3-2008

Determining the Surface-to-Air Missile Requirement for Western and Southern Part of the Turkish Air Defense System

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DETERMINING THE SURFACE-TO-AIR MISSILE REQUIREMENT FOR WESTERN AND SOUTHERN PART OF THE TURKISH AIR DEFENSE SYSTEM

THESIS

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AFIT/GOR/ENS/08-01

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AFIT/GOR/ENS/08-01

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THESIS

Presented to the Faculty

Department of Operational Sciences

Graduate School of Engineering and Management

Air Force Institute of Technology

Air University

Air Education and Training Command

In Partial Fulfillment of the Requirements for the

Degree of Master of Science in Operations Research

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March 2008

AFIT/GOR/ENS/08-01

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Abstract

An air defense system is vital for countries to protect their homelands. Today, air defense systems consist of integrated systems such as early warning radars, fighter aircraft, airborne early warning aircraft and surface-to-air missile (SAM) systems. The Turkish air defense system does not have long range SAM systems. Turkey plans to procure SAM systems to protect her borders.

This research develops two location optimization models to optimally locate SAM sites to defend specified areas of the nation. One of the models finds the minimum number of SAM sites to cover the specified area; the other finds the maximum coverage for a given number of SAM sites. The model is formulated as an integer program, and the LINGO 10 software package is used to solve the model. Three candidate SAM systems are examined. All models use the maximum range of each SAM system. Solutions are presented for the decision makers to examine. Sensitivity analysis is used to explore how much the optimal solution(s) change given fluctuations in input values.

The main objective of this research is to provide the Turkish Air Force coverage information regarding the three candidate SAM systems. This research also provides a model and an approach that can be used to examine other candidate systems. The results and models presented in this research should facilitate development of a more efficient and effective air defense system to support Turkey’s homeland defense.

AFIT/GOR/ENS/08-01

I would like to dedicate this thesis to heroic pilots of 191^{st} and 192^{nd} Squadrons who serves their country. It is a great privilege and honor for me being in this family.

Acknowledgments

This research does not contain the official policy of the Turkish Government or Turkish Air Force about selecting surface-to-air missiles for the Turkish Air Defense System. All the candidate missiles are chosen by me. The maps to show the results are only for demonstration, not for implementation. I am solely responsible for all the comments and critiques in this research.

I would like to express sincere appreciation to my thesis advisor, Dr. James T. Moore for taking the challenge of guiding me through this thesis effort. This thesis would not have been complete without his expert advice and unfailing patience. I would also like to thank Maj August G. Roesener, for the support provided to me in this endeavor.

I am indebted to the Turkish Air Force and the great Turkish nation for providing me this wonderful Master's program opportunity. My words of gratitude can not express the feelings I carry for them.

Finally, I would like to thank my family for raising and supporting me all my life. My gratitude will be forever.

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DETERMINING THE SURFACE-TO-AIR MISSILE REQUIREMENT FOR WESTERN AND SOUTHERN PART OF THE TURKISH AIR DEFENSE SYSTEM

I. Introduction

1.1 Relevant Information about Turkey

Turkey is one of the largest countries in the region where Asia and Europe meet. She shares nearly 3000 kilometers of border with 8 different countries. The region where Turkey is located has had major wars in recent years. Stability of the nations in this region is always a problem. For a country, protecting its own borders means protecting its people. Today, the most important border protection is supplied by a good air defense system.

1.1.1 Location and Neighbors of Turkey

Turkey lies at a strategic "crossroads" where two continents, Europe and Asia meet, and also where cultures and civilizations come together.

Geographically, Turkey is roughly rectangular in shape with a width of approximately 550 kilometers and a length of approximately 1500 kilometers. The country is located in the northern hemisphere at a longitude of 36 degrees N to 42 degrees N and a latitude of 26 degrees E to 45 degrees E. The total land borders of Turkey are 2949 kilometers with two European and six Asian countries. Figure 1 shows the general location of Turkey.

> The land border to the northeast with Georgia is 276 kilometers long; the border to the east with Armenia is 328 kilometers long and that with Azerbaijan (Nakhichevan) is 18 kilometers long. The land border to the southeast with Iran is 560 kilometers long; the border to the south with Iraq is 384 kilometers long, and

that with Syria is 911 kilometers long, which took its present form in 1939, when the Republic of Hatay joined Turkey. Turkey's borders on the European continent consist of a 203-kilometers frontier with Greece and a 269-kilometers border with Bulgaria. (Embassy of the Republic of Turkey, 2008).

Because of its location, Turkey has functioned as a bridge between the East and West throughout history. The lands of Turkey are the birthplace of many great civilizations. The mainland of Anatolia (Turkey) has always found favor throughout history because of its geographical location. It has also been prominent as a center of commerce because of its land connections to three continents and the sea surrounding it on three sides (Embassy of the Republic of Turkey, 2008).

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Figure 1. Map of Turkey and Her Neighbors (The World Factbook, 2008)

1.1.2 Turkish Politics & Policy

During the independence war between 1919 and 1922, the first Turkish Grand National Assembly accepted the first constitution of Turkey. After having won a victory in the independence war, the Turkish Republic was declared on 29 October 1923. Turkey, as a newborn country, immediately embarked on a reform in all walks of life. She proceeded to establish good relations and cooperation with the West, and based her political and legal systems on modern, secular models.

> The goal as expressed by Mustafa Kemal Atatürk, the leader of the nationalist movement and first President of the Republic, was "to reach the level of contemporary civilization." And to achieve this aim, a doctrine for foreign relations was formulated that has held true to this day; in the words of Atatürk, "Peace at home, Peace in the world." The primary objectives of Turkish foreign policy are to establish and to develop friendly relations with all countries, in particular with neighboring ones; to promote and to take part in regional and international cooperation; and to resolve disputes through peaceful means and to contribute to regional peace, stability, security and prosperity. (Republic of Turkey Ministry of Foreign Affairs, 2008).

The government of Turkey functions in accordance with the constitution of 1982. According to the first four articles of the 1982 constitution, the Republic of Turkey is "a democratic, secular and social state governed by the rule of law, respecting human rights and loyal to the political philosophy of Kemal Atatürk, who was the Republic's founding father. The Turkish State, with its territory and nation, is an indivisible entity."

"Turkey was officially recognized as a candidate state destined to join the European Union on the basis of the same criteria applied to the other candidates". (Republic of Turkey Ministry of Foreign Affairs, 2008). As a candidate of European

Union, Turkey is fully committed to democracy, respect for human rights and fundamental freedoms, the rule of law and a free market economy.

Turkey is a member of many international and regional organizations such as the United Nations, the Council of Europe and the North Atlantic Treaty Organization (Republic of Turkey Ministry of Foreign Affairs, 2008).

1.1.3 Relations with United States of America

Turkey's membership in NATO in 1952 is a milestone for the relationship between two countries. Turkey formed a strong friendship, partnership and alliance with the United States (U.S.) during the Cold War years. The security aspect of the relationship became more pronounced, particularly following the Truman Doctrine. The strategic relations between Turkey and the U.S. gathered both breadth and depth over the years and continued to move forward with shared values.

Because of the geopolitical location of Turkey, the Turkish-American relationship is also essential for the maintenance of peace, stability and democracy in the world. Hence, Turkey and the U.S. continue to closely coordinate, cooperate and consult with each other about regional conflicts and combating international terrorism. Based on these concepts, Turkey and the U.S. cooperated in the Gulf War, Somalia, Bosnia-Herzegovina, Kosovo, Afghanistan and lately, in the war against international terrorism in the aftermath of September 11, 2001 attacks in the USA.

> Turkey and the U.S. are two major countries in their own rights. The United States is the only remaining superpower in the world, while Turkey is a key country in moderating tensions in her immediate region, which might otherwise have serious repercussions for the international community. Turkey is also an

> investment gateway into the vast and lucrative markets around her. In conclusion, despite occasional difficulties, the bilateral ties of friendship and partnership are firmly and deeply rooted. The two countries may have some differing approaches in defining certain notions, but the values and ideals that they subscribe to are identical. Therefore, the Turkish-American relationship is also essential for the maintenance of peace, stability and prosperity as well as the preservation of freedom and democracy in the wider geographies of mutual concern. (Embassy of the Republic of Turkey, 2008).

As Atatürk noted in the 1920s to a visiting American delegation in Ankara, “the U.S. is an old democracy in a new continent, and Turkey is a new democracy in an old continent.” (Embassy of the Republic of Turkey, 2008).

1.2 Background

1.2.1 Air Defense

Aerial threats existed before the invention of the airplane. “Even before the Wright Brothers flew at Kitty Hawk in 1903, there was a concept of air defense.” (Crabtree, 1994:1). In 1783 in France, man first left the earth by artificial means using a balloon. The use of balloons opened the era of the air defense. France used balloons in war for observing enemy lines. This led to the first active air defense as Austrians attempted to prevent the French from using balloons to gather intelligence by shooting at the balloons (Crabtree, 1994:1-10).

Past experiences and today’s wars show that air defense is vital for a nation’s security. Armed forces without air power and air defense systems are not able to defend their own borders and populace from an external aggressor. “As long as there are nation states which value the freedom of their citizens and their right to decide their own future, there will be others who are equally determined to undermine peace and stability for their own ends" (Elsam, 1989:79).

Air defense is a combination of major components like air defense fighter aircraft, ground early warning radar, airborne early warning radar, surface-to-air missile (SAM) and short range air defense (SHORAD) systems. Most of the NATO countries use command, control and communication (C3) to bring together these disparate components into a cohesive whole. In an air defense context, the problem is the same: to detect the incoming raiders in sufficient time and to launch the defending fighters to destroy them before they reach their targets (Elsam, 1989:67-74).

1.2.1.1 Air Defense Concepts

Today there are some major concepts which are used in air defense doctrine. The important concepts for air defense are explained in the following sections.

Counter-Air, Counter-Space:

"Counter-air and counter-space consist of operations to attain and maintain a desired degree of air superiority by the destruction or neutralization of enemy air and space power." (Crawford and Moon, 2000:269). Some countries may be able to conduct military operations without air and space superiority, but, for most countries, air and space superiority is the sine qua non. Therefore, having air power and dominance of the air and space domain will be essential in future warfare.

Counter-Land:

“Counter-land operations are those conducted to attain and maintain a desired degree of superiority over surface operations by the destruction or neutralization of enemy surface forces.” (Crawford and Moon, 2000:269). To dominate the surface environment and to prevent the opponent from doing the same is the main objective of the aggressors.

Strategic Attack:

“Strategic attack is the operation intended to directly achieve strategic effects by striking at the enemy’s centers of gravity.” (Crawford and Moon, 2000:269). The objectives of strategic attack include producing effects to demoralize the enemy’s leadership, military forces, and populations, thus affecting an adversary’s capability to continue the conflict.

Counter-Information:

Counter-information usually involves destroying, degrading or limiting enemy information capabilities for the establishment of information superiority.

Surveillance and Reconnaissance:

“Surveillance is observing air, space and surface by visual, aural, electronic, photographic or other means. Reconnaissance is observing a specific area for obtaining specific information about the activities and resources of an adversary.” (Crawford and Moon, 2000:271). Today, space-based surveillance assets are essential to national defense (Crawford and Moon, 2000:267-271).

1.2.1.2 Importance of Air defense

To understand the importance of the air defense, consider some recent major conflicts like Gulf War I and the Bosnia War. Of course, air defense was also important in World War I and II, but it reaches its peak point in recent wars.

“During Gulf War I, it is generally acknowledged that the United States conducted most of its offense through the use of air power. When the war was over, there were only 100 hours of ground forces operations.” (Crawford and Moon, 2000:296). Coalition Forces conducted selective and systematic attacks on Iraqi command, control, communication systems, Scud missile sites and air defense targets. “Having lost its command centers and its nerve system, Iraq became strategically disabled. The United States then focused its air power on the Iraqi supply line and executed compounding damage.” (Crawford and Moon, 2000:296-297). In the Bosnia War, NATO did not need to conduct land operations. Instead, NATO used its air power to strike the strategic command and control systems as the Coalition Forces and the US did in Iraq. Considering the Iraq and Yugoslavia view point from the air defense aspect, if they had had a strong air defense system, they might not have lost the war so easily.

The other important aspect of Gulf War I is the missile defense systems. The Iraqi Army launched numerous Scud missiles at Saudi Arabia and Israel; these strikes highlighted the importance of the interception of ballistic missiles. The United States used Patriot missile defense systems to protect Saudi Arabia, Israel and other allies.

These recent wars demonstrate the need to have both air superiority and a high quality air defense system.

1.2.2 Turkish Air Defense System

Because of Turkey's location and its political and military ties, it maintains a strong vibrant military force. Turkey is in the middle of a strategically important region in the Middle East and eastern Mediterranean Sea, and all of its neighbors have potent weapon capabilities. On the south and east borders, Iran, Iraq and Syria have Russian made aircraft and ballistic missiles. It is currently suspected that Iran is developing nuclear power which might lead the region to a more complicated and unbalanced future. In the Aegean Sea, Turkey has some conflicts with Greece over airspace. Even though Greece is also a NATO member, she focuses the use of most of her military power on the Aegean region. Greece locates air bases and missile defense systems on the Aegean islands which are not supposed to house military personnel and weapon systems according to the Lausanne agreement.

Under these circumstances, Turkey keeps her readiness to react with a counter attack. Most of her air defense systems consist of air defense fighter aircraft, ground early warning radars and short range air defense (SHORAD) systems. Turkey also has some anti-aircraft missiles such as the I-HAWK, MIM 14-B Nike Hercules, Rapier missiles, and Stinger missiles. Turkey has a limited number of these missiles; this capacity is not enough to protect her borders and various strategically important points.

Because of the insufficiency of the air defense system, Turkey is planning to enhance her future military capability. Turkey is modernizing her air defense F-16 aircraft and has become the seventh international partner in the Joint Strike Fighter (JSF) Project. Turkey is expected to order 100 F-35A "CTOL/Air Force versions" at a reported cost of $11 billion. She also has ordered four Boeing 737 Airborne Early Warning &

Control (AEW&C) aircraft to enhance her air space control. These aircraft will enter the Turkish Air Force (TUAF) inventory in 2008 (Turkish Air Force, 2008). Of course, these enhancing steps are important, but without a SAM system, the air defense system would be incomplete. As a result, Turkey is investigating the most suitable anti-aircraft and missile defense system for her air defense. Several candidate SAM systems are available; the Turkish government will decide which SAM system to buy in the near future. In this research, three SAM systems are studied. This selection does not show the official policy of the Turkish Air Force or Turkish Government about selecting SAM systems for the Turkish Air Defense System.

1.2.3 Problem Statement

Currently, the TUAF protects its homeland only with air defense fighter aircraft. Turkey wants to locate SAM sites to protect its Aegean and Western Mediterranean borders from intruders. The major considerations are type and number of SAM systems and their location. The TUAF first wants to obtain full protection on the Aegean and Western Mediterranean border of the homeland.

There are some possible candidate points where the SAM sites can be located and certain demand points which must be served. Each candidate point has logistical, geographical and strategic values which are established by the Decision Support Group (DSG) in the TUAF. All the demand points created to determine the air defense umbrella must be covered. Given this scenario, the problem is to cover all the demand points by locating the minimum number of SAM systems.

The problem addressed by this study is summarized by the following questions. Given that there are three available candidate SAM systems to locate, what is the minimum number of SAM sites for each candidate SAM systems to cover all the Aegean and Mediterranean border of Turkey to satisfy air defense requirements? Where should we locate the SAM sites to minimize the number of sites and optimize the coverage? Answering these questions would help the TUAF make good decisions when purchasing a new SAM system. The results would show them how many sites they need to establish to accomplish their mission.

1.2.4 Assumptions

The following simplifying assumptions are made:

1. The views expressed in this research are those of the author and do not reflect the official policy or position of the Turkish Air Force or the Turkish Government. The author is solely responsible for any comments or critiques in this research.
2. Each type of SAM system is modeled separately. Since the basing cost of each type of SAM system is assumed the same, the basing cost is not included in this problem.
3. Demand points and candidates points are generic. They were located in the region to define the air defense area that required coverage.
4. This research examines only SAM battery location selection. It does not deal with personnel, equipment, design, or training issues.
5. The number of potential SAM battery locations and demand points are finite.

6. Demand is satisfied, or covered, if it is within the effective range of a SAM battery location.
7. All demand points in the air defense area must be covered.
8. Each demand point may be covered by more than one SAM battery.
9. No political aspect is considered while selecting candidate SAM systems.
10. Only the ranges of the SAM missiles are considered in modeling the problem.
11. The maximum effective ranges used for modeling were taken from open source publications and may not show the actual effective range of the missiles. The accepted missiles range used in this research are:

    S-300 is 150 km, Patriot PAC-2 is 160 km, and Arrow-2 is 70 km.
12. Altitude limits of the missiles are not considered in modeling.

1.2.5 Research Objectives

The purpose of this research is to determine how many SAM systems Turkey needs to cover her Aegean and Western Mediterranean borders to create an effective air defense umbrella. An additional objective is to determine the best available location points for the SAM sites.

1.3 Overview

Chapter 1 provides relevant information about the Turkish Republic and background for the importance of the air defense and its concepts. Additionally, the research objective was addressed and a scope for this thesis was outlined. Chapter 2 gives a literature review for sources of information helpful in addressing the problem. In addition, specifications and general information about three candidate SAM systems are given in chapter 2. Chapter 3 explains how the problem was approached and modeled. Chapter 4 provides a test plan, test results, and analysis of test results. Chapter 5 provides the conclusion and summarizes the research effort and results and states recommendations for future research.

II. Literature Review

2.1 General

In this chapter, general information and specifications about candidate SAM systems are presented. Jane's Land Based Defense 2006-2007 publication is used for the detailed information. Additionally, evolution of the location type problems, solution techniques and approaches are addressed. The review presents information from theses, journal articles and textbooks.

2.2 Candidate SAM Systems for Turkish Air Defense

This research examines three candidate SAM systems which are MIM-104 Patriot, S-300 (NATO SA-10 'Grumble') and Arrow-2.

