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Estimating Budget Relationships with a Leontief Input-Output Model

Guenever L. R. Shariff

ESTIMATING BUDGET RELATIONSHIPS

WITH A LEONTIEF INPUT-OUTPUT

MODEL

THESIS

Guenever LR Shariff, First Lieutenant, USAF

AFIT/GAQ/ENS/00D-01

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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Guenever L.R. Shariff, B.S., M.S.

First Lieutenant, USAF

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Guenever L.R. Shariff, B.S., M.S.
First Lieutenant, USAF

Approved:

Mark A. Gallagher | 12 Dec 00
Lt Col Mark A. Gallagher (Advisor) | Date

William K. Stockman | 14 DEC 00
Lt Col William K. Stockman (Reader) | Date

Al W for | 14 DEC 00
Lt Col Raymond R. Hill (Reader) | Date

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Guenever L.R. Shariff

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Abstract

Forty years ago, the Office of the Secretary of Defense proposed using the Leontief input-output model to assess tradeoffs in the Department of Defense's (DoD) budget. We demonstrate that the Leontief input-output model can assess tradeoffs in the Air Force's budget.

To increase one part of the Air Force's budget, we need to know the interrelationships between that budget area and the other areas. In this research, we look at different methods of how the functional areas might interact. We demonstrate our methodology on two data sets – DoD and the Air Force aggregate budget data. By looking at how the functional areas interact, we hope to be able to find a sound methodology that will provide assistance to Air Force leadership for determining appropriate levels of funding.

Estimating Budget Relationships

With A

Leontief Input-Output Model

1. Introduction

Dr. Wassily Leontief (1906-1999) developed his macro-economic input-output model to show the interaction between economic sectors in the 1940s. He correctly predicted that the end of World War II would result in a significant increase in demand, rather than excess labor that other economists predicted. His model has been applied to a wide range of "economies," ranging from international, national, and regional; to individual businesses, schools, and hospitals. Dr. Leontief won the Nobel Prize for Economics in 1973 for his work.

In the late 1960's, the Office of the Secretary of Defense's (OSD) Systems Analysis office (now called Program Analysis and Evaluation) realized that the Department of Defense's (DoD) annual budget could be modeled using a Leontief input-output model. They proposed that this model be used to provide tentative fiscal guidance (TFG), which is an initial proposal of how future defense budgets might look (Patton et al, 365).

Today, using a Leontief input-output model, similar to Patton et al's Electric Five-Year Defense Program, may help provide the capability to test fiscal reasonableness of alternative force structures rapidly. More importantly, it enables tradeoff comparisons in different types of expenditures (Patton et al, 367).

We apply this estimation approach to functional areas within the Air Force's budget. We estimate the relationships between the functional areas [i.e. intelligence, operations and maintenance (O&M), military construction, etc.] of this specialized economy. The model indicates the amount of funding in the various support and combat areas to maximize effective combat spending (Patton et al, 365).

When expenditures on one type of weapon system are increased, all of the supporting equipment and personnel must be increased. Indirect related costs, such as transportation, intelligence, and communications, also must increase, but these are not often currently estimated. We conceptualize these relationships with the Leontief input-output model to assess total system impacts in terms of costs. We answer the research question: Can the Leontief input-output model provide aggregate relationships of Air Force budget relationships? In modeling the DoD budget using a Leontief input-output model, we estimate parameters using historical budget data covering the last thirty years compiled from the Automated Budget Interactive Data Environment System (ABIDES). This research builds a model of the Air Force as an economy that conceptualizes the relationships between the various functional areas at an aggregate level. This approach may also be used to indicate areas in which there are unusually high rates of spending, or areas in which funding does not appear to be sufficient to support related budget areas.

This research provides Air Force leadership with an approach to show tradeoffs between the current Air Force expenditures and new system spending rates. For example, if the Air Force increases procurement expenditures, it also needs to increase the accompanying maintenance, intelligence, transportation, and communications support. This model does not substitute for detailed program office estimates.

In the next chapter, we will discuss the background of the Leontief input-output tables, and the basic structure of the static and dynamic models. Chapter Three details our methodology to estimate the coefficients for the Leontief input-output tables using a minimization scheme. Chapter Four presents the methodology and the results that we obtained using a regression for estimation. Chapter Five concludes this research and suggests other potential applications.

2. Leontief Input-Output Model

The French economist François Quesnay conceptualized the idea of a detailed accounting system of inter-industry activities. In 1758, he published "Tableau Économique" which was a diagrammatic representation that showed how expenditures could be systematically traced through an economy (Miller and Blair, 1). Wassily Leontief won the 1973 Nobel Prize in Economics for extending this representation to develop the "Leontief input-output model," which estimates the interactions between different industries in an economy. His purpose in creating this theory was to "provide a simplified picture of real systems" (Survey of Current Business, 10).

The Leontief input-output model can be adapted to systems of varying complexity. One form is a linear programming problem (this is the static open model that is discussed below). The input-output model is often used to determine the impact of flow interactions for single corporations, regions, nations, and even the world's economy. Because of its flexibility, mathematicians and economists use the model to study diverse problems ranging from combat modeling (Snodgrass 2000) to environmental pollution.

In this chapter we shall discuss the basic static open Leontief input-output model, the assumptions for that model, the consumption possibility curve, the open and closed models, the dynamic model, and the fixed coefficients of production.

2.1 Leontief Input-Output Model

Any country's economy is an "interwoven fabric." Simplified examples include the interrelationships within a single industry, say the printing and publishing industry, which depends on the paper and allied products industry, which in turn depends on the lumber and trucking industries (Leontief, 6). The example that Leontief uses to explain his model is the United States economy. He divides our economy into forty-two major sectors grouped into four areas: production, distribution, transportation, and consumption. When set into matrix format, the rows reflect the output of each sector to the other sectors. Columns show how each sector obtains the inputs (goods and services) that it needs from the other sectors. In other words, the columns represent how a particular sector uses the inputs of the other sectors (including inputs that it needs from its own industry) (Wu and Coppins, 208). This matrix reflects the flow of trade between the different sectors of the economy.

Wu and Coppins illustrate with an economy with only three industries (A, B, and C), shown in Table 1 (Wu and Coppins 1981). The columns reflect the inputs to the economy, and the rows show the outputs. For example, in the second column of the first row, industry B requires an input of fifteen units from A, four units from itself, and eight units from C. An agricultural example of consumption within the producing industry is seed corn used to plant the next year's crop.

Table 1: Three-Industry Example

	Inputs				
Outputs	**A**	**B**	**C**	**Final Demand**	**Total**
A	10	15	25	15	65
B	5	4	18	23	50
C	22	8	77	18	125

The units in any row may be any aggregate measure of that industry's output. Measures may include tons, truckloads, or value. In this research, we always measure all the outputs in dollars.

The nine main entries in Table 1 show the flow between the three industries. Industry A's output of $65 used $10 by A, $15 by B, $25 by C, and the final demand is $15. Final demand includes such economic categories as exports, government purchases, inventory accumulation, and payments to households. Since these are exported to members of the final demand sector, they are outside the production economy (Wu and Coppins, 208).

The next step in the analysis of Table 1 is to develop a table of technology coefficients. These coefficients give the amount of input required from each industry to generate one unit of output from that industry for a given production technology. Each element in Table 1 may be represented with

x_{ij} = output (in dollars) of industry i used by industry j. (i, j = A, B, C),

X_i = total output from each industry, i = A, B, C, and

y_i = final demand for each industry's output, i = A, B, C .

Therefore, the output of industry A used by industry B is represented by $x_{AB} = \$15$.

Let a_{ij} equal the number of dollars' worth of industry i's output required by industry j to produce one dollar's worth of output from industry j. The inputs $x_{Aj} + x_{Bj} + x_{Cj}$ are required to produce the output of industry j. Dividing x_{ij} by X_i normalizes the value to the input from industry i for each dollar of industry j output. This yields the technology coefficients,

$$a_{ij} = \frac{x_{ij}}{X_j}. \quad \textbf{(2-1)}$$

For example,

$$a_{BA} = \$10/\$65 = 0.154,$$

which is the dollar amount of industry B's output required to produce one dollar's worth of industry A's output. Therefore, the total dollar value of inputs from all three industries required to produce one dollar's worth of industry A's output is 0.154 + 0.077 + 0.338 = 0.569. Table 2 shows the matrix of technology coefficients (a_{ij}) which are based on equation 2-1 (Wu and Coppins, 209).

Table 2: Technology Coefficients

	Purchasing Industry		
Producing Industry	**A**	**B**	**C**
A	0.154	0.300	0.200
B	0.077	0.080	0.144
C	0.338	0.160	0.616

If there are no significant changes in the production technology, the coefficients remain constant over time.

For the economy in the three-industry example, the general system of equations is

$$X_A \geq y_A + a_{AA}X_A + a_{AB}X_B + a_{AC}X_C \quad \textbf{(2-2)}$$

$$X_B \geq y_B + a_{BA}X_A + a_{BB}X_B + a_{BC}X_C$$

$$X_C \geq y_C + a_{CA}X_A + a_{CB}X_B + a_{CC}X_C \ .$$

These three equations show that the output from industry A may go to final demand (y_A) or become inputs for the three industries. These equations can then be written in matrix notation as

$$\mathbf{X} = \mathbf{y} + \mathbf{AX} \tag{2-3}$$

or equivalently,

$$\mathbf{y} = (\mathbf{I} - \mathbf{A})\mathbf{X}$$

where I is the identity matrix, and for a three-sector application,

$$\mathbf{y} = \begin{bmatrix} y_A \\ y_B \\ y_C \end{bmatrix},$$

$$\mathbf{A} = [a_{ij}],$$

and

$$\mathbf{X} = \begin{bmatrix} x_A \\ x_B \\ x_C \end{bmatrix}.$$

The matrix $(\mathbf{I} - \mathbf{A})$ is known as the Leontief matrix (Wu and Coppins, 210). The Hawkins-Simon condition requires that

> If an economic system in which each sector functions by absorbing output of other sectors directly and indirectly is to be able not only to sustain itself but also to make positive delivery to final demand, each one of the smaller and smaller subsystems contained in it must necessarily be capable of doing so too. (Leontief, 26)

Dorfman, Samuelson, and Solow give the following explanation of the Hawkins-Simon condition, which ensures viable production:

> If we add up the direct and indirect inputs of coal that go into a ton of output (the coal to make coal, the coal to make coal to make coal, the coal to make steel to make coal, the coal to make steel to make coal to make coal, the coal to make steel to make steel to make coal, etc., ad infinitum), that all this will be less than one ton. Clearly if a ton of coal "contains,"

directly and indirectly, more than a ton of coal, self-contained production is not viable. (Dorfman et al, 215)

Mathematically, the Hawkins-Simon condition requires the determinant of the matrix and all submatrices $(\mathbf{I}-\mathbf{A})$ to be positive, and hence the Leontief matrix is non-singular. A non-singular matrix, by definition, is both square and invertible. Thus, we may rewrite equation 2-3 as

$$\mathbf{X} = (\mathbf{I}-\mathbf{A})^{-1}\mathbf{y}\,. \tag{2- 4}$$

Now, determining the necessary production to support final demand mathematically reduces to finding $\mathbf{X} \geq 0$ such that $(\mathbf{I}-\mathbf{A})\mathbf{X} = \mathbf{y}$. For the three-industry economy, we have

$$(\mathbf{I}-\mathbf{A})^{-1} = \begin{bmatrix} 1.727 & 0.77 & 1.186 \\ 0.409 & 1.345 & 0.715 \\ 1.696 & 1.241 & 3.953 \end{bmatrix}.$$

The 1.727 in the first position of $(\mathbf{I}-\mathbf{A})^{-1}$ means that for every dollar's worth of industry A's products delivered to the final demand sector, the total intra-industry transactions add up to require an additional 72.7 cents (Wu and Coppins, 211).

