (* If next component is a clique of no more than 10 targets,
    accept as a package and go to next component. If the clique is bigger than 10,
    partition it into the fewest number of packages of cardinality 10 or less. *)
  If[CliqueQ[g, First[Components[g]]], Strikes +=
    Ceiling[Length[First[Components[g]]]/10.]; Components[g] = Rest[Components[g]];]
  Continue[];
  (* Select first component for consideration and delete it from the list *)
  Bob[g] = InduceSubgraph[g, First[Components[g]]];
  Components[g] = Rest[Components[g]];
  (* Repeatedly select a vertex of order one, and accept it and its neighbor as
    a package. The remaining graph may now have multiple components *)
  Bob[g] = DeleteDegreeOneVertices[Bob[g]];
  If[Bob[g] == Graph[{}, {}], Continue[]];
  (* If Bob now has multiple components,
    Bob will be recursed through the same procedure as the original graph *)
  If[(Not[ConnectedQ[Bob[g]]]), CountStrikes[Bob[g]]];
  (* Conclude Bob is connected with minimum vertex degree at least 2. *)
  Shirley = GraphComplement[Bob[g]];
  (* Color its complement as the smallest number of independent sets in Shirley,
    which is the smallest number of cliques in Bob *)
  Strikes += Max[VertexColoring[Shirley]];]
Main Program

Strikes = 0;
g = DefineGraph[Distance[Targets]];
ShowLabeledGraph[g];
CountStrikes[g];
Strikes

10
2
7
14
11
12
16
18
5
15
20
3

13
6
17
19

10
4

# Multi-objective Evaluation of Target Sets in a Logistics Network

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**Abstract:**
This thesis addresses the selection of target sets to disrupt an adversary's logistics network in a wartime environment. In the presence of many objectives—such as reducing maximum flow, lengthening routes, avoiding collateral damage, all at minimal risk to our pilots—the problem of determining the best target set is complex. Previous efforts have not adequately considered the value of the remaining network functionality after target destruction. In addition, current network targeting procedures optimize target sets with respect to only a single metric. This thesis uses a multi-objective decision analysis framework capturing actual targeting decision-maker values and preferences to evaluate and analyze 10 alternative target sets. Sensitivity analysis and persistency analysis on the results give insight as to how to select better target sets to meet stated strategic objectives.
Bibliography