2.2.1 MIM-104 Patriot Air Defense System

The patriot is arguably one of the best SAM systems in the world. It is a long-range, all-altitude, all-weather air defense system designed to counter tactical ballistic missiles, cruise missiles and advanced aircraft. The missile's guidance and control electronics and ground equipment for the missile system are manufactured by the Raytheon Company. The missile's airframe and canister are manufactured by Lockheed Martin under contract to Raytheon. The original Patriot Air Defense Missile System dates back to the 1970s. The first launch occurred in February 1970 and after successfully additional tests, the US Army declared Patriot to be fully operational in 1984.

After declaration of fully operational, the US Army has a continuous Patriot Improvement Research and Development program under way to keep Patriot performance effective against a changing threat. Based on this development program, first modifications to the system were applied in 1986. These modifications were dubbed Patriot Anti-tactical Missile Capability-1 (PAC-1) and involved software, radar, and missile trajectory upgrades. These new Patriot systems were deployed with the US Army in Europe during 1988. The PAC-2 modification involves further software changes and a new missile warhead casing with enhanced explosives and fuzing system. The Patriot PAC-2 systems were first deployed in 1991 for the Gulf War I. There were 53 Scud missiles fired from Iraq into Patriot-defended areas. The Patriot PAC-2 intercepted 51 of them.

The Patriot PAC-3 system was fielded with improvements and modifications to the radar and communication systems. It also provided significant improvement in target Classification, Discrimination and Identification ability. In 2004, new improved PAC-3 missiles were successfully tested. The PAC-3 missile intercepted 17 targets in 19 opportunities during its testing phase. These tests' results represent the most successful testing of any air and missile defense interceptor of this complexity. Patriot PAC-3 missile sites were tested once again by Iraq's forces in Operation Iraqi Freedom (OIF) to provide the critical services of shooting down enemy Scud missiles and protecting soldiers and civilians from a missile attack.

In addition to the US, the Patriot is in service in Germany, Greece, Israel, Japan, Jordan, Kuwait, the Netherlands, Saudi Arabia and Taiwan (O'Halloran, 2006:377-381).

This research examines the Patriot PAC-2 since it has a long effective range. The Patriot PAC-2 is also more appropriate to use it against intruders like surface attack aircraft. For the specifications of the Patriot PAC-2, one may refer to Table 1. Figure 2 shows a typical Patriot SAM system.

Table 1. Specifications of the Candidate SAM Systems (O'Halloran, 2006)

Missiles	Patriot PAC-2	S-300 PMU-1	Arrow-2
MAX RANGE	160 KM	150 KM	70 KM
MIN RANGE	3 KM	3 KM	UNKNOWN
MAX SPEED	5 MACH	6 MACH	9 MACH
MAX ALTITUDE	24 KM	30 KM	50 KM
MIN ALTITUDE	60 M	25 M	8 KM

Figure 2. Patriot SAM System (Redstone, 2008)

2.2.2 S-300 (NATO SA-10 ‘Grumble’) Air Defense System

In 1967, Russian Almaz Central Design Bureau began the development of the S-300P air defense system which is regarded as one of the most effective all-altitude regional air defense systems in the world. There are many different versions of S-300 missiles. The only export models of the S-300 SAM systems are S-300 PMU, S-300 PMU1, and S-300 PMU2.

The S-10 Grumble missile (which is the NATO designation of the S-300P) has three basic versions: b, c and d. Each version has a different missile; ranges of the missiles are between 90 to 200 km. “The maximum number of targets engaged by the enhanced battery remains at six with up to a max of 12 missiles at any one time being guided simultaneously. The battery deployment time is five minutes, and firing rate is one missile every three seconds from a launcher.” (O’Halloran 2006:171). A standard S-300 battery has a maximum total of 32 available missiles. Almost all the S-300 family SAM systems use Clam Shell low-altitude detection radar, Flap Lid acquisition and tracking radar and Big Bird long range detection radar.

In addition to Russia, the S-300 SAM system is in service in Belarus, China, Croatia, Greece, Hungary, India, Kazakhstan, Syria, Ukraine, Vietnam, Bulgaria and Iran (O’Halloran 2006:168-176). For the specifications of the S-300 PMU-1, one may refer to Table 1. Figure 3 shows a typical S-300 SAM system.

Figure 3. S-300 SAM System (Sinodefence, 2008)

2.2.3 Arrow- 2 Air Defense System

The Arrow weapon system has been developed by the Israel Aircraft Industries and is in operation with the Israeli Defense Forces. In 1988, the US Department of Defense Strategic Defense Initiative placed a contract on the Electronics Division of Israel Aircraft Industries to build and test the Chetz-1 (Hebrew name for Arrow-1) Anti Tactical Ballistic Missile (ATBM) system. After successful tests, the system entered full-scale development and production.

The second stage of development known as the Arrow-2 began in 1993. The Arrow-2 is a two-stage weapon with a solid propellant rocket motor booster and sustainer. The weight of the Arrow-1 was approximately 2000 kilogram. The Arrow-2 missile is 700 kilogram less than that of the Arrow-1. The Arrow-2 missile has a 70 kilometer maximum effective range with an altitude limit up to 50 kilometers. In the development phase of the Arrow-2, the US Department of Defense (DoD) paid 72% and the Israeli Missile Defense Organization paid 28% of the total cost (O'Halloran, 2006:321). Further Arrow Weapon System developments have been planned and will be funded by Israel and the U.S.

The Arrow Weapon System uses EL/M2080 L-band early warning and fire control radar. The code name of this radar is "Green Pine." The Green Pine radar can detect targets at ranges up to about 500 kilometers and is able to track targets up to speeds over 3,000 m/s and can guide an Arrow missile to within 4 meters of the target.

Today, only Israel has Arrow-2 SAM systems. There are some countries interested in this weapon system (including Turkey). India has placed an order for the Green Pine radar for use with India's air defense system against ballistic missiles. It is estimated that India will use the Green Pine radar with its Russian Federation supplied anti-ballistic missile capable systems. India can cover all of Pakistan's military command centers and bases with the Green Pine's detection range up to 500 kilometers (O'Halloran, 2006:321-323). For the specifications of the Arrow-2, one may refer to Table 1. Figure 4 shows a typical Arrow-2 SAM system.

Figure 4. Arrow-2 SAM System (Israeli-Weapons, 2008)

2.3 Evolution of Location Problems

“Location, location, and location. These words have been uttered since the infancy of man’s drive to optimally locate supply centers responding to required demand.” (Eberlan 2004:26). For example, there are some vital public service needs such as fire departments and ambulatory services in cities which must be located close enough to demand centers to provide timely service; failure to do so could result in damage to property or even death. In addition to these vital public services, there are some commercial and economical services which require appropriate positioning. For example, the oil companies want to locate gas stations in the best appropriate places to increase their profits.

Location analysis is not only useful to public and private commercial enterprises, but it is also relevant for use in the military. There may even be more location problem applications in the military than in the civilian sector. From a public service and resource dispersion stand point, it is essential that military bases be optimally located. Considering this research's problem, it is also important that the Turkish Air Force locate SAM sites at the most advantageous locations to minimize the number of resources required while maximizing performance of the overall SAM sites network to achieve the most efficient use of homeland defense resources. Since there are many different types of location problems, "for more than 120 years, mathematicians, analysts, operation researchers, and management science scholars have tried to devise algorithms and techniques to identify optimal locations given a wide variety of problem parameters, resource constraints, and model objectives." (Eberlan 2004:26).

This research's problem is also one of the common location problems. Due to the limited budget constraints, the problem in this research is also a vital location problem for the Turkish government. The literature offers a variety of solution techniques for these different kinds of location problems.

The first formally introduced location problem involved locating a single warehouse and was first presented in 1909 by Alfied Weber. The objective was to minimize the total travel distance between the warehouse and a set of spatially distributed customers. In the 1950s and 1960s, a number of authors considered the problem of facility layout and design. These efforts consisted primarily of a number of separate applications that were not tied together by a unified theory. Hakimi (1965) considered the general problem of locating one or more facilities on a network to minimize the travel distances in the network; this research sparked the interest in location problems. Previously, considerable research has been carried out in the field of location theory (Brandeau, M., Chiu, S. 1989).

Basing or coverage type problems are also treated as location problems. “The goal in location problems is to locate service facilities to minimize some cost function or to maximize the amount of demand for service that can be satisfied.” (Basdemir, 2000:16). The concept of coverage is a well-known alternative while specific point-to-point distances are used. The norm of partitioning inter-point distances based on some standard distance has been employed extensively in location literature for over thirty years. Location models fit into two broad categories based on whether coverage is required or optimized (Koksalan, M., Sural, H., Kirca, O. 1995).

2.4 Location Problem Types

The p-median problem type is the first location problem type presented in this section. The p-median problem theory dates back to 1960s. Locating multiple facilities presents the need to allocate demand to the respective locations. The decision of where to locate the facilities and where to allocate the demand simultaneously was the beginning of location-allocation modeling (Ghosh and Rushton, 1987). Cooper (1963) developed the classic facility location problem on a plane which minimizes costs for a multiple location network. Cooper used a heuristic approach to minimize shipping costs for a multiple facility location problem. This approach was named the p-median problem. The p-median problems are used to find the location of p supply centers while minimizing the demand weighted aggregate distance. Hakimi (1965) and ReVelle and Swain (1970) extended the p-median problems to solve a network with discrete locations. “The development of the network formulation of the p-median problem greatly extended the range of situations in which location-allocation models could be applied” (Ghosh and Rushton, 1987). The evolution of the p-median problem did not only allow the application of location-allocation techniques to a greater number of circumstances, but it also drove the development of more efficient algorithms for solving location problems (Eberlan, 2004:30).

The p-center or minimax problem and the set covering location problem are the two major model innovations, responding to the need to formulate models addressing maximum distance objectives. The p-center or minimax problem was first developed by Hakimi (1965). There are several variations to solve the p-center problems. In the vertex p-center problem, the location of facility sites is restricted to the nodes of the network. On the other hand, the absolute p-center problem allows facilities to be located along the arcs (Current, 2002). The vertex p-center and the absolute p-center problems can be solved as a capacitated or uncapacitated location problem.

Since both problems locate facilities within a critical distance of demand nodes, the location covering problems can adequately handle resource constraints. The set covering location problem (SCLP) is developed by Toregas et al. (1971). SCLP determines the minimum number and location of facilities within a specified distance or time constraint from the demand sites. The problem solution gives the minimum number and locations of facilities to cover the demand points. The SCLP allocates each demand node to at least one facility. It is not essential that the demands always be allocated to the closest facility. Each solution location point can cover a specific demand point if it is within the range. There are many combinations for covering demand in the SCLP. There are also different methods for solving the problem (Eberlan, 2004:33).

The maximal covering location problem (MCLP) exogenously restricts the number of facilities located by a pre-determined fixed number. It maximizes the amount of demand that can be covered within the desired or critical distance. Mandatory closeness, budget or some other pre-determined constraints can also be included in MCLP (Current, 1992). The MCLP was developed by Church and ReVelle (1974).

The maximal expected covering location problem (MEXCLP) is another version of MCLP. “The MEXCLP has been used extensively in analyzing locations for public service facilities. The MEXCLP accounts for the possibility that a covered demand point is not serviced since all facilities capable of covering the demand are engaged serving other demands.” (Daskin, 1983).

For this research’s problem, the best model is the SCLP since all the demand points need to be covered. On the other hand, MCLP modeling is more appropriate to maximize the number of covered demand points when there is an insufficient number of SAM missile batteries to completely cover an area . Additionally, since it is impossible to cover all the demand points by locating Arrow-2 SAM systems, MCLP modeling is used to maximize coverage of located Arrow-2 sites. Chapter 3 and Chapter 4 include the models used in this research.

Table 2 shows the relationships between SCLP, MCLP and P-center type of location problems.

Table 2: Relationships between SCLP, MCLP and P-CENTER (Daskin, 1995)

Problem	Number of Facilities	% Demand Coverage	Coverage Distance
SCLP	Objective Function Minimum	100%	(EXOGENOUS)
MCLP	(EXOGENOUS)	Objective Function Maximum	(EXOGENOUS)
P-CENTER	(EXOGENOUS)	100%	Objective Function Minimum

2.5 Solution Techniques and Tools

Both optimization and heuristic techniques can be applied when the solution procedures are considered. For example, three main heuristic algorithms have been developed for the p-median problems; these are the Greedy Algorithm, the Drop Algorithm, and the Interchange Algorithm. The Greedy Algorithm was developed by Kuehn and Hamburger (1963) to locate facilities incrementally by least cost until p facilities are located. The Drop Algorithm was developed by Feldman, Lehrer and Ray (1966). This algorithm starts with facilities located at all possible sites and iteratively drops the facility at each stage with the least impact on the objective function (Ghosh and Rushton, 1987). The Interchange Algorithm was developed by Teitz and Bart (1968). This algorithm was built around the selection of p sites and an original minimum objective function value computed from the sites. Then, sites not in the set are iteratively substituted for each site in the set and the objective function value is recalculated. The substitution continues until the value of the objective function is minimized.

The evolution of the p-median problem gave rise to the development of new heuristics. It also resulted in the greater application of mathematical programming methods such as linear programming (LP). ReVelle and Swain (1970) recommend using a branch-and-bound algorithm for finding the optimal integer solution when the decision variables in location problems take on fractional quantities. Lagrangean relaxation has been shown to yield success in such applications as well. (Daskin,1995).

LP optimization, matrix row reduction, a combination of both, or cutting planes can be used to solve the SCLP (ReVelle and Williams, 2002). The LP relaxation of the traditional set covering problem often results in an all integer solution (Current, 2002). If the LP relaxation of the SCLP results in a fractional solution, Current, Daskin, and Schilling (2002) recommend using a Branch and Bound algorithm to obtain an all integer solution. For MCLP and MEXCLP, the same solution techniques that were used in SCLP may be applied.

The Simplex Algorithm with branch-and-bound is the primary algorithm used today. There are many commercially available linear solvers, such as CPLEX, LINDO, LINGO and Excel Solver. This research's problem was coded as an integer model in LINDO format and solved with LINGO 10 software.

2.6 Similar Problems

Since location is a common problem in civilian and military applications, there are many problems similar to this research's problem. Three of these problems were examined to take advantage of their approaches.

The first problem is the application of SCLP and MCLP to locating emergency warning sirens in a Midwestern city. Current and O'Kelly (1992) had two siren types available, each having different costs and covering radii. Using a modified version of the set covering location model, they analyzed the cost implications of several policy options being considered by the city’s planners. Results of the study indicated that location covering models could be powerful and efficient tools in the design of such systems, and their use could lead to significant cost savings. In addition, such models provide decision makers the flexibility to examine the inherent costs associated with various policy options (Current and O’Kelly, 1992).

The second problem is the application of MCLP to locating search and rescue (SAR) sites. Basdemir (2000) modeled the problem of finding the optimum SAR locations as a MCLP. He added additional constraints and set standards on various issues in the regions. Main emphasis was given to finding the minimum number of SAR locations that achieved maximum coverage in the operational area. Bonus values that indicate the importance of covering demand points were also included for analysis purposes. He examined several scenarios and presented results and analysis.

The third problem is the application of SCLP and P-median modeling to locating alert sites. Eberlan (2004) developed a location optimization model to locate alert sites post-11 September 2001 to cover areas of interest in the Continental United States (CONUS). The model finds the minimum number of alert sites, minimum aggregate network distance, and minimized maximum distance given a range of aircraft launch times and speeds. This research provides air defense planners a tool to use in formulating an optimal alert network (Eberlan, 2004).

As seen, all three efforts provide decision makers the flexibility to examine the various policy and location options.

2.7 Summary

In this chapter, the general information about candidate SAM systems were reviewed. Additionally, the evolution, methods and approaches for the location type problems were presented. This chapter also reviewed problems which are similar to this research's problem. In Chapter 3, the methodology of this research is described and then presented in a mathematical formulation.

III. Methodology

3.1 Problem Description and Objectives

Turkey wants to locate SAM sites to protect its Aegean and Western Mediterranean borders from airborne intruders. The major considerations are type and number of SAM systems and their location. The TUAF first wants to obtain full protection on the Aegean and Western Mediterranean border of homeland.

There are some possible candidate points where the SAM sites can be located and certain demand points which must be covered or protected. Each candidate point has logistical, geographical and strategic values which are established by the Decision Support Group (DSG). All the demand points created to determine the air defense umbrella must be covered.

Given this scenario, the problem is to cover all the demand points by locating the minimum number of SAM systems. This research has four main objectives:

1. Minimize number of SAM sites.
2. Cover all demand points with at least one SAM site.
3. Maximize the objective function with the bonus points.
4. Determine the best available location points for the SAM sites.

Minimizing the required number of SAM sites and covering all the demand points are the first or overarching requirements. The other objectives are equally important in the overall model.

3.2 Data

Data was provided to this study by personnel at the DSG in the TUAF headquarters. DSG is a special unit in the TUAF which leads some of the important projects. The overall research objective was also suggested by DSG.

3.3 Critical Model Parameters

In this research, there are two main model parameters which are "Candidate Points" and "Demand Points." The possible location points for the SAM sites are named "Candidate Points" and the points which must be covered by the SAM sites are named "Demand Points."

3.3.1 Candidate Point Selection

Since each candidate SAM missile has a different and limited effective range and since there is a relatively large area to cover, most of the candidate points were selected near the coast of the Aegean and Mediterranean Seas. Hundreds of candidate points could be generated, but DSG personnel established some criteria that each candidate point must meet.

In order to be considered a suitable candidate for a SAM site, the following criteria must be met:

1. A candidate point must be close enough to a military facility to receive logistical support. Since air defense is a responsibility of TUAF, these facilities are preferred to be Air Force facilities. The more preferable facilities are the main jet, airlift and tanker bases.

2. A candidate point must be in a geographically acceptable location that maximizes its effective range, for example, a high point or plain surface.