Primary factors are basic resources available to the "economy" or system being modeled. The three-industry model, currently, does not include primary factors. Economists often assume that labor is the only primary factor in the economy. Wu and Coppins state that other than primary factors, "all other productive factors are produced by the economy" (Wu and Coppins, 211). If we let "industry" supply the primary factor, the amount of labor needed by industry j to produce one unit of output can be represented by the variable a_{oj}. Given total output $\mathbf{X}$, the total labor required is

$$X_o = a_o\mathbf{X}\,, \tag{2- 5}$$

where

$$a_o = [a_{oA}, a_{oB}, a_{oC}] \,. \tag{2- 6}$$

Incorporating the labor requirement into the Leontief input-output model results in the following change to the previous equation, and transforms it into equation (2- 7).

$$X_o = \mathbf{a_o}\,(I - A)^{-1}\mathbf{y}\,, \tag{2- 7}$$

and therefore, given that labor is a scarce resource, the bill of goods is producible only if

$$\mathbf{a_0(I - A)^{-1}y \geq 1} \,. \tag{2- 8}$$

For the three-industry example, if the bill of final goods to be produced is $\mathbf{y} = [15, 23, 48]^T$, the production is given by equation (2- 5), given the total labor with

$$\mathbf{X_o} = [10 \quad 7.5 \quad 15] \begin{bmatrix} 1.727 & 0.77 & 1.186 \\ 0.409 & 1.345 & 0.715 \\ 1.696 & 1.241 & 3.953 \end{bmatrix} \begin{bmatrix} 15 \\ 23 \\ 18 \end{bmatrix} = 2900 \text{ units of labor.}$$

An alternate way of viewing the "*simple*" Leontief input-output model is as a linear program, where we minimize labor required to produce the final goods.

Minimize $\quad f = \mathbf{a_0X}$

Subject to $\quad \mathbf{(I - A)X \geq y}$ and

$\quad \mathbf{X} \geq 0.$

The optimal solution to this problem is

$$\mathbf{X = (I - A)^{-1}\, y} \text{ with } f = \mathbf{a_0(I - A)^{-1}y}\,.$$

In summary, the application of the Leontief input-output model requires dividing the system into sectors and, for every sector's production, determining the constant rate of consumption from every other sector. We can use the resulting linear system of equations to determine each sector's production from a given set of primary factors--in this case, only labor.

2.2 Assumptions of the Leontief Input-Output Model

The Leontief input-output model makes three basic assumptions. First, input resources to an industry are homogeneous. For example, in terms of budgeting, *any* money is "money," or in logistics supply, all fuels are equal, whether JP-8, diesel, or 87-octane MOGAS. An analyst may mitigate this assumption, if needed for the analysis, by modeling the system at a higher fidelity. For example, the primary fuel factor may be divided into the various specific fuels of interest.

The second assumption--and the strongest--is that the coefficients of production are fixed. Specifically, each sector's output requires a certain minimum input of each commodity per unit of output. This minimum may be zero, when a particular sector's production does not directly use another sector's output. Certain sectors may require some of their own output to be directly consumed in future production, such as the classic example of "seed corn" necessary to grow more corn.

Since the technology coefficient, a_{ij}, equals the required minimum input of commodity i per unit of commodity j, the total possible production from a sector may be determined based on the available resources for that sector and these technology coefficients. For the three-industry model where i and j equal A, B, and C, Sector A's output is constrained by

$$X_A \le \min\left(\frac{x_{AA}}{a_{AA}},\frac{x_{BA}}{a_{BA}},\frac{x_{CA}}{a_{CA}},\frac{X_{OA}}{r_{AA}}\right), \qquad \textbf{(2-9)}$$

where a_{oa} is equal to the amount of resource i consumed per unit of sector j's output; p_{ij} is the amount of resource i consumed by resource j. Since X_A equals the smallest of the four ratios, X_A must be less than or equal to all of the ratios:

$$x_{AA} \geq a_{AA}X_A,\ x_{BA} \geq a_{BA}X_A,\ x_{CA} \geq a_{CA}X_A,\ p_{AA} \geq r_{AA}X_A\,.$$

Dorfman states that in any system not involving free goods, the equality will hold for all these ratios because in an efficient economy all of the ratios will be driven to the minimum (Dorfman et al, 210). In other words, an efficient system will not produce goods (at some expense) that are not used in other production or consumed. Therefore, the inequality in equation 2-7 may be replaced with equality.

The third assumption is generalized returns to scale: If the intensity or the gross inputs are doubled or halved then the net outputs will either be doubled or cut in half (Dorfman et al, 220). If each required input x_{ij} (including primary factors) for sector i is multiplied by a constant, the corresponding X_i is also multiplied by that constant, thus yielding constant returns to scale. Constant returns exist as production increases until a binding constraint is reached. Once production in a sector is limited by some input, any increase in the other inputs to that sector beyond that point is excess.

2.3 Consumption Possibility Schedule

The consumption possibility schedule is a valuable tool used in economics. It illustrates the feasible demands of the modeled economy, based on capacity constraints and limited primary factor resources. In economic terms, it can be thought of as a social transformation curve (Dorfman et al, 223).

The availability and distribution of the primary factors determine the consumption possibility schedule. The only primary factor in the three-industry example is labor. Therefore, the formula for distributing labor is

$$\mathbf{P} = \mathbf{A}\mathbf{X}, \tag{2- 10}$$

where **P** is a column vector of primary resources available to the economy, and **A** is the matrix of technology coefficients. Equation (2- 4 gives the production required to produce a given bill of goods. In order to create the consumption possibility curve we must the restate the model; it is shown below. For the three-industry example, this yields the form

$$\mathbf{X} = \mathbf{a}\mathbf{X}+\mathbf{y},$$

where **X** is an *m* dimensional column vector of gross outputs for each sector, **y** is an *m*-dimensional column vector that represents consumption, and **a** is an *m* x *m* square matrix of input coefficients. This is shown below.

$$\begin{bmatrix} 0.154 & 0.300 & 0.200 \\ 0.077 & 0.080 & 0.144 \\ 0.338 & 0.160 & 0.616 \end{bmatrix} \begin{bmatrix} X_A \\ X_B \\ X_C \end{bmatrix} + \begin{bmatrix} y_A \\ y_B \\ y_C \end{bmatrix} = \begin{bmatrix} X_A \\ X_B \\ X_C \end{bmatrix}, \text{ and}$$

$$\begin{bmatrix} 50.01 & 27.005 & 106.97 \end{bmatrix} \begin{bmatrix} y_A \\ y_B \\ y_C \end{bmatrix} = P_A \ .$$

Then,

$$X_A = 50.01X_A + 27.005X_B + 106.97X_C,$$

where X_A represents net output of commodity A. Substituting from equation 2-9, we get

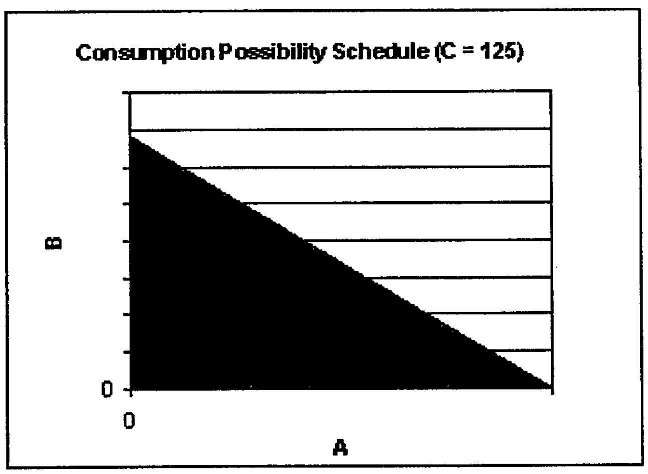
$$P_A = 50.01(1.727y_A + 0.77y_B + 1.186y_C) + 27.005(0.409y_A + 1.345y_B + 0.715y_C) + 106.97(1.696y_A + 1.241y_B + 3.853y_C).$$

This shows that the final possible output is limited by the available inputs.

In a model with only two industries and one primary factor, the consumption possibility curve is a straight line. The three-industry example has a three-dimensional consumption possibility frontier.

It can be shown in two-dimensions by limiting one of the outputs to a constant and graphing the other two.

Figure 1: Consumption Possibility Schedule



The line represents the maximum amount of inputs that are available, and the shaded area below the line is the region of feasible production. In this example, the maximum available labor is 2,900 hours. This consumption possibility curve has only one primary factor associated with it (labor), and thus is a straight line.

In the general case of a Leontief input-output model, each primary factor of production has a straight line associated with it, and the region of feasible production becomes a convex polygon.

With two or more primary factors, the consumption possibility frontier is piecewise linear, and the assumption of generalized returns applies. In defense of constant returns to scale, Dorfman, Samuelson, and Solow state:

> It is only when we insist on infinitesimal substitution, on continuously varying marginal rates of transformation, on sensitivity of factor proportions to all price variations no matter how slight, that we have to give up the polygonal frontier for the neoclassical smooth curve. (Dorfman et al, 349)

2.4 Dynamic Leontief Input-Output Models

The dynamic Leontief input-output model accounts for production and consumption of goods and services over periods of time, recognizing that production and consumption cannot occur simultaneously. Dorfman and Leontief's preferred method of solving dynamic models is to convert them into a series of static models that are related through the stock of the prior period. A dynamic model is converted to a static model when each sector for each period becomes a separate sector variable. For example, in the three-industry example, if it is solved over two periods, instead of having twelve variables, there will be twenty-four sector variables.

2.5 Capital Stock

A stock is something that is accumulated in prior periods and then used in a later period, creating a lag time. In economic terms, capital stock is capital investment. Generally these are items that are going to be used by a firm, or economy, for a long period of time, items such as buildings, machinery, or land. The first step in adding capital stock to the model is to create the function $S_n(t)$, which is the stock of commodity (or primary factor) n at time t (and $n = 1, 2, 3$). If it is convenient, as a mathematical bookkeeping measure, labor can be counted as a commodity and represented by S (Dorfman et al, 282).

Explaining how X_i is produced requires the introduction of S_{ij}. S_{ij} is the flow of capital stocks. The first subscript describes the physical nature of the commodity concerned, and the second subscript refers to the industry in which the good is employed. Therefore, $S_{AA} + S_{AB} + S_{AC}$ is the economy's stock of capital in the form of commodity A.

S_{AA}, S_{BA}, and S_{CA} describe the capital structure of the industry producing the first commodity. In a simple form of the Leontief input-output model, capital stocks should be thought of as being present for production purposes, but they are not used up by production occurring in the current period (Dorfman et al, 284).

If each grade of capital stock is homogenous. For example, dollars, the total stock is equal to the sum of the separate allocations of the stock among different industries, for example

$$S_i(t) \geq S_{iA}(t) + S_{iB}(t) + S_{iC}(t). \tag{2-11}$$

Writing this as an inequality means that there can be excess capacity in terms of the capital good i. This, however, does not show rates of change of the two sides of the inequality. When equation (2-11) is an inequality, economically the two sides are independent, and have independent rates of growth. Practically, and mathematically, while the two sides may have independent growth rates, the fact that the sides are independent is ignored. Representing stock this way yields the following system of equations for the general Leontief input-output problem:

$$\begin{aligned}
&x_A \geq a_{AA}x_A + a_{AB}x_B + a_{AC}x_C + \Delta S_A + C_A \\
&x_B \geq a_{BA}x_A + a_{BB}x_B + a_{BC}x_C + \Delta S_B + C_B \\
&x_C \geq a_{CA}x_A + a_{CB}x_B + a_{CC}x_C + \Delta S_C + C_C \\
&x_i, a_{ij}, S_{i,t}, C_i \geq 0 \\
&i, j = A, B, \text{or } C
\end{aligned} \tag{2- 12}$$

However, this does not allow for use of the stock later, nor does it allow the accumulated capital stock to increase the efficiency of the system over time. Therefore, adding the stock from the current period, and subtracting the stock from the previous period, will allow for negative increases in the total stock level; in general, it will allow for the stock that was accumulated in previous periods to be used in future periods. This changes the model to

$$
\begin{aligned}
& x_A \geq a_{AA}x_A + a_{AB}x_B + a_{AC}x_C + (S_{A,t} - S_{A,t-1}) + C_A \\
& x_B \geq a_{BA}x_A + a_{BB}x_B + a_{BC}x_C + (S_{B,t} - S_{B,t-1}) + C_B \\
& x_C \geq a_{CA}x_A + a_{CB}x_B + a_{CC}x_C + (S_{C,t} - S_{A,t-1}) + C_C, \\
& x_i, a_{ij}, S_{i,t}, C_i \geq 0 \\
& i, j = A, B, \text{or } C
\end{aligned}
\qquad \textbf{(2-13)}
$$

and t is the time period.

