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Search Theory and U-Boats in the Bay of Biscay

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THESIS

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Abstract

Threats to our nation's resources and forces are becoming increasingly lethal and mobile. Therefore, our ability to locate and interdict these threats is more important than ever. Search theory is one tool that is vital to countering the increasing threat. This research presents a multi-agent simulation, built around the allied search for U-boats in the Bay of Biscay during World War II, which extends several classic search theory algorithms. Comparison of techniques is based on the effectiveness of finding high-valued, mobile assets. A JAVA-based multi-agent simulation model is designed, built and tested, and used to demonstrate the existence of differing emergent behaviors between search patterns currently used by the United States military.

SEARCH THEORY AND U-BOATS IN THE BAY OF BISCAY

I. Introduction

1.1 General Issue

Given the relative increase in military weapons lethality and mobility during World War II, it became clear to the allies that they needed to invest scientific effort into the process of search. From this effort arose the concept of search as a unique organism, possessing its own structural properties and obeying its own physical laws. Nowhere were the consequences of studying search more critical than the battle of the seas. Prior to World War II, references to search theory in the scientific literature were scarce. Since that time, the volume of published material on the subject would fill many bookshelves.

To date, most search theory study has focused either on analytical models of specific situations requiring rigid assumptions, or, as in the case of search and rescue, operational experiments aimed at obtaining detection probabilities for a variety of scenarios. Analytical search theory results provide bounds on empirical results. This research introduces an agent-based simulation approach to the subject of offensive search operations in combat. Generally, the value of combat simulation is measured in terms of insights gained through experimentation. Agent-based simulation enables insights with regards to the emergent behavior of the individual combatants, groups of combatants, or the system as a whole. Emergent behavior for the purposes of this research is system behavior, not explicitly programmed, arising from local interactions between agents. Such behavior with respect to search effectiveness is investigated within the context of a historical case study involving offensive search.

1.2 Background

The U-boat war from World War II provides a valuable case study for the application of search theory in that detailed analyses on this long-lasting facet of the great conflict is not only available, but can be verified historically. The allies, most notably the British, given the geographical fact that the island of Great Britain was highly dependent upon merchant shipping for much-needed supplies, were greatly concerned about effectively countering the U-boat threat. If unchecked in the North Atlantic, U-boats had the ability to impart potentially crippling losses to international shipping. One location with a particularly high concentration of U-boats was the Bay of Biscay, through which U-boats transited between occupied French ports and the North Atlantic. The U-boats were most vulnerable during their Bay transits due to the coastal constraints of England and Ireland to the North and Spain to the South.

British allied command realized that conducting effective airborne offensive search operations in the Bay could hold the key to offsetting the threat. The allies combined analytical insights, operational experience, and intelligence information to form a basic search patrol methodology applied in the Bay of Biscay to include the first offensive search pattern—barrier patrol search. Since it was believed that aircraft maximized sighting distance by approaching at 45 degrees to U-boat tracks, this search pattern involved running NW-SE and NE-SW across some assigned coverage area (U-boats generally maintained east-westerly transit routes to cross the Bay).

1.3 The Problem

Little, if any, work has quantitatively examined differences between search patterns in airborne offensive search scenarios. The search and rescue community has defined several separate search patterns from which a selection is made depending upon the situation at hand, five of which can be adapted for the offensive search situation evident during the U-boat war. The Defense Modeling and Simulation Office (DMSO) sponsored this research as part of an ongoing interest to study the virtue of possible emergent behaviors in agent-based combat simulation. Given the insights gained to date from agent-based simulation in a wide variety of disciplines, it is proposed that search theory be re-examined within an agent-based framework. An agent-based model representing the U-boat war was built to determine whether or not variations in the search pattern alone impact search efficiency, all else being equal.

1.4 Research Objectives

The objective of this research is to develop and employ a methodology for empirically quantifying the effects of different search patterns on search efficiency. Part of this process involves developing an object-oriented simulation in JAVA. There are many advantages to utilizing an agent-based approach here, not the least of which are that many random processes evident in the scenario will be well represented and that "agents" have independent goals, resources, and threads of execution.

1.5 Scope of Research

This research focuses on multiple searchers seeking to detect multiple targets within the context of the historical scenario described herein. Agent and simulation design data was compiled according to the following hierarchy: 1) historical fact as found directly from sources credited to allied and German participants; 2) published studies directly related to the offensive search in the Bay; 3) data derived from raw numbers in one or more of the preceding sources; and 4) good judgment (operational expertise) when the three previous sources fail or contradict one another. Also regarding design data, we must mention that a special debt of gratitude is owed Dr. Brian McCue whose insights and historical materials related to search theory and the U-boat war were invaluable. By maintaining statistical similarity with regards to U-boat density in the Bay and the number of hours flown by search aircraft, the problem of measuring search efficiency using separate search patterns is reduced to observing differences in the average number of U-boats sighted per month.

1.6 Contribution of Research

To the best of our knowledge, the effects of using different search patterns on search efficiency have yet to be quantified, other than notionally. This study provides background on search theory and describes a JAVA model used to assess the impact different search patterns have on U-boat sightings. Our hope is that readers will see that the structure of search itself can have a direct impact on desired outcomes, perhaps in ways not ostensibly intuitive (i.e., emergent behaviors through agent-based simulation). Military operations that stand to benefit from airborne offensive search pattern analysis include drug interdiction, broad area searches for arms control treaty violations, and hunting for mobile scud missile launchers, terrorist combat groups, smugglers, and opposing forces in rugged terrain.

II. Literature Review

2.1 Agent-Based Simulation

"'The real voyage of discovery,' noted Marcel Proust, 'lies not in finding new landscapes, but in having new eyes.' This thought drives our interest in the nonlinear sciences and their relationship to the profession of arms" (Hoffman and Horne, 1998:preface). General Charles C. Krulak, Commandant of the United States Marine Corps, penned these words underscoring the importance of studying the intangibles of warfare with emergent non-classical methods. One such method involves the use of software "agents" within the framework of statistical simulation.

2.1.1 Applications of Agents.

An agent is an entity, real or virtual, that perceives and acts in its environment (Russell and Norvig, 1995:7). Specifically, an agent is characterized as a physical or virtual entity having a partial representation of its environment which is capable of perceiving that environment in some limited sense and acting within it; agents can interact with other agents, and have a set of internal tendencies or goals that guide their behavior in an attempt to satisfy these goals given its resources, abilities, and perceptions (i.e., autonomous; adapted from Ferber, 1999). The concept of "software agents" is therefore derived from examining this definition of agents within the context of software development. Recently, much attention has been given to agent-based simulation in an attempt to identify emergent behaviors and adaptations that are likely to result from agent interaction within the context of simulation. Thus, the utility of agent-based simulation goes far beyond applications in the profession of arms to such disciplines as the social sciences (see Bonabeau, 2002). Professionals from many disciplines are interested in gaining insight into how cooperative and competitive agent behavior affects a real-world system (the whole versus the sum of the parts) and whether or not individual agent behavior has changed as a result.

> This characteristic of … analyzing the interaction systems that exist between agents is what distinguishes multi-agent systems from the more classical systemic approaches, in that preference is given to emergence, and action and interaction are considered as the motor elements in the structuring of a system taken as a whole." (Ferber, 1999:4-5)

Further, unlike the majority of literature emphasizing cooperative agents, combat agents are designed to purposefully compete with elements (i.e., other combat agents) that seek to prevent them from attaining their goals. The notion of such "antagonistic interaction" can certainly be analyzed as a possible "interaction system" mentioned above.

Clearly, military leaders seek to understand the nature of combat at any level that might offer an opportunity to ultimately better influence the outcome of such conflict. Few educational opportunities are better suited toward this motive than the study of historical military conflict about which a great many facts have already been recorded and analyzed. Such situations present a source against which new methods to observe and describe combat behavior can be compared. The international conflict arguably most responsible for the birth of the discipline of operations research was the U-boat war of World War II (Morse and Kimball, 1954:3). The wealth of data and analyses that exist regarding the U-boat conflict makes it an ideal benchmark.

2.2 Operational Context for Analysis; Hunting U-boats in World War II

The main objectives of operations research were originally defined as the "prediction of the effects of new weapons and tactics" (Waddington, 1973:26). One of the chief concerns for the allies in World War II was how to effectively counter the U-boat threat. Once in the North Atlantic, U-boats imparted terrible losses to international shipping; Hitler himself stated, "U-boats will win the war" (Waddington, 1973:38). The degree of concern held by the allies was perhaps best stated by British Prime Minister Sir Winston Churchill when he said, "the only thing that ever really frightened me during the war was the U-boat peril" (Churchill, 1949:598). One such U-boat location was the Bay of Biscay in which captured French ports were used to stage U-boat operations against allied shipping in the North Atlantic. The U-boats were most vulnerable when transiting the Bay of Biscay either embarking from or returning to their ports on the west coast of France. British allied command was the first to realize that conducting effective offensive search operations in the Bay could play an important role in terms of offsetting the threat.

The Royal Air Force (RAF) Coastal Command formed the Operational Research Section (ORS) early in the war; a body of scientists expected to work closely with military high command, the first duties of which involved analyzing the U-boat situation. ORS addressed various aspects of the anti-U-boat campaign, not all of which involved anti-submarine tactics in the Bay. Rather, the original organization of ORS into four groups testified to the ubiquitous approach taken to counter the U-boat threat.

Those groups were:

- Anti-U-boat operations;
- Anti-shipping operations, photo reconnaissance operations, and weapons;
- Planned flying and maintenance; and
- Weather and navigation. (Waddington, 1973:19)

In view of the large area of operations involved as well as the technology and platforms available at the time, it was determined that search operations for U-boats could only effectively take place either in the Bay of Biscay or near allied merchant convoys. Beyond the Bay, the North Atlantic simply involved too much area in which a U-boat could hide, curtailing the ability of allied aircraft to find them. The Bay itself constituted 130,000 square miles of searchable area, and extended from the northern coast of Spain in the South to the coast of France in the East on to England and Ireland in the North. Allied search effort was applied both to convoys and to the Bay.

Operations research efforts began by analyzing various raw data sets with the intent to form predictive and explanatory models. Enemy patterns of behavior were identified; an amalgam of ORS insights, operational experience, and intelligence information produced a basic search patrol methodology applied in the Bay of Biscay (HBMSO, 1943:98-101). Aircraft on patrol flew to a specific bearing and covered a predefined area extending from the bearing for a fixed number of hours. The state of the weather, the number of hours of daylight, and the range of the aircraft regulated the duration of each patrol; therefore, patrol durations were not laid down in a hard and fast manner (even though they were technically fixed). Repeated coverage of an area was not only possible, but often occurred. Though the patrol area was marked on a chart, the “essence of the problem” was for the navigator to find the area and keep the aircraft within search limits.

Figure 1. Briefing for an Anti-Submarine Patrol (HBMSO, 1943:100)

It was discovered early on that aircraft maximized sighting distance by approaching at 45 degrees to U-boat tracks. Therefore, most search patterns ran either NW-SE or NE-SW across some assigned coverage area since they generally assumed east-westerly transit routes for U-boats crossing the Bay. Analyses also revealed that the U-boat distribution for 1942 and 1943 in the Bay could be modeled as a Poisson field, even though the U-boats began to employ "pack tactics" in September of 1943. Pack tactics involved the grouping of at least two U-boats, both to increase their ability to protect each other by increasing the number of available lookouts and anti-aircraft guns, and to increase their lethality to merchant shipping convoys. Such tactics did not appear to affect U-boat distribution to the point that it warranted further analysis, so this distributional assumption continued to hold (Waddington, 1973:235).

With regard to a specific search pattern, RAF Coastal Command had instituted use of a "crossover barrier patrol system" for search aircraft shortly before the war. Barrier patrols yielded the best distribution of flights—uniformly spaced parallel sweeps that allowed searching aircraft to always remain at an acute angle relative to the U-boat tracks (see Figure 2 below). As it happens, operations research analysts later determined that track spacing could be chosen arbitrarily without an effect on search efficiency (though it was said that track spacing might have had an effect on the amount of search resources used) (Koopman, 1999:204-205), (OASG, 1977:142-164).