3. A candidate point must protect some strategically important places such as cities or military bases.

There are many sites that can be included as candidate points since they meet these criteria. DSG personnel used a systematic approach to select the best candidate points. They gave mathematical bonus values to all the candidate points based on their geographical, strategic and logistical importance.

Geographical bonus value generation is based on the candidate point's geographic advantages. If the candidate point is in a mountainous region and it is at a low level place, it has a lower geographical bonus value. Otherwise, it has a higher geographical bonus value.

Strategic bonus value generation is based on the candidate point's strategic advantages. If the candidate point is near high value ground assets such as big cities, refineries and important military bases, it has a higher strategic bonus value. Otherwise, it has a lower strategic bonus value.

Logistical bonus value generation is based on the candidate point's closeness to a military bases or facility where it can easily get logistic supplies. If the candidate point is close to a military facility, it has a higher logistic bonus value. Otherwise, it has a lower logistic bonus value.

After establishing these bonus values, DSG decided a minimum weighted value that each candidate point must have. They eliminated low value candidate points that could not meet the predetermined desired value. They selected 100 candidate points for this research. The logistics, geography and strategic values for each candidate point are presented in Appendix B.

One of the most important values for the candidate points is coverage data since it defines how large an area that candidate SAM site can cover. Each of the candidate points has different effective radii for each candidate SAM missile. The coverage distance is examined for 85 NM (Patriot PAC-2), 81 NM (S-300 PMU-1) and 40 NM (Arrow-2) radii. These values are approximate values that are derived from Jane's Land-Based Air Defense 2006-2007. Coverage distance data is shown in Table 3.

Table 3. Coverage Distance Data

Missiles	Patriot PAC-2	S-300 PMU2	Arrow-2
DISTANCE KM	160 KM	150 KM	70 KM
DISTANCE NM	85 NM	81 NM	40 NM

3.3.2 Demand Point Selection

Demand points are as important as the candidate points, since demand points define the air defense area that requires coverage. Only the coordinates of the desired air defense area are supplied by the DSG personnel. In order to cover the desired air defense area, 200 demand points are generated. FalconView, version 3.2.0, a special mapping system, is used to see the air defense area coordinates, demand, and candidate points. The research model is easily constructed using this program, since all the points, regions, and coverage distances can be easily viewed.

No special methodology was used to select the demand points. Demand points were evenly spaced over the area. The only consideration is to define and cover all the air defense area indicated by the coordinates effectively. All 200 demand points were located on the area with equal importance. Each demand point also has an importance bonus value that indicates the priority of that location point. The DSG personnel assigned bonus values to the demand points according to their closeness to the possible intruder locations or closeness to strategic places in the homeland. The demand points closer to the intruder locations or closer to the strategic places in the homeland have higher bonus values. These demand values were used as objective function coefficients in the MCLP model to insure the coverage of the most preferred demand points. The bonus values for each demand point are presented in Appendix B.

The digital maps of the air defense region are supplied by the TUAF. The general view of candidate and demand points is shown in Figure 5.

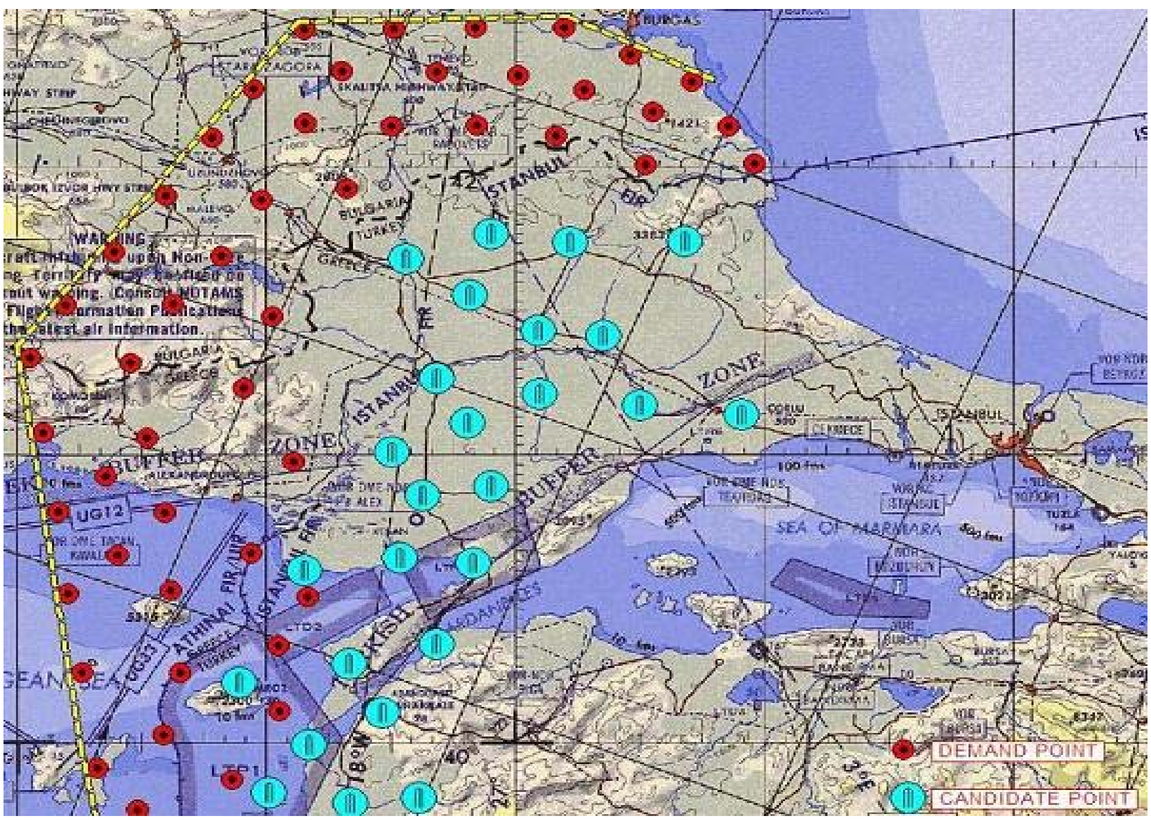

Figure 5. General View of Candidate and Demand Points

3.4 Selection of Solution Technique

There are two primary techniques available for solving location problems; optimization and heuristics. In this research, mathematical optimization was selected since an optimal solution is needed; a heuristic does not guarantee optimality. Additionally, this size of this problem does not cause algorithmic completion times to be significantly large.

After selection of mathematical programming as the desired optimization technique, it was decided to use integer programming as the mathematical programming method. Integer programming is the best fit since the formulations of the location modeling methods are already in integer programming format and there cannot be fractional demand (Current, 2002). After selecting optimization as the preferred solution technique, a specific location modeling method was selected.

3.5 Selection of Location Modeling Method

One of the most important aspects of any location research effort is location modeling method selection. Some of the location modeling techniques are discussed in Chapter 2. In this research, the best fit model is SCLP, since SCLP modeling satisfies all the research objectives—namely minimizing the number of the SAM sites and covering all the demand points. The second location modeling method used in this research is MCLP. The MCLP modeling was used to maximize the number of covered demand points when there were an insufficient number of SAM missile batteries to completely cover the area. It was also used in sensitivity analysis. After calculating the minimum number of SAM sites needed to achieve complete coverage, this minimum number was decreased by one and used in the MCLP model. Additionally, since it is impossible to cover all the demand points by locating Arrow-2 missiles, MCLP modeling was used to maximize coverage of located Arrow-2 sites.

3.5.1 The Mathematical Formulation of SCLP

The original SCLP was developed by Toregas et al. (1971); however, the formulation used in this research comes from ReVelle and Williams (2002). A typical SCLP is formulated mathematically as:

$$\text{MINIMIZE} \quad \sum_{j \in J} x_j \qquad (1)$$

$$\text{SUBJECT TO:} \quad \sum_{j \in N_i} x_j \geq 1 \qquad \forall i \in I \qquad (2)$$

$$x_j \in \{0,1\} \qquad \forall j \in J \qquad (3)$$

where:

i, I = the index and set of demand points;

j,J = the index and set of candidate points;

d_{ij} = the distance from each demand point i to each candidate facility j;

S = maximum covering distance

$N_i = \{\{j \in J \mid d_{ij} \leq S\} \ \forall \ i \in I\}$ is the set of candidate facilities j within the coverage distance S of demand point i;

$x_j \in \{0,1\}$. It is 1 if a facility is located at site j, and 0 otherwise.

The objective function (1) minimizes the number of selected facilities needed to cover each demand point. Constraint (2) requires that each demand point must be covered by at least one candidate facility within S distance. Constraint (3) is the integrality constraint on the decision variables.

3.5.2 The Research Model Formulation of SCLP

In this research, two SCLP model formulations were used. The two models are related to each other. First, the research used the above pure SCLP model formulation to obtain the number of SAM sites needed to cover all the demand points. Second, the objective function was changed to “Maximize” and used candidate point bonus values as objective function coefficients. Finally, an additional constraint was added to prevent the model from exceeding a specified number of SAM sites.

The modified model formulation is:

MAXIMIZE $$\sum_{j \in J} c_j x_j \qquad (4)$$

SUBJECT TO: $$\sum_{j \in N_i} x_j \geq 1 \qquad \forall\, i \in I \qquad (5)$$

$$\sum_{j \in J} x_j \leq P \qquad (6)$$

$$x_j \in \{0,1\} \qquad \forall\, j \in J \qquad (7)$$

where:

i, I = the index and set of demand points;

j,J = the index and set of candidate points;

d_{ij} = the distance from each demand point i to each candidate SAM site j;

S = maximum covering distance

c_j = weighted bonus point value for $\forall\, j \in J$.

$N_i = \{\{j \in J \mid d_{ij} \leq S\}\ \forall\ i \in I\}$ is the set of candidate SAM sites j within the coverage distance S of demand point i;

$x_j \in \{0,1\}$. It is 1 if a SAM site is located at site j, and 0 otherwise.

P = the number of SAM sites that can be occupied.

The objective function (4) maximizes the bonus value of the occupied candidate sites. Constraint (5) requires that each demand point must be covered by at least one candidate facility within S distance. If there are not enough SAM sites, this constraint can cause infeasibility. Constraint (6) specifies the number of SAM sites that can be occupied. Constraint (7) is the integrality constraint for the decision variables. The complete mathematical form of this model is in Appendix C.

3.5.3 The Mathematical Formulation of MCLP

The MCLP exogenously restricts the number of facilities located by a pre-determined fixed number. It maximizes the amount of demand that can be covered within the desired or critical distance. The MCLP was developed by Church and ReVelle (1974). A typical MCLP is formulated mathematically as:

MAXIMIZE $$\sum_{i \in I} a_i y_i \qquad (8)$$

SUBJECT TO: $$\sum_{j \in N_i} x_j \geq y_i \qquad \forall\ i \in I \qquad (9)$$

$$\sum_{j \in J} x_j \leq P \qquad (10)$$

$$x_j \in \{0,1\} \qquad \forall\, j \in J \tag{11}$$

$$y_i \in \{0,1\} \qquad \forall\, i \in I \tag{12}$$

where:

i, I = the index and set of demand points;

j,J = the index and set of candidate points;

d_{ij} = the distance from each demand point i to each candidate facility j;

S = maximum covering distance

P = the number of facility location sites that can be occupied.

a_i = bonus point value of demand points $\forall\, i \in I$.

$N_i = \{\{j \in J \mid d_{ij} \leq S\} \;\; \forall\, i \in I\}$ is the set of candidate facilities j within the coverage distance S of demand point i;

$x_j \in \{0,1\}$. It is 1 if a facility is located at site j, and 0 otherwise.

$y_i \in \{0,1\}$. It is 1 if demand point at i is covered, and 0 otherwise.

The objective function (8) maximizes the sum of the bonus point values of the demand points that are covered. Constraint (9) determines the demand points that the candidate SAM sites cover. Constraint (10) requires that number of SAM sites occupied must be less than or equal to the specified number of SAMs. In other words, it indicates how many candidate points may be selected as SAM sites. Constraints (11) and (12) are the integrality constraints for the decision variables.

3.5.4 The Research Model Formulation of MCLP

After obtaining the minimum number of SAM sites needed to cover all the demand points, the MCLP model is used to explore the impact of fewer SAM batteries. In this research, a modified MCLP model is used to investigate the impact of fewer SAM sites while achieving a specified level of bonus points for the SAM sites occupied.

The modified model formulation is:

MAXIMIZE $$\sum_{i \in I} a_i y_i \tag{13}$$

SUBJECT TO: $$\sum_{j \in N_i} x_j \geq y_i \qquad \forall\ i \in I \tag{14}$$

$$\sum_{j \in J} x_j \leq P \tag{15}$$

$$\sum_{j \in J} c_j x_j \geq W \tag{16}$$

$$x_j \in \{0,1\} \qquad \forall\ j \in J \tag{17}$$

$$y_i \in \{0,1\} \qquad \forall\ i \in I \tag{18}$$

where:

i, I = the index and set of demand points;

j,J = the index and set of candidate points;

d_{ij} = the distance from each demand point i to each candidate facility j;

S = maximum covering distance

P = the number of facility location sites that can be occupied.

W = the pre-determined weighted candidate points bonus value to reach.

a_i = bonus point value of demand points $\forall\ i \in I$.

c_j = weighted bonus point value for candidate points $\forall\ j \in J$.

$N_i = \{\{j \in J \mid d_{ij} \leq S\}\ \forall\ i \in I\}$ is the set of candidate facilities j within the coverage distance S of demand point i;

$x_j \in \{0,1\}$. It is 1 if a facility is located at site j, and 0 otherwise.

$y_i \in \{0,1\}$. It is 1 if demand point at i is covered, and 0 otherwise.

The objective function (13) maximizes the bonus values of the covered demand points. Constraint (14) determines the demand points covered by the candidate SAM sites. Constraint (15) requires that the number of SAM sites must be less than or equal to the specified number of SAM sites. In other words, it indicates how many candidate points may be selected as SAM sites. Constraint (16) requires that the selected candidate SAM sites provide a pre-determined fixed weighted bonus value. Constraints (17) and (18) are the integrality constraints for the decision variables.

3.6 Calculation of Bonus Values for Candidate Points

For the MCLP model, the given demand point bonus values are used directly as objective function coefficients. Although each candidate SAM location has three different bonus values, a single weighted bonus value is used in the SCLP and MCLP models as coefficients.

Each candidate SAM site has three different bonus point values that are strategic, geographical and logistical bonus points. The single weighted bonus point is calculated by summation of 15% of the geographical bonus point, 25% of the strategic bonus point and 60% of the logistical bonus point. These percentage values were supplied by the DSG personnel. An example of generated single bonus points is shown in Table 4.

Table 4. Bonus Value Calculation for Candidate Points

Bonus Points				
Candidate Points	Logistics (60%)	Strategic (25%)	Geography (15%)	Weighted Points
X1	100	100	100	100
X2	90	95	100	92.75
X3	90	80	75	85.25
X4	70	70	95	73.75
X5	98	80	95	93.05
X6	80	80	95	82.25
X7	100	90	100	97.5
X8	80	97	100	87.25
X9	95	97	100	96.25
X10	90	95	100	92.75

For illustrative purposes, consider candidate point X2:

$$\text{X2 Weighted Bonus Point} = \{(90 \times 0.60) + (95 \times 0.25) + (100 \times 0.15)\} = 92.75$$

3.7 Model Restrictions and Important Assumptions

In SCLP and MCLP models, the most important restriction is the candidate SAM effective ranges. In the model, only the effective ranges are considered in the formulation. The other characteristics, for instance minimum effective range, maximum and minimum altitude and missile velocity, were not included. Three different candidate SAM systems are considered; thus, there are three different effective ranges and three different mathematical formulations. The Patriot PAC-2 and S-300 PMU-1 SAM systems have large enough effective ranges to cover all the demand points in the given geographical regions. The Arrow-2 SAM system does not have enough range to cover all the demand points. Even if all the Arrow-2 missile sites are occupied, it is impossible to cover all the demand points because some of the demand points are more than 40 nm from all potential SAM sites. Since the Arrow-2 missile does not have enough effective range, the SCLP will not produce feasible solutions. As a result, only the MCLP model formulation will be used for the Arrow-2.

The most important assumptions that affect the model are:

1. Each type of SAM system is modeled separately. Since the cost of locating each type of SAM system is assumed the same, the cost is not included in this problem.
2. The study examines only the number of the SAM sites and their location selection. It does not deal with personnel, equipment, design, or training issues.
3. The number of potential SAM battery locations is finite.
4. Demand is satisfied, or covered, if it is within the effective range of a SAM battery location.
5. Each demand point may be covered by more than one SAM battery.
6. Only the ranges of the SAM missiles are considered in covering the demand points

3.8 Software Used for the Model

LINGO 10 is used to solve the SCLP and MCLP program models. This licensed software can handle unlimited integer variables and unlimited constraints. LINGO gives a separate report output that shows the details with respect to the solution. LINGO has functions to generate different formats of the formulation (i.e., algebraic, MPS, LINDO, spreadsheet formats). In this research, a LINDO format is used since it has natural representations that are similar to mathematical notation; it is also much faster and easier to read the code. The complete LINDO formats of all the SCLP and MCLP models are in Appendix C.

3.9 Flow of the Solution, Analysis and Conclusion Process

The step by step solution and analysis process is shown below:

Step 1:

The first step constructs a pure SCLP model for Patriot PAC-2 and S-300 PMU-1 candidate SAM systems. There are two different formulations that include 85 NM coverage for Patriot PAC-2 and 81 NM coverage for S-300 PMU-1.

Step 2:

In the second step, each pure SCLP model is solved to get the minimum number of SAM sites that cover all the demand points in the desired air defense area. After this step, the SAM site limiting numbers are known and can be used in the modified SCLP model.

Step 3:

The third step constructs a modified SCLP model with bonus points in the objective function. The additional SAM site limiting constraint prevents the model from exceeding the limit on the number of SAM sites.