2.6 Open and Closed Models

Leontief input-output models may be classified as either open or closed. The open model's final products are exported to a final demand sector (in Economics, the final demand sector is usually households). The three-industry model is an example of an open model. In a closed model, all outputs are inputs to other sectors. No consumption exists outside of the system--all consumption occurs within the economy. Economists call the closed model an endogenous system. According to Leontief, a static system cannot be closed because endogenous exportation requires consideration of the structural relationships, and because inputs and outputs occur in different periods of time (Leontief, 27); ergo, production and consumption cannot occur simultaneously.

2.7 Fixed Coefficients of Production

To make this model economically correct, a method is needed to explain the behavior of the system over time, the choices among the alternative methods of production, and allocation of scarce resources. Conditions need to be added to model the economy's efficiency. Leontief's solution to this problem was to assume fixed coefficients of production and only one way of producing each output.

The Leontief production function requires fixed minimum amounts of a_{ij} to produce $x_{ij}(t)$; and lagged capital stock amounts b_{ij} for each unit of $x_j(t)$. b_{ij} is the flow coefficient of flow input per unit of flow output. It is the capital coefficient representing stock input per unit of flow output; we use it as the Research and Development, Testing and Evaluation (R&D) lag values. For example, the development of a new airplane has built-in lag time associated with it. There is time between when the Concept Exploration phase begins and when Production can start. This means that b, unlike a, depends on time; however, the a_{ij}s occur at the same time. A production function for the three-industry example with capital stock is:

$$X_A = F^1(X_{AA}, x_{BA}, x_{CA}, S_{AA}, S_{BA}, S_{CA}) = \min\left(\frac{x_{AA}}{a_{AA}}, \frac{x_{BA}}{a_{BA}}, \frac{x_{CA}}{a_{CA}}, \frac{S_{AA}}{b_{AA}}, \frac{S_{BA}}{b_{BA}}, \frac{S_{CA}}{b_{CA}}\right). \tag{2-14}$$

The production functions for the other industries are similar. The a_{ij}s and the b_{ij}s must be non-negative, since negative consumption is unrealistic. The a_{ij}s must also satisfy the Hawkins-Simon condition, which is if the capital coefficients (a_{ij} and b_{ij}) are all set to zero, then this dynamic system is equivalent to a static system with a stock sector. The capital requirements divide the net yield (which provided consumption in the static model) between consumption and gross investment.

According to Dorfman, in time t, the dynamic Leontief input-output model for three functional areas can be set up as the following system of equations:

$$
\begin{aligned}
&x_A \geq a_{AA}x_A + a_{AB}x_B + a_{AC}x_C + \Delta S_A + C_A \\
&x_B \geq a_{BA}x_A + a_{BB}x_B + a_{BC}x_C + \Delta S_B + C_B \\
&x_C \geq a_{CA}x_A + a_{CB}x_B + a_{CC}x_C + \Delta S_C + C_C \\
&S_A \geq b_{AA} + b_{AB}x_B + b_{AC}x_C \\
&S_B \geq b_{BA} + b_{BB}x_B + b_{BC}x_C \\
&S_C \geq b_{CA} + b_{CB}x_B + b_{CC}x_C \\
&x_i, a_{ij}, \Delta S_i, C_i, S_i, b_{ij} \geq 0 \\
&i, j = A, B, \text{or } C
\end{aligned}
\qquad \textbf{(2- 15)}
$$

where ΔS_i is equal to $S_i(t+1) - S_i(t)$.

2.8 Net Production Model

So far, we have presented the input-output model based on total sector production. Leontief also developed a net production model where each sector's self-consumption is accounted before distribution of output (Leontief). Let

$$\overline{X}_i = X_i(1-a_{ii}). \qquad \textbf{(2- 16)}$$

Further, the technology matrix for the net model has elements

$\bar{A} = [\bar{a}_{ij}] = 0,\ i=j$

$$\frac{a_{ij}}{1-aj_j},\ i \neq j \qquad \textbf{(2- 17)}$$

Making the appropriate substitutions into equation 2-2, we may write equation 2-3 equivalently as

$$\overline{X} = y + \overline{AX}\ . \qquad \textbf{(2- 18)}$$

In this chapter, we can derive all the other relationships for the net production model by substituting $\overline{X}$ for X and $\overline{A}$ for A. Since the net production model reduces the technology coefficients to estimates, we apply it in our research.

This chapter presents variations of the Leontief input-output model. We apply these to model budget relationships in the next two chapters.

3 Estimating a Leontief Input-Output Model for the Defense Budget

The discussion up to this point has assumed that all of the information to construct the Leontief input-output model is readily available; however; not all of the data needed to estimate model parameters that provide aggregate relationships of the defense budget are readily available. Unlike economic systems, we have the dollar output in each functional area of the budget, but the inter-sector transfers that relate the sectors are not available. This chapter documents several failed attempts to estimate the technology coefficients.

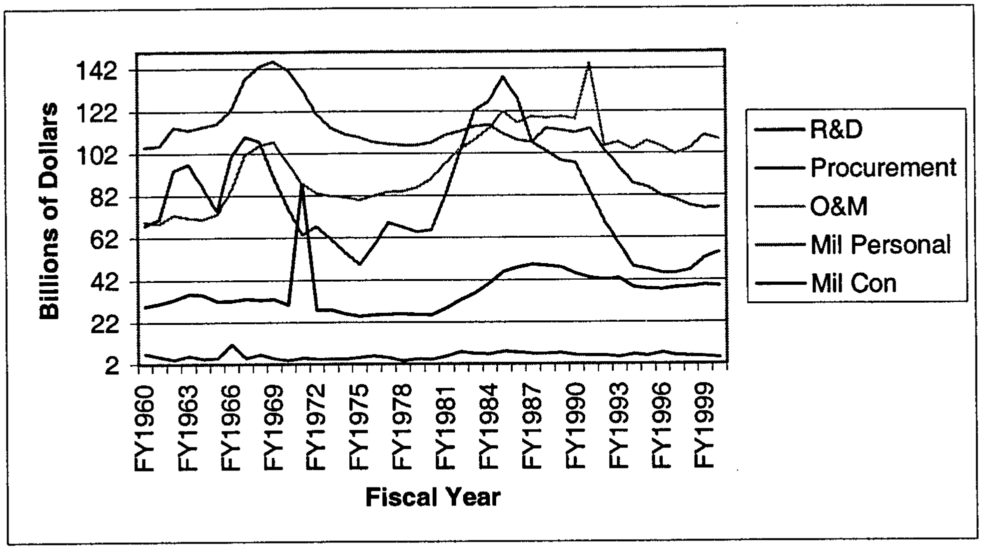
Once we develop an appropriate model using this data, we use Air Force budget data to develop a model that will show the portion of the budget that optimally should be expended on Research and Development (R&D) to maximize the Operations and Maintenance budget (O&M). We have chosen to maximize this portion the budget because it represents the operational United States Air Force, and is its "teeth" (GAO, 1).

3.1 Developing the Input-Output Model

The model for the Department of Defense budget has five functional areas – Operations and Maintenance (O&M), Military Construction (MilCon), Military Personnel (MilPer), Procurement (Proc) and Research Development, Testing and Evaluation (R&D). We let each of these functional areas represent a sector, and we assess their interactions with the Leontief input-output model. We use the aggregate budget data from the last forty years to estimate model parameters. We obtained the data shown in

Figure 2 from the Air Force Magazine. Appendix A shows the individual data points in constant FY 2000 billions of dollars.

Figure 2: Department of Defense Aggregate Budget Data 1960-2000



Our model is based on the static net-production model discussed in Chapter 2, and shown below for a single fiscal year.

$$
\begin{aligned}
&X_1 = e_1 + a_{11}X_1 + a_{12}X_2 + \ldots + a_{1n}X_n \\
&X_2 = e_2 + a_{21}X_1 + a_{22}X_2 + \ldots + a_{2n}X_n \\
&X_n = e_c + a_{n1}X_1 + a_{n2}X_2 + \ldots + \\
&X_i, a_{ij}, e_1 \geq 0 \\
&i, j = 1,2,\ldots,n
\end{aligned}
\qquad \textbf{(3- 1)}
$$

The variable e_i represents excess production in sector i.

In later estimation approaches, we use the dynamic Leontief input-output model to estimate the technology coeffiecents (a_{ij}). We estimated the technology based on the idea of minumum ratios of minimum input of some commodity i per unit of commodity j taken across the entire industry. This directly leads to the LINGO program that we wrote to estimate the technology coeffiecients-a_{ij}s-for a given set of fiscal data over a period of years. This program is shown in Appendix B; the data set that it uses is shown in Appendix A. The goal of this program is to minimize the sum of the errors over the five functional areas; these are excess output. Then using this program we obtain an initial set of technology coefficients (a_{ij}s).

We use several versions of this program; the first one that we use puts zero in the a_{ij} position that relates MilCon and MilPer, because most military construction is contracted out to civilian firms. Military construction is not related to the other functional areas because it comes out of a separate section of the DoD budget. We also use the net production model, which sets the values on the diagonal of the matrix–the a_{ii}s to zero, because otherwise we would obtain the trivial solution. These zeros are shown in bold face in Table 1. The locations of the initial a_{ij}s are shown in Table 3.

Table 3: Initial Technology Coefficient Locations

a_{ij}	R&D	Proc	O&M	MilPer	MilCon
R&D	**0**	0	0.22948	0	0.22948
Proc	0.08797	**0**	0.14498	0.32480	0
O&M	0.16414	0	**0**	0.49225	1.63929
MilPer	0.91165	0.19681	0	**0**	5.24399
MilCon	0.02682	0.02668	0.00973	0	**0**

When the initial technology coeffiecents are plotted, it is clear that not all of the a_{ij} locations are logical. For example, O&M funding should not be directly related to R&D funding. Therefore, we set many of the technology coefficients to zero. The reason for many of the zero positions is they are related, but only with a lag period. For example, a program that starts in R&D progresses to Procurement, and then becomes a full-fledged program in O&M--thus the Procurement budget is probably going to have more of a direct effect on the O&M budget than on the R&D budget.

The positions that are forced to zero are shown in Table 4, along with the location and values of the new a_{ij}s that are determined by the LINGO program.