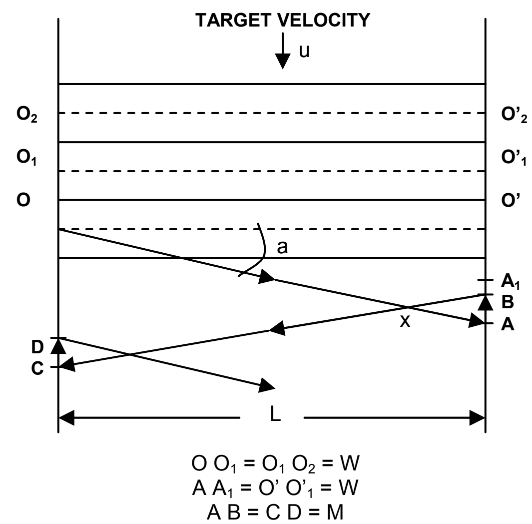

Figure 2. Illustration of Crossover Barrier Patrol System (Koopman, 1999)

Aircraft that actually attacked a U-boat were not available for search effort elsewhere during that sortie as all weapons were expended in hopes of damaging or destroying the target. Aircraft flying search patrols almost always flew alone, and during a sortie in which an aircraft had sighted a U-boat and dropped weapons in hopes of damaging or sinking the craft, the aircraft would have maintained area presence for purposes of battle damage assessment (BDA). Most search aircraft were equipped with a camera for BDA, and were keenly interested in sighting such things as survivors, surface debris, secondary explosions, oil slicks, or even sustained streams of bubbles. Even so, BDA was difficult, and the aircrew only knew for sure whether or not the U-boat had sunk if survivors were recovered (HBMSO, 1943:98-101).

After aircraft involved in the sighting and attack of U-boats returned to base, the aircrew debriefings were "searching and severe." Statistics for different patrols were used to keep track of the routes followed by U-boats although it was pointed out that timing of patrols was far too stereotyped (Waddington, 1973:235-236).

2.3 Search Theory Concepts, Terms and Classical Approaches

Search theory has played an important role in military operations. It is within the context of agent-based simulation that this study will focus on applications of search theory. Organized search theory and the discipline of operations research were born at the same time and indeed share a common lineage, the necessity of securing the survival of the allied nations faced with the threat of Nazi Germany during World War II. The passage of time has not changed the need for the military to be involved in search theory; if anything, the requirement for continuing research in this area has increased. McCue observes candidly that given recent advances in defense technology, "the operations of war are operations of search" (McCue, 1990:168). A distinction here must be made between search and rescue (SAR) efforts and (previously mentioned) offensive search operations.

An excellent survey of search theory literature is available (Benkoski, Monticino, and Weisinger, 1991). For this research, pertinent material addresses the following:

- Emphasize search planning, not search modeling;
- Take a tactical, not strategic, viewpoint;
- Assume search involves uncertainty;
- Aim more at obtaining initial detections, than at fusing multiple detections;
- Involve a moving target; and
- Involve a non-cooperative target. (Benkoski, Monticino, and Weisinger, 1991)

2.3.1 Search and Rescue in World War II.

The difference between SAR and offensive search operations is best characterized by the nature of the target. By definition, SAR involves a cooperative or neutral target (as in the case of a seagoing craft in distress and lacking the means to draw attention). The aforementioned survey addresses SAR as well as offensive search. Ironically, SAR had its formalized beginnings in the early 1940's. In fact, for the Royal Air Force, SAR began as "an improvisation" in World War II (MacMillan, 1950:58).

The concept of SAR also involves distress prevention where possible. Coastal Command aircraft would warn allied ships prior to their entering a minefield or running aground during thick fog, for example (via signaling or even by firing machine gun bursts across the bows of such sea craft) (HBMSO, 1943:114). Where search was involved, history is replete with examples. Such cases usually began with a request for assistance by the vessel itself or by a witness to its distress, and often included defensive air cover against hostile forces to enable rescue operations. Sometimes, as in the case of the British submarines "Triad" and "Triumph" as well as the HMS "Kelly," the vessel was towed safely to port. Other times, as in the cases of the Norwegian vessel "Tropic Sea," the S.S. "Kensington Court," and the "City of Benares" ocean liner, only the crew and/or passengers were rescued (the latter craft carrying children from Great Britain to Canada; 46 survivors were rescued) (HBMSO, 1943:115-118). Many of these rescues were made possible by the fact that Coastal Command had "flying boats" in their inventory (i.e., Sunderlands as depicted in Figure 3).

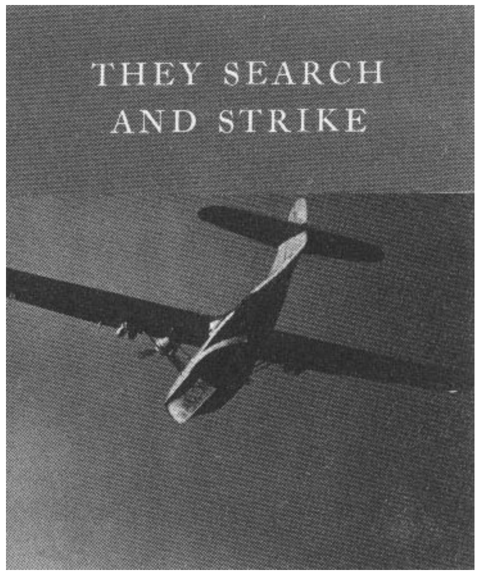

Figure 3. British Poster Showing a Sunderland Flying Boat (HBMSO, 1943:5)

More often than not, however, aircraft notified allied ships in the vicinity, which in turn rescued the survivors. Early SAR operations were thus, for the most part, reactive and involved little in the way of search planning unless the allies knew survivors existed and had an idea as to their location.

2.3.2 Search Possesses Structure of Its Own.

Koopman describes the operation of search as "an organic whole having a structure of its own—more than the sum of its parts" (Koopman, 1999:2). Such language is often used to describe agent-based modeling and emergent behavior. He underscores the necessity to study search structure for its real importance and most effective performance, without overemphasizing immediate practical answers to questions regarding how to plan a search; "rules without scientific explanation" are to be avoided. Credible research is paramount for a discipline used so ubiquitously for such things as mineral deposits, police operations, pattern recognition, disease or contamination, medical diagnostics, and markets (Koopman, 1999:12-13). For example, though anti-submarine warfare (ASW) is conducted differently today than in World War II, search techniques used in historical ASW have potential application to these other areas (modern anti-submarine warfare tactics mainly involve surveillance around convoys by sonar-equipped warships, helicopters, and inshore mobile units; that is not to say offensive search operations similar to what was used in the Bay of Biscay would not recur, only that more modern examples of such tactics are not prevalent) (OASG, 1977:191). In other words, examples of search from World War II are good illustrations in that they are available, detailed (in many cases), and can be verified historically.

2.3.3 Recent Uses of Search.

Search techniques in many areas have changed as technology has grown. The search for mineral deposits, for example, now involves such items as airborne laser-based remote sensing systems that "map" an area. This is accomplished by measuring various spectral emissions and thermal properties of a given area through a range of light frequencies accounting for atmospheric, altitude, and instrument effects. Promising results have been produced for minerals such as quartz, clay/feldspars, garnet, talc, dolomite, and amphibole to name a few (Mortensen, 1996:22-23). Another example involves the search for a relationship between disease risk and potential point sources for such diseases. Utilizing specialized databases and standard statistical techniques such as parametric modeling, researchers can explore a variety of potential correlations between certain locations and disease risk (Diggle et. al., 1997). Professionals in the field of economics employ search techniques as well. Models exist that predict a worker's optimal job search strategy in a particular market as a function of supply, demand, wage determination, unemployment durations, etc. (Rauh, 1997:128-153). Even SAR techniques now include such things as satellite involvement and ground station networks as a means of relaying distress alert and location information to rescue authorities (Cospas-Sarsat, 1998).

Perhaps one of the more logical extensions of offensive search operations involves drug interdiction. As opposed to flying specialized search patterns in order to ambush elusive, non-cooperative targets, authorities place an emphasis on the fusion of a number of elements including intelligence information, radar warning networks, military aircraft, and specialized interdiction police ground teams. An example occurred on November 18, 1992, when a twin turbo-prop Convair 580 carrying $2.7 billion (Canadian) worth of cocaine flew non-stop from Colombia to a deserted airstrip in Quebec. U.S. Customs and the North American Defense Command (NORAD) had it under surveillance almost from the moment it took off. Four CF-18 Canadian fighter jets were eventually scrambled to track the aircraft as it neared the Canadian coast, and four CH-136 Twin Huey helicopters carrying a Royal Canadian Mounted Police interdiction team met the aircraft upon landing and promptly arrested the occupants. Obviously, the key search component in this scenario and for interdiction operations in general, was the airspace surveillance radar network (Hughes, 1993:48-51).

Broad area searches for arms control treaty violations provide yet another example involving an application of offensive search operations. Since using aerial assets (i.e., aircraft or satellites) to search an entire country for illegal military equipment is not cost-effective, a suitable alternative might be to employ aerial surveillance based upon known search theory principles including prior information about where and for what to look. In 1991, a study sponsored by the United States Congress examined the conditions under which aerial monitoring would make a significant contribution to arms control verification (U.S. Congress, 1991). Most of the quantitative analyses documented in the congressional report are based upon classical search theory concepts mentioned elsewhere in this research effort.

2.3.4 Concepts Defined—Mathematical Foundation for Search Theory.

For purposes of this paper, the object of interest is the "target," while the "searcher" is concerned with finding the object of interest. McCue introduces a mathematical foundation for search theory by stating that "instantaneous sighting probabilities form a sighting potential: potentials integrate to form a lateral range curve, whose integral is the sweep width" (McCue, 1990:68).

The concept of a "lateral range curve" in reference to a specific sensor is a graph of the probability of detection (POD) against the perpendicular distance from the sensor relative track to the target (which is the same as the object's distance from the sensor to the closest point of approach). Figure 4 is an example lateral range curve.

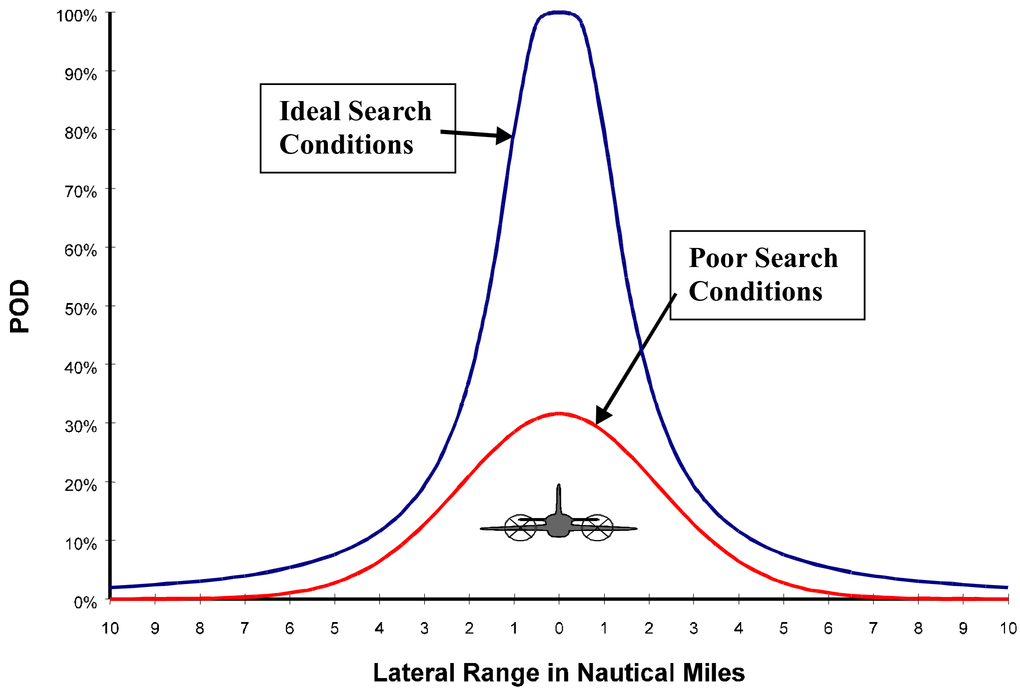

Figure 4. Sample Lateral Range Curves (Fundamentals of SAR, 2002)

The "sweep width" is the area underneath the lateral range curve, and is used as a measure of search effectiveness of a given sensor (Koopman, 1999:65). It is easy to tabulate, and in fact the National Search and Rescue Manual gives sweep widths for a variety of sensors in a variety of environments (NSRC, 2000:App.G). These tables are periodically updated via simulation exercises with the respective sensors (Edwards et. al., 1980). Inherent in the concept of "sweep width" is use of the definite range law, the basis of which is that no probability exists to detect targets outside the specified range, while targets within the specified range are detected with certainty. The [effective search] sweep rate is the mean number of targets detected per unit time (Koopman, 1999:66).