Step 4:

The two modified models are solved to get two different maximized objective function values for each candidate SAM systems. The solution may have new location points that are different from the first model since the objective function is now maximizing the bonus point values.

Step 5:

After application of all candidates SAM sites to the models and consolidating the results, the solutions are used as a starting point for the MCLP models. In each run, the required minimum number of SAM sites for complete coverage is decreased and maximal coverage for that decreased value is found. This process provides a means to compare the percentage of demand points that can be covered for each number of SAM sites below the required number. Even if Arrow-2 SAM sites are located on the borders, it is impossible to cover all the demand points since Arrow-2 missile has an effective range of 40 NM. Only the MCLP modeling is used for the Arrow-2 SAM system.

3.10 Summary

In this chapter, a general presentation of the methodology was given. The objectives of this research effort were presented along with the assumptions used to meet those objectives. The critical model parameters, such as candidate and demand points, were introduced, and their selection techniques were explained. The solution technique and mathematical formulations of this research's models were presented with detailed descriptions. Finally, flow of the solution, analysis and conclusion process was described step by step. Chapter 4 presents the results and sensitivity analysis.

IV. Results and Analysis

4.1 Introduction

This chapter summarizes and compares the results of the SCLP and MCLP models formulated in Chapter 3. Each model is run for three basic candidate SAM missiles: Patriot PAC-2, S-300 PMU-1 and Arrow-2 (if applicable). The only consideration is effective ranges of these missiles. In the SCLP solution, the minimum number of SAM sites needed to cover the desired air defense area is found. The modified SCLP solution is determined in order to locate SAM sites at the best available places with maximum candidate bonus values. The solution number obtained from the SCLP is used as a starting point for the MCLP model. The MCLP model is also run using fewer SAM sites than are required for the SCLP; the number of SAM sites available iteratively decreases by 1 and the MCLP model solution is found. The purpose of this analysis is exploring the impact of fewer SAM sites. In each run, the coverage of the SAM missiles is maximized by finding the best locations for them. Additionally, the coverage percentage with fewer SAM sites is determined. The bonus values of the demand points are used in the objective function of the MCLP models to encourage covering more important demand points. After running the models separately for each candidate SAM system, the results are compared. A sensitivity analysis is conducted for each SAM system separately to show the impact of changing certain model constraints.

Consecutively, the three separate solutions are presented in this chapter. The results for the three candidate SAM systems can be easily viewed, and using these results, the best SAM systems for the Turkish Air Defense System can be selected.

4.2 Solutions for Each Candidate SAM Systems

There are three candidate SAM systems chosen for this research. Both SCLP and MCLP models are used for each candidate SAM systems. All results are shown by SAM type.

4.2.1 Solutions for Patriot PAC-2

Both SCLP and MCLP models are constructed based on the maximum range of the Patriot Pac-2 missile. The value of the maximum range is 85 NM as stated in Jane's Land-Based Air Defense 2006-2007 publication.

For the SCLP model, the objective is to find the minimum required number of SAM sites needed to cover all the demand points. In order to find this value, the pure SCLP model is run. As indicated by the solution to this model, the minimum required number of Patriot PAC-2 sites to cover all the demand points must be at least six. When this number is used as the right hand side (RHS) of the SAM site limiting constraint (6) in the modified SCLP model, the selected candidate locations are X6, X12, X38, X45, X73 and X98 with the objective function value of 588; this value is the maximized sum of candidate bonus values. As expected, the modified SCLP model provides a larger sum of candidate bonus values. The number of double covered demand points obtained from the modified SCLP is greater than that found by the pure SCLP model. This is an advantage since six sites are still located but more regions are protected by two different SAM sites. The solution report for the pure and modified SCLP models is shown in Table 5.

Table 5. SCLP Solution Report for Patriot PAC-2

Model	Number of Sam Sites	Selected Sam Sites	Objective Function Value	Candidate Bonus Value / Average	Double Covered Points	Total Covered Points / Coverage Rates
PURE SCLP	6	X13, X26, X35, X44, X51, X65	*MIN* 6	514 / 85.63	23	200 / 100%
MODIFIED SCLP	6	X6, X12, X38, X45, X73, X98	*MAX* 588	588 / 98	32	200 / 100%

The objective of the MCLP model is to maximize the coverage of the located SAM sites and the covered demand point bonus values. Table 6 shows the solution report for the MCLP model.

Table 6. MCLP Solution Report for Patriot PAC-2

Number of Sam Sites	Selected Sam Sites	Objective Function Value	Candidate Bonus Value / Average	Double Covered Points	Total Covered Points / Coverage Rates
6	X6, X13, X27, X35, X44, X51	*19035*	532.95 / 88.83	27	200 / 100%
5	X6, X15, X29, X37, X48	*18142*	445.55 / 89.11	11	190 / 95%
4	X26, X38, X48,X77	*16487*	348.5 / 87.13	0	173/ 86.5%
3	X26, X38, X77	*13170*	268.8 / 89.60	0	136/ 68%
2	X26, X77	*9080*	171.8 / 85.90	0	92/ 46%
1	X26	*4600*	88.60 / 88.60	0	46/ 23%

The MCLP model is run six times for Patriot PAC-2. In the first run, a coverage of 100% is obtained with six selected SAM sites and this solution is an alternative solution for the complete coverage of the air defense area. As expected, total bonus value of selected SAM sites in the MCLP solution is smaller than the solution in the modified SCLP solution. This is caused by the difference between the objective functions. The number of covered demand points and their bonus values are maximized in the MCLP.

On the other hand in modified SCLP model, total bonus value of selected SAM sites are maximized. Comparison of these two solutions is shown in Figure 6.

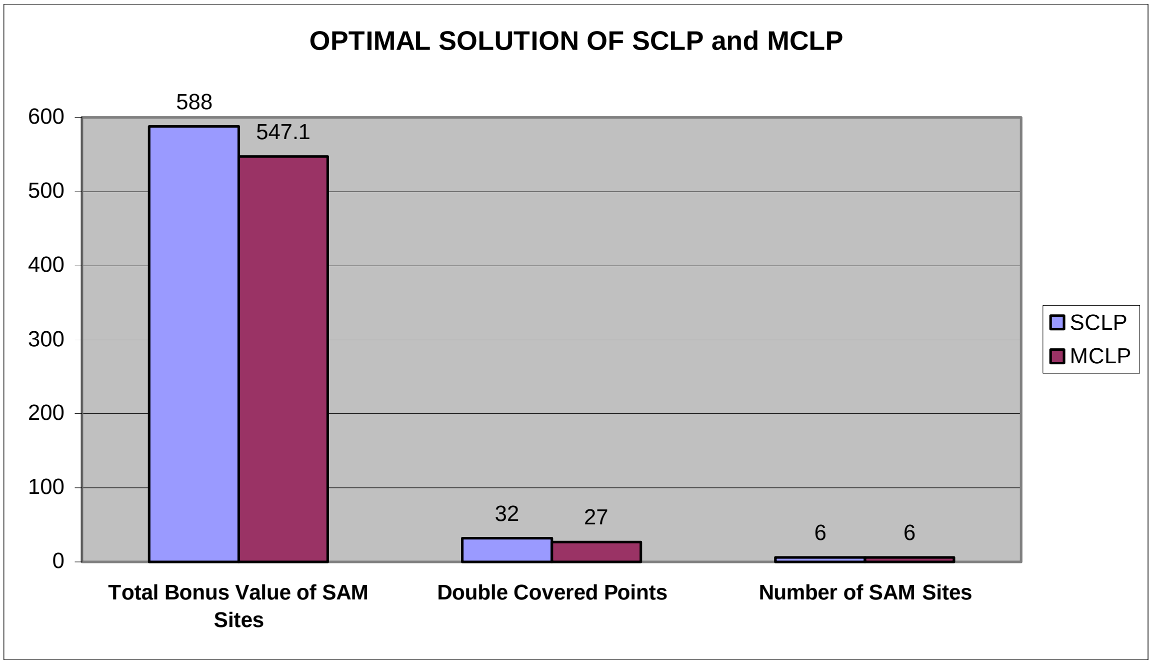

Figure 6. Comparison of MCLP and SCLP's Best Solutions

The number of double covered demand points decreases when the number of selected SAM sites decreases. Having fewer double covered demand points can be an advantage when there is a budget constraint. This is because unnecessary double covered demand points may cause a decrease in the total coverage percentage when there are insufficient SAM sites due to a budget constraint.

A single SAM site has a coverage rate of 23% and each additional SAM site gains approximately an extra 20%. Figure 7 shows the relationship between the number of SAM sites and the coverage rate.

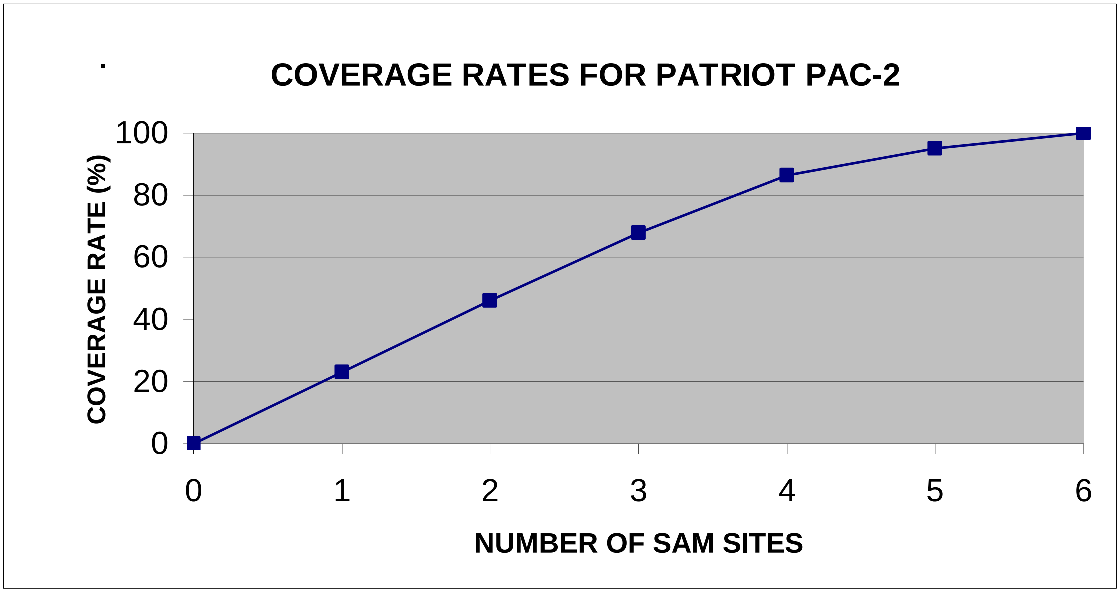

Figure 7. Relationship between Number of SAM Sites and Coverage Rate

4.2.1.1 Conclusion

If Patriot PAC-2 is selected to protect the western and southern borders of Turkey, at least six SAM sites are needed to totally cover the desired air defense area. The candidate location points X6, X12, X38, X45, X73, and X98 would be the best choice with an average weighted bonus value of 98. If there is a budget constraint and six SAM sites cannot be located, the coverage rates of the SAM sites should be maximized. Table 6 shows the best choice of locations with a fewer number of SAM sites. The difference in coverage between six and five SAM sites is 5% but the difference between the other consecutive numbers of SAM sites is approximately 20%. Although the model's candidate bonus value constraint forces the model to provide at least an average 80 of points, all solutions average over 85. Figure 8 shows the approximate location of the SAM sites and their coverage. This representation is only <u>for demonstration, not for implementation.</u>

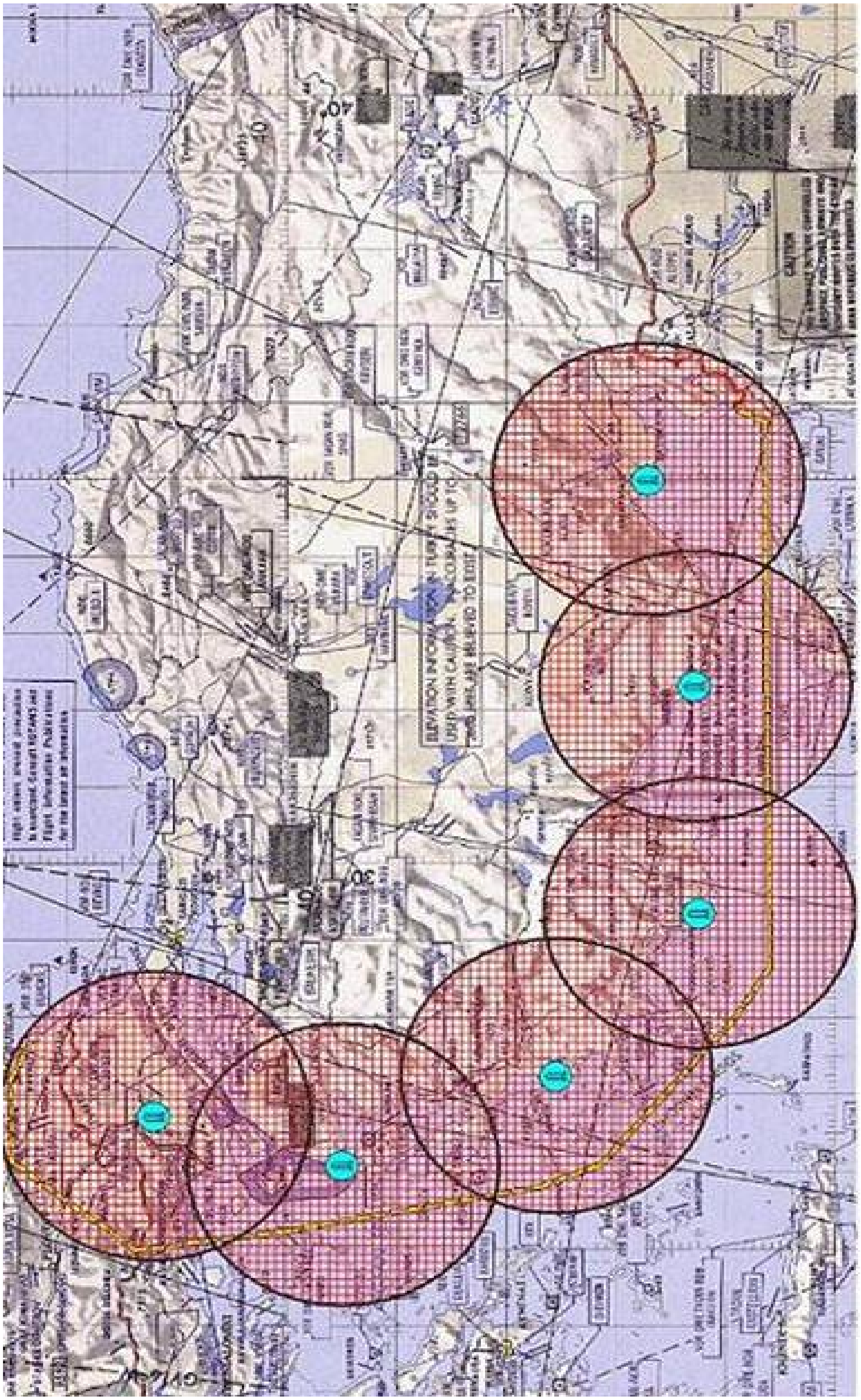

Figure 8. Demonstration of Patriot PAC-2 Solution

4.2.1.2 Sensitivity Analysis

Using sensitivity analysis, the affect of constraint and basic variable changes on solution results are examined. Sensitivity analysis is applied to both the SCLP and the MCLP models.

SCLP Model Sensitivity Analysis

First, solution candidate points are excluded one at a time to see the impact on the optimal solution. Table 7 shows these solutions.

Table 7. Forcing Basic Variables to Become Zero in SCLP Model

Excluded Candidate Point	New Selected SAM Site	Old Objective Function Value	New Objective Function Value	Difference	Coverage Rate
X6	X7, X12, X38, X45, X73, X98	588	585.5	2.5	100%
X12	X6, X15, X38, X45, X73, X98	588	579.25	8.75	100%
X38	X6, X12, X36, X73, X92, X98	588	570.45	17.55	100%
X45	X6, X12, X38, X73, X92, X98	588	583.8	4.2	100%
X73	X6, X12, X27, X36, X92, X98	588	568.25	19.75	100%
X98	X6, X12, X38, X45, X52, X73	588	588	0	100%

As seen in Table 7, forcing variable X73 to become zero affects the solution the most. This change decreases the objective function value by 19.75 points. Forcing variable X98 to become zero has no effect on the optimal solution and this change provides an alternative solution with the selected candidate points X6, X12, X38, X45, X52 and X73.

MCLP Model Sensitivity Analysis

In the MCLP model, the same analysis is accomplished as that of the SCLP model. Only the solution given the 100% coverage is used for this analysis. First the solution candidate points are removed one at a time to see the effects on the optimal solution. Table 8 shows the output of these solutions.

Table 8. Forcing Basic Variables to Become Zero in MCLP Model

Excluded Candidate Point	New Selected SAM Site	Objective Function Value Change	Total Bonus Value Change of Selected SAM Site	Coverage Rate
X6	X8, X26, X35, X44, X51, X64	0	- 17.6	100%
X13	X6, X8, X26, X36, X44, X51	0	+ 10.80	100%
X27	X6, X15, X40, X45, X57, X73	0	+ 2.3	100%
X35	X6, X13, X27, X36, X92, X98	0	+ 13.7	100%
X44	X7, X11, X26, X36, X46, X53	0	- 1.75	100%
X51	X6, X10, X26, X36, X53, X93	0	+ 10.05	100%

As seen in Table 8, the total bonus values of candidate SAM sites increases when some of the basic variables are forced to become zero. This shows that by selecting appropriate candidate SAM sites, weighted total candidate bonus values could be improved. When variables X13, X27, X35 and X51 are forced to become zero, the new solutions are better. On the other hand, if X6 and X44 are forced to become zero, the new solutions are worse. This shows that candidates X6 and X44 are most sensitive.