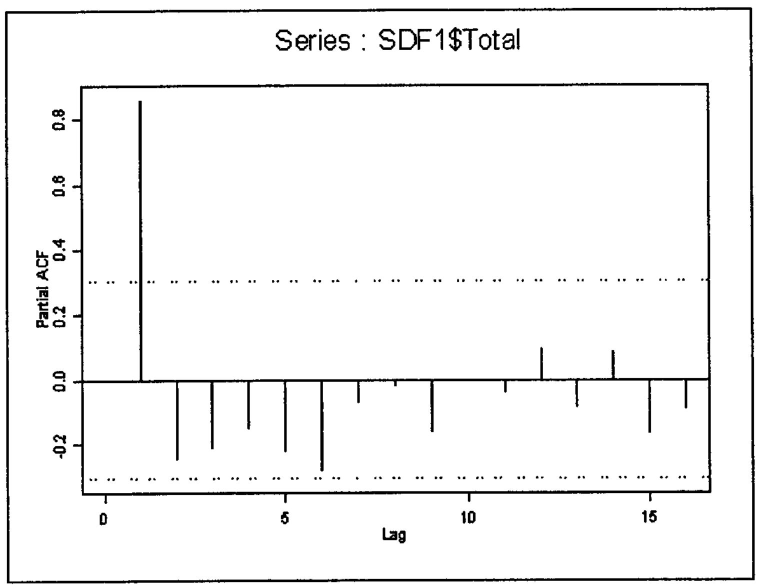
Table 4: New Technology Coefficients

a_{ij}	R&D	Proc	O&M	MilPer	MilCon
R&D	**0**	0	0.35307	0	0
Proc	**0**	**0**	0	0.169549	11.53205
O&M	**0**	**0**	**0**	**0**	16.82378
MilPer	0	0	1.08157	**0**	**0**
MilCon	**0**	**0**	**0**	**0**	**0**

The next step is to complete a correlation study to accurately determine where the relationships between the functional areas lie. Using the CORREL function in MS Excel to correlate the areas between each other, we arrive at the correlation matrix shown in Appendix C. We also must look at the Partial Auto-Correlation Functions (PACF) for the functional areas to determine the amount of lag time that is significant. The PACFs are obtained using SPLUS, a statistics software package with time-series functions. A lagged relationship is one that occurs over more than one time period. For example, R&D projects generally take a long period of time. If an R&D project is started today, it will generally take several years to complete, and the funding for that project will be spread out over all the projected years. Thus, this relationship will occur at a lag. The PACFs for the individual functional areas, show different lags are significant. R&D and MilCon have no significant lag period, Procurement and MilPers have two, and lastly O&M has one significant lag period. These figures are shown in Appendix C.

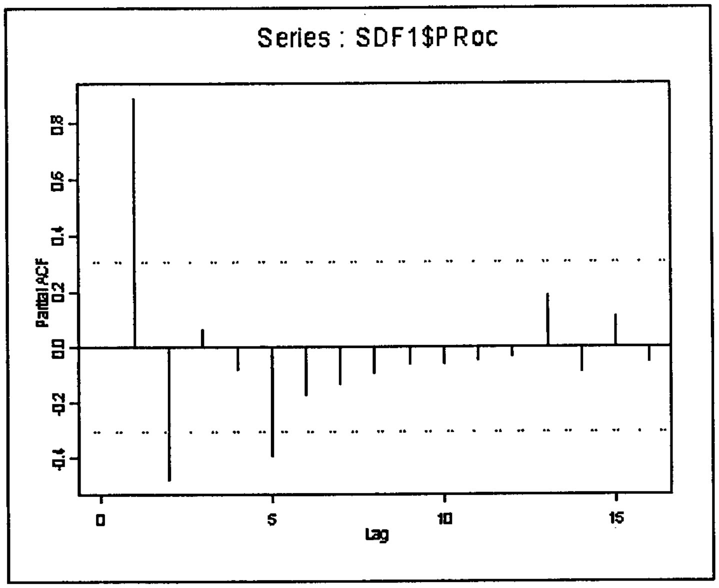
The total budget authority is a compilation of the five functional areas. The PACF, as computed by SPLUS, is shown in Figure 2, and it shows the significant lags that are dominant for the data set. Significance in the PACFs is shown by rises above the dashed line, or for negatives values, drops below it. For the total DoD budget authority, only the first lag is significant; this is shown in Figure 3 by the spike rising above the dashed line.

Figure 3: PACF for the DoD Total Budget Authority



An example of a PACF with two significant lags is procurement. This is shown in Figure 4.

Figure 4: PACF for the DoD Procurement Budget Authority



This functional area is significant at two lagged time periods because in the first time period, the PACF rises above the significant point; and in the second time period it is negatively auto-correlated, and drops below the significance line. The second significant negative spike is not being used because currently we are only using two lags. The partial auto-correlation functions will form the a_{ii}s of the Leontief equations. The rest of the PACFs are shown in Appendix C.

We also look at the cross-correlations between the five functional areas to determine how many lags are significant. In order to complete the multi-variate cross-correlations, we use the statistics package SAS. The SAS program we use to compute the cross-correlation between R&D and O&M is shown in Appendix E.

The output from that program is displayed in Figure 5, which shows that the cross-correlation between R&D budgetary authority and O&M budgetary authority is significant at the second lag.

Figure 5: SAS Output Showing Cross-Correlation between R&D and O&M.

```
                 Cross-Correlations between R&D and O&M

Lag  Covariance  Correlation  -1 9 8 7 6 5 4 3 2 1 0 1 2 3 4 5 6 7 8 9 1

 0   -7.324522    -.06719    |              .     *|   .              |
 1    4.790914    0.04395    |              .      |*  .              |
 2   25.539035    0.23426    |              .      |*****.            |
 3   10.197211    0.09354    |              .      |**  .             |
 4   18.558876    0.17024    |              .      |***  .            |
 5   -8.079528    -.07411    |              .     *|   .              |
 6  -19.181926    -.17595    |            .   ****|   .               |
 7   -8.447385    -.07749    |             .    **|   .
 8    0.739949    0.00679    |             .      |   .               |
 9    4.163540    0.03819    |             .      |*  .               |
10   -8.986959    -.08244    |             .    **|   .               |

                      "." marks two standard errors
```

Due to the inherent flexibility in the Leontief input-output tables, the individual lags can be built into each of the data series so that more clarity can be achieved in the model. Therefore, instead of basing the lags in the entire model on the total DoD budget authority, we can base the lags on the individual significant lags for each functional area. In our model we use the least lagged values to explain as much of the data as possible, then progress to the next lag (where that is applicable). Another nice feature of using the Leontief input-output tables is that the error can be tracked by sector; this will allow us to identify areas where the model is not doing a good job, and to adjust only that section instead of having to fix the entire model.

The new Leontief model that we use can be represented by equation 3- 2. This series of equations is based on Table 13, which shows the locations of the new a_{ij}s. In the interest of space, all of the a_{ii}s, that are set to zero are not included. However, since lagged a_{ii}s will not force the linear program to the trivial solution, these are included. We also assume that consumption is equal to zero; therefore, these are removed from equation 3- 2. The variables that have a prime symbol (') associated with them represent one lagged time period; two prime symbols ('') means two lagged time periods.

$$x_{RD} \geq a_{RD,RD'}x_{RD'} + a_{RD,RD''}x_{RD''} + a_{RD,Proc}x_{Proc} + a_{RD,OM}x_{OM} + a_{RD,MilPer}x_{MilPer} + a_{RD,Proc''}x_{Proc''} + a_{RD,OM''}x_{OM''} + a_{RD,MilPer'}x_{MilPer'} + a_{RD,MilPer''}x_{MilPer''}$$

$$x_{Proc} \geq a_{Proc,Proc'}x_{Proc'} + a_{Proc,Proc''}x_{RD''} + a_{Proc,MilPer}x_{MilPer}$$

$$x_{OM} \geq a_{OM,Proc}x_{Proc} + a_{OM,Proc'}x_{Proc'} + a_{OM,OM'}x_{OM'} + a_{OM,OM''}x_{OM''} + a_{OM,MilPer''}x_{MilPer''}$$

$$x_{MilPer} \geq a_{MilPer,OM}x_{OM} + a_{MilPer,MilPer'}x_{MilPer'} + a_{MilPer,MilPer''}x_{MilPer''}$$

$$x_{MilCon} \geq_{MilCon,MilCon'} x_{MilCon'} +_{MilCon,MilCon''} x_{MilCon''}$$

$$x_i, a_{ij}, S_{i,t}, C_i \geq 0$$
$$i, j = RD, Proc, OM, MilPer, \text{ or } MilCon \qquad \textbf{3- 2}$$

Based on cross-correlations which provide new locations of the a_{ij}s, each cell in Table 4 has three numbers; the first one is the a_{ij} that is not lagged, the second one is the a_{ij} at one lag, and the third is the a_{ij} at two lags. Zeros that are shown in bold face are those that have been set to zero in the LINGO program. The LINGO program in Appendix D reflects these modifications to the program shown in Appendix A. The program in Appendix D results in the technology coeffiecients shown in Table 4.

Table 4: New Technology Coefficients with Lag Values

	R&D	**Proc**	**O&M**	**MilPer**	**MilCon**
R&D	0/0/0.047	**0/0/**0.003	**0/0/**0.2724	0/0/0	**0/0/0**
Proc	**0/0/0**	0/0.822/0	**0/0/0**	0/0/0	**0/0/0**
O&M	**0/0/0**	0.226/0/0	**0/**0.253/0.451	**0/0/0**	**0/0/0**
MilPer	**0/0/0**	**0/0/0**	0/0/0	**0/**0.911/0	**0/0/0**
MilCon	**0/**0.0236**/0**	**0/0/0**	**0/0/0**	**0/0/0**	**0/**0.266/0.365

3.2 Estimating Techonology Coefficients Based on Residuals

We next use a different estimation method. Instead of LINGO and a linear program format, we choose Microsoft Excel and estimate the a_{ij} coffiecients from the correlation between the residuals and the data points. This estimation method allows us to estimate the functional areas individually. This is more easily accomplished than attempting to estimate a_{ij} coefficeients of this specialized economy in one step.

The first step in using this new estimation scheme is to set up Excel worksheets, one for each of the functional areas. First we establish the worksheets with areas for data points, residuals, correlations, and estimated a_{ij} values. After this,we bring in the values for a_{ij}, carefully checking to make certain that they don't make values of the residuals negative. If a residual becomes negative when a proposed a_{ij} is brought into the tableau, then a red flag is sent up, which indicates that this is too large. The values are picked by the amount of correlation by year that is held between the residuals and the sector data. The a_{ij} values with the highest correlation are tried first. This process is continued across all five of the functional areas. The a_{ij} values that are obtained are shown in Table 5.

Table 5: a_{ij} Values Obtained Using Microsoft Excel.

$a_{ij}/aa_{ij}/aaa_{ij}$	**R&D**	**Proc**	**O&M**	**MilPer**	**MilCon**
R&D	0/0/0	0/0/0	0.2836/0.2292/0	0/0/0	0/0/0
Proc	0/0/0	0/0.7448/0	0/0/0	0/0/0	0/0/0
O&M	0/0.3957/0	0/0/0	0/0.7036/0	0/0/0	0/0/0
MilPer	0/0/0	0/0/0	0/0/0	0/0.8950/0	0/0/0
MilCon	0/0/0	0/0/0	0/0/0.03893	0/0/0	0/0/0

There is an important difference between this estimation scheme and the previous estimation schemes; here the lag values are forward lag values instead of backwards lags. We have decided that it is more logical, for the purpose of this thesis, for this year's budget to affect next year and the following year's a_{ij} values, instead of having it affect the two previous years. Therefore, we cannot compare the results of this estimation with those of the previous estimation. Instead we must reestimate the a_{ij} values that were obtained using LINGO to use the forward lag. The rewritten LINGO program estimating the a_{ij} coefficients is shown in Appendix F. The results of running this program are shown in Table 6.

Table 6: a_{ij} Coefficients From LINGO With Forward Lags

	R&D	**Proc**	**O&M**	**MilPer**	**MilCon**
R&D	0/0.149/0.176	0/0/0	0/0/0.1486	0/0/0	0/0/0
Proc	0/0/0	0/0.571/0	0/0/0	0.150/0/0	0/0/0
O&M	0/0/0	0/0/0	0/0.785/0.046	0/0/0	0/0/0
MilPer	0/0/0	0/0/0	0.1132/0/0	0/0.825/0	0/0/0
MilCon	0/0/0	0/0/0	0/0/0	0/0/0	0/0.318/0

Comparing the values obtained through the two different estimation techniques, we see that both the position and the values of the a_{ij} coeffiecients are similar.