The idea of "exhaustive search" involves complete coverage of an area, and is used in many instances as an upper bound on search effectiveness (Washburn, 1989:1-2). Questions regarding the smallest amount of track length for complete coverage or the largest amount of track length in the search region without overlap can be addressed using so-called "raster scan" (lawnmower approach), spiral-in, or spiral-out patterns, depending upon the situation. Classically, exhaustive search assumes that detection is certain since early models assumed the target always stayed within the search area. Such an assumption as a matter of course cannot be made in the case of searching for U-boats at sea, since the U-boats were known to enter and leave the search area.

"Random search" as a concept plays a central role in search theory as its use places a lower bound on the probability of detection (likened to searching small regions of interest as "confetti on the ground") (Washburn, 1989:2-4). Since it has been shown to follow an exponential distribution (probability of sighting = $1 - e^{-(\text{search effort/area})}$, the cumulative distribution function for an exponential distribution; McCue, 1990:166), random search is characterized by the "memoryless property" of that distribution. This is consistent with intuition in that the length of time a searcher has been looking to detect a target has no bearing on subsequent detection probability.

The "inverse cube law" is defined as a reasonable compromise between random and exhaustive search (Washburn, 1989:2-5). The law states that the probability of detection by a searcher seeking a target is inversely proportional to the cube of the separation distance. Vision, active radar, and radar detectors are all considered inverse cube law search devices. The "first search curve" listed in the National Search and Rescue Manual, from which other curves are derived, is the inverse cube law. The derivation of the inverse cube law is characterized by four equations, a combination of infinite sums and integrals, that can be evaluated in closed form; this fact led Washburn to state that the law is "therefore possibly holy" (Washburn, 1989:2-15), and can be used even when the underlying assumptions are not directly verifiable. It is chiefly through the use of this law, and Koopman's forestalling theorem (McCue, 1990:74), that expressions for *a priori* detection and counter-detection probabilities can be derived for searchers and targets based upon realistic ranges of the equipment used and the distance between searcher and target (such as in a simulation). Also, the use of this law in the Bay of Biscay agent-based simulation forces the empirical results to be bounded by the analytical search theory results.

2.3.5 Regarding Classical Approaches.

Some problems associated with models based solely on geometric analysis are that:

- Using the definite range law in practice can be unrealistic, as search should be associated with at least some degree of randomness;
- Real-world navigation is not perfect (especially when multiple searchers are involved; there is a difference between planning and execution); and
- The target's intent to evade may lead to unpredictable movement on the part of the target (Washburn, 1989:2-1).

Models based on detection-rates tend to be more robust than geometric models, as the events of detection in non-overlapping time intervals are assumed independent. Search models also have to strike a balance between efficiency and completeness of coverage. This is perhaps illustrated best through the "circle-packing" concept in which tightly packed adjacent circles represent search coverage areas; no matter how big the circles, there will either be duplication of effort or area that remains unsearched. Likewise, when patrolling a channel, how tightly should the searching surface craft turn as it proceeds? There is always this tradeoff to consider (Washburn, 1989:1-4).

2.3.6 The Role of Computer Simulation in Classical Search Theory.

Koopman agrees that computer simulation has its role, but these roles are largely relegated to a minor role in *Search and Screening* (Koopman, 1999). Such devices are good to the extent that they may "lighten the work," but he warns that there still exists the need for solid underlying mathematics. Computer simulation should never be used as a means to shortcut scientific rigor. Techniques should be based on "painstaking tactical, physical, and mathematical analysis." The danger of self-deception in simulation is all the more insidious because the concepts are buried in a computer program, sheltered from rational criticism (Koopman, 1999:252).

On the other hand, operations research analysts involved in search theory during World War II often used simulation for various purposes. Some of these uses involved evaluating operational measures such as sweep rate, counter-measures, and probability of detection curves for a variety of search devices as a function of range to target (Morse and Kimball, 1954:38, 98, 134). In his article on Koopman's life, Morse even states that Kimball utilized simulation "to improve search procedures" (Morse, 1982:421). In fact, one of the advantages of using computer simulation as opposed to other analytical models is that simulation allows one to relax mathematical assumptions required for those models.

2.4 More Modern Approaches to Search Theory

Many types of glimpse models (i.e., models where target detection is attempted in discrete "snapshots" in time such as a radar beam sweeping a circular area outside the search platform) have been derived using geometric and stochastic methods. Such models, though well established analytically, tend to be highly dependent upon rigid assumptions regarding searcher and target movement and position. Models exist, for example, for fleeing targets for which the searcher has obtained a fix at a certain point in time (see Ishida and Korf, 1995), areas where a mobile target is known to be uniformly located, and stationary (see Zhu and Oommen, 1997) or randomly moving targets (such as a lost hiker). Research has been done to choose optimal search paths upon which to allocate search effort in order to find non-cooperative targets; these models assume the existence of a network of discrete arcs (see Musman, et. al., 1997, and Hohzaki and Iida, 2001). There appears to be a seemingly endless series of problems that must be addressed situationally when employing analytical methods. The solution pattern for such problems has classically involved starting with a series of assumptions and deriving a sweep width based upon a distribution of effort (Washburn, 1989:1-22).

Other examples of such search models based on simplifying assumptions include multiple targets distributed in the searchable area according to a Poisson distribution. If this assumption is valid, for example, the searcher then knows the sensor is completely characterized by its sweep width from the standpoint of search efficiency (Washburn, 1989:8-4). Since all directions are equivalent in a Poisson field, for example, it is impossible to orient oneself in such a field for search optimization purposes.

Stochastic models based on “signal excess” equations, which tend to be complicated, involve rigid assumptions about signal behavior. Most such detection models, Washburn concludes, yield little improvement over the definite range law since signal excess is so sensitive to range to target that detection probabilities will still either be very close to zero or very close to one (as in the case of the definite range law) (Washburn, 1989:3-7).

If the search area can be partitioned into regions, each having some probability of containing the target (based upon subject matter expert input), the amount of search effort per region can be optimized using Everett’s theorem (constrained optimization) (Washburn, 1989:5-2). One necessary assumption is that the target is stationary within one of the regions.

In what Washburn refers to as “myopic search with discrete looks,” a greedy algorithm is employed to optimize the amount of effort applied to the different regions comprising a total area of search (Washburn, 1989:5-14). Again, it is assumed that the target is stationary within one of the regions.

Search problems where a target takes evasive action in order to forestall detection (as in anti-submarine warfare) seldom yield to analysis; in such cases simulation is said to be a good alternative (Washburn, 1989:2-15). Another interesting approach to the types of problems involving an evading opponent uses minimax strategy (game theory) in order to maximize the probability of detection for the searcher (see Dambreville and Le Cadre, 2001). A potential complication with this minimax strategy might be that varying strategies for multiple searchers and targets could quickly become too computationally complex to solve using any methods currently available.

Analytical models tend to be complex, the real world even more so. A littoral environment (open ocean) is especially variable in ways not easy to describe.

> It is therefore tempting to describe search capability through experiments where the searcher performs a specified maneuver in an attempt at detection, rather than trying to discover fundamental parameters of the environment and then reasoning deductively. (Washburn, 1989:4-1)

This is the very idea behind lateral range curves, and hints at the possible utility of using simulation.

2.4.1 The Role of Computer Simulation in Modern Search Theory.

Perhaps the best instance of computer simulation as it relates to search theory is the Computer-Assisted Search Planning System (CASP) introduced by the United States Coast Guard in 1974. An overview of CASP is given in Haley and Stone [1980] and in Richardson and Discenza [1980]. "CASP has been used at the Air Force Central Rescue Headquarters at Scott AFB, Illinois, to help plan and coordinate missions for lost airplanes within the continental United States" (Richardson and Discenza, 1980:661). This system, based in part on Monte Carlo simulation, was devised specifically for search and rescue (i.e., target is stationary or subject to random drifts based on weather and littoral currents and is not trying to evade the searcher). CASP is characterized by a probability map display where each search grid square has a probability of target location. Its underlying structure is a markov process with three-dimensional state space consisting of variables representing latitude, longitude, and search failure probability. CASP generates an initial probability distribution, and updates the display taking into account wind and current information, as well as negative and false positive search results. Given the assumptions upon which it is based, namely that it does not account for evading targets or for targets entering and leaving the search area, this system could not realistically be applied to the situation addressed in this research effort.

As an example of a larger modeling effort, the Coast Guard's Research and Development Center is involved in the Maritime Operations Simulation (MarOpsSim), a discrete event simulation tool designed to support Coast Guard mission areas and acquisition strategies. Specific work includes SAR research aimed at studying and prioritizing "various alternatives and possibilities for improved search planning models, technology, tactics, and doctrine" (Downer, 1999). To date, MarOpsSim has yielded promising results in detecting targets consistent with known lateral range curves, generalizing detection capability in certain instances, accurately representing SAR planning tactics, and accurately analyzing resource and target motion.

There is also ongoing work in the SAR arena investigating the use of software agents interfacing between a CASP-like Coast Guard network server and an ocean simulation server. The result of this research is near real-time projected trajectory information of survivor whereabouts upon which the Coast Guard can act ("real-time" meaning from the time the Coast Guard receives the call for help; see Wilson, Burroughs, Kumar, and Sucharitaves, 2001).

Another example using Monte Carlo simulation to optimize a search pattern is also given in Haley and Stone [1980] where search aircraft seek to detect and identify surface targets for surveillance purposes. Ironically, the only search pattern analyzed (by varying measurements of the legs involved) is the heretofore-mentioned crossover barrier patrol system used by RAF Coastal Command in World War II.

2.4.2 Computer Simulation to Determine Optimum Search Paths.

Washburn warns against preoccupation with discovering search paths that are optimal for the searcher as such solutions are rare in search theory (Washburn, 1989:1-1). Even if an optimal solution is found, the path might not be easy to follow; in fact, a "path" is not a convenient mathematical object at all, he contends. The object of interest is more likely to be a distribution of effort. Even so, the search and rescue working group report produced by Haley and Stone [1980] comments on search pattern analysis. Specifically,

> Modern aircraft electronics and microcomputer devices are capable of automatic control of aircraft and can be programmed to execute detailed flight patterns. This capability permits more accurate execution of existing patterns, but more importantly, gives the opportunity to introduce new and unusual search patterns previously disregarded as beyond the capabilities of human pilots. Are there some types of search paths which might be more effective for search? Can a path-by-path optimization algorithm be constructed? Would such an optimization be useful for unusual sensor characteristics such as radar, magnetic anomaly detectors or emergency locator searches? Analytical studies should be undertaken and any resulting unusual patterns should be exercised in modern aircraft under automatic control. Certain of these unusual search circumstances may find pathwise optimization more nearly optimal than standard patterns. (Haley and Stone, 1980:56)

The United States Coast Guard teaches their personnel that choosing an appropriate search pattern involves many factors and is highly dependent upon the given scenario (Fundamentals of SAR, 2002). The National Search and Rescue Manual currently lists five search pattern types of interest in this research effort. These are the Parallel, Creeping Line, Square, Sector, and Barrier [crossover patrol system] patterns (NSRC, 2000:App.G). These patterns will be compared and contrasted with respect to the number of U-boats sighted by aircraft within the context of agent-based computer simulation to determine whether or not differences in these value measures exist between the patterns considered in the context of the historical case studied herein. If, as Washburn contends, such comparisons are analytically intractable, it stands to reason that this type of problem is a candidate for our agent-based simulation approach.