Table 6 and Figure 7 show the result of changing the number of SAM sites in MCLP model. The RHS of the related constraint is decreased one at a time until it is one. As seen in Figure 7, coverage rate decreases nearly linearly while number of SAM sites decreases.

The last change is increasing RHS of the candidate SAM sites bonus value requirement constraint. The Model objective is meeting the value of at least average 80 points for each run. This value is increased five points in each run. Table 9 shows the output of these solutions.

Table 9. Changing RHS of Additional Constraint in MCLP Model

Number of SAM Sites	Coverage Rate Per RHS				
	80	85	90	95	100
6	100%	100%	100%	100%	91.5%
5	95%	95%	95%	93.5%	88.5%
4	86.5%	86.5%	86%	83%	78.5%
3	68%	68%	67.5%	67.5%	61.5%
2	46%	46%	45%	45%	44%
1	23%	23%	22.5%	22.5%	22.5%

According to results, the 100% coverage is still provided by increasing the RHS value to 95 for six SAM sites. This shows that, by using appropriate candidate SAM sites, higher weighted total bonus values still could be obtained while keeping coverage at 100%.

4.2.2 Solutions for S-300 PMU-1

The maximum range value used to construct both SCLP and MCLP models for the S-300 PMU-1 is 81 NM. Although the range is only 4 NM less than that of the Patriot PAC-2, vastly different solutions are obtained. This shows the importance of the location problem; small changes can have a large impact on the solutions.

When the pure SCLP model is run to find the minimum number of SAM sites needed to cover all the demand points, a value of seven is obtained. Thus, the minimum required number of S-300 PMU-1 sites to cover all the demand points is at least seven. When this number is used as the RHS of the SAM site limiting constraint in the modified SCLP model, the selected candidate locations are X6, X12, X22, X30, X38, X45 and X53 with a candidate bonus values of 692.05. As expected, the modified SCLP model provides a larger bonus value. The double covered demand points obtained from the modified SCLP is less than the pure SCLP model, but the modified SCLP model solution covers more area in the western part of the air defense region. More area could be covered than desired air defense area over the Aegean Sea. The solution reports for the pure and modified SCLP models are shown in Table 10.

Table 10. SCLP Solution Report for S-300 PMU-1

Model	Number of Sam Sites	Selected Sam Sites	Objective Function Value	Candidate Bonus Value / Average	Double Covered Points	Total Covered Points / Coverage Rates
PURE SCLP	7	X6, X8, X25, X29, X38, X47, X98	*MIN* 7	634.75 / 90.68	68	200 / 100%
MODIFIED SCLP	7	X6, X12, X22, X30, X38, X45, X53	*MAX* 692.05	692.05 / 98.86	53	200 / 100%

The objective for the MCLP model is to maximize both coverage of located SAM sites and assigned demand point bonus values. Table 11 shows the solution report for the MCLP model. The MCLP model is run seven times for S-300 PMU-1. In the first run, a coverage level of 100% is obtained with seven selected SAM sites; this solution is an alternative solution for the complete coverage of the air defense area. As expected, total bonus value of selected SAM sites in the MCLP solution is smaller than the solution in modified SCLP. This is because of the difference between the objective functions. Comparison of these two solutions is shown in Figure 9.

Table 11. MCLP Solution Report for S-300 PMU-1

Number of Sam Sites	Selected Sam Sites	Objective Function Value	Candidate Bonus Value / Average	Double Covered Points	Total Covered Points / Coverage Rates
7	X7, X12, X26, X34, X39, X53, X96	*19035*	641.35 / 91.62	43	200 / 100%
6	X6, X12, X47, X53, X73, X87	*18935*	526.45 / 87.74	19	196 / 98%
5	X6, X12, X37, X48, X73	*17477*	448.9 / 89.78	10	185/ 92.5%
4	X6, X25, X38, X48	*15517*	366.15 / 91.53	0	163/ 81.5%
3	X6, X25, X38	*12465*	286.45 / 95.48	0	129/ 64.5%
2	X6, X25	*8655*	189.45 / 94.73	0	88/ 44%
1	X25	*4480*	89.45 / 89.45	0	45/ 22.5%

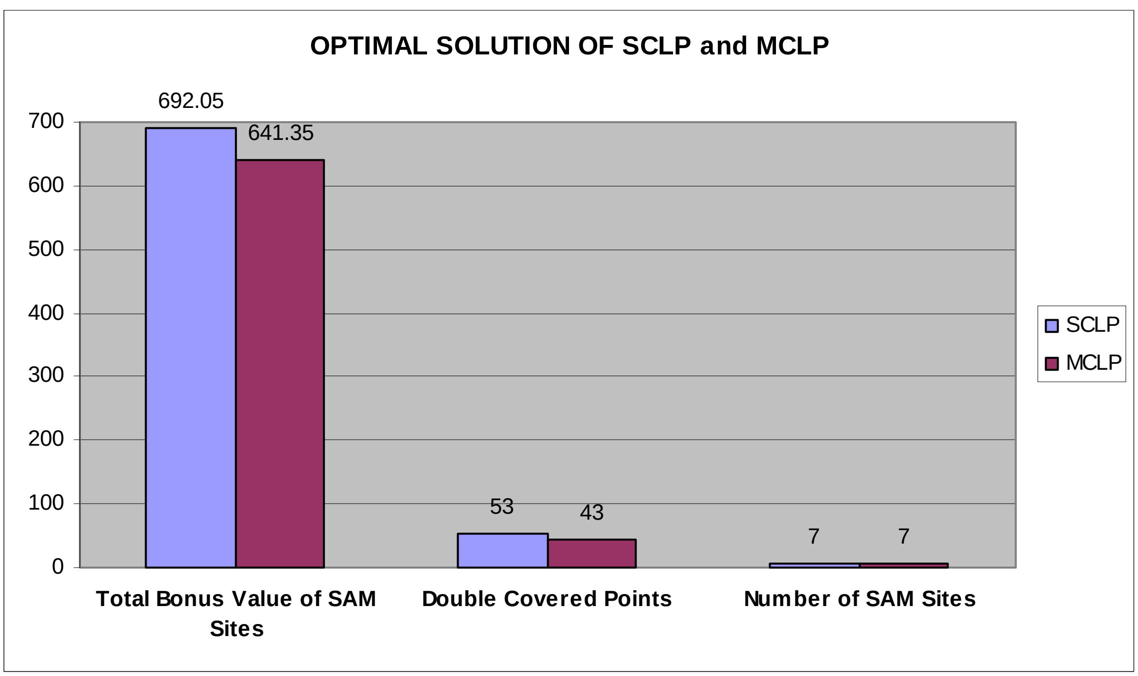

Figure 9. Comparison of SCLP and MCLP's Best Solutions

The number of double covered demand points decreases when the number of selected SAM sites decreases. Having fewer double covered demand points is an advantage when there is a budget constraint on SAM system spending. This is because unnecessary double covered demand points may cause a decrease of the total percentage of coverage when there are insufficient SAM sites due to a budget constraint.

A single SAM site has a coverage rate of 22.5%. Coverage rates increase approximately linearly until the number of SAM sites to locate is four. After four SAM sites, the rate of increase slows down. This is because of the effect of double covered points. There are no double covered points until the number of SAM sites equals five. Figure 10 shows the relationship between the number of SAM sites and coverage rate.

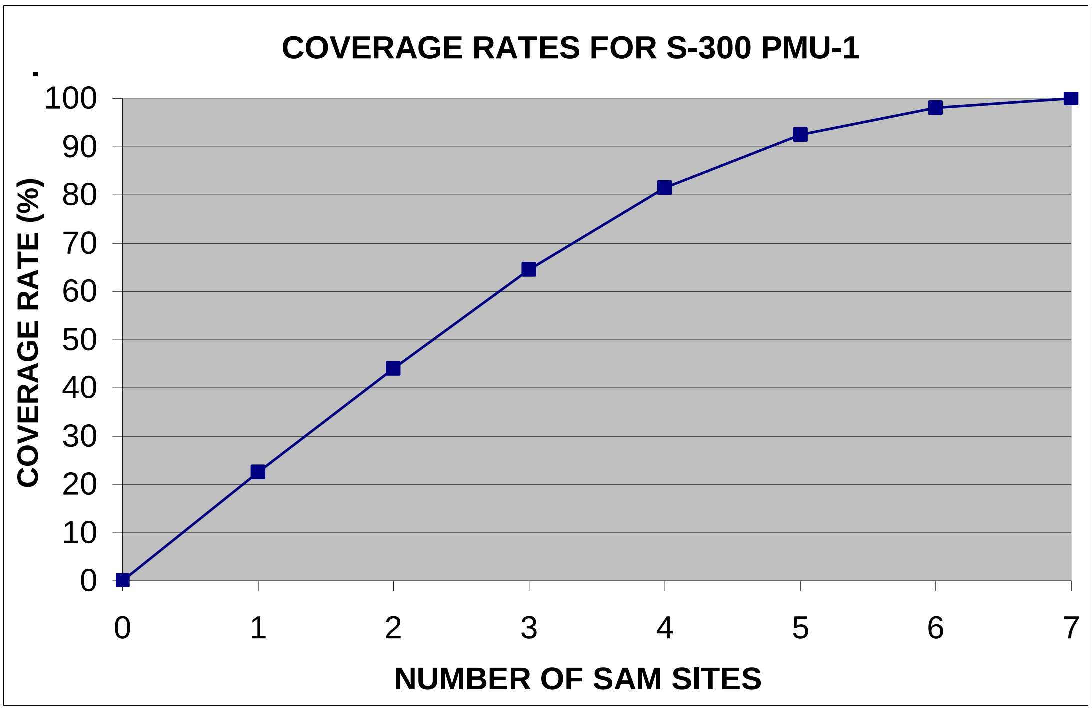

Figure 10. Relationship between Number of SAM Sites and Coverage Rate

4.2.2.1 Conclusion

At least seven S-300 PMU-1 sites are needed to cover the desired air defense area spread over the Aegean and Mediterranean Sea. The candidate location points X6, X12, X22, X30, X38, X45 and X53 would be the best choices with an average weighted candidate bonus value of 99 per site. If the budget is a constraint on buying seven SAM sites, the coverage rates of SAM sites should be maximized. Table 11 shows the best choices of locations with fewer SAM sites than required to cover all the demand points. The difference between seven and six SAM sites is only 2%. The other differences are greater. Although the model's candidate bonus value constraint forces the model to meet an average of at least 80, all solutions exceed 85. Figure 11 shows the approximate location of the SAM sites and their coverage. This representation is only <u>for demonstration, not for implementation.</u>

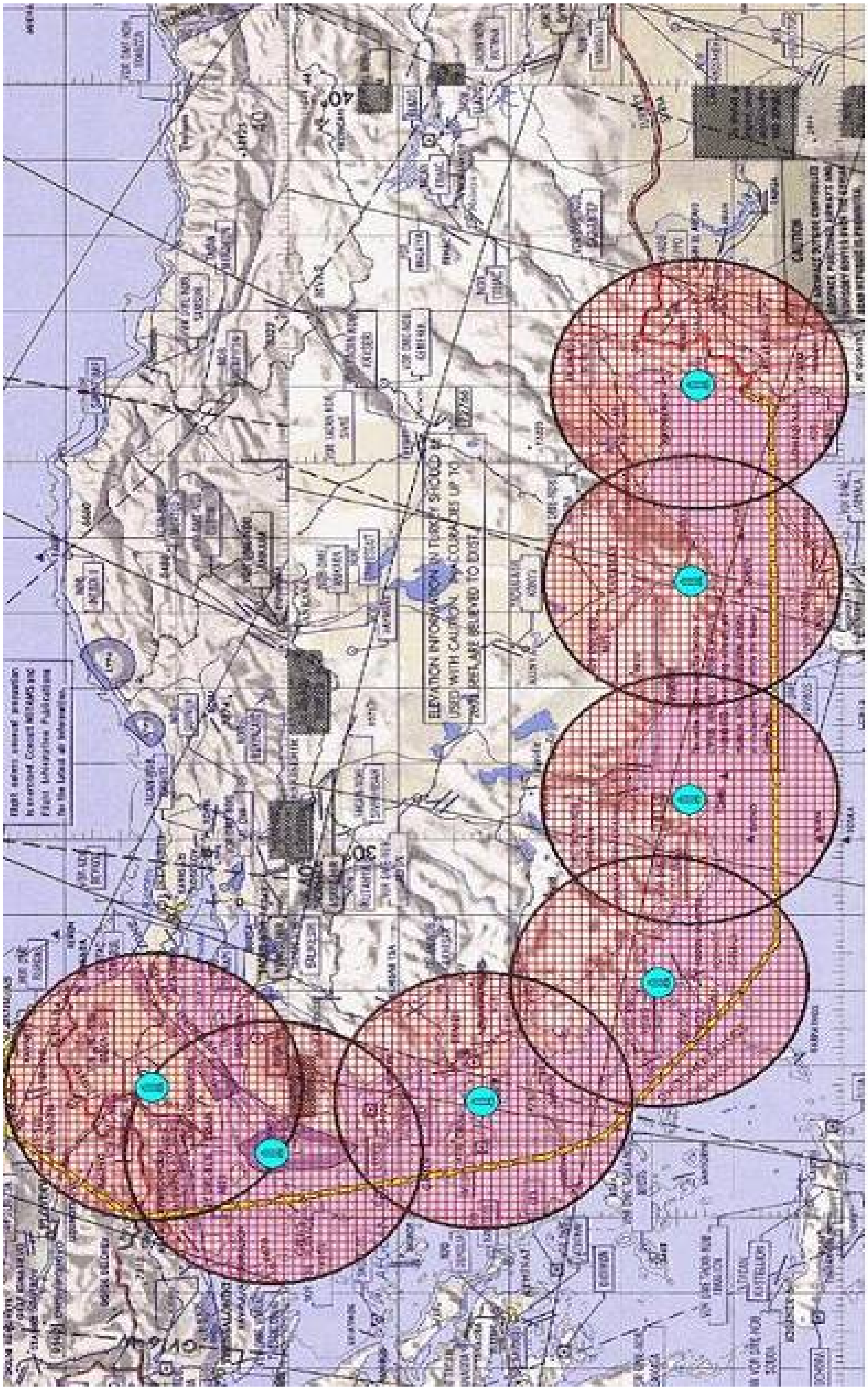

Figure 11. Demonstration of S-300 PMU-2 Solution

4.2.2.2 Sensitivity Analysis

Sensitivity analysis is applied to both SCLP and MCLP models.

SCLP Model Sensitivity Analysis

First, solution candidate points are removed one at a time to see their effects on the optimal solution. Table 12 shows the results.

Table 12. Forcing Basic Variables to Become Zero in SCLP Model

Excluded Candidate Point	New Selected SAM Site	Old Objective Function Value	New Objective Function Value	Difference	Coverage Rate
X6	X7, X12, X22, X30, X38, X45, X53	692.05	689.55	2.5	100%
X12	X6, X9, X22, X30, X38, X45, X53	692.05	689.50	2.55	100%
X22	X6, X12, X20, X30, X38, X45, X53	692.05	690.85	1.2	100%
X30	X6, X12, X22, X28, X38, X45, X53	692.05	690.55	1.5	100%
X38	X6, X12, X27, X34, X45, X53, X90	692.05	678.30	4.2	100%
X45	X6, X12, X22, X30, X38, X53, X96	692.05	673.80	13.75	100%
X53	X6, X12, X22, X30, X38, X45, X58	692.05	677.85	14.2	100%

As seen in Table 12, forcing variables X53 and X45 to become zero has maximum effect on the optimal solution. The objective function value decreases about 14 points. These two variables are more sensitive than the others.

MCLP Model Sensitivity Analysis

Only the solution yielding 100% coverage is used for this analysis. First, solution candidate points are removed one at a time to see their effects on the optimal solution. Table 13 shows the results.

Table 13. Forcing Basic Variables to Become Zero in MCLP Model

Excluded Candidate Point	New Selected SAM Sites	Objective Function Value Change	Total Bonus Value Change of Selected SAM Site	Coverage Rate
X7	X3, X19, X28, X38, X58, X61, X94	0	- 10.10	100%
X12	X7, X10, X25, X41, X47, X53, X70	0	- 60.65	100%
X26	X2, X24, X30, X41, X45, X58, X61	0	- 27.20	100%
X34	X7, X12, X31, X49, X55, X58, X69	0	- 17.85	100%
X39	X7, X20, X47, X53, X72, X85, X87	0	- 29.15	100%
X53	X7, X13, X27, X85, X87, X93, X98	0	- 27.40	100%
X96	X7, X18, X29, X38, X58, X79, X94	0	- 2.70	100%

Table 13 shows that, when each of the basic variable is forced to become zero, all the new total bonus values of selected SAM sites decreases. The most sensitive variable is X12 with a reduction of 60.65 points and the least is X96 with 2.70 points.

Table 11 and Figure 10 show the result of changing the number of SAM sites in the MCLP model. The RHS of the related constraint is decreased one at a time until it reaches a value of one. As seen in Figure 8, the coverage rate decreases nearly linearly and with high percentage when the number of SAM sites is below four. On the other hand, while it is above four, the rate of decrease is relatively low.

The last change explored involves increasing the RHS of the candidate SAM sites bonus value requirement constraint. This value is increased five points in each run. Table 14 shows these results.

Table 14: Changing RHS of Additional Constraint in MCLP Model

Number of SAM Sites	Coverage Rate Per RHS				
	80	85	90	95	100
7	100%	100%	100%	100%	89%
6	98%	98%	98%	98%	89%
5	92.5%	92.5%	92.5%	91%	85%
4	81.5%	81.5%	81.5%	79%	74%
3	64.5%	64.5%	64.5%	64.5%	59%
2	44%	44%	44%	43%	42%
1	22.5%	22.5%	22.5%	22.5%	22.5%

The same coverage rate is provided for each number of SAM sites up to a RHS value of 90. This shows that by using appropriate candidate SAM sites, higher weighted total candidate bonus values still could be obtained while maintaining the same coverage rate. According to the results, if seven S-300 PMU-1 sites are located for air defense, 100% coverage still could be obtained when increasing the RHS value to 95.