Next we add two extra lag values to the Procurement functional area. Doing this changes the nature of the chart so that the second row contains five different values. The first three values are the same as in previous estimations. The fourth and fifth values are the next forward lagged values.

However, the a_{ij} values are calculated separately by functional area; thus, only the Procurement line of Table 6 will change. The new a_{ij} values are shown below in Table 7.

Table 7: a_{ij} Values Obtained Using Microsoft Excel With More Forward Lags

	R&D	Proc	O&M	MilPer	MilCon
R&D	0/0/0	0/0/0	0.284/0.23/0	0/0/0	0/0/0
Proc	0/0/0/0/0	0/0.748/0/0/0	0/0/0/0/0	0/0/0/0/0	0/0/0/0/0
O&M	0/0.396/0	0/0/0	0/0.704/0	0/0/0	0/0/0
MilPer	0/0/0	0/0/0	0/0/0	0/0.895/0	0/0/0
MilCon	0/0/0	0/0/0	0/0/0.039	0/0/0	0/0/0

Looking at the values for a_{ij} shown in Table 7, we realize that these have the same problem as the values that are obtained using LINGO–a lack of connections between the functional areas. This means that when any one functional area is at its maximun, all of the remaining areas must be zero.

Since we know that, in reality, there are connections between these functional areas, we next add a constraint to the Microsoft Excel spreadsheet to measure the connectivity. Then we check the combinations of a_{ij}s that maximize the connections between the functional areas. An example is the connection between R&D and Procurement--R&D programs lead to Procurement programs; thus, this is a direct connection.

We believe that pathways do exist in the Air Force. If this is correct, we should be able to use a combination of minimizing the error and maximizing the connectivity to obtain a set of a_{ij} values that explain how the DoD uses its budget money. Using this modified estimation scheme, we obtain the values shown in Table 8.

Table 8: a_{ij} Values Modified MS Excel Estimation Scheme.

	R&D	**Proc**	**O&M**	**MilPer**	**MilCon**
R&D	0/0/0	0/0/0	0.2836/0/0	0/0/0	0/0/0
Proc	0/0/0	0/0.099/0	0.3365/0/0.0320	0/0/0	0/0/0
O&M	0/0.1/0.7	0/0/0	0/0.4/0	0.2/0/0	0/0/0
MilPer	0/0/0	0.2/0/0.1	0.231/0.2/0	0/0.2/0	0/0/0
MilCon	0/0/0	0/0/0	0/0/0.03893	0/0/0	0/0/0

These a_{ij}s appear to be reasonable: There are very few unexpected connections, the values are not extremly high or low, and there is significantly more connectivity than in many of the previously attempted estimations.

We next place these a_{ij} values into a linear program and estimate how much of the budget should be be spent on R&D in order to maximize O&M. The critical assumption of this linear program is that the budget is in steady state. The LINGO program is shown in Appendix G.

It is designed to maximumize the amount of O&M as a percentage of the DoD budget. The results of Appendix G are shown in Table 9.

Table 9: Proposed Optimal Funding Levels for the DoD budget

Functional Area	**Proposed Funding (%)**	**Average Historical Funding (%)**
R&D	11.7	11.1
Procurement	16.8	24.2
O&M	41.3	29.7
Military Personal	28.6	33.2
Military Construction	1.6	1.8

Comparing the predicted levels of funding to the historical averages shows that most of the areas are similar, and that a tradeoff exists between Procurement and O&M. This is a descriptive model, so recreating numbers that are similar to the historical averages acts as a justification for this model.

From here we change data sets to the Air Force's budget data, use the modified estimation scheme, and then place those results into LINGO. When we place the Air Force's budget data and the appropriate technology coefficients into the LINGO model, we are unable to reproduce anything similar to the historical averages.

We theorize that this is true because this estimation scheme looks for the minimization of the error. However, when this is more closely examined, it appears that the lowest point is an outlier; because of this, the estimation scheme will have problems finding appropriate technology coefficients. A different estimation system, one that is not based on low outliers, is the next one used in an attempt to establish technology coefficients for the Air Force; this estimation trial is detailed in Chapter Four.

4. Using Regression to Estimate the Technology Coefficients

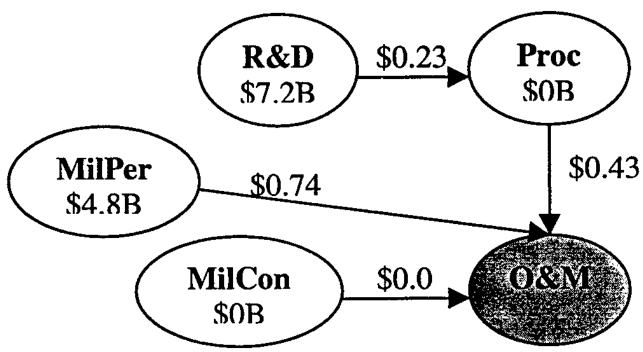
In the last chapter, we attempted to use correlation to come up with the a_{ij} coefficients for the Leontief input-output table. In this chapter, we detail a new estimation scheme that uses regression to come up with the technology coefficients. This estimation scheme uses the same breakout of the aggregate Air Force budget (Procurement, Operations and Maintenance, Military Personnel, Military Construction, and Research and Development) that we used in previous chapters.

We also model the budget data with eight sectors--Strategic Forces and General Purpose Forces, Special Operations, Intelligence and Communications, Research and Development, Air National Guard and Air Force Reserve (ANG&AFR), Central Supply and Maintenance, and Training Medical and General Personnel. Two other sectors (Administration and Associated Costs, and Support of Other Nations) in this budget breakout are relatively small and do not appear to be related to the other areas. Therefore, we did not include them.

4.1 Regression Estimation Scheme on the Five-Sector Problem

We use regression to model the distribution of net output among the other sectors. Each row in equation 2-2 is a separate regression equation. The budget for each fiscal year is treated as the dependent variable, and the other sector budgets are the independent variables. The resulting intercept represents the final demand, y_i, and the resulting regression coefficients are the estimated technology coefficients, a_{ij}. Relationships are accepted if several conditions are met. First, the relationship needs to be one that could happen in the military. For example, MilCon does not support R&D. Second, the relationship needs to be statistically significant. Finally, the intercept and coefficients should be non-negative. If two or more regressions meet these criteria, we choose the regression that has the highest R^2-value. In Figure 6 the amounts under R&D and MilPer represent the final demands. These may be considered fixed costs that do not vary based on the receiving sectors. The arrows between sectors represent the amounts spent per each dollar in the other areas. For example, for each dollar spent in Proc budget, an additional 23 cents is required in the R&D budget. This is the final set of relationships that we have chosen for the Air Force.

Figure 6: Air Force Budget Relationships in the Five Functional Area Problem



The Proc relationship with O&M has a negative intercept. In effect, the first $16.7 billion of O&M does not appear to require Proc support.

The Leontief input-output model that we use is a net flow model. We solve the linear program for every fiscal year using the estimated technology coefficients. Each problem has one additional constraint--the total budget not be exceeded. The program that we use to solve these problems is in Appendix H and is written for fiscal year 1962.

We now look at the results for the unbounded model; the binding constraint on this model is that the total budget for that fiscal year must not be exceeded. For each fiscal year, we solve the model as a linear program that maximizes O&M.

When we compare the model results to the actual budgets, we see that the overall averages for each area are nearly the same. The variability in the model's results is considerably less than the actual data. The model results indicate that the budget percentage for R&D and MilCon are steady at approximately 14% and 2% (respectively) of the total Air Force budget.

As the total budget increases from seventy-seven billion dollars to one hundred forty-three billion dollars, the percent spent on MilPer changes from approximately 32% to approximately 25% and O&M from 30% to 25%. These changes are shown in Appendix I. During the same time period, Procurement increases from 22% to 35%. This is a descriptive--not a causal model. It reflects acquisition policy over the last thirty years.

If the budget continues to remain in the range of seventy to eighty billion dollars per fiscal year, this model indicates a challenge for Air Force leadership. Procurement will remain relatively low as a percentage of the total budget and in dollar amount as well. Increasing Procurement requires a corresponding increase in Research and Development. In order to change how a lower budget would react, the Air Force would have to change the acquisition process drastically to reflect how the United States military funding has changed.

4.2 Eight Sector Technology Coefficient Scheme

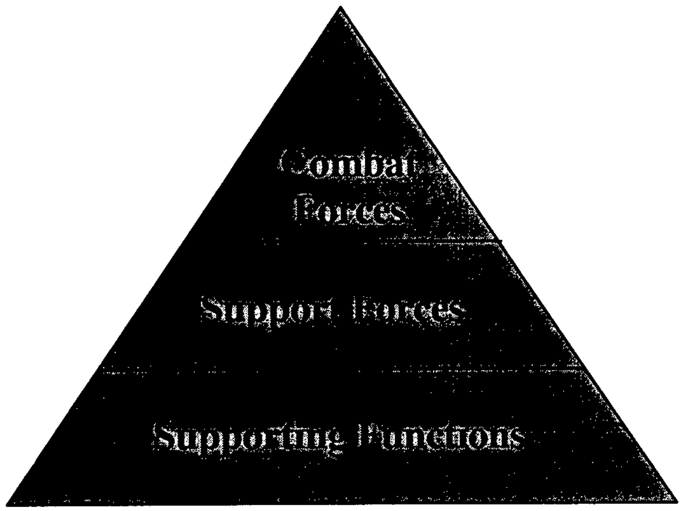
This estimation scheme is based on the same idea as the one detailed in the previous section. The only difference is that this one uses a different compilation of budget data from ABIDES. In this scheme, we broke the ABIDES data into nine sectors--Strategic Forces, General Purpose Forces, Special Operations, Intelligence and Communications, Research and Development, Air National Guard and Air Force Reserve (ANG&AFR), Central Supply and Maintenance, and Training Medical and General Personnel. Also included in this ABIDES breakout, although we don't use them, are Support of Other Nations, and Administration and Associated Costs.

The first step after getting the data, and looking at the nine sectors, is to identify the appropriate relationships. We use the same technique as before and come up with relationships that cannot statistically occur due to multicolinearity. The General Purpose Forces and Strategic Forces are very highly correlated; therefore, their impact cannot be separated. Our next step is to change the budget breakout. Thus, we combine General Purpose Forces and Strategic Forces into one new functional area–Forces.

We divide the budget into three areas--Combat Forces, Supporting Forces, and Supporting Functions. Combat Forces is at the top of the pyramid shown in Figure Seven, and consists of Forces (General Purpose and Strategic Forces) and Special Operations, neither of which support any other sector. Supporting Forces consists of Intelligence and Communications, and Airlift. They support themselves and Combat Forces.

Finally, Supporting Functions consists of the Air National Guard/Air Force Reserve, Research and Development, Central Supply and Maintenance, and finally, Medical and General Personnel. The Supporting Functions can support all of the other budget areas.

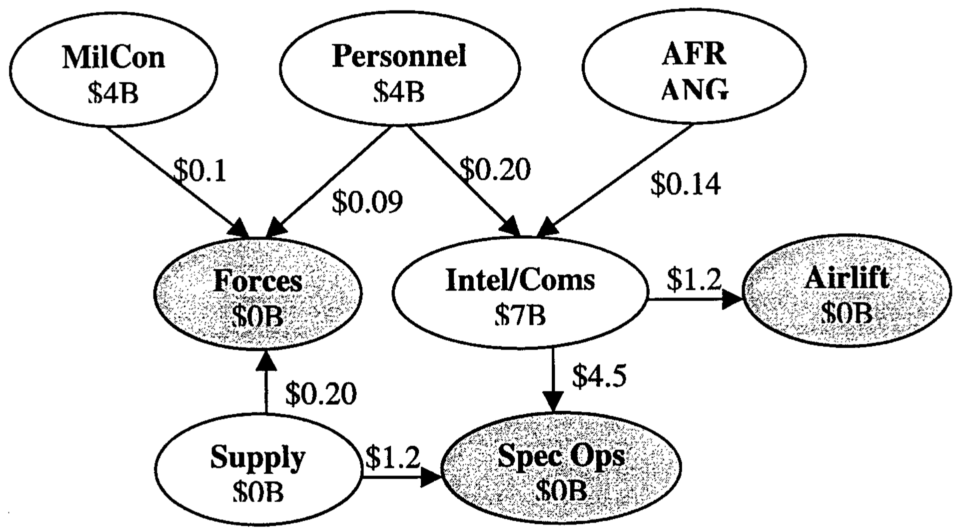
Figure 7: Functional Area Relationships



Initially, we thought that Airlift would support Combat Forces; however, it turns out not to support any other sector in this model. We assume that it doesn't support Air Force Combat Forces because the Air Force's Airlift capacity probably supports the Army.