III. Methodology

3.1 Introduction

The purpose of this research is to present a multi-agent simulation, built around the allied search for U-boats in the Bay of Biscay during World War II, which extends several classic search theory algorithms. Comparison of techniques is based on the effectiveness of finding high-valued, mobile assets. A JAVA multi-agent simulation model was designed, built and tested, and used to demonstrate the existence of differing emergent behaviors between search patterns currently used by the United States military. This chapter addresses specifics of the study demonstrating the modern search patterns used for offensive search operations and the particulars of modeling the historical scenario.

3.2 Search Patterns Defined

Each of the five search patterns from the National Search and Rescue Manual are described in more detail. For each, a figure is provided depicting the pattern and the key assumptions are provided. Each figure includes a commence search pattern (CSP) point.

When the point of last contact with the target (datum) is not known with a high degree of certainty and the search area is large, either the parallel (Figure 5) or the creeping line (Figure 6) search is preferable. The parallel pattern is most desirable when the target is equally likely to occupy any part of the search area.

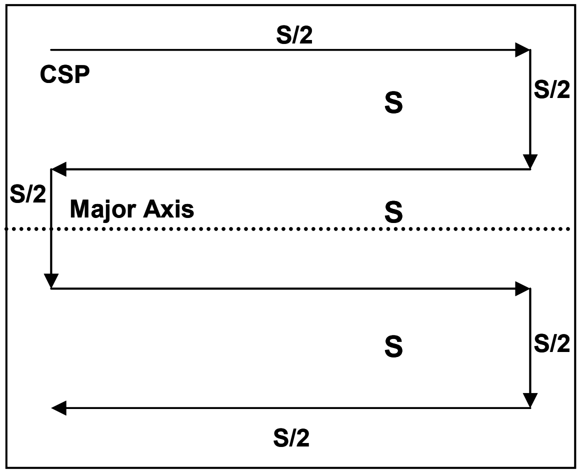

Figure 5. Parallel Search Pattern (Fundamentals of SAR, 2002)

The creeping line pattern, on the other hand, is typically employed when the target is more likely to be in one end of the search area than the other.

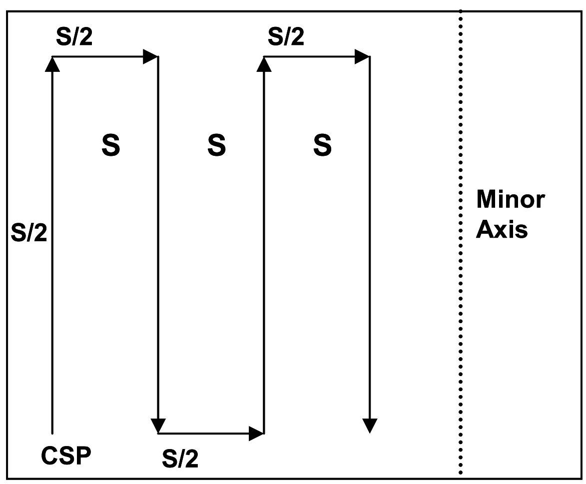

Figure 6. Creeping Line Search Pattern (Fundamentals of SAR, 2002)

When the point of last contact is well known or established within close limits, the square (Figure 7) and sector (Figure 8) search patterns are preferable. The square pattern is used when uniform coverage of the search area is desired, while the sector search is used in scenarios where the target is difficult to detect.

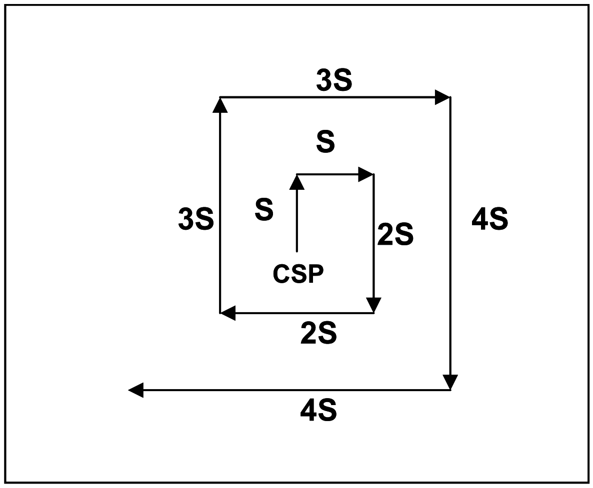

Figure 7. Square Search Pattern (Fundamentals of SAR, 2002)

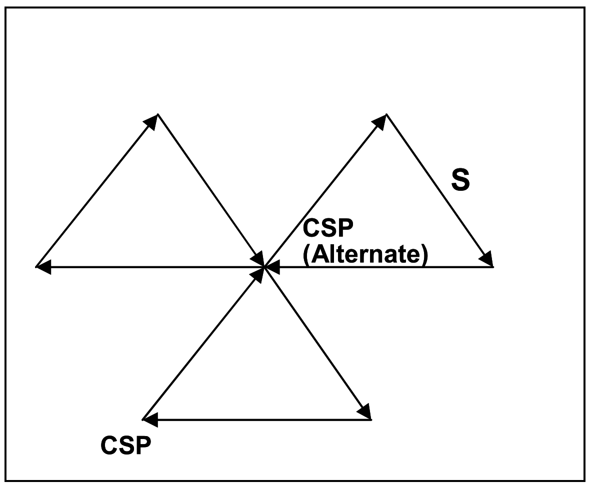

Figure 8. Sector Search Pattern (Fundamentals of SAR, 2002)

Finally, when the target is fast moving or when a strong current is present in the search area, the barrier patrol search pattern (Figure 9) is preferred.

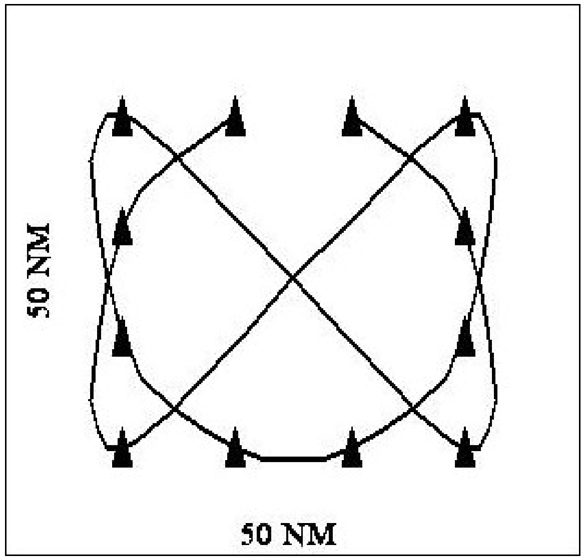

Figure 9. Barrier Patrol Search Pattern (Fundamentals of SAR, 2002)

3.3 Simulation Assumptions

As previously mentioned, the Bay of Biscay simulation was written in JAVA and run on a 2-GHz Pentium 4 PC with 256 MB of RAM running a Windows 2000 operating system. Agent and simulation design data was compiled according to the following hierarchy: 1) historical fact as found directly from sources credited to allied and German participants; 2) published studies directly related to the offensive search in the bay; 3) data derived from raw numbers in one or more of the preceding sources; and 4) good judgment (operational expertise) when the three previous sources fail or contradict one another.

Within the simulation, “day” is defined as the time between nautical dawn and nautical dusk (i.e., the sun is above -12º with respect to the horizon). Detection sensors used by aircraft and U-boats conform to the inverse cube law, and each of the aircraft and U-boat agents is independent. For each iteration, a 12-month warm-up period was used followed by six months of data collection. The six-month period (October, 1942, through

March, 1943) used in the scenario was chosen based upon homogeneous use of detection devices.

Initially, 70 U-boats are in place, and replacements enter the Bay from the North Atlantic in numbers consistent with history; 32 in October, 27 in November, 11 in December, 14 in January, 14 in February, and 25 in March (McCue, 1990). The U-boats start off uniformly distributed in the Bay with half heading to the North Atlantic and half heading to their homeport. Each U-boat is assigned to one of five homeports on the west coast of France, the total number of U-boats being distributed evenly among the ports. Also, each U-boat leaves port with 30 days of supplies and returns from operations in the North Atlantic so as to arrive back in port with no supplies remaining. U-boats move at 10 knots surfaced and 2.5 knots submerged and must spend a minimum of 3 hours surfaced for each 100 nautical miles (NM) traveled to fully recharge their batteries. Refueling at sea is implicitly accomplished by allowing a 0.25 probability of each U-boat agent extending time in the North Atlantic by 30 days. U-boats will submerge immediately upon detecting an allied aircraft.

Forty aircraft operate from Plymouth, England, and will standoff from the coast of France to avoid enemy air patrols and escorts. Also, there is no attrition due to accident or anti-aircraft defenses. Aircraft speed is 120 knots, and each aircraft will fly up to 70% of its fuel load or until it has expended its munitions, whichever occurs first. An aircraft can detect a U-boat only when the U-boat is surfaced, and will attack the U-boat upon detection, expending its entire payload of munitions. Maintenance and weather cancellations occur before take-off only, and aircraft sortie take-off times are randomly scheduled to occur once in a 24-hour period while maintaining a minimum of 12-hours between landing and take-off for each aircraft. A screenshot of the simulation is included below (see Figure 10). This figure not only shows aircraft and U-boat entities, but also coastlines and the area of offensive search operations for the aircraft (the search grid, denoted by the large rectangle in the middle of the figure).

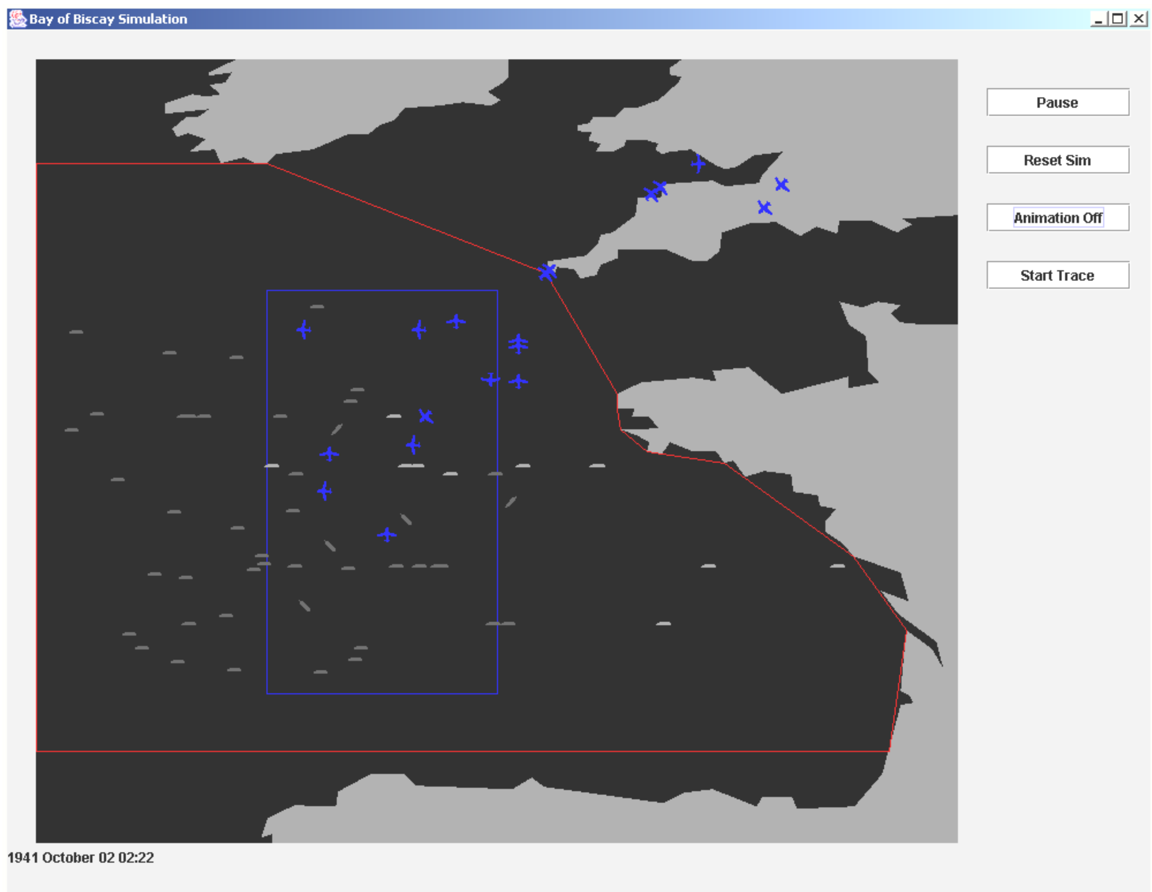

Figure 10. Screenshot of Bay of Biscay Simulation Graphics

IV. Results

4.1 Overview

This chapter describes the results associated with two scenarios, each having different search region sizes. For both scenarios, aircraft search a 200 x 350 NM^2 area, hereafter referred to as the search grid. In the first scenario, the search grid is subdivided into 50 x 50 NM^2 non-overlapping squares, hereafter referred to as search regions. At least one aircraft per day is assigned to search each region. In the second scenario, the search grid is subdivided into 100 x 100 NM^2 overlapping grids; aside from this difference, all assumptions mentioned in the previous chapter are valid for both scenarios. The search patterns were varied between the five described previously.