4.2.3 Solutions for Arrow-2

The maximum range value used to construct both SCLP and MCLP models for Arrow-2 is 40 NM. Only the MCLP model is used for Arrow-2 since its maximum range is not large enough to cover all the demand points. Even if all the sites are located on the national border of Turkey, it is impossible to reach most of the points. Since the SCLP model can not find a feasible solution, only the MCLP model is used to study the Arrow-2. The goal is to find the maximum coverage with a minimum number of SAM sites. Table 15 shows the solution report of the MCLP model.

Table 15. MCLP Solution Report for Arrow-2

Number of Sam Sites	Selected Sam Sites	Objective Function Value	Candidate Bonus Value / Average	Double Covered Points	Total Covered Points / Coverage Rates
30	-	*15727*	2695.5 / 89.85	-	165 / 82.5%
25	-	*15727*	2232. 5 / 89.3	-	165 / 82.5%
19	X3, X7, X8, X11, X12,X21, X26, X29, X31, X36, X38, X46, X49, X53, X55, X58, X61, X91, X97	*15727*	1696.95 / 89.31	86	165 / 82.5%
18	***X3, X7, X8, X12,X21, X26, X29, X31, X36, X38, X46, X49, X53, X55, X58, X61, X91, X97***	***15727***	***1612.5 / 89.58***	***76***	***165 / 82.5%***
17	X3, X7, X8, X12,X21, X25, X29, X31, X36, X38, X46, X48, X53, X55, X58, X91, X94	*15627*	1525.3 / 89.72	64	164 / 82%
16	X3, X7, X8, X12,X21, X26, X29, X31, X36, X38, X46, X48, X53, X55, X91, X94	*15457*	1442.4 / 90.15	57	162/ 81%
14	X3, X7, X8, X12,X21, X25, X29, X31, X36, X46, X49, X53, X55, X97	*15097*	1248.2 / 89.12	41	158/ 79%
11	X3, X8, X12, X21, X29, X31, X36, X48, X53, X55, X93	*13851*	983.35 / 89.40	16	145 / 72.5%
7	X3, X8, X21, X30, X38, X48, X93	*10543*	627.35 / 89.62	0	110 / 55%
3	X8, X21, X29	*5850*	281.25 / 93.75	0	59 / 29.5%
1	X21	*2650*	96.40 / 96.40	0	22 / 13.5%

Table 15 shows that a minimum of eighteen Arrow-2 SAM sites are required to achieve maximum coverage. There are no double covered points obtained with up to eight SAM sites. Over eight SAM sites, the double covered points increase and additionally there are some triple covered points. The coverage difference between eighteen and seventeen SAM sites is only 0.5%. The other coverage differences are also small between consecutive numbers of SAM sites. Although the model's candidate bonus value constraint forces the model to provide at least an average of 80, all the solutions have over 85 candidate bonus points per site.

If the demand point bonus values are not included to force the model to cover more important points, the maximum coverage rate is 82.5%.

A single SAM site has a coverage rate of 13.5%. Figure 12 shows the relationship between the number of SAM sites and coverage rate.

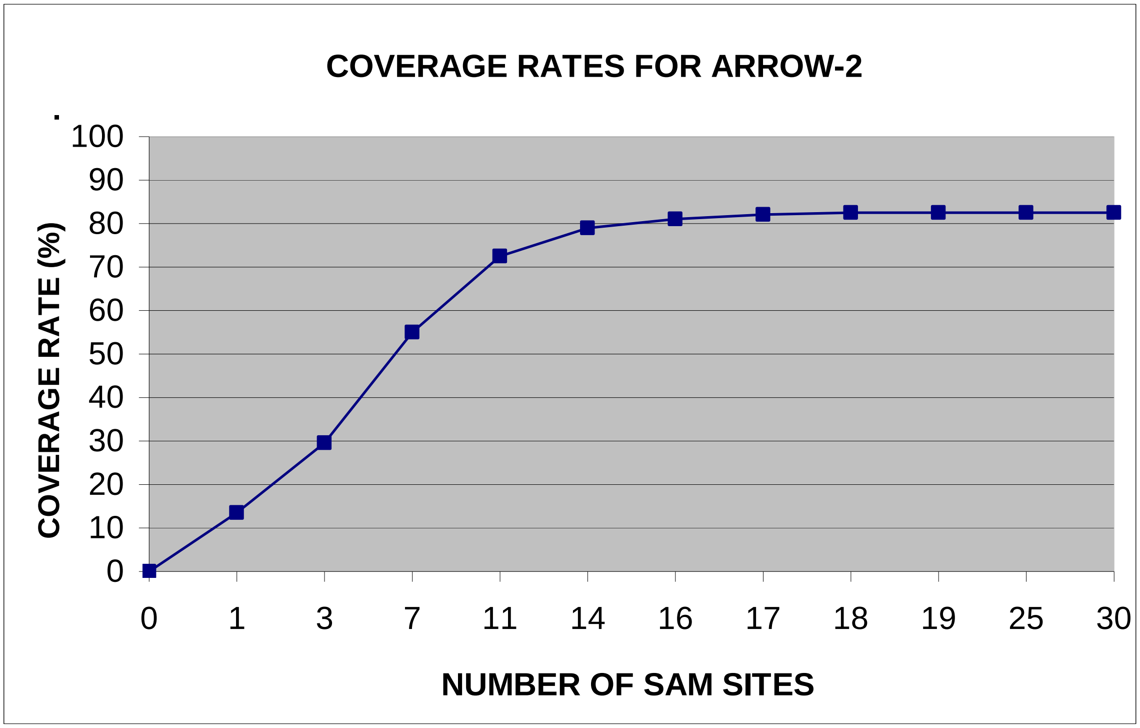

Figure 12. Relationship between Number of SAM Sites and Coverage Rate

4.2.3.1 Conclusion

A minimum of eighteen Arrow-2 sites are needed to achieve maximum coverage over the desired air defense area spread over the Aegean and Mediterranean Sea. The candidate location points X3, X7, X8, X12, X21, X26, X29, X31, X36, X38, X46, X49, X53, X55, X58, X61, X91 and X97 would be the best choice with average weighted candidate bonus value of 89.58. Figure 13 shows the approximate location of the SAM sites and their coverage. This representation is only <u>for demonstration, not for implementation.</u>

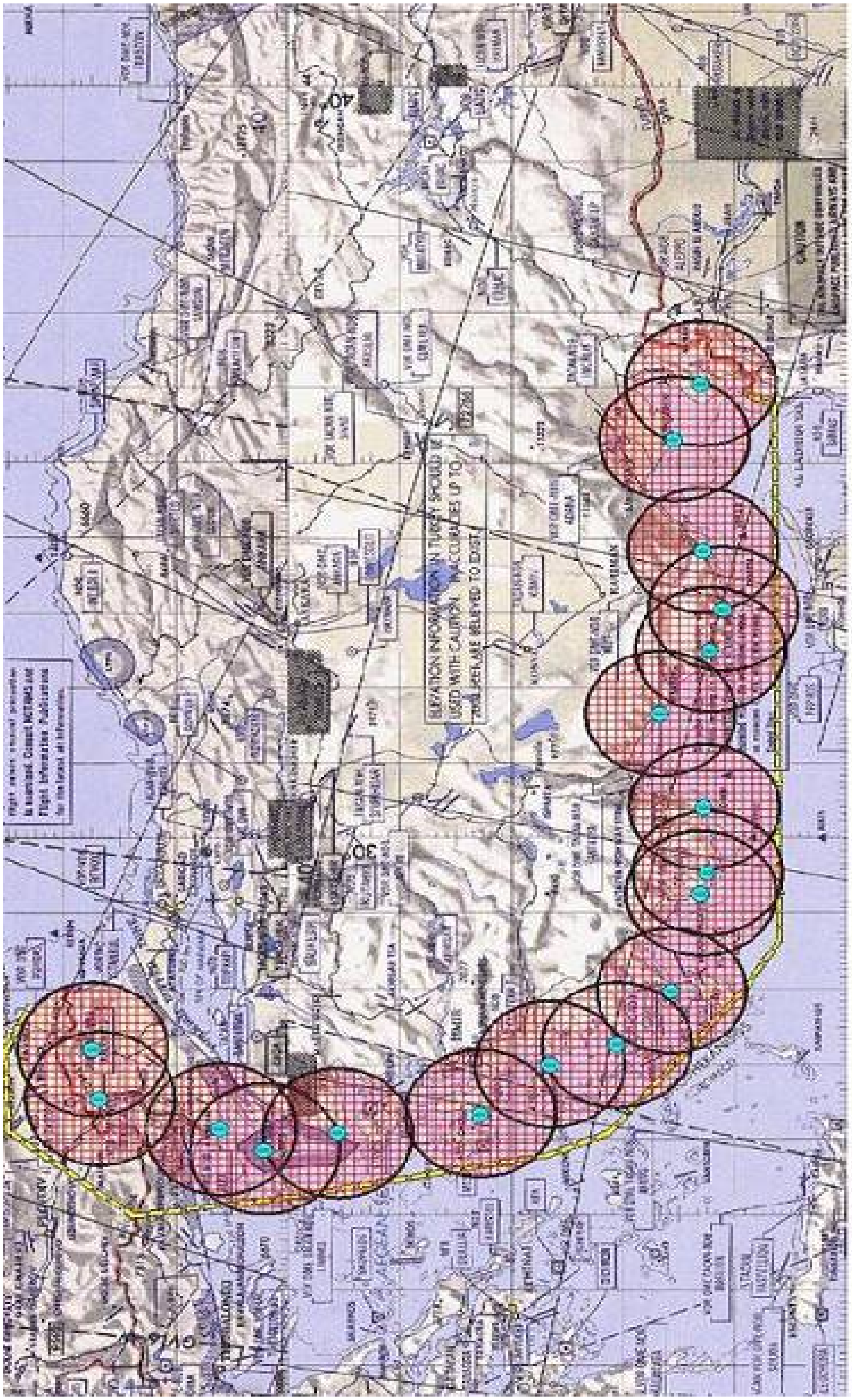

Figure 13. Demonstration of Arrow-2 Solution

4.2.3.2 Sensitivity Analysis

Sensitivity analysis is applied to MCLP model for Arrow-2.

MCLP Model Sensitivity Analysis

Only the solution giving the 82.5% coverage with eighteen SAM sites is used for this analysis. First the solution candidate points are removed one at a time to see their effects on the optimal solution. Table 16 shows the output of these solutions.

Table 16. Forcing Basic Variables to Become Zero in MCLP Model

Excluded Candidate Point	New Selected SAM Sites	Objective Function Value Change	Total Bonus Value Change of Selected SAM Site	Coverage Rate
X3	X4, X7, X8, X12,X21, X25, X29, X31, X36, X38, X46, X49, X53, X55, X58, X61, X91, X97	- 95	- 10.65	82.5%
X7	X4, X8, X12, X21, X25, X29, X31, X36, X38, X43, X46, X48, X49, X53, X55, X58, X61, X64	-285	- 37.15	81.5%
X8	X3, X7, X11, X12, X21, X26, X29, X31, X36, X38, X46, X49, X53, X55, X58, X61, X91, X97	- 200	- 2.8	81.5%
X12	X3, X7, X8, X11, X17, X21,X26, X29, X31, X36, X38, X46, X49, X53, X55, X58, X91, X97	- 300	- 7.65	81%
X21	X3, X7, X8, X12, X20,X24, X25, X29, X31, X36, X38, X48, X49, X53, X55, X58, X91, X93	- 100	+ 6	82%
X26	X3, X7, X8, X12, X21,X24, X29, X31, X36, X38, X46, X48, X53, X55, X58, X61, X91, X94	0	+3.7	82.5%
X29	X3, X7, X8, X12, X21,X24, X29, X31, X36, X38, X46, X48, X53, X55, X58, X61, X91, X94	- 100	+ 5.25	81.5%
X31	X3, X7, X8, X12, X21,X26, X30, X34, X36, X38, X46, X48, X53, X55, X58, X61, X91, X94	- 200	+ 8.25	81.5%
X36	X3, X7, X8, X12, X21,X25, X29, X31, X35, X38, X43, X46, X49, X53, X55, X58, X61, X97	- 100	+ 5	82%
X38	X3, X7, X8, X12, X21, X26, X29, X31, X36, X37, X43, X46, X48, X49, X53, X55, X58, X61	- 90	+ 21.55	82.5%
X46	X3, X7, X8, X12, X21, X26, X29, X31, X36, X38, X43, X48, X53, X55, X58, X61, X93, X94	0	+ 0.15	82.5%
X49	X3, X7, X8, X12, X21, X26, X29, X31, X36, X38, X48, X53, X55, X58, X61, X91, X93, X94	0	+ 3.85	82.5%
X53	X3, X7, X8, X12, X21, X26, X29, X31, X36, X38, X43, X46, X49, X55, X58, X61, X97, X98	- 100	+ 0.05	82%
X55	X3, X7, X8, X12, X21, X26, X29, X31, X36, X37, X38, X43, X46, X48, X49, X53, X58, X61	- 180	- 4.25	81.5%
X58	X3, X7, X8, X11, X12, X21, X26, X29, X31, X36, X38, X46, X49, X53, X55, X61, X91, X97	- 170	+ 2.40	82%
X61	X3, X7, X8, X11, X12, X21, X25, X29, X31, X36, X38, X43, X46, X49, X53, X55, X58, X97	- 100	- 9.30	82.5%
X91	X3, X7, X8, X12, X21, X25, X29, X31, X36, X38, X43, X46, X49, X53, X55, X58, X61, X97	0	- 2.85	82.5%
X97	X3, X7, X8, X12, X21, X26, X29, X31, X36, X38, X48, X53, X55, X58, X61, X91, X93	0	+ 3.85	82.5%

Table 16 shows that, when each of the basic variables is forced to become zero, both the objective function value and total candidate bonus value change. The most sensitive variable is X12 with a decrease of 300 points in objective function value, 7.65 points in total bonus value and 1.5% in coverage rate. The least sensitive variable is X46 with no change in objective function value and coverage rate value. Excluding X46 only effects total bonus value by 0.15 point increase.

Table 15 shows the results of changing the number of SAM sites in the MCLP model. The RHS of the SAM site limiting constraint (15) is changed with values up to thirty. Maximum coverage rate is 82.5%. With more than eighteen SAM sites, no improvement in coverage rate is obtained. Figure 12 shows the relationship between number of SAM sites and coverage rates.

The last change explored is increasing the RHS of the candidate SAM sites bonus value requirement constraint (16). This value is increased five points each run. Table 17 shows the output of these solutions.

Table 17. Changing RHS of Additional Constraint in MCLP Model

Number of SAM Sites	Coverage Rate Per RHS				
	80	85	90	95	100
18	82.5%	82.5%	82%	77.5%	INFEASIBLE
17	82%	82%	81.5%	77%	INFEASIBLE
16	81%	81%	81%	76%	INFEASIBLE
14	79%	79%	79%	75%	INFEASIBLE
11	72.5%	72.5%	72%	68.5%	INFEASIBLE
7	55%	55%	55%	54%	28.5%
3	29.5%	29.5%	29.5%	28.5%	21.5%
1	13.5%	13.5%	13.5%	13.5%	12%

The 82.5% coverage rate is still provided between bonus value points 85 and 90. This shows that by using appropriate candidate SAM sites, the total candidate bonus values of selected SAM sites still could be increased while achieving the same coverage. The maximum RHS value that still could provide an 82.5% coverage rate is 1617.2 (average 89.84) and the new optimal solution has SAM sites X3, X7, X8, X12, X21, X25, X29, X31, X36, X38, X48, X53, X55, X58, X61, X91, X93 and X94.

4.3 Combined Demonstrations of Solutions

In this section of the research, some combined graphical demonstrations of the three candidate SAM missiles' results are given.

Table 20 shows the best solution data of three candidate SAM missiles. Figure 14 shows the graphical demonstration of coverage rate, average bonus points and number of SAM sites.

Table 18. Best Solutions of Each Candidate Missiles

Candidate Missiles	Number of SAM Sites	Selected SAM Sites	Objective Function Value	Average Candidate Bonus Value	Coverage Rate
PATRIOT PAC-2	6	X6, X12, X38, X45, X73, X98	588	98	100%
S-300 PMU-1	7	X6, X12, X22, X30, X38, X45, X53	692.05	98.86	100%
ARROW-2	18	X3, X7, X8, X12, X21, X25, X29, X31, X36, X38, X48, X53, X55, X58, X61, X91, X93, X94.	15727	89.84	82.5%

As seen in Table 18, the candidates X6, X12, X38 and X45 are selected as SAM sites for both Patriot PAC-2 and S-300 PMU-1. The candidates X12 and X38 are selected as SAM sites for all three missile types.

Figure 14 shows that only Patriot PAC-2 and S-300 PMU-1 can cover the entire air defense area. Arrow-2 can not cover all the demand points no matter how many sites are occupied. All three candidates have good average values for logistically, geographically and strategic weighted bonus point values. Patriot PAC-2 requires the minimum number of SAM sites to cover the desired area.