By delineating the relationships before we complete the regression, we are able to eliminate some relationships that do not appear to be able to occur in the Air Force today. For example, there does not appear to be a direct relationship between Medical and General Personnel, and Central Supply and Maintenance. Therefore, we are able to eliminate the indirect relationships and concentrate on direct ones. When we complete the regressions, examine the p-values, R^2-values, and ensure positive intercepts, we obtain the relationship tree that is shown in Figure 8.

Figure 8: Relationship Tree in the Eight-Sector Problem



In Figure 8, the fixed minimum in the sector is below the name.

We use LINGO to solve the problem, and the program that we use to solve it is in Appendix J. The results from this program (FY1980-FY89) are shown below the program in Appendix J. This model forces some of the functional areas to zero because it is not efficient to place resources into them. For example, when maximizing the sum of Forces, Special Operations, and Airlift, Forces requires fewer inputs than other combinations; therefore, all available resources are placed in Forces. In order to place one dollar into Intelligence and Communications, it requires twenty cents of Personnel; however, for that dollar to be placed into Forces, it only requires ten cents worth of Personnel. This model seeks to act in the most efficient manner possible; therefore, it is not prudent to place money into the more expensive functional areas. Hence, Forces are maximized while Special Operations and Airlift are set to zero.

5. Conclusions and Follow-On Research

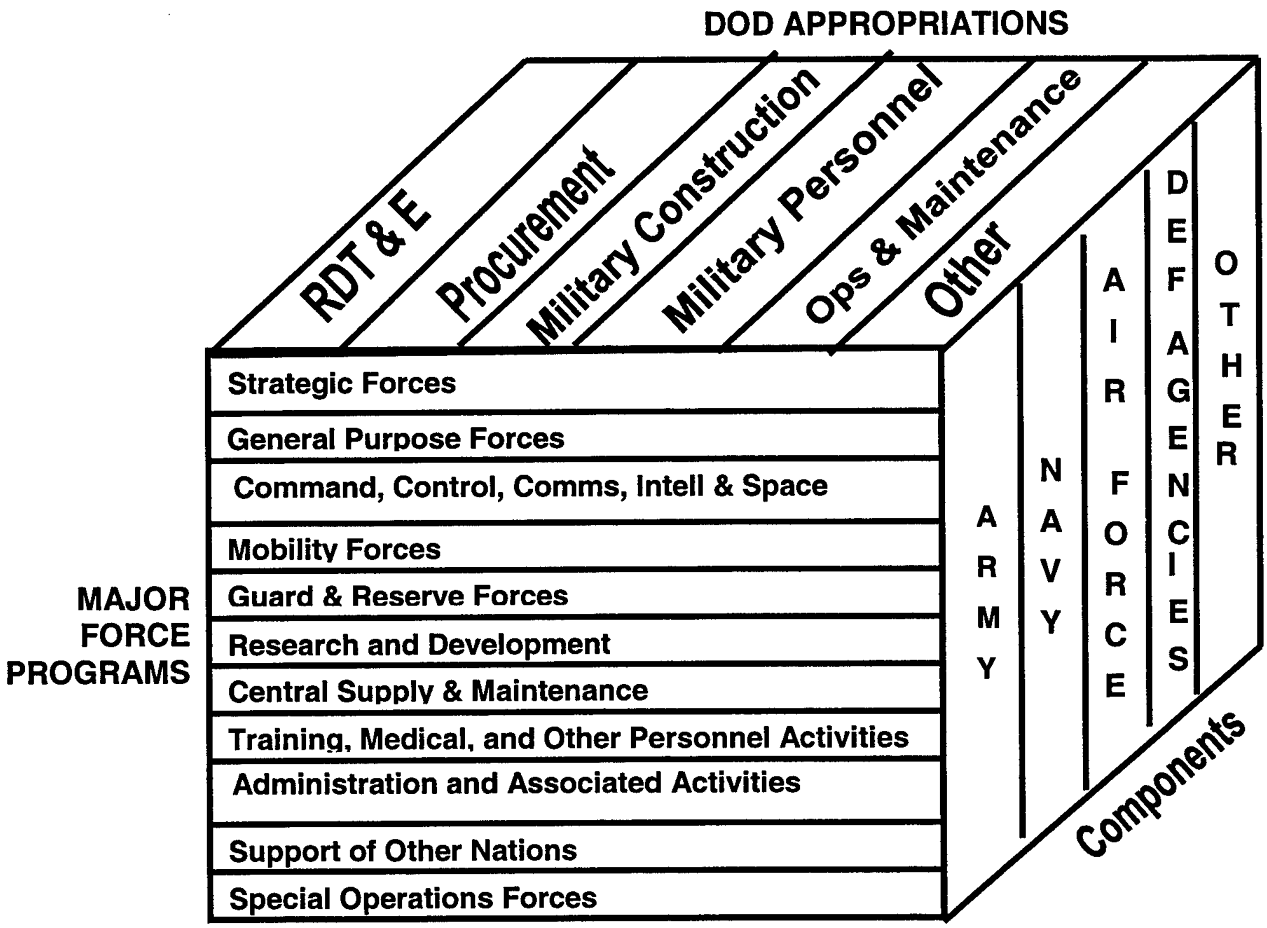
We have tried two very dissimilar estimation techniques--minimizing the error and regression--while trying to estimate the technology coefficients that could be used in a Leontief input-output model for the Air Force budget problem.

The DoD data set is significantly larger than the Air Force data set--it contains twenty extra years of data. This might explain why the DoD aggregate budget problem works better. A possible area of future research suggested by our study is to is to use our second estimation technique on an expanded Air Force data set.

Another area of future research could be to expand the eight-sector model by incorporating the Army's budget data into it. Since the Army and the Air Force often work jointly, it would be interesting to see if there are inter-relationships between the two services, especially the reimbursement portion of each service's budget. Another interesting way to expand this model would be to add the Navy into the Air Force budget problem. There is a known relationship between the two military forces--the Navy quite often provides high-volume sealift and other services to the Air Force.

Therefore, this is a two-step process: solve either the Army's or the Navy's aggregate budget problem, and then relate the services to each other. The relationships between the services are shown in Figure 9.

Figure 9: Future Years Defense Program



Bibliography

Dorfman, Robert; Samuelson, Paul A; Solow, Robert M. Linear Programming and Economic Analysis. 2nd ed. New York: Dover Publications, 1987.

Dudney, Robert S. "Defense Spending Illusions and Realities." Air Force Magazine: Journal of the Air Force. April 2000, pp. 62-66.

Hillier, Frederick S; Lieberman, Gerald J. Introduction to Operations Research. 1st ed. San Francisco: Holden-Day, 1967.

Leontief, Wassily. Input-Output Economics. 2nd ed. New York: Oxford University Press, 1986.

Miller, Ronald E. and Blair, Peter D. Input-Output Analysis Foundations and Extensions. Englewood Cliffs: Prentice-Hall, 1985.

Patton, George T; Snyder, Jr. Christopher L; Szynkowski. Planning, Programming and Budgeting Under the New Laird/Packard Fiscal Constraints: The Role of the Electric FYDP System. Office of the Secretary of Defense: Systems Analysis, 1971.

Snodgrass, Anthony. Modeling Strategic Effects Using the Input-Output Model: A Thesis. Air Force Institute of Technology: 2000.

United States General Accounting Office. Defense Budget: Analysis of Operations and Maintenance Accounts for 1985-2001. General Accounting Office, 1997.

Wassily Leontief and His Contributions to Economic Accounting. Survey of Current Business. Vol. 79 Issue 3, March 1999, pp. 9-12

Wu, Nesa; Coppins, Richard. Linear Programming and Extensions. McGraw-Hill Book Company: 1981.

Appendix A: Department of Defense Aggregate Budget Data (FY00$B)

Year	R&D	Proc	O&M	Mil Pers	Mil Con
FY1960	29.7	68.1	69.8	105.3	7.1
FY1961	30.9	71.1	68.9	105.7	5.5
FY1962	32.8	94.1	72.9	114.6	4.1
FY1963	35.5	97	71.5	113.1	6.1
FY1964	35.1	86.6	71.1	114.7	4.7
FY1965	32	74.7	73.2	116.2	5
FY1966	32.2	100.3	84.7	122.7	11.6
FY1967	33.1	110	101.6	137.1	4.9
FY1968	32.6	107.9	105.9	143.1	6.6
FY1969	33	90.7	107.8	145.5	4.8
FY1970	30.4	76.3	97.5	141.5	3.8
FY1971	87.7	63.5	87.7	132.6	4.9
FY1972	28	67.5	83.5	121.2	4.5
FY1973	28	61.4	82	114.3	4.4
FY1974	26.2	55.3	81.4	111.5	4.5
FY1975	24.9	49.5	80.1	109.9	5.2
FY1976	25.5	57.8	81.9	107.6	5.9
FY1977	25.9	69.3	83.9	106.7	5.1
FY1978	26	67.1	84	106.1	3.5
FY1979	25.7	64.8	85.7	106.1	4.5
FY1980	25.5	65.8	89.9	107.3	4.1
FY1981	28.6	82.2	97.3	110.8	5.7
FY1982	32.7	103.3	104.2	112.7	7.9
FY1983	35.8	122.3	108.5	114.9	7
FY1984	40.6	126.6	113.8	115.5	6.8
FY1985	45.9	138.1	121.8	111.5	8.1
FY1986	48	128.1	116.5	108.3	7.5
FY1987	49.4	107.5	119.5	107.5	7
FY1988	48.7	103.1	118.7	113.9	7
FY1989	48	98.7	119.5	113	7.2
FY1990	44.9	97.7	118.3	111.9	6.2
FY1991	43	83.7	144.4	113.8	6.1
FY1992	42.2	69.6	105.7	103.7	6.1
FY1993	42.9	59	107.4	95.8	5.2
FY1994	38.5	48.5	103.9	87.8	6.7
FY1995	37.7	47.1	107.9	86	5.9
FY1996	37.5	45.2	105.3	82	7.4
FY1997	38.4	45.2	101.7	80.2	6
FY1998	38.7	46.7	104.3	77.2	5.8
FY1999	39.6	52.5	110.7	75.8	5.6
FY2000	39.1	55.1	108.8	76.1	4.9

Appendix B: a_{ij} generation in LINGO.