4.2 Scenario One, Non-Overlapping Search Regions

The simulation was run with 20 iterations per search pattern with monthly statistics collected on a variety of value measures (see Appendix A). Simulation output was analyzed using the SAS JMP statistical software package.

Table 1. Scenario One Means Comparison—All Pairs (20 Iterations)

Search Pattern			Mean Sightings
Square	A		106.9
Creeping Line	A	B	98.3
Barrier Patrol		B	96.4
Sector		B	91.9
Parallel		B	91.7

For the sake of comparison, simultaneous confidence intervals of the means were generated using all search patterns. The results are shown in Table 1. Letters in columns 2 and 3 signify statistical equivalence. Rows with common letters indicate no statistical difference between search methods. For instance, the square and barrier patrol patterns are statistically equivalent (both have “A”), and barrier patrol and creeping line are equivalent (both have “B”). However, square and creeping line are not equivalent since they do not share a common letter.

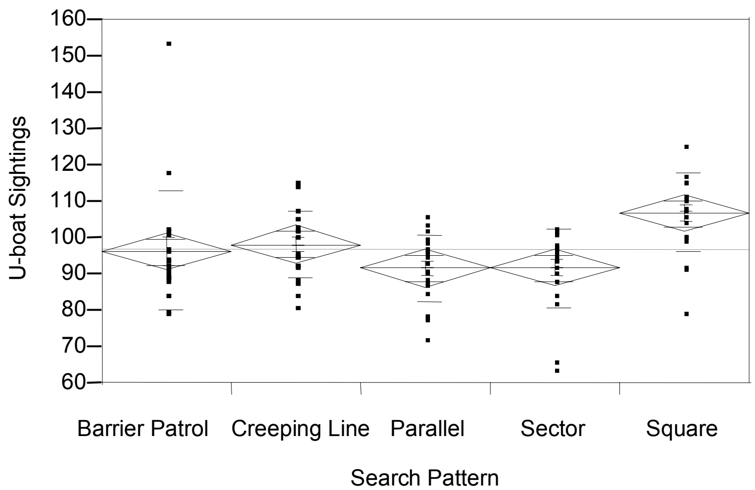

Figure 11. Scenario One Analysis of Sightings by Pattern (20 Iterations)

The most important value measure is “U-boat sightings.” Figure 11 shows a graphical representation of simultaneous confidence intervals for each search pattern (the JMP software package uses Tukey’s Highly Significant Difference, or HSD, test to compare all means). Simulation output was analyzed at the $\alpha = 0.05$ level (i.e., a 95% simultaneous confidence interval).

Therefore, according to the model, the two most successful patterns with regard to sightings on U-boats are the square pattern and the creeping line pattern. At this point, it was suspected that running the model with a greater number of iterations would reveal the square pattern to be the most successful with respect to sightings of U-boats. Value measures regarding flying hours were also collected for each pattern to determine whether or not that measure of effectiveness (MOE) affected the simulation results. Quantifying efficiency involves knowledge of both search resources used and the average density of target craft (Morse and Kimball, 1954:39). In fact, the operational sweep rate is characterized by the following equation:

$$\text{SWEEP RATE} = \frac{\text{U-BOAT SIGHTINGS}}{\text{(U-BOATS PER SQUARE MILE)} \times \text{FLYING HOURS}}$$

Next, the simulation was run with 30 iterations per search pattern, only on the two patterns found to be most successful with regards to sightings of U-boats and the Barrier Patrol pattern, due to its historical significance as having been the pattern actually used in the Bay. Again, monthly statistics were collected on a variety of MOEs (see Appendix B), and simulation output was analyzed using the SAS JMP statistical software package. Figure 12 shows a graphical representation of simultaneous confidence intervals for each search pattern (again, JMP uses Tukey's HSD test to compare all means).

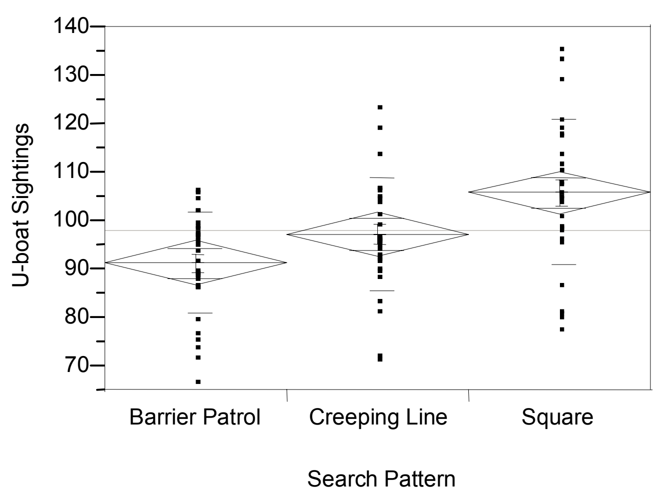

Figure 12. Scenario One Analysis of Sightings by Pattern (30 Iterations)

Simulation output was analyzed once again at the $\alpha = 0.05$ level (i.e., a 95% simultaneous confidence interval). This data is summarized in Table 2. As in previous tables, rows with common letters indicate no statistical difference between search methods.

Table 2. Scenario One Means Comparison—All Pairs (30 Iterations)

Search Patterns			**Mean Sightings**
Square	A		105.9
Creeping Line		B	97.3
Barrier Patrol		B	91.4

Therefore, according to the model, the most successful pattern with regard to sightings on U-boats is the square pattern.

4.3 Scenario Two, Overlapping Search Regions

As previously mentioned, the search grid dimensions remain unchanged between scenarios (200 x 350 NM^2); each of the search regions for scenario two, however, are four times the size of the individual search regions for scenario one. Figure 13 details the arrangement of the search regions. In the figure, the number of each of the 18 search regions is placed in the top left corner of the respective region. The middle regions (2, 5, 8, 11, 14, and 17) are shown on a separate graph for clarity, but in actuality the two grids are superimposed over a single search area.

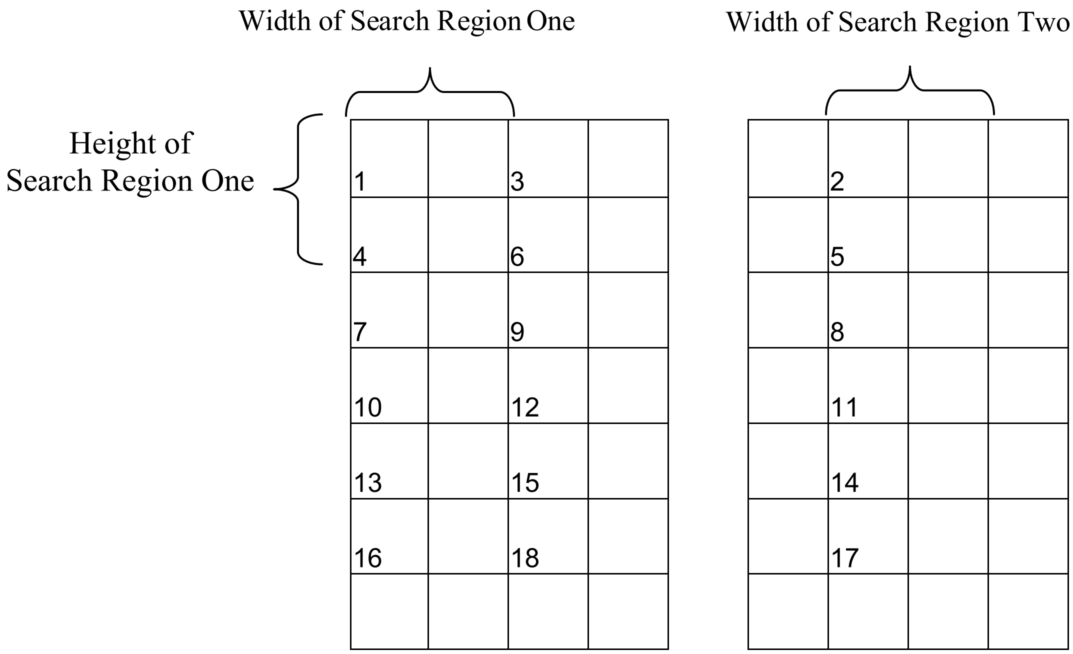

Figure 13. Scenario Two—Overlapping Search Regions (Separated for Clarity)

30 iterations of the Bay of Biscay model were run for each of the five search patterns to detect potential differences in output MOEs. The results are displayed in Figure 14.

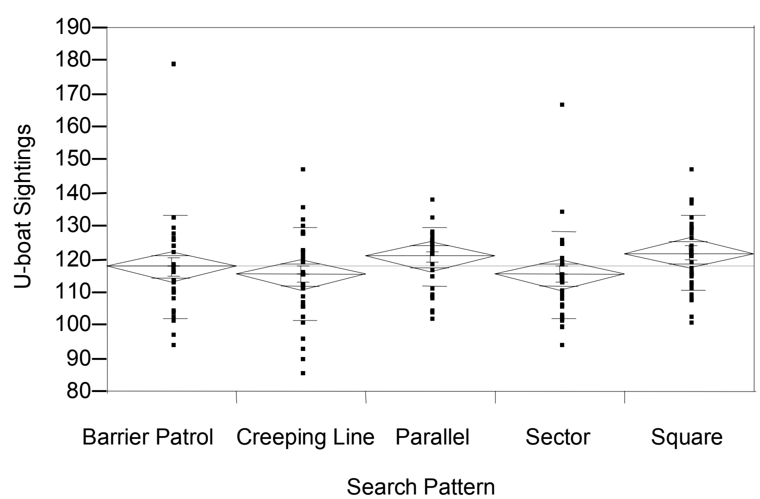

Figure 14. Scenario Two Analysis of Sightings by Pattern (30 Iterations)

Scenario one's MOEs were again collected for scenario two (see Appendix C). "U-boat sightings" is the main value measure used to ascertain differences in search patterns. Simulation output was analyzed using the SAS JMP statistical software package (Tukey's HSD "all means" comparison). Table 3 shows the results for sightings.

Table 3. Scenario Two Means Comparison—All Pairs (30 Iterations)

Search Pattern		**Mean Sightings**
Square	A	122.1
Parallel	A	121.0
Barrier Patrol	A	118.0
Sector	A	115.6
Creeping Line	A	115.6

As is evident from the table, there are no statistical differences between search patterns. This is an expected result, since the effects of structural differences between search patterns on MOEs should be minimized when the aircraft routes are allowed to overlap.