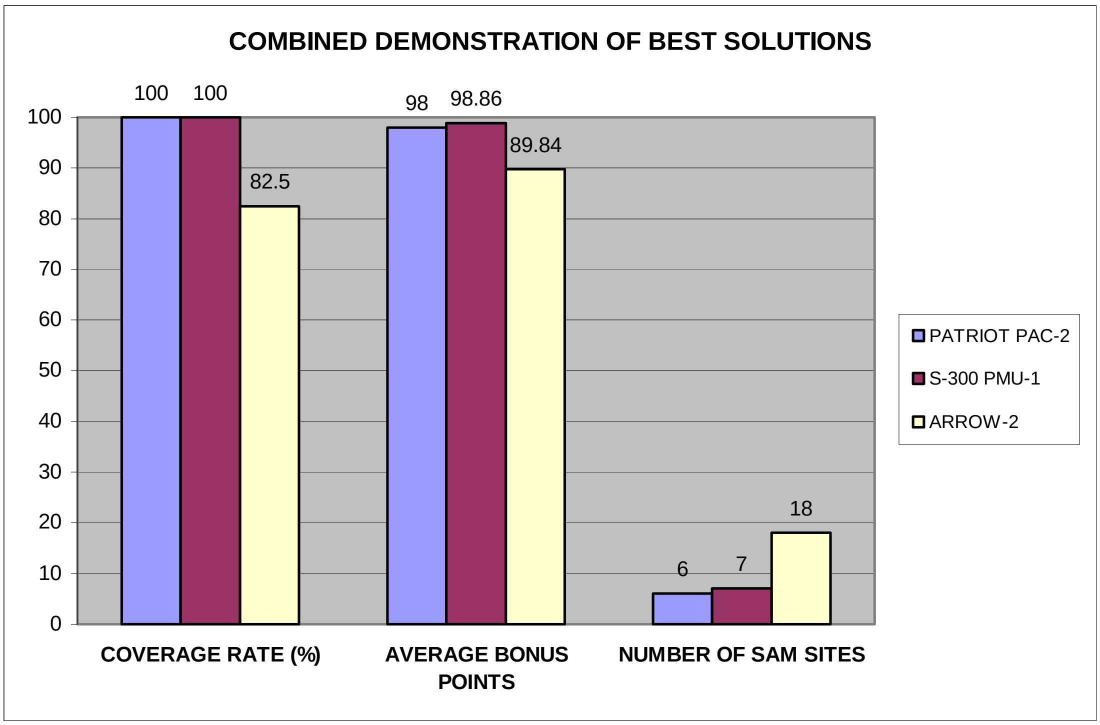

Figure 14. Combined Demonstration of Best Solutions

Figure 15 shows the relationship between number of SAM sites and coverage rates for each candidate SAM missile. All three candidates have approximately linear increment in coverage per missile until they reach their maximum coverage.

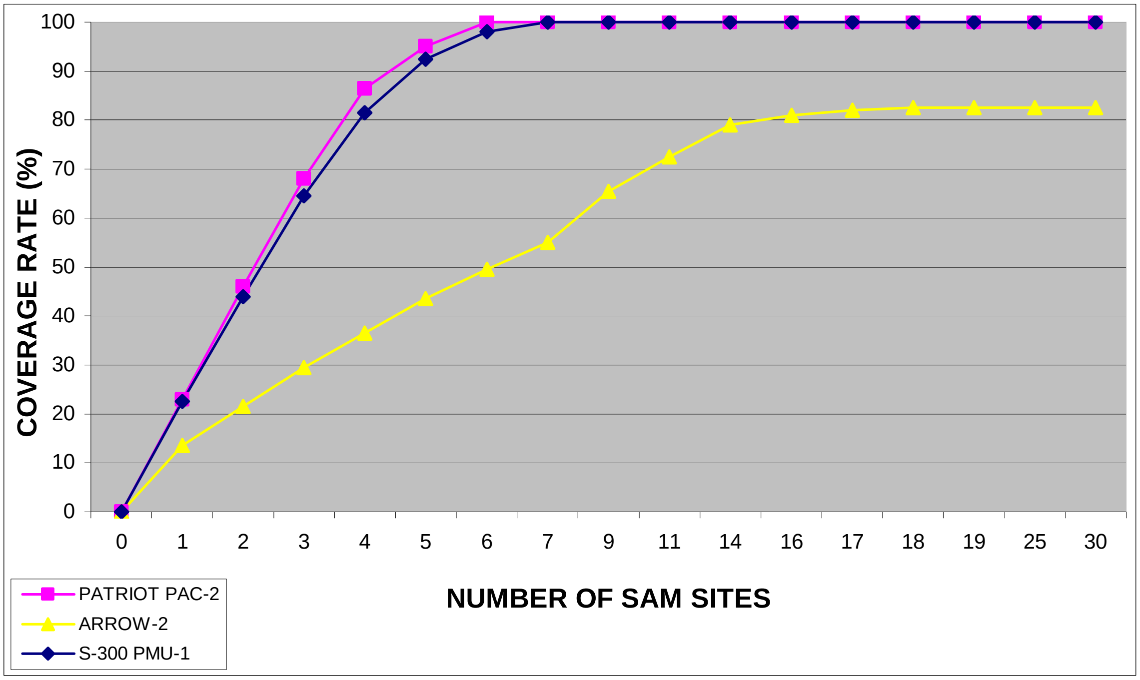

Figure 15. Relationship between Number of SAM Sites and Coverage Rates

4.4 Summary

This research objective is not to decide which candidate SAM system should be purchased. Instead, the research shows coverage to the decision makers. It also provides them with the best location points with maximum strategic, logistical and geographical bonus values.

This chapter gives the solution results for the three candidate SAM missiles. The candidate solution results are presented separately. Sensitivity analysis is applied separately to each candidate as well. A report of the best optimal solutions is presented at the end of Chapter 4.

V. Conclusions and Recommendations

5.1 Summary

Chapter 1 presents the problem background and general information about Turkey. The importance of air defense and its major concepts are briefly reviewed. General information and specifications of three candidates SAM missiles are also presented in Chapter 1. Chapter 2 presents the literature review for this research. Some specific location type models and their implementation areas are listed. SCLP and MCLP models are used to solve this research's problem. These two models' mathematical formulations and the methodology used in this research are presented in Chapter 3. Chapter 3 also presents the modified SCLP and MCLP formulations used for each candidate SAM missile. Chapter 4 presents the optimal solutions and sensitivity analysis for each model and candidate SAM missile separately. A combined demonstration of solutions is also presented in Chapter 4. Chapter 5 presents conclusions and recommendations. Suggestions for future research are also presented.

5.2 Research Conclusions and Contributions

Location problems are very common in operations research (OR). There are numerous military and civilian applications of these problems. Locating SAM sites is one of these applications. The SCLP model is used to determine the minimum number of SAM sites to cover the demand points as well as the location for these sites. The MCLP model, in contrast, is used for a fewer number of SAM sites than required to cover the entire desired air defense area. In this model, the objective is maximizing the coverage of

SAM sites by locating them at appropriate locations. Both models were coded in the program language LINDO and solved using LINGO 10. All the models give optimal solutions in under two seconds. Model parameters are easily changed and new constraints can be added.

This research effort provides an approach for dealing with locating a facility having a certain coverage value. Only three candidate SAM missiles were chosen for this research. The model successfully found the minimum number of SAM sites needed as well as their preferred location for the Patriot PAC-2 and S-300 PMU-1. Although there is only a small difference between ranges of Patriot PAC-2 and S-300 PMU-1 (based on the ranges stated in Jane's Land-Based Air Defense 2006-2007 publication), one less Patriot PAC-2 SAM site is needed than S-300 PMU-1. Locating an additional SAM site may be very costly. This result shows the importance and benefits of using location problem approaches. Since it is impossible to entirely cover the desired area with Arrow-2 SAM sites, the maximum coverage with minimum SAM sites was determined. Solutions for the number of SAM sites below the required number to cover all desired air defense area are also presented. These solutions provide decision makers with information they could use if there is a budget constraint on SAM system purchases. Chapter 4 presents these solutions in detail.

This research is based on several assumptions. The most important assumption is the maximum ranges of the candidate SAM missiles. Since these values were taken from an unclassified open source document, analysts should redesign the SCLP and MCLP models after determining the exact ranges. This can be done by using the methodology presented in Chapter 3.

The SCLP and MCLP models used in this research can be used for many other applications. Since the only consideration is maximum range of candidate SAM missiles for our models, the approaches could be used for other applications: locating radar sites, communication antennas and Quick Response Alert aircraft. Decision makers easily may change the bonus values and add new constraints to see the effects of new data.

5.3 Recommendations for Future Research

There are many aspects of buying a military weapon system. Some basing issues such as SAM battery cost, facility construction cost, budget limits, personnel training, utilities and other issues that are considered for on-base operations were not covered in this research. All these issues can be included in the models as additional constraints. Some other specifications of candidate SAM missiles, such as minimum and maximum effective altitudes, probability of kill and minimum ranges, could also be modeled. These issues can be explored with deterministic approaches. One could make a decision tree approach by adding political aspects.

The candidate SAM missiles of this research are considered for only protecting borders from intruders. Turkey has neighbors who have surface to surface ballistic missiles and potentially have (or are developing) nuclear missiles. One could redesign this research's models to locate missile defense systems.

Only three candidate SAM missiles were chosen for this research. Other systems that may be candidates could be included.

This research includes only protection of western and southern part of Turkey. The eastern and especially southeastern part of Turkey could be modeled in future researches.

Turkey has ordered four Boeing 737 Airborne Early Warning & Control (AEW&C) aircraft to enhance her air space control. These aircraft will enter the TUAF inventory in 2008. One could use this research's approach with some modifications to determine the best orbit point locations of these aircraft.

In conclusion, although this research model solves optimum location for three candidate SAM systems, it can be easily applied to other location problems. Some heuristic approaches also could be used depending on the number of variables and time required to find an optimal solution.

Appendix A: Acronyms

AEW&C	Airborne Early Warning & Control
ATBM	Anti Tactical Ballistic Missile
C3	Command Control and Communication
CONUS	Continental United States
DSG	Decision Support Group
JSF	Joint Strike Fighter
LP	Linear Program
MEXCLP	Maximal Expected Covering Location Problem
MCLP	Maximal Covering Location Problem
NATO	North Atlantic Treaty Organization
OIF	Operation Iraqi Freedom
PAC	Patriot Advanced Capability
RHS	Right Hand Side
SAM	Surface to Air Missile
SAR	Search and Rescue
SCLP	Set Covering Location Problem
SHORAD	Short Range Air Defense
SSAM	Surface to Surface Air Missiles
TUAF	Turkish Air Force
US	United States

Appendix B: Bonus Values of Demand and Candidate Points

Bonus Values of Candidate Points				
Candidate Points	**Logistics (60%)**	**Strategic (25%)**	**Geography (15%)**	**Weighted Values**
X1	100	100	100	100
X2	90	95	100	92.75
X3	90	80	75	85.25
X4	70	70	95	73.75
X5	98	80	95	93.05
X6	80	80	95	100
X7	100	90	100	97.5
X8	80	97	100	87.25
X9	95	97	100	96.25
X10	90	95	100	92.75
X11	77	93	100	84.45
X12	98	100	100	98.8
X13	77	70	90	77.2
X14	93	77	80	87.05
X15	95	80	90	90.5
X16	90	75	90	86.25
X17	96	100	100	97.6
X18	100	100	100	100
X19	94	100	96	95.8
X20	98	100	100	98.8
X21	94	100	100	96.4
X22	100	100	100	100
X23	100	94	92	97.3
X24	91	92	94	91.7
X25	89	89	92	89.45
X26	88	88	92	88.6
X27	90	90	90	90
X28	100	100	90	98.5
X29	96	100	100	97.6
X30	100	100	100	100
X31	95	100	100	97
X32	96	89	100	94.85
X33	85	78	94	84.6
X34	100	100	100	100
X35	85	80	70	81.5
X36	80	85	96	83.65
X37	75	77	93	78.2
X38	95	100	100	97
X39	77	75	90	78.45
X40	50	65	70	56.75

Bonus Values of Candidate Points				
Candidate Points	**Logistics (60%)**	**Strategic (25%)**	**Geography (15%)**	**Weighted Values**
X41	50	65	70	56.75
X42	100	100	90	98.5
X43	90	96	90	91.5
X44	90	96	90	91.5
X45	100	100	100	100
X46	80	80	85	80.75
X47	55	75	78	63.45
X48	77	80	90	79.7
X49	76	83	90	79.85
X50	80	95	94	85.85
X51	90	98	95	92.75
X52	100	100	100	100
X53	95	100	95	96.25
X54	95	100	95	96.25
X55	75	80	98	79.7
X56	95	100	95	96.25
X57	95	100	95	96.25
X58	75	90	97	82.05
X59	78	94	85	83.05
X60	95	80	90	90.5
X61	90	90	96	90.9
X62	77	70	90	77.2
X63	90	100	100	94
X64	70	70	95	73.75
X65	80	80	95	82.25
X66	80	80	95	82.25
X67	87	93	96	89.85
X68	90	96	90	91.5
X69	88	88	92	88.6
X70	85	78	94	84.6
X71	80	92	98	85.7
X72	85	92	98	88.7
X73	90	94	98	92.2
X74	90	95	100	92.75
X75	90	80	75	85.25
X76	98	80	92	92.6
X77	87	70	90	83.2
X78	89	72	90	84.9
X79	80	85	90	82.75
X80	100	90	83	94.95

Bonus Values of Candidate Points				
Candidate Points	**Logistics (60%)**	**Strategic (25%)**	**Geography (15%)**	**Weighted Values**
X81	90	97	100	93.25
X82	90	100	100	94
X83	90	95	100	92.75
X84	77	90	95	82.95
X85	90	97	90	91.75
X86	85	90	87	86.55
X87	70	75	100	75.75
X88	70	75	100	75.75
X89	70	75	100	75.75
X90	90	100	95	93.25
X91	93	100	96	95.2
X92	94	100	96	95.8
X93	80	75	100	81.75
X94	80	75	100	81.75
X95	80	75	100	81.75
X96	80	75	100	81.75
X97	75	75	100	78.75
X98	100	100	100	100
X99	90	75	90	86.25
X100	100	100	96	99.4

Bonus Values of Demand Points							
Demand Points	Bonus Values	Demand Points	Bonus Values	Demand Points	Bonus Values	Demand Points	Bonus Values
Y1	95	Y51	100	Y101	90	Y151	93
Y2	95	Y52	100	Y102	90	Y152	93
Y3	95	Y53	100	Y103	90	Y153	93
Y4	95	Y54	100	Y104	90	Y154	93
Y5	95	Y55	100	Y105	90	Y155	93
Y6	95	Y56	100	Y106	90	Y156	93
Y7	95	Y57	100	Y107	90	Y157	93
Y8	95	Y58	100	Y108	90	Y158	93
Y9	95	Y59	100	Y109	90	Y159	93
Y10	95	Y60	100	Y110	90	Y160	93
Y11	95	Y61	100	Y111	90	Y161	85
Y12	95	Y62	100	Y112	90	Y162	85
Y13	95	Y63	100	Y113	90	Y163	85
Y14	95	Y64	100	Y114	90	Y164	85
Y15	95	Y65	100	Y115	90	Y165	85
Y16	95	Y66	100	Y116	90	Y166	85
Y17	95	Y67	100	Y117	90	Y167	85
Y18	95	Y68	100	Y118	90	Y168	85
Y19	95	Y69	100	Y119	90	Y169	85
Y20	95	Y70	100	Y120	90	Y170	85
Y21	95	Y71	100	Y121	90	Y171	85
Y22	95	Y72	100	Y122	90	Y172	85
Y23	95	Y73	100	Y123	90	Y173	85
Y24	95	Y74	100	Y124	90	Y174	85
Y25	95	Y75	100	Y125	90	Y175	85
Y26	100	Y76	100	Y126	90	Y176	85
Y27	100	Y77	100	Y127	90	Y177	85
Y28	100	Y78	100	Y128	90	Y178	85
Y29	100	Y79	100	Y129	90	Y179	85
Y30	100	Y80	100	Y130	90	Y180	85
Y31	100	Y81	100	Y131	90	Y181	100
Y32	100	Y82	100	Y132	90	Y182	100
Y33	100	Y83	100	Y133	90	Y183	100
Y34	100	Y84	100	Y134	90	Y184	100
Y35	100	Y85	100	Y135	90	Y185	100
Y36	100	Y86	100	Y136	90	Y186	100
Y37	100	Y87	100	Y137	90	Y187	100
Y38	100	Y88	100	Y138	90	Y188	100
Y39	100	Y89	100	Y139	90	Y189	100
Y40	100	Y90	100	Y140	90	Y190	100
Y41	100	Y91	100	Y141	93	Y191	100
Y42	100	Y92	100	Y142	93	Y192	100
Y43	100	Y93	100	Y143	93	Y193	100
Y44	100	Y94	100	Y144	93	Y194	100
Y45	100	Y95	100	Y145	93	Y195	100
Y46	100	Y96	100	Y146	93	Y196	100
Y47	100	Y97	100	Y147	93	Y197	100
Y48	100	Y98	100	Y148	93	Y198	100
Y49	100	Y99	100	Y149	93	Y199	100
Y50	100	Y100	100	Y150	93	Y200	100

Appendix C: Mathematical Models

Modified SCLP Model for Patriot PAC-2

MAX

100X1 + 92.75X2 + 85.25X3 + 73.75X4 + 93.05X5 + 100X6 + 97.5X7 + 87.25X8 + 96.25X9 + 92.75X10 + 84.45X11 + 98.8X12 + 77.2X13 + 87.05X14 + 90.05X15 + 86.25X16 + 97.6X17 + 100X18 + 95.8X19 + 98.8X20 + 96.4X21 + 100X22 + 97.3X23 + 91.7X24 + 89.45X25 + 88.6X26 + 90X27 + 98.5X28 + 97.6X29 + 100X30 + 97X31 + 94.85X32 + 84.6X33 + 100X34 + 81.5X35 + 83.65X36 + 78.2X37 + 97X38 + 78.45X39 + 56.75X40 + 56.75X41 + 98.5X42 + 91.5X43 + 91.5X44 + 100X45 + 80.75X46 + 63.45X47 + 79.7X48 + 79.85X49 + 85.85X50 + 92.75X51 + 100X52 + 96.25X53 + 96.25X54 + 79.7X55 + 96.25X56 + 96.25X57 + 82.05X58 + 83.05X59 + 90.5X60 + 90.9X61 + 77.2X62 + 94X63 + 73.75X64 + 82.25X65 + 82.25X66 + 89.85X67 + 91.5X68 + 88.6X69 + 84.6X70 + 85.7X71 + 88.7X72 + 92.2X73 + 92.75X74 + 85.25X75 + 92.6X76 + 83.2X77 + 84.9X78 + 82.75X79 + 94.95X80 + 93.25X81 + 94X82 + 92.75X83 + 82.95X84 +91.75X85 + 86.55X86 + 75.75X87 + 75.75X88 + 75.75X89 + 93.25X90 + 95.2X91 + 95.8X92 + 81.75X93 + 81.75X94 + 81.75X95 + 81.75X96 + 78.75X97 + 100X98 + 86.25X99 + 99.4X100