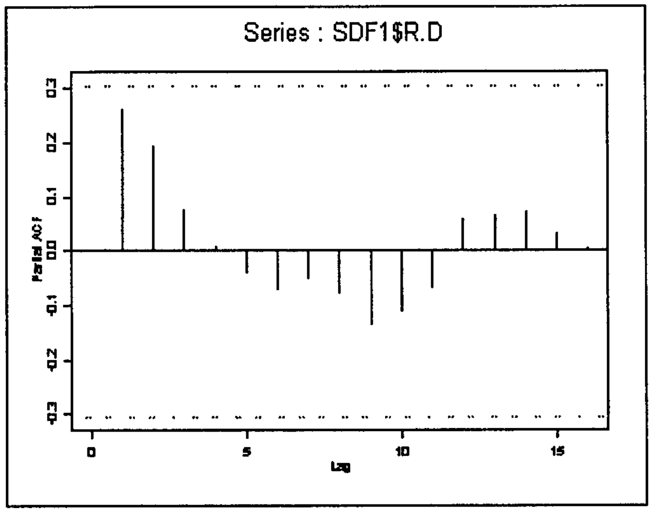
```
!A Leontief Input-Output model of the DoD budget;
SETS:
! Sectors are the five major functional areas of the DoD budget authorization;
	SECTORS/ OM RD MILPER PROC MILCON/: FUNDS;
	YEARS /  FY1960 FY1961 FY1962 FY1963 FY1964 FY1965 FY1966 FY1967
FY1968 FY1969 FY1970 FY1971 FY1972 FY1973 FY1974 FY1975 FY1976 FY1977
FY1978 FY1979 FY1980 FY1981 FY1982 FY1983 FY1984 FY1985 FY1986 FY1987
FY1988 FY1989 FY1990 FY1991 FY1992 FY1993 FY1994 FY1995 FY1996 FY1997
FY1998 FY1999 FY2000/: CONSUMPTION;
	BYSECTORS (SECTORS, SECTORS): A;
	BYYEAR (SECTORS, YEARS): BUDGET, ERRORS;
ENDSETS

!The objective function = minimize the sum of the errors;
MIN = @SUM( BYYEAR(I, J): ERRORS(I, J));
!Constraints;
@FOR( BYYEAR(I, J):
	@SUM( SECTORS(K): BUDGET(K, J)*A(I, K)) – ERRORS(I, J) = BUDGET(I,
J);
@SUM(SECTORS(I):A(I, I)) = 0;

!The data;
DATA:
!Import the data from MS Excel;
	CONSUMPTION = @OLE( 'D:\THESIS\PARAMETER ESTIMATION\
AIJ_ESTIMATION.XLS');
	BUDGET = @OLE( 'D:\THESIS\PARAMETER ESTIMATION\
AIJ_ESTIMATION.XLS');
```

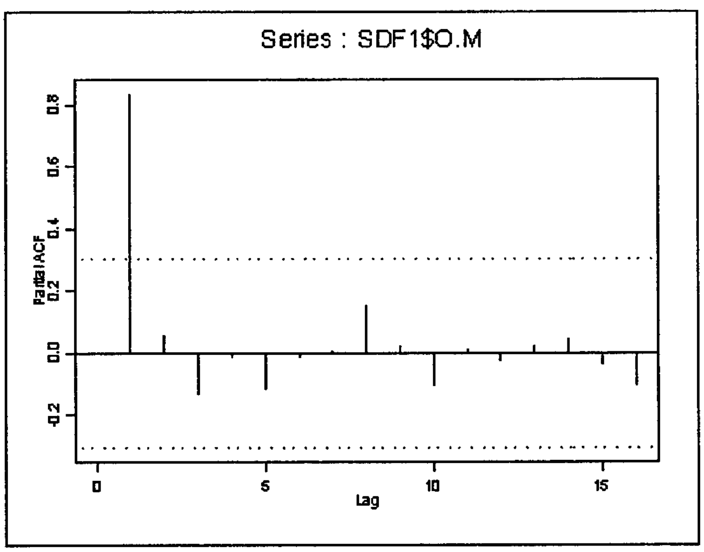
Appendix C: Correlation Study Results

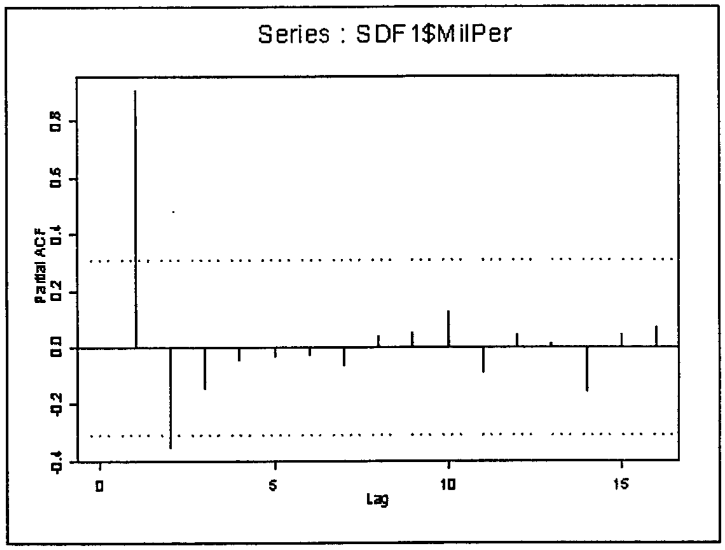
Auto-Correlation Table for DoD Budget Data 1960-2000

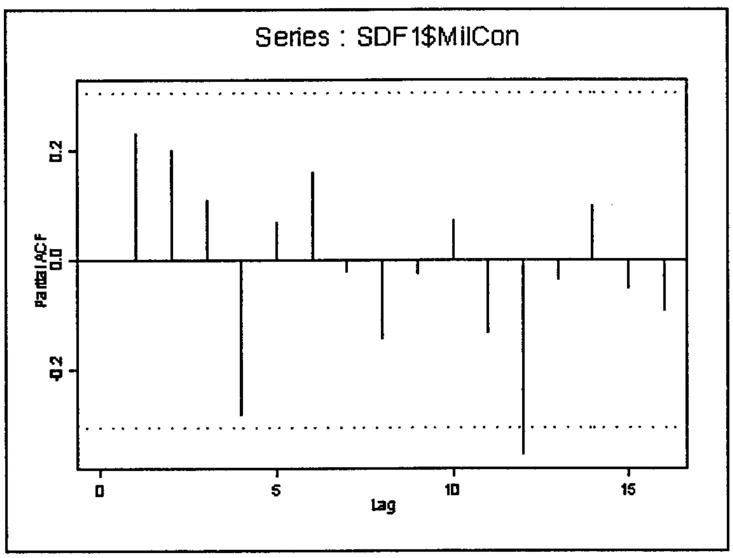
Lag	R&D	PROC	O&M	MilPer	MilCon	Total
1	0.261726	0.903172	0.867917	0.955533	0.236027	0.878177
2	0.252122	0.715044	0.779279	0.852023	0.247649	0.702451
3	0.172154	0.527604	0.647332	0.710826	0.198293	0.48431
4	0.104928	0.358104	0.541745	0.549515	-0.15948	0.252483
5	0.037375	0.142577	0.411675	0.387877	0.04612	-0.00307
6	-0.02793	-0.12245	0.309345	0.231518	0.083073	-0.28426
7	-0.05206	-0.38807	0.210471	0.086859	-0.03509	-0.50972
8	-0.10535	-0.59912	0.19576	-0.01513	-0.00128	-0.6546
9	-0.18686	-0.73515	0.182091	-0.05988	-0.0463	-0.75502
10	-0.20412	-0.80026	0.189002	-0.01648	-0.06782	-0.78329
11	-0.19327	-0.81679	0.189292	0.064607	-0.18443	-0.73315
12	-0.15348	-0.7781	0.202955	0.185056	-0.43131	-0.58561
13	-0.0735	-0.63665	0.246987	0.304414	-0.28902	-0.4058
14	0.041915	-0.44038	0.320939	0.362933	-0.3272	-0.17465
15	0.104215	-0.21793	0.382165	0.398228	-0.24588	0.028619
16	0.163569	0.023715	0.452124	0.434845	0.070082	0.240915
17	0.181514	0.2781	0.518883	0.485084	-0.05422	0.459232
18	0.210282	0.548067	0.551782	0.533395	-0.13559	0.642175
19	0.168586	0.729971	0.527887	0.57507	0.263641	0.740841
20	0.154019	0.770661	0.489635	0.590378	0.188168	**0.748956**
21	0.162515	0.751033	0.478152	0.544959	0.206162	0.602269
22	0.22474	0.700953	0.426302	0.425317	0.315246	0.465145
23	-0.01928	0.584032	0.16873	0.23725	0.216982	0.251734

Partial Auto-Correlation Function Tables for the DoD Functional Areas






Series : SDF1$MilPer
Partial ACF
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
0
5
10
15
Lag


Series : SDF1$MilCon
Partial ACF
0.2
0.0
-0.2
0
5
10
15
Lag

Appendix D: a_{ij} generation in LINGO (Three Lags)

```
MODEL:
!A Leontief Input-Output Model of the DoD Budget;

SETS:
!Sectors are the five major areas of budget authorizations;
  SECTORS/ RD PROC OM MILPER MILCON/: FUNDS;
  YEARS/ FY1960 FY1961 FY1962 FY1963 FY1964 FY1965 FY1966 FY1977
FY1968 FY1969 FY1970 FY1971 FY1972 FY1973 FY1974 FY1975 FY1976 FY1977
FY1978 FY1979 FY1980 FY1981 FY1982 FY1983 FY1984 FY1985 FY1986 FY1987
FY1988 FY1989 FY1990 FY1991 FY1992 FY1993 FY1994 FY1995 FY1996 FY1997
FY1998 FY1999 FY2000/;
  BYSECTORS (SECTORS, SECTORS): A, AA, AAA;
  BYYEAR (SECTORS, YEARS): BUDGET, ERRORS;
ENDSETS

!The objective function -> minimize error;
MIN = @SUM( BYYEAR(I, J): ERRORS(I,J));
!Constraints;
        !i=years, j=sectors;
@FOR( BYYEAR(I, J)| J #GE# 3:
   @SUM(SECTORS(K):A(I,K)*BUDGET(K,
J))+@SUM(SECTORS(K):AA(I,K)*BUDGET(K,J-
1))+@SUM(SECTORS(K):AAA(I,K)*BUDGET(K,J-2))+ERRORS(I,J) =
BUDGET(I,J));

@SUM(SECTORS(I):A(I,I)) = 0;

!AAA is the second lag period;
AAA(MILPER,MILPER) =0;
!R&D;
A(RD,PROC)=0;
AA(RD,PROC)=0;
A(RD,OM)=0;
AA(RD,OM)=0;
A(RD, MILCON)=0;
AA(RD,MILCON)=0;
AAA(RD,MILCON)=0;
!Procurement;
A( PROC, RD)=0;
AA( PROC, RD)=0;
AAA( PROC, RD)=0;
A(PROC, OM)=0;
AA(PROC, OM)=0;
```

```
AAA(PROC, OM)=0;
AA(PROC, MILPER)=0;
AAA(PROC, MILPER)=0;
A(PROC, MILCON)=0;
AA(PROC, MILCON)=0;
AAA(PROC, MILCON)=0;
!O&M;
A( OM, RD)=0;
AA( OM, RD)=0;
AAA( OM, RD)=0;
AAA( OM, PROC)=0;
A( OM, MILPER)=0;
AA( OM, MILPER)=0;
A(OM, MILCON)=0;
AA(OM, MILCON)=0;
AAA(OM, MILCON)=0;
!Military Personnel;
A(MILPER, RD)=0;
AA(MILPER, RD)=0;
AAA(MILPER, RD)=0;
A(MILPER, PROC)=0;
AA(MILPER, PROC)=0;
AAA(MILPER, PROC)=0;
AA(MILPER, OM)=0;
AAA(MILPER, OM)=0;
A( MILPER, MILCON)=0;
AA( MILPER, MILCON)=0;
AAA( MILPER, MILCON)=0;
!Military Construction;
@SUM(SECTORS(I):A(MILCON,I)) = 0;
AA(MILCON, RD)=0;
AAA(MILCON, RD)=0;
AA(MILCON,PROC)=0;
AAA(MILCON,PROC)=0;
AA(MILCON, OM)=0;
AAA(MILCON, OM)=0;
AA(MILCON, MILPER)=0;
AAA(MILCON, MILPER)=0;
```

```
!the data;
DATA:
!Import the data from MS Excel;
   BUDGET= @OLE( 'D:\AFIT\THESIS\AIJ
ESTIMATION\AIJ_ESTIMATION.XLS');
ENDDATA
       END
```