The fact that the magnitudes of the mean number of sightings in scenario two are at least 10 sightings higher than those of scenario one is a counterintuitive result. Actually, scenario two represents a less-efficient search methodology of the search grid due to duplication of effort (the aircraft have no knowledge of U-boat routing). One possible explanation for this result stems from the fact that the model does not allow the U-boats to modify their transit strategies relative to aircraft activity, other than to submerge once they see the aircraft. Since actual U-boat routes in the model involve shortest paths between homeports and their North Atlantic hunting grounds, the overlapping search methodology may allow more aircraft access to U-boat routes than the non-overlapping search methodology.

V. Conclusion

Differing the search patterns used by search aircraft has an effect on the efficiency of search operations. Though this research does not intend to make specific claims as to which search pattern is "best" in any general sense, it is worth noting that in the scenario with non-overlapping search regions, the square pattern stood out as the one that produced the most U-boat sightings in the model. The creeping line pattern came in second, though was only statistically equivalent to the square pattern in the experiment characterized by 20 iterations.

What is perhaps more remarkable is that certain assumptions upon which these particular search pattern choices are typically made (by the search and rescue community) have been relaxed. Given that the different runs involve the same number of aircraft, allow them to cover the same area, and the aircraft possess no explicit advantage with regards to probability of detection, there should be no advantage to using one search pattern over another in our model.

In the scenario where the search regions were allowed to overlap, there were no statistical differences between search patterns. As one observes the model animation and attempts to focus on the search grid as a whole (as opposed to individual aircraft), there is no discernable pattern evident in the cluster of aircraft as they search for U-boats. Ostensibly, a consequence of differing the search patterns should be that inherent advantages or disadvantages any one pattern has over any other with respect to U-boat sightings are mitigated when the aircraft routes are allowed to overlap. Therefore, this is an expected result.

The higher average numbers of U-boat sightings in scenario two relative to scenario one are not an expected result. It is suspected that this is partly due to the non-reactive behavior of U-boats to aircraft. Since in scenario two a higher number of aircraft have access to the static routes used by the U-boats, the numbers of U-boat sightings increase in the midst of this seemingly less-efficient search methodology.

5.1 Recommendations for Future Research

Though this research identified the use of agent-based simulation as a tool for revealing emergent behavior in the theory of search, further studies can be done to define potential root causes of this behavior. It would appear from some of the results that the patterns that cause aircraft to spend a greater amount of time flying 90 degrees to the target track obtain a higher number of target detections. If that is the case, why then does the parallel search pattern perform as well as any other pattern in scenario two? Is it because one or more of the regions allowed a search track to be in such proximity to one of the target tracks that an equivalent number of sightings were inevitable? Only by updating the code to display a map to reveal the locations of all sightings could one answer that question; the resulting "sighting pattern" analysis might reveal further insights.

The agent-based nature of this model could be further utilized to gain insights into agent behavior by allowing agents to react in more complicated ways to other agents in the simulation. Search pattern analysis could then be performed in the presence of such behaviors as aircraft and U-boat "flocking" according to some predefined triggers (i.e., hard allied intelligence on U-boat locations or U-boats traveling in packs for defensive purposes), U-boats attacking aircraft with anti-aircraft guns, certain U-boats designated as refueling boats (whose losses would have an obvious impact on the number of refuelings possible in the North Atlantic), U-boats avoiding perceived "danger zones" based upon relatively recent sightings and kills of other U-boats, etc. Cross-referencing could also be accomplished using other agent-based software packages to identify emergent behaviors and possible root causes.

Appendix A: Non-overlapping Scenario Data Tables (20 Iterations)

For this and subsequent appendices, "U-boat Sightings" refers to the number of U-boats sighted by aircraft while "Aircraft Sightings" refers to the number of aircraft sighted by U-boats. Also, "Time to Cross Bay" refers to the average number of hours it takes each U-boat to cross the Bay of Biscay while "Time in the Operational Zone" refers to the average number of hours each U-boat spends in the North Atlantic.

Barrier Patrol Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	80	1.67	7,070.93	320.17	238.67	9.69	87.95	589.52
2	80.17	2.67	7,574.33	263.67	198.83	14.37	85.63	593.1
3	92.67	1.33	8,593.82	137.67	239.5	26.38	85.4	568.79
4	101.33	2.17	8,526.80	127.5	242.33	30.65	86.91	591.27
5	154.33	3	8,560.56	109.83	240.67	30.82	89.82	609.47
6	90.33	2.5	8,446.85	138.83	245.17	22.49	89.12	608.79
7	94	2.83	7,916.47	208.5	241	15.68	86.64	632.3
8	93.17	1.67	8,531.68	150.83	251.67	26.96	89.29	600.6
9	94.5	2.33	8,691.86	127.17	231.17	31.23	85.91	625.02
10	97.67	1	8,638.40	120.83	241	31.06	82.96	590.75
11	102.83	1.5	8,535.87	143.33	245.67	32.33	87.77	611.42
12	96.83	2.5	8,903.18	99.17	258.67	34.25	82.13	585.46
13	102.5	2.5	8,636.96	117.67	219.33	32.41	87.05	620.9
14	84.5	1.67	7,656.21	240.5	243.33	13.83	88.11	562.87
15	85	1.83	7,787.33	233	217.67	15.37	80.37	606.24
16	89.5	1.67	8,195.57	171.33	226.67	26.41	85.05	601.23
17	89.67	1	8,357.78	151.17	233	23.41	87.53	600.32
18	91.33	1.67	8,661.16	121.17	224.67	27.37	83.39	585.19
19	118.5	2.17	8,734.02	107.17	253.17	30.6	89.02	607.78
20	88.67	1.67	8,199.88	182	245.33	19.23	87.85	600.39
Mean	96.37	1.97	8,310.98	163.58	236.88	24.73	86.39	599.57

Parallel Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	72.5	0.5	7,104.99	313.5	208.67	9.73	87.97	597.14
2	79.17	2.17	7,559.05	270.17	195.33	14.97	85.6	607.04
3	90.83	2.5	8,655.50	137.17	228.83	27.7	85.79	604.89
4	97	2.33	8,529.41	133.33	226	29.64	87.67	579.07
5	91.5	1.33	8,787.34	119.17	226.83	31.94	85.79	619.37
6	96.5	1.33	8,696.69	111.17	221.83	32.36	88.33	642.01
7	106.17	1.67	7,665.07	238.83	233.83	10.52	90.84	604.17
8	88.33	1.17	7,767.50	236.83	217.67	14.89	85.07	588.78
9	87.67	1.83	8,354.35	160.67	232.17	25.32	85.09	595.96
10	102.33	2.33	8,673.24	116.67	258.33	31.56	85.59	597
11	95.17	2	8,530.82	140.67	237.5	30.2	86.43	591.62
12	104.17	1.17	8,855.88	102.83	245.67	31.45	84.47	624.83
13	97.5	2.67	8,658.87	115	230.5	31.14	92.19	589.93
14	93.67	3.33	8,835.23	103	217	31.89	90.48	643.74
15	100.17	2.83	8,856.91	96.17	249.83	30.02	86.7	598.01
16	78.5	2	7,674.57	250.33	215.17	16.61	87.2	593.68
17	89.33	0.83	8,069.84	203.83	223.17	16.68	84.63	585.31
18	78.17	2.17	7,870.41	224.67	234.33	10.79	87.64	583.23
19	85.33	2.5	8,099.28	201.83	215.17	14.96	86.72	592.66
20	99.33	2.67	8,613.98	135	244.67	27.75	86.17	604.07
Mean	91.67	1.97	8,292.95	170.54	228.12	23.51	87.02	602.13

Creeping Line Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	81.17	1.33	7,170.53	312.17	229.67	9.81	87.96	591.09
2	88.33	2.17	7,584.97	268	210.17	13.86	86.59	592.72
3	102.67	1.5	8,522.14	151.33	265.33	26.29	87.58	562.35
4	108.33	1.67	8,560.42	124.17	245.5	29.6	86.49	617.52
5	102.67	1.67	8,820.51	113.83	237.5	31.73	82.88	602.47
6	103.5	2.33	8,743.01	112.17	256.67	31.98	87.48	622.61
7	92.5	1.17	7,820.41	223.5	228.5	12.87	87.84	599.07
8	84.83	2.33	7,929.04	230.67	203.5	16.35	84.89	607.63
9	96.5	2.17	8,347.95	163.33	240.17	26.33	86.83	599.13
10	93.33	2.17	8,695.98	125.5	223.33	31.47	87.17	584.41
11	93.17	1.83	8,596.19	138.67	221.83	30.91	87.1	622.88
12	106	2.33	8,765.72	112	228.67	30.96	87.17	555.9
13	95.5	3.17	8,685.07	123.83	237	32	84.43	599.11
14	114.5	2.67	8,863.49	105.5	246.83	32.44	89.07	590.99
15	115.67	2.5	8,841.06	102.17	263.17	32.86	94.08	570.44
16	95.17	2.5	7,691.47	251.5	208.33	17.35	88.76	579.38
17	89	0.83	7,807.45	230.83	260.5	10.72	89.44	602.09
18	95.83	2.33	8,007.87	207	237.17	15.25	82.48	595.32
19	100.83	1.33	8,633.80	137.67	251.33	26.3	87.84	593.68
20	106	2.67	8,711.48	113.67	238.17	30.78	85.89	585.55
Mean	98.28	2.03	8,339.93	167.38	236.67	24.49	87.1	593.72

Square Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	80	1.5	7,130.64	316.67	254.5	9.76	87.98	597.92
2	92.33	3	7,622.49	265.33	214.5	13.65	84.93	609.86
3	104.83	2.5	8,649.26	143	253.33	27.5	88.65	613.61
4	100.67	2.67	8,673.18	119.67	227.83	29.66	85.79	614.31
5	112	2	8,786.42	113.67	228.33	30.67	90.2	606.93
6	115.83	2.67	8,773.41	103.67	239.17	29.9	86.3	624.31
7	108.83	2	8,086.92	193.17	263.5	17.69	89.03	630.35
8	99.5	2.5	8,326.73	175.17	231	23.79	89.72	621.95
9	100.83	3.33	8,570.65	139.67	242.83	29.32	89.16	565.92
10	110.67	1.5	8,742.46	111.83	244.5	31.26	85.48	602.09
11	108.33	2.33	8,638.72	139.83	239.17	28.21	92.19	607.18
12	111	2	8,027.93	200.83	255.17	17.45	91.05	614.06
13	92.17	2	7,432.31	284	210.83	14.79	82.88	628
14	112	2.5	8,383.06	162.17	247.17	24.94	90.17	603.54
15	111.17	1.5	8,262.84	183.17	253.17	20.57	88.55	595.45
16	106.33	2.67	8,389.80	159.17	253.33	25.48	88.09	587.8
17	111.67	3	8,783.97	114.67	246.83	30.32	91.63	624.65
18	126	2.17	8,828.93	101.5	255.5	29.86	86.32	595.53
19	115.67	3.83	8,914.36	99.5	266.67	29.69	83.39	631.21
20	117.67	1.83	8,994.52	93.83	263.33	27.47	87.16	619.25
Mean	106.87	2.38	8,400.93	161.02	244.53	24.6	87.93	609.7

Sector Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	64	1.33	7,178.67	315.33	211.17	9.83	88.08	599.41
2	84.5	2.33	7,585.11	263.83	226.17	13.84	86.73	616.28
3	88.5	1.67	8,592.45	143.83	270.33	25.98	88.82	600.52
4	66.33	1.33	7,502.50	265.67	229.5	10.26	88.53	613.86
5	97.17	2	8,043.81	209.33	261.5	14.87	86.96	595.16
6	90.83	2.33	8,516.47	153.5	235.17	25.11	84.47	618.9
7	97.67	2	8,792.71	110.33	243.67	31.16	86.2	622.95
8	95.67	2.17	8,673.02	124	249	31.24	83.96	584.95
9	101.33	1.83	8,623.13	131.5	236	27.3	89.21	595.86
10	88.67	1	8,220.66	180	266	21.48	87.8	596.4
11	94.33	1.83	8,022.98	212.17	241.67	23.98	88.93	586.56
12	98	2.33	8,676.99	118	231	29.29	87.14	588.55
13	102.5	3.17	8,615.36	128.83	245	31.91	93.95	590.22
14	103	4.33	8,788.61	120	230.5	30.67	91.53	626.22
15	94.67	1.5	8,493.03	154.33	252.17	26.4	88.88	551.1
16	82.67	1.33	8,037.06	214	228.17	21.85	84.65	589.55
17	101.67	1.67	8,778.86	112.67	264.5	29.93	88.11	601.13
18	94.33	2.33	8,924.41	99	234.33	33	87.27	610.2
19	92.5	2.67	8,840.61	103.33	243.67	32.08	89.61	614.97
20	98.67	1.83	8,859.77	101.5	241	31.41	89.64	593
Mean	91.85	2.05	8,388.31	163.06	242.03	25.08	88.02	599.79