Appendix E: SAS Program for Calculating Cross Correlations.

```
data sample;
input RD OM;
cards;
29.7  68.1
30.9  71.1
32.8  94.1
35.5  97
35.1  86.6
32    74.7
32.2  100.3
33.1  110
32.6  107.9
33    90.7
30.4  76.3
87.7  63.5
28    67.5
28    61.4
26.2  55.3
24.9  49.5
25.5  57.8
25.9  69.3
26    67.1
25.7  64.8
25.5  65.8
28.6  82.2
32.7  103.3
35.8  122.3
40.6  126.6
45.9  138.1
48    128.1
49.4  107.5
48.7  103.1
48    98.7
44.9  97.7
43    83.7
42.2  69.6
42.9  59
38.5  48.5
37.7  47.1
37.5  45.2
38.4  45.2
38.7  46.7
39.6  52.5
```

```
39.1    55.1;
proc arima;
identify var=RD(1) noprint;
estimate p=2 q=0 noconstant;
identify var=OM(1) crosscor=(RD(1)) NLAG=10;
    run;
```

Appendix F: LINGO Program for Estimating a_{ij} Coefficients with Forwards Lags.

```
MODEL:
!A Leontief Input-Output Model of the DoD Budget;

SETS:
!Sectors are the five major areas of budget authorizations;
  SECTORS/ RD PROC OM MILPER MILCON/: FUNDS;
  YEARS/ FY1960 FY1961 FY1962 FY1963 FY1964 FY1965 FY1966 FY1977
FY1968 FY1969 FY1970 FY1971 FY1972 FY1973 FY1974 FY1975 FY1976 FY1977
FY1978 FY1979 FY1980 FY1981 FY1982 FY1983 FY1984 FY1985 FY1986 FY1987
FY1988 FY1989 FY1990 FY1991 FY1992 FY1993 FY1994 FY1995 FY1996 FY1997
FY1998 FY1999 FY2000/;
  BYSECTORS (SECTORS, SECTORS): A, AA, AAA;
  BYYEAR (SECTORS, YEARS): BUDGET, ERRORS;
ENDSETS

!The objective function -> minimize error;
MIN = @SUM( BYYEAR(I, J): ERRORS(I,J));
!Constraints;
        !i=years, j=sectors;

!The difference between this program and the one shown in Appendix D is highlighted in
boldface type;


@FOR( BYYEAR(I, J)| J #LE# 39:
   @SUM(SECTORS(K):A(I,K)*BUDGET(K,
J))+@SUM(SECTORS(K):AA(I,K)*BUDGET(K,J+1))+@SUM(SECTORS(K):AAA(I
,K)*BUDGET(K,J+2))+ERRORS(I,J) = BUDGET(I,J));

@SUM(SECTORS(I):A(I,I)) = 0;

!AAA is the second lag period;
AAA(MILPER,MILPER) =0;
!R&D;
A(RD,PROC)=0;
AA(RD,PROC)=0;
A(RD,OM)=0;
AA(RD,OM)=0;
A(RD, MILCON)=0;
AA(RD,MILCON)=0;
AAA(RD,MILCON)=0;
!Procurement;
A( PROC, RD)=0;
```

```
AA( PROC, RD)=0;
AAA( PROC, RD)=0;
A(PROC, OM)=0;
AA(PROC, OM)=0;
AAA(PROC, OM)=0;
AA(PROC, MILPER)=0;
AAA(PROC, MILPER)=0;
A(PROC, MILCON)=0;
AA(PROC, MILCON)=0;
AAA(PROC, MILCON)=0;
!O&M;
A( OM, RD)=0;
AA( OM, RD)=0;
AAA( OM, RD)=0;
AAA( OM, PROC)=0;
A( OM, MILPER)=0;
AA( OM, MILPER)=0;
A(OM, MILCON)=0;
AA(OM, MILCON)=0;
AAA(OM, MILCON)=0;
!Military Personnel;
A(MILPER, RD)=0;
AA(MILPER, RD)=0;
AAA(MILPER, RD)=0;
A(MILPER, PROC)=0;
AA(MILPER, PROC)=0;
AAA(MILPER, PROC)=0;
AA(MILPER, OM)=0;
AAA(MILPER, OM)=0;
A( MILPER, MILCON)=0;
AA( MILPER, MILCON)=0;
AAA( MILPER, MILCON)=0;
!Military Construction;
@SUM(SECTORS(I):A(MILCON,I)) = 0;
AA(MILCON, RD)=0;
AAA(MILCON, RD)=0;
AA(MILCON,PROC)=0;
AAA(MILCON,PROC)=0;
AA(MILCON, OM)=0;
AAA(MILCON, OM)=0;
AA(MILCON, MILPER)=0;
AAA(MILCON, MILPER)=0;

!the data;
DATA:
```

```
!Import the data from MS Excel;
  BUDGET= @OLE( 'D:\AFIT\THESIS\AIJ
ESTIMATION\AIJ_ESTIMATION.XLS');
      ENDDATA
      END
```

Appendix G: LINGO Program to Estimate Optimal Budget Levels

```
Model:
Max = x3;
(0.6686+0.194+0.011)*x2 + (.0457+0.0329+0.03315)*x3 + (0.043+0.0297+0.02932)*x4
<= x1;
(0+0.00492)*x1 + (0+0.02887)*x2 + (0.5289+0.0659)*x3 <= x2;
(0.023+0.1209+0.0231)*x1 + (0.026+0.2414+0.034)*x2 + (0+0.189+0)*x3 +
(0.071+0.0504+0.047)*x4 + (0.0023+00.00268)*x5 <=  x3;
(0.0140+0.0122+0.0135)*x1 + (0.1238+0+0.00146)*x2 + (0.4451+0.041+0.04188)*x3 +
(0.0013+0+0)*x4 + (0.0122+.0011+0.0160)*x5 <= x4;
(0.00965+0.00965+0.00965)*x1 + (0.02278+0+0)*x2 + (0.0013+0+0)*x5 <= x5;

x1 + x2 + x3 + x4 + x5 = 100;
end
```

Appendix H: LINGO Program for Five-Sector Regression Problem

This model is for the 1962 fiscal year problem.

```
Model:
Max = OM;
RD - 0.227708*PROC >= 7.233544;
PROC - 2.622166*(OM-16.6678) >= 0;
MILPER - 0.845477*OM >= 4.893739;
MILCON - 0.055221*PROC >= 0;
RD + PROC + OM + MILPER + MILCON <= 108.2;
end
```

Appendix I: Sorted and Arrayed Five-Sector Regression Model Results

Sorted Unbounded Percentages						
Fiscal Year	R&D	Proc	O&M	Mil Per	Mil Con	Total Budget Dollars (Millions)
1997	14.5%	22.3%	30.1%	31.8%	1.2%	77.1
1998	14.4%	23.1%	29.8%	31.4%	1.3%	79.4
1979	14.3%	23.3%	29.7%	31.2%	1.5%	80.1
1975	14.4%	23.4%	29.7%	31.2%	1.3%	80.2
1996	14.3%	23.5%	29.7%	31.2%	1.3%	80.3
1995	14.3%	23.7%	29.6%	31.1%	1.3%	81.0
1976	14.3%	23.7%	29.6%	31.1%	1.3%	81.0
1978	14.3%	23.8%	29.5%	31.0%	1.3%	81.5
1974	14.3%	24.3%	29.4%	30.7%	1.3%	83.0
1977	14.2%	24.5%	29.3%	30.6%	1.4%	83.4
1994	14.2%	24.5%	29.3%	30.6%	1.4%	83.7
1980	14.2%	24.7%	29.2%	30.5%	1.4%	84.2
1993	14.5%	24.0%	29.5%	30.5%	1.5%	87.7
1973	14.1%	26.2%	28.6%	29.7%	1.4%	89.4
1972	13.9%	27.1%	28.3%	29.1%	1.5%	93.1
1981	13.9%	27.5%	28.1%	28.9%	1.5%	94.6
1992	13.9%	28.1%	27.9%	28.6%	1.5%	96.9
1971	13.8%	28.3%	27.8%	28.5%	1.6%	98.1
1970	13.6%	30.3%	27.0%	27.4%	1.7%	107.9
1982	13.6%	30.3%	27.0%	27.4%	1.7%	108.0
1991	13.6%	30.4%	27.0%	27.3%	1.7%	108.2
1965	13.5%	31.4%	26.6%	26.8%	1.7%	114.3
1983	13.4%	31.8%	26.4%	26.6%	1.8%	116.2
1990	13.4%	31.9%	26.4%	26.5%	1.8%	117.3
1964	13.4%	32.4%	26.2%	26.2%	1.8%	120.4
1988	13.4%	32.5%	26.1%	26.1%	1.8%	121.3
1989	13.3%	32.9%	26.0%	26.0%	1.8%	123.7
1969	13.3%	33.1%	25.9%	25.8%	1.8%	125.4
1962	13.3%	33.2%	25.9%	25.8%	1.8%	125.7
1963	13.3%	33.3%	25.8%	25.7%	1.8%	127.0
1968	13.3%	33.5%	25.8%	25.6%	1.8%	128.2
1984	13.3%	33.6%	25.7%	25.5%	1.9%	129.1
1966	13.3%	33.6%	25.7%	25.5%	1.9%	129.3
1967	13.2%	33.8%	25.7%	25.5%	1.9%	130.2
1987	13.2%	34.0%	25.6%	25.3%	1.9%	132.4
1986	13.2%	34.6%	25.4%	25.0%	1.9%	136.9
1985	13.1%	35.3%	25.1%	24.6%	1.9%	143.5

Appendix J: LINGO Program to Solve the 3-Area 8-Sector Problem

```
MODEL:
MAX = forces + specops +airlift;
RD - 0.16*FORCES >=4211391;
TRAIN - 0.095*FORCES - 0.201* INTEL >= 3142027;
SUPPLY - 0.2001*FORCES - 1.26* SPECOPS >= 45707;
INTEL - 4.59*SPECOPS - 1.24*AIRLIFT >= 7036931;
ANG - 0.144*INTEL >= 4396755;
RD + TRAIN + SUPPLY + INTEL + ANG + FORCES + SPECOPS + AIRLIFT <=
133633029;

END
```

The Results from the LINGO program above:

Fiscal Year	[illegible]	[illegible]	Intel& Comms	Airlift&Sealift	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1980	43,270,800	0	7,036,931	0	5,410,073	11,134,720	8,704,194	8,667,176	84,223,894
1981	50,464,180	0	7,036,931	0	5,410,073	12,285,660	10,143,590	9,350,547	94,690,981
1982	59,767,630	0	7,036,931	0	5,410,073	13,774,210	12,005,210	10,234,380	108,228,434
1983	66,214,230	0	7,036,931	0	5,410,073	14,805,670	13,295,170	10,846,800	117,608,874
1984	75,807,080	0	7,036,931	0	5,410,073	16,340,520	15,214,700	11,758,120	131,567,424
1985	85,220,570	0	7,036,931	0	5,410,073	17,846,680	17,098,340	12,652,400	145,264,994
1986	80,431,700	0	7,036,931	0	5,410,073	17,080,460	16,140,090	12,197,460	138,296,714
1987	77,226,640	0	7,036,931	0	5,410,073	16,567,650	15,498,760	11,892,980	133,633,034
1988	69,745,550	0	7,036,931	0	5,410,073	15,370,680	14,001,790	11,182,280	122,747,304

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14. ABSTRACT

Forty years ago, the Office of the Secretary of Defense proposed using the Leontief input-output model to assess tradeoffs in the Department of Defense's (DoD) budget. We demonstrate that the Leontief input-output model can assess tradeoffs in the Air Force's budget.

To increase one part of the Air Force's budget, we need to know the interrelationships between that budget area and the other areas. In this research, we look at different methods of how the functional areas might interact. We demonstrate our methodology on two data sets – DoD and the Air Force aggregate budget data. By looking at how the functional areas interact, we hope to be able to find a sound methodology that will provide assistance to Air Force leadership for determining appropriate levels of funding.

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