Appendix B: Non-overlapping Scenario Data Tables (30 Iterations)

Barrier Patrol Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	77.17	1.33	7,124.51	316.83	226	9.75	87.97	583.21
2	86.67	1.67	7,535.58	276.67	247.83	13.71	86.53	568.21
3	88.5	1.67	8,534.08	154.67	225.5	25.32	85.93	620.81
4	98.83	2.33	8,566.77	133.33	251	30.37	91.17	574.57
5	106.33	1.33	8,809.96	111.83	253.67	31.97	86.07	599.12
6	102.67	3	8,770.49	107.17	247.33	31.29	88.09	619.82
7	107	2	8,853.46	95.33	258	32.82	88.46	595.86
8	96.5	2.67	8,814.57	112.33	219.83	33.91	87.69	611.63
9	105.33	3	8,848.22	104.33	248.17	31.25	89.38	603.69
10	76.17	2	7,652.94	247	243.67	14.52	87.38	586.25
11	80.33	1.5	7,299.22	299.33	226.67	13.25	85.18	617.6
12	90	1.83	8,511.87	154.17	234.83	25.5	87.05	605.23
13	89.5	2.17	8,402.58	156	235.83	29.16	82.48	581.01
14	97.17	1.67	8,746.72	124.67	233.83	30.47	83.71	600.96
15	72.17	0.5	7,773.27	248	250.17	14.07	88.37	589.4
16	74.5	2.33	7,554.38	266.33	198.83	15.41	86.53	627.27
17	95.5	1.83	8,574.74	138.17	252.83	27.36	86.42	580.33
18	92.5	1.83	8,760.63	112.5	238.33	31.7	89.77	601.01
19	87.17	3.17	8,091.66	196.5	226.33	16.53	85.58	576.68
20	86.83	1.67	8,208.03	187.17	218	19.95	86.69	622.87
21	97.83	1.83	8,365.44	165.67	243.33	27.06	91.48	620.96
22	98.33	2.83	8,719.76	112.17	250.5	30.38	88	637.6
23	94.5	2	8,370.16	160.67	234.67	19.19	87.2	597.78
24	99.33	2.17	8,730.64	126.33	265.67	25.43	87.73	641.57
25	89.5	1.67	8,499.94	158.5	244.67	27.16	87.07	589.44
26	89.67	2.5	7,824.41	237	231.17	15.3	84.28	586.9
27	90.33	2.83	8,575.41	139.83	242.67	26.31	83.31	590.84
28	102.83	2	8,757.91	118	256.33	30.57	89.5	600.64
29	100.33	1.33	8,727.82	112.5	262	32.28	90.92	591.62
30	67.17	1.83	7,463.87	276.33	231.17	10.73	87.85	596.2
Mean	91.36	2.02	8,315.63	171.64	239.96	24.09	87.26	600.64

Creeping Line Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	72	1.67	7,084.80	321.67	212	9.71	89.39	598.74
2	96.33	2.5	7,588.94	269.17	224	14.36	84.31	582.75
3	95.5	1.67	8,699.51	141.33	227	27.5	86.53	569.27
4	92.17	2.67	8,685.79	120.5	238	31.46	89.33	590.31
5	119.67	2.17	8,701.63	114.17	266.83	29.58	89.03	586.7
6	97	2	8,805.38	104.5	212.33	33.13	85.44	595.6
7	107.17	1.83	8,737.88	104.17	245.83	27.93	84.85	578.6
8	105.83	3.17	8,811.72	105.67	241.67	29.09	87.43	583.19
9	96.83	3.33	8,829.27	105.67	219.33	31.27	85.28	602.35
10	114.5	2.67	8,796.37	99.83	267	31.92	88.2	606.24
11	72.67	0.67	7,344.14	306	194.83	12.91	88.64	582.26
12	96.17	2.17	7,971.93	215.17	222.33	17.28	87.62	584.92
13	84	2.83	8,334.26	169.67	210.5	26.72	83.39	568.21
14	99.67	2.83	8,656.02	119.33	232	30.77	89.06	616.98
15	105.33	2.17	8,746.63	107.5	239.33	31.95	88.64	591.58
16	81.83	2.67	7,564.67	268.5	214	15.41	90.84	572.86
17	107.33	1.33	8,243.51	171.5	253.33	19.19	87.93	612.66
18	106.67	1.5	8,844.08	118.67	267.67	28.85	89.27	580.54
19	92.67	2.17	8,624.78	138.17	234.5	22.94	86.79	605.47
20	93.67	2.5	8,191.33	183	220.5	19.36	84.74	594.27
21	101.67	2.33	8,461.01	157.33	233.33	28.47	85.79	592.21
22	104.17	3	8,800.89	113	241.17	32.44	84.36	601.63
23	97.5	3.17	8,788.26	100.33	241.5	32.62	85.98	615.4
24	93.33	1.33	8,058.61	200	257	13.25	88.75	584.17
25	95	1.33	7,831.93	231.5	219.83	16.9	87.87	590.17
26	90.17	3	8,527.89	143.5	229.17	26.77	89.05	598.34
27	124	3.33	8,676.22	112.83	265	30.2	86.51	593.9
28	88.83	2	7,799.52	241.83	257.67	11.71	88.68	576.17
29	90.83	2.33	7,916.89	220.33	239.33	15.81	88.19	636.37
30	96.33	2	8,439.62	161.33	225	26.73	87.22	568.32
Mean	97.29	2.28	8,352.11	165.54	235.07	24.21	87.3	592.01

Square Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	81.5	1	7,108.86	321	245.5	9.73	88.01	607.06
2	78.33	2.33	7,588.70	276	187.67	14.13	83.6	607.57
3	101.33	2.83	8,657.69	133.83	259	27.46	90.13	618.88
4	118.67	2.33	8,602.42	130.5	249.5	30.95	90.47	606.78
5	119.67	2	8,759.80	109.67	262.83	31.31	88.21	586.86
6	111.17	2.5	8,987.82	96.5	237.67	33.68	88.42	583.29
7	114.5	3.5	8,868.95	95.33	225.33	32.7	85.1	630.19
8	129.83	2.67	8,841.47	103.83	275	30.29	89.63	583.19
9	136	4.83	8,691.57	116.33	267.5	32.9	90.17	626.97
10	133.83	2.17	8,802.31	90.17	268.33	31.02	86.57	571.62
11	81.83	1.67	7,059.65	336	229.83	9.95	88.58	589.17
12	108.5	2.5	7,817.67	220	255.17	15.14	88.86	596.75
13	98.5	3.33	8,208.46	189	225.33	25.05	86.23	594.12
14	111.17	2.83	8,640	125.67	257.83	29.56	86.63	590.9
15	108.33	2	8,621.09	117	243.33	30.12	87.66	585.03
16	87.17	1.83	7,336.05	289	242.5	11.8	87.29	589.85
17	104.33	2.83	8,049.20	190.5	231.83	16.83	87.99	577.74
18	121.33	2.83	8,690.23	119	263.5	27.88	89.08	592.99
19	96	1.83	7,797.35	241.33	263.67	10.7	87.55	576.51
20	99.17	1.67	8,083.86	202.5	251.83	15.7	86.41	586.61
21	96.83	3.67	8,266.37	172.83	241.5	27.19	85.45	584.01
22	118.33	1.83	8,635.96	113.67	242.5	28.47	85.72	596.7
23	112.33	2.5	8,881.26	101.33	259.33	32.39	86.57	610.39
24	105.17	2	8,527.77	150.83	274.67	20.39	87.92	607.63
25	99	3.17	8,310	175.33	208.5	24.93	86.27	602.4
26	105.67	4.17	8,668.33	127.17	271.17	28.38	87.84	588.23
27	99.33	2.5	8,173.83	190.17	232.67	18.92	87.95	624.89
28	106.33	2.17	8,310.63	168.5	233.67	21.05	88.76	619.25
29	111	2.5	8,735.84	118.5	246.83	29.58	86.24	575.29
30	80.5	1.67	7,396.56	280.83	214.83	10.94	90.48	604.28
Mean	105.86	2.52	8,303.99	170.08	245.63	23.64	87.66	597.17

Appendix C: Overlapping Scenario Data Tables (30 Iterations)

Barrier Patrol Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	94.83	1.5	7,121.26	318.33	276.5	9.76	89.3	583.78
2	110.67	2.83	7,529.88	270.33	255.67	14.08	84.64	595.84
3	118.83	4	8,638.48	135.17	289.83	26.77	85.65	596.67
4	102.17	3.17	7,477.64	263.83	301.83	10.93	90.22	591.75
5	130.67	2	8,029.57	199.33	278.67	17.79	89.62	601.66
6	126.83	3	8,595.92	131.17	311.5	27.36	84.74	596.52
7	114.33	2.83	7,881.68	219.67	291.17	13.87	87.42	624.45
8	98.17	2.83	8,123.47	204.5	234.5	19.48	85.3	617.82
9	128.67	3.17	8,605.98	143	321.83	27.71	84.98	603.93
10	104.83	2.17	7,937.74	211.67	295.5	15.5	88.31	595.58
11	111.83	1.67	7,695.59	254.67	250	18.25	83.45	603.94
12	117.83	2	7,856.54	227.67	315.17	14.07	86.99	592.84
13	113.83	2.17	7,594.63	253.33	272.17	19.18	86.85	599.87
14	116.67	4.67	8,575.55	136.33	281.67	28.51	85.67	614.48
15	133.5	2.83	7,661.77	250.83	257.83	12.52	89.76	618.44
16	114.67	2.83	7,559.99	257.17	277.67	17.08	87.99	577.94
17	114.17	2.33	8,681.82	128.17	297.17	28.74	90.78	594.24
18	180.17	4	8,605.80	113.17	298.5	28.7	90	589.63
19	103.5	2.17	7,855.48	231.17	289	12.94	89.17	620.18
20	108.83	2.17	8,118.88	187	261.33	17.92	83.16	620.04
21	110.67	2.83	8,256.91	172.17	293.83	26.82	84.47	579.85
22	133.5	3.67	8,799.02	114.83	315.33	28.38	89.33	609.18
23	123	3.33	8,797.57	107.83	294.17	28.28	87.7	621.93
24	125.17	3	8,771.41	99.33	288.67	32.53	86.1	603.09
25	124.67	3.33	8,661.27	114.17	274.67	29.4	87.21	594.21
26	105.5	2	7,868.84	216	298.33	13.03	87.44	593.71
27	114.83	2.67	7,951.73	206	272.83	15.51	89	610.32
28	119.17	3	8,544.27	136	323.5	27.5	90.76	583.82
29	127.5	2.17	8,696.61	121.83	308.17	30.52	87.48	644
30	111.17	3	7,883.78	233.83	278	16.27	88.12	589
Mean	118.01	2.78	8,145.97	188.62	286.83	20.98	87.39	602.29

Parallel Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	104.83	2	7,135.42	313.5	269.17	9.76	87.54	585.98
2	109.83	2.83	7,260.36	295.5	260.33	10.86	87.68	597.58
3	123.67	2.67	7,964.13	216.83	262.17	14.52	86.64	594.11
4	128.33	3.17	8,422.69	158	282.33	26.85	88.92	579.01
5	115.67	2.17	7,791.97	240.33	307.67	10.7	87.45	574.74
6	112	2.67	7,933.85	217.17	253	14.95	84.39	587.39
7	126.5	2.5	8,541	135.5	269.83	26.86	84.15	598.44
8	126.33	4	8,526.51	143.67	265	27.47	82.59	588.25
9	122.17	2.5	8,125.18	188.5	267.83	21.98	87.83	617.11
10	117.5	2.83	7,716.61	237.67	300.67	11.07	90.07	597.22
11	109.33	2.33	7,507.34	284.17	248.67	15.45	88.92	565.61
12	139.17	2.17	8,599.88	138.67	314	27.72	91.22	582.86
13	125.83	2.33	6,966.95	333	259.17	9.72	91.72	571.94
14	110.17	3.5	7,818.31	232.17	245.67	15.28	86.72	590.91
15	126.17	3	8,419.68	151.5	268.5	26.16	85.32	652.76
16	127	2.83	8,550.72	138.17	252	30.89	87.68	609.99
17	115.67	2.17	7,848.29	217.67	298.83	11.81	88.76	618.48
18	122.17	3.5	8,063.57	203.83	264	15.89	86.21	645.21
19	129.17	3.17	8,536.23	135.5	277.17	27.76	85.87	578.49
20	126	2.33	8,828.93	108.17	265	31.28	85.87	604.4
21	127.67	2.5	8,409.09	149	258.67	27.97	85.53	606.78
22	124.33	2.33	7,906.75	212.67	255	15.61	82.33	593.88
23	105.5	2.33	7,940.17	224.33	286.17	11.91	87.54	627.5
24	122.33	3.33	8,331.62	174	264.17	16.15	84.16	595.54
25	125.17	2.33	8,449.27	153.33	281.5	28.1	85.88	594.82
26	123.83	3.17	8,820.40	118.33	272.5	31.54	83.64	621.09
27	126.67	3.17	8,030.34	202.17	315.17	15.54	87.52	577.74
28	119.33	3.17	8,024.61	205.33	262.5	16.31	87.15	617.12
29	133.67	2.67	8,555.87	135.67	285	27.41	89.97	605.43
30	103.17	1.83	7,563.97	267	261.67	12.07	89.99	611.94
Mean	120.97	2.72	8,086.32	197.71	272.44	19.65	86.98	599.74

Creeping Line Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	86.67	2.33	7,185.09	320.83	257	9.83	90.57	583.95
2	93.67	2.67	7,415.93	290.17	271.17	10.63	87.58	613.39
3	103.83	3	8,000.01	219.5	255.5	15.75	86.51	618.52
4	111.83	3.17	8,428.31	157.17	263	26.53	87.38	633.76
5	101.83	1.5	7,806.22	241.33	307	10.97	90.58	623.26
6	116.83	1.67	7,969.29	217.5	287.17	15.24	86.06	592.32
7	113.33	3	8,615.98	130.33	277.33	26.6	84.58	587.97
8	124	2.67	8,801.45	119.5	288.67	30.28	88.32	632.14
9	106.83	3	7,902.92	234.67	287.17	14.69	87.75	620.86
10	106.67	1.83	8,142.18	186	268	19.54	82.18	604.6
11	97	1.5	7,051.88	329.83	299.17	10.48	90.31	566.54
12	148	2.17	7,932.38	214.83	265	15.18	90.87	612.76
13	121.67	2.17	8,258.44	185.33	286.33	26.75	83.49	582.64
14	113.33	2	7,921.82	222.5	303.67	16.19	86.4	582.57
15	116.17	2.5	7,860.74	240.17	295.67	14.59	84.76	597.9
16	108	2.67	8,339.68	173.83	261.83	26.74	85.78	585.55
17	129.17	2.33	8,788.44	112.33	307.5	31.72	89.57	605.89
18	131.17	2.33	7,790.87	223.83	293.67	11.43	90.9	608.48
19	116.5	2.33	8,056.41	208.17	280.17	16.05	90.23	588.5
20	132.83	2.33	8,622.31	133.17	295.17	27.39	91.13	596.2
21	116.5	3	8,557.64	147	281	30.38	84.84	596.25
22	128.83	2.67	8,869.27	104.17	304.5	30.7	83.57	590.78
23	109.5	1.83	7,931.82	219	312.33	11.96	88.12	560.5
24	120.17	2	8,287.08	176.83	300.5	15.98	84.14	621.33
25	113	2.67	8,568.79	145.83	277	26.82	84.06	595.67
26	123.17	3	8,784.17	119.17	293.17	30.36	84.16	590.04
27	136.83	4	8,812.35	102.5	318.5	30.64	85.35	616.24
28	120.83	2.5	8,624.19	130.33	290.5	25.19	88.91	577.6
29	128.67	2.67	8,416.36	152.33	310.5	25.73	86.06	578.46
30	90.5	2	7,569.61	276.67	261	11.18	87.71	616.65
Mean	115.58	2.45	8,177.05	191.16	286.64	20.52	87.06	599.38

Square Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	103.83	1.33	7,088.41	322	297.17	9.71	90.53	602.16
2	125	4	7,514.69	266.17	272.5	14.05	88.29	585.73
3	120.17	2	8,698.86	141.33	301.5	28.27	86.83	625.36
4	128.5	3	8,649.14	120.17	288.67	30.01	84.92	610.15
5	148	3.67	8,732.17	112.17	313.83	28.36	87.51	564.24
6	112.33	2	7,768.36	232	327.33	11.87	87.26	579.03
7	122	2.33	7,951.81	203.17	292.33	15.9	84.72	611.18
8	129.83	2	8,523.72	139.83	340.33	27.91	85.09	582.36
9	110.5	2.67	7,739.57	245.33	290.5	14.9	88.19	603.54
10	122.67	1.5	8,095.78	196	279.5	19.3	88.05	603.25
11	114	1.33	8,290.64	179.17	294.67	27.31	84.72	569.43
12	127.33	4.67	8,799.04	114.67	289.33	30.29	82.62	597.02
13	108.5	2.17	7,051.01	327.5	282.67	10.46	85.83	604.88
14	118.5	3.33	7,844.98	240.17	280.33	15.22	86.6	598.42
15	130.33	3.33	8,505.84	140.17	314.5	27.74	87.57	571.53
16	133.17	2.17	8,570.66	142.17	304.67	29.32	89.71	562.1
17	117	2	7,813.82	229.33	338.83	10.72	89.77	615.25
18	109	2.5	8,069.99	206.17	256.17	14.23	88.4	622.43
19	123.5	2.83	8,640.89	135.5	297.17	26.63	87.93	600.22
20	138.67	1.67	8,792.14	111.17	316.17	29.24	88.04	611.98
21	102	1.17	7,509.02	272	289.33	13.32	87.82	593.72
22	131.83	2.5	8,114.93	198.17	308.83	17.76	85.57	593.98
23	138	2	8,575.78	126.67	322.33	27.13	86.08	600.77
24	115.5	2.67	8,102.39	197	317.5	12.01	87.95	576.98
25	116.5	1.83	7,635.64	255.5	281.83	12.87	85.96	591.24
26	130.17	1.67	8,526.56	148.5	335.17	27.33	87.39	603.93
27	138.67	3.33	8,777.14	104.17	316.83	31.72	89.26	617.89
28	113.17	2.5	7,914.20	216.17	306.67	14.91	87.7	609.66
29	116.5	2.33	8,075.84	194.33	283	20.61	84.88	602.18
30	117	1.5	7,637.63	255.83	289.5	14.16	88.63	592.37
Mean	122.07	2.4	8,133.69	192.42	300.97	20.44	87.13	596.77

Sector Pattern Output (Mean Per Month)

Rep |	U-boat Sightings	U-boats Killed	Aircraft Sortie Hours	Sorties Cancelled	Aircraft Sightings	Aircraft Over Bay	Time to Cross Bay	Time in the Operation Zone
1	95	1.17	7,210.70	313.83	271.67	9.87	89	583.56
2	119.17	2.33	7,627.79	262.67	279	14.65	84.74	582.46
3	115	3.5	8,713.15	137.67	290	25.96	87.93	602.35
4	167.67	4	8,362.10	153.33	302.67	25.12	91.49	579.37
5	116.33	2	8,131.41	201.67	268.67	17.03	82.65	593.84
6	119.17	3.5	8,681.76	130.33	288	28.21	85.56	614.19
7	113.83	2.5	7,897.68	208.17	312.17	15.07	87.76	580.95
8	115	2.83	8,050.84	205.5	282	18.65	85.95	595.83
9	116.17	3.33	8,647.79	135.33	299.5	27.51	90.92	619.96
10	125.83	3.33	8,490.68	151.83	300.33	21.67	89.91	609.61
11	116.5	3	8,056.61	209.33	279	20.67	88.07	613.69
12	113.67	2.83	7,925.99	220	326.17	14.32	87.71	570.34
13	103.83	3.83	7,504.02	283.33	238.17	15.14	84.33	593.94
14	106.5	2.5	7,806.23	256.83	283.17	11.84	87.51	601.94
15	113.83	2.67	7,938.16	218.33	262.83	16.04	86.88	598.53
16	121	2.83	8,343.42	170.83	285	25.4	87.78	611.57
17	135.17	2.5	8,755.85	119.33	304.83	28.97	88.43	624.94
18	119.67	2.17	7,890.46	224.5	336.5	11.9	88.37	588.64
19	111.17	2.5	8,190.46	200.33	264	16.26	82.36	575.93
20	109.17	1.67	7,841.10	240.83	303.83	10.75	87.09	609.28
21	107.17	2.67	7,626.34	266.17	247.67	14.8	82.8	573.48
22	104.33	3.5	7,893.19	234.17	299.33	12.24	86.4	607.47
23	103.17	1.5	7,920.27	231.67	307.17	11.07	87.42	592.27
24	114.33	2.83	8,159.63	194	291.17	12.72	83.72	586.35
25	110.33	2	7,670.39	250	267.33	14.83	85.09	597.32
26	119.83	3.67	8,271.18	196.83	266.67	19.36	88.13	602.23
27	125.83	3.17	8,542.21	146	293.83	27.92	90.55	605.91
28	126.83	2	8,723.85	121.17	286.17	29.71	87.61	594
29	100.17	2.67	7,672.63	255	285.83	11.92	87.36	583.92
30	102.17	2.33	7,756.27	249.67	255.83	13.62	85.58	610.1
Mean	115.59	2.71	8,076.74	206.29	285.95	18.11	86.97	596.8

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Vita

Captain R. Gregory Carl entered the University of California, San Diego, in September of 1983. He graduated in June of 1988 with a Bachelor of Arts degree in Mathematics (Applied). After working seven years in the private sector, he attended Officer Training School and was commissioned a second lieutenant in the United States Air Force in August of 1995. He was assigned to the 5th Communications Squadron, 5th Support Group, 5th Bomb Wing, Minot AFB, North Dakota, where he worked as the base network chief and commander of the 5th Support Flight until January of 1998. He then was assigned as chief of protocol for the 5th Bomb Wing, where he was responsible for numerous high-visibility visits and events.

In January 1999, he was assigned to the Air Force Institute of Technology at Wright-Patterson AFB, Ohio, as executive officer to the commandant. He served in that capacity until entering the AFIT Graduate School of Engineering and Management in August of 2001. During the fall of 2001, he completed Squadron Officer School by correspondence and received a Certificate in Statistical Theory and Methods from Colorado State University for coursework he had completed prior to becoming an AFIT student. Upon graduation, he will be assigned to the Missile Defense Agency, Schriever AFB, Colorado.

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14. ABSTRACT

Threats to our nation's resources and forces are becoming increasingly lethal and mobile. Therefore, our ability to locate and interdict these threats is more important than ever. Search theory is one tool that is vital to countering the increasing threat. This research presents a multi-agent simulation, built around the allied search for U-boats in the Bay of Biscay during World War II, which extends several classic search theory algorithms. Comparison of techniques is based on the effectiveness of finding high-valued, mobile assets. A JAVA-based multi-agent simulation model is designed, built and tested, and used to demonstrate the existence of differing emergent behaviors between search patterns currently used by the United States military.

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