ST

1] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 >=1

2] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 >=1

3] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 >=1

4] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 +X77 >=1

5] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 +X77 >=1

6] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X74 >=1

7] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 >=1

8] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X76 + X77 + X78 >=1

9] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X76 + X77 + X78 >=1

10] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X76 + X77 + X78 >=1

11] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 + X78 >=1

12] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X77 + X78 >=1

13] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X77 + X78 >=1

14] X2 + X3 + X5 + X6 + X7 + X64 + X65 +X66 + X74 + X77 + X78 >=1

15] X2 + X3 + X4 + X5 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X76 + X77 + X78 >=1

16] X2 + X3 + X4 + X5 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X76 + X77 + X78 >=1

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29] X5 + X6 + X7 + X61 + X64 + X65 + X72 + X77 + X78 >=1

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32] X3 + X2 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X13 + X61 + X63 + X64 +

X65 + X67 + X66 + X71 +X72 + X74 + X76 + X77 + X78 >=1

33] X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X61 + X63 + X64 + X65 + X67 + X66 + X71 +X72 + X74 + X75 + X76 + X77 + X78 >=1

34] X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X61 + X62 + X63 + X64 + X65 + X67 + X66 + X71 +X72 + X74 + X75 + X76 + X77 + X78 + X79 >=1

35] X5 + X6 + X7 + X8 + X9 + X10 + X11 + X13 + X61 + X63 +X67 + X71 + X72 + X77 + X76 + X78 >=1

36] X2 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X61 + X62 + X63 + X64 +X65 + X66 + X67 + X71 + X72 + X77 + X76 + X78 + X79 >=1

37] X5 + X6 + X8 + X9 + X10 + X11 + X12 + X13 + X61 + X62 + X63 + X67 + X71 + X72 +X76 + X78 + X77 + X79 >=1

38] X2 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X15 + X61 + X62 + X63 +X65 + X66 + X67 + X71 + X72 + X76 + X77 + X78 + X79 >=1

39] X5 + X8 + X9 + X10 + X11 + X12 + X13 + X15 + X61 + X62 + X63 + X67 + X71 + X72 +X76 + X78 + X79 >=1

40] X5 + X6 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X60 + X61 + X62 + X63 +X67 + X71 + X72 + X76 + X77 + X78 + X79 >=1

41] X5 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X60 + X61 + X62 + X63 + X67 + X71 + X72 + X78 + X79 >=1

42] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X60 + X61 + X62 + X63 + X67 + X72 + X79 +X99 >=1

43] X5 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19

+ X20 +X60 + X61 + X62 + X63 + X67 + X71 + X72 + X76 + X77 + X78 + X79 + X80 + X99 >=1

44] X5 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X59 + X60 + X61 + X62 + X63 + X67 + X71 + X72 + X79 + X80 + X81 + X99 + X100>=1

45] X5 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X60 + X61 + X62 + X63 + X67 + X71 + X72 + X79 + X99>=1

46] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X59 + X60 + X61 + X62 + X63 + X67 + X69 + X79 + X81 + X80 + X82 + X99 + X100 >=1

47] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X59 + X60 +X61 + X62 + X63 + X67 + X72 + X79 + X80 + X81 + X99 + X100 >=1

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49] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X23 + X22 + X24 + X25 + X59 + X60 + X62 + X63 + X79 + X82 + X80 + X81 + X99 +X100>=1

50] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X23 + X22 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X63 + X79 + X99 + X100 >=1

51] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X23 + X22 + X24 + X25 + X26 + X27 +X59 + X60 + X62 + X63 + X69 +

X79 + X80 +X81 + X82 + X99 + X100 >=1

52] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X23 + X22 + X24 + X25 + X27 + X59 + X60 + X62 + X63 + X67 + X69 + X79 + X80 + X81 + X82 + X83 + X99 + X100 >=1

53] X9 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X23 + X22 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X63 + X69 + X79 + X80 + X81 + X82 + X83 + X84 + X99 + X100 >=1

54] X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X59 + X79 + X60 + X62 + X69 + X82 + X99 + X100 >=1

55] X9 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X63 + X69 + X79 + X82 + X99 + X100>=1

56] X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X63 + X69 + X79 + X80 + X81 + X82 + X83 + X84 + X99 + X100>=1

57] X12 + X11 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X69 + X73 + X79 + X80 + X81 +X82 + X83 + X84 + X85 + X99 + X100 >=1

58] X12 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X69 + X82 >=1

59] X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X69 + X79 + X81 + X82 +

X83 + X99 + X100>=1

60] X12 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 +

X25 + X26 + X27 + X28 + X29 + X59 + X60 + X62 + X69 + X73 + X80 + X81 +

X82 + X83 + X84 + X85 + X99 + X100 >=1

61] X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 +

X59 + X60 + X69 +X73 + X82>=1

62] X12 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 +

X26 +X27 + X59 + X60 + X62 + X69 + X82 + X83 + X84 + X73 + X100 >=1

63] X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 +

X29 + X59 + X69 + X73 + X83 >=1

64] X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 +

X28 + X29 + X59 + X60 + X69 + X73 + X82 + X83 + X84 + X85 +X100 >=1

65] X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 +

X26 + X27 + X28 + X29 + X33 + X59 + X60 + X69 + X73 + X81 + X82 + X83 +

X84 + X85 + X99 + X100 >=1

66] X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 +

X29 + X30 + X33 + X59 + X69 + X73 + X82 + X83 + X84 + X85 +X100 >=1

67] X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 +

X28 + X29 + X69 + X73 +X83 + X84 + X85 >=1

68] X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 +

X28 +X29 + X30 + X33 + X59 + X60 + X69 + X73 + X81 + X82 + X83 + X84 +

X85 + X100 >=1

69] X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X33 +

X69 + X73 + X83 + X84 + X85 >=1

70] X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X33 + X59 + X69 + X73 + X82 + X83 + X84 + X85 >=1

71] X7 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X33 + X59 + X69 + X70 + X73 + X82 + X83 + X84 + X85 + X100 >=1

72] X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X33 + X59 + X69 + X70 + X73 + X82 + X83 + X84 + X85 >=1

73] X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 +X31 + X32 + X33 + X59 + X69 + X70 + X73 + X82 + X83 + X84 + X85 + X86 >=1

74] X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X33 + X69 + X70 + X73 + X82 + X83 + X84 + X85 >=1

75] X21 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X33 + X69 + X73 + X83 + X84 + X85 >=1

76] X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X69 + X70 + X73 + X82 + X83 + X84 + X85 >=1

77] X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X69 + X70 + X73 + X82 + X83 + X84 + X85 + X86 >=1

78] X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X69 + X70 + X73 + X83 + X84 + X85 >=1

79] X21 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X33 + X69 + X73 + X83 + X84 + X85 >=1

80] X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X69 + X70 + X73 + X83 + X84 + X85 >=1

81] X21 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X69 + X70 + X73 + X84 + X85 + X86>=1

82] X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X69 + X70 + X73 + X84 + X85 >=1

83] X21 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X69 + X70 + X73 + X83 +X84 + X85>=1

84] X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X69 + X70 + X73 + X83 +X84 + X85 + X86 >=1

85] X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X70 + X73 + X84 + X85 + X86 >=1

86] X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X69 + X70 + X73 + X83 + X84 + X85 + X86 >=1

87] X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X69 + X70 + X73 + X83 + X84 + X85 + X86 >=1

88] X26 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X70 + X73 + X84 + X85 + X86>=1

89] X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X70 + X73 + X84 + X85 + X86 >=1

90] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X70 + X73 + X85 + X86 >=1

91] X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X40 +

X70 + X73 + X83 + X84 + X85 + X86 +X87 >=1

92] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X40 + X70 + X73 + X84 + X85 + X86 +X87 >=1

93] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X38 + X40 + X41 + X70 + X73 + X84 + X85 + X86 +X87 +X88 >=1

94] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X70 + X73 + X84 + X85 + X86 +X87 >=1

95] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X38 + X40 + X70 + X86 + X87 >=1

96] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X38 + X40 + X41 + X70 + X86 + X87 + X88 >=1

97] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X70 + X85 + X86 + X87 + X88 >=1

98] X30 + X31 + X32 + X33 + X34 + X35 + X36 + X38 + X40 + X41 + X70 + X86 + X87 >=1

99] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X70 + X85 + X86 + X87 + X88 + X89 + X90 >=1

100] X28 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X70 + X86 + X87 + X88 >=1

101] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X86 + X87 + X88 >=1

102] X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X70 + X86 + X87 + X88 + X89 + X90 >=1

103] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X55 + X70 + X86 + X87 + X88 + X89 >=1

104] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X55 + X70 + X86 + X87 + X88 + X89 + X90 >=1

105] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X55 + X86 + X87 + X88 +X89 >=1

106] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X86 + X87 + X88 + X89 +X90 >=1

107] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X55 + X68 + X86 + X87 + X88 + X89 +X90 >=1

108] X32 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X55 + X86 + X87 + X88 + X89 +X90>=1

109] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X87 + X88 + X89 >=1

110] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X87 + X88 + X89 + X90 >=1

111] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X87 + X88 + X89 + X90 >=1

112] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X55 + X68 + X87 + X86 + X88 + X89 +X90 +X91 >=1

113] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X55 + X68 + X87 + X88 + X89 + X90 >=1

114] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X55 + X68 +

X87 + X88 + X89 + X90 +X91 >=1

115] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X55 + X87 + X88 + X89 + X90 >=1

116] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X55 + X68 + X87 + X88 + X89 + X90 >=1

117] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X55 + X68 + X87 + X88 + X89 + X90 >=1

118] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 >=1

119] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 >=1

120] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X55 + X68 + X87 + X88 + X89 +X90 + X91 >=1

121] X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X55 + X68 + X87 + X88 + X89 +X90 + X91>=1

122] X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 >=1

123] X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X47 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 +X93 >=1

124] X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X47 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 +X93 >=1

125] X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X47 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 +X93 >=1

126] X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 +X93 >=1

127] X37 + X38 + X39 + X41 + X42 + X43 + X44 + X45 + X46 + X47 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 +X93 >=1

128] X37 + X38 + X39 + X41 + X42 + X43 + X44 + X45 + X46 + X47 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 +X93 + X94 + X96 >=1

129] X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X45 + X46 + X47 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 +X93 + X96 >=1

130] X37 + X39 + X41 + X42 + X43 + X44 + X45 + X46 + X47 + X55 + X68 + X88 + X89 +X90 + X91 + X92 +X93 + X94 + X96 >=1

131] X37 + X39 + X40 + X42 + X43 + X44 + X45 + X46 + X47 + X49 + X55 + X68 + X88 + X89 + X90 + X91 + X92 +X93 + X94 + X96 >=1

132] X37 + X38 + X39 + X42 + X43 + X41 + X44 + X45 + X46 + X47 + X55 + X68 + X88 + X89 +X90 + X91 + X92 +X93 + X94 + X96 >=1

133] X38 + X37 + X39 + X41 + X42 + X43 + X44 + X45 + X46 + X47 + X55 + X68 + X88 + X89 +X90 + X91 + X92 +X93 + X96 >=1

134] X37 + X39 + X42 + X43 + X44 + X45 + X46 + X47 + X49 + X55 + X68 + X89 +X90 + X91 + X92 +X93 + X94 + X96 >=1

135] X39 + X42 + X43 + X44 + X45 + X46 + X47 + X48 + X49 + X55 + X68 + X89 + X90 + X91 + X92 +X93 + X94 + X95 + X96 >=1

136] X43 + X44 + X45 + X46 + X47 + X48 + X49 + X68 + X91 + X92 +X93 + X94 + X95 + X96 >=1

137] X37 + X39 + X42 + X43 + X44 + X45 + X46 + X47 + X49 + X55 + X68 + X89

+X91 + X92 +X93 + X94 + X96 >=1

138] X39 + X43 + X44 + X45 + X46 + X47 + X49 + X55 + X68 + X89 + X91 + X92

+X93 + X94 + X96 >=1

139] X43 + X44 + X45 + X46 + X47 + X48 + X49 + X91 + X92 +X93 + X94 + X95 +

X96 + X97 >=1

140] X43 + X44 + X45 + X46 + X47 + X48 + X68 + X49 + X91 + X92 +X93 + X94 +

X95 + X96 >=1

141] X43 + X44 + X45 + X46 + X47 + X49 + X68 + X91 + X92 +X93 + X94 + X95 +

X96>=1

142] X43 + X44 + X45 + X46 + X47 + X48 + X49 + X91 + X92 +X93 + X94 + X95 +

X96 >=1

143] X44 + X45 + X46 + X47 + X48 + X49 + X91 + X92 +X93 + X94 + X95 + X96 +

X97 >=1

144] X44 + X45 + X46 + X47 + X48 + X49 + X92 +X93 + X94 + X95 + X96 + X97>=1

145] X44 + X45 + X46 + X47 + X48 + X49 + X50 + X91 + X92 +X93 + X94 + X95 +

X96 + X97 >=1

146] X44 + X45 + X46 + X47 + X48 + X49 + X50 + X92 +X93 + X94 + X95 + X96 +

X97 >=1

147] X44 + X45 + X46 + X47 + X48 + X49 + X50 + X92 +X93 + X94 + X95 + X96 +

X97 >=1

148] X44 + X45 + X46 + X47 + X48 + X49 + X50 + X51 + X92 +X93 + X94 + X95 +

X96 + X97 + X98 >=1

149] X45 + X46 + X47 + X48 + X49 + X50 + X51 + X92 +X93 + X94 + X95 + X96 +

X97>=1

150] X45 + X46 + X47 + X48 + X49 + X50 + X51 +X53 + X92 + X93 + X94 + X95 + X96 + X97 + X98>=1

151] X45 + X46 + X47 + X48 + X49 + X50 + X51 + X53 + X92 + X93 + X94 + X95 + X96 + X97 + X98>=1

152] X45 + X46 + X47 + X48 + X49 + X50 + X51 + X92 + X93 + X94 + X95 + X96 + X97 >=1

153] X45 + X46 + X47 + X48 + X49 + X50 + X51 + X93 + X94 + X95 + X96 + X97 + X98 >=1

154] X45 + X47 + X48 + X49 + X50 + X51 + X52 + X53 + X56 + X58 + X93 + X94 + X95 + X96 + X97 + X98 >=1

155] X48 + X49 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X94 + X95 + X96 + X97 + X98 >=1

156] X48 + X49 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

157] X45 + X47 + X48 + X49 + X50 + X51 + X53 + X93 + X94 + X95 + X96 + X97 + X98 >=1

158] X45 + X47 + X48 + X49 + X50 + X51 + X53 + X94 + X95 + X96 + X97 + X98 >=1

159] X45 + X48 + X49 + X50 + X51 + X52 + X53 + X58 + X94 + X95 + X96 + X97 + X98 >=1

160] X45 + X47 + X48 + X49 + X50 + X51 + X52 + X53 + X93 + X94 + X95 + X96 + X97 + X98 >=1

161] X45 + X48 + X49 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X94 + X95 + X96 + X97 + X98 >=1

162] X48 + X49 + X50 + X51 + X52 + X53 + X56 + X57 + X58 + X94 + X95 + X96 + X97 + X98>=1

163] X45 + X48 + X49 + X50 + X51 + X53 + X58 + X94 + X95 + X96 + X97 + X98 >=1

164] X48 + X49 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

165] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

166] X48 + X49 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

167] X48 + X49 + X50 + X51 + X52 + X53 + X56 + X57 + X58 + X94 + X95 + X97 + X98 >=1

168] X48 + X49 + X50 + X51 + X52 + X53 + X57 + X58 + X95 + X97 + X98 >=1

169] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

170] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X97 + X98 >=1

171] X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X98 >=1

172] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

173] X48 + X50 + X51 + X52 + X53 + X54 +X56 + X57 + X58 + X95 + X97 + X98 >=1

174] X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X97 + X98 >=1

175] X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X97 + X98 >=1

176] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X97 + X98 >=1

177] X50 + X51 + X52 + X53 + X54 + X57 + X58 + X97 + X98 >=1

178] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X97 + X98 >=1

179] X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X98 >=1

180] X51 + X52 + X53 + X54 + X56 + X57 + X58 + X98 >=1

181] X6 + X5 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X60 + X61 + X62 +X63 + X66 + X67 + X71 + X72 + X76 + X77 + X78 + X79 + X80 + X99 >=1

182] X2 + X6 + X7 + X5 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X60 + X61 + X62 +X63 + X64 + X65 + X66 + X67 + X71 + X72 + X74 + X75 + X76 + X77 + X78 + X79 + X80 + X99 >=1

183] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X20 + X21 + X23 + X22 + X24 + X25 + X59 + X60 + X61 + X62 + X63 + X67 + X69 + X71 + X72 + X79 + X80 + X81 + X82 + X99 + X100 >=1

184] X8 + X9 + X10 + X11 + X12 + X13 + X15 + X60 + X61 + X62 + X63 + X67 + X71 + X72 + X76 + X77 + X78 + X79 +X99 >=1

185] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X19 + X20 + X21 + X22 + X59 + X60 + X61 + X62 + X63 + X67 + X79 + X99 >=1

186] X5 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X59 + X69 + X70 + X73 + X80 + X83 + X84 + X85 + X86 >=1

187] X12 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 +

X26 + X27 + X59 + X60 + X62 + X69 + X73 + X82 + X83 + X100 >=1

188] X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 +X59 + X69 + X73 + X82 + X83 + X84 + X85 >=1

189] X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X33 + X59 + X60 + X69 + X73 + X82 + X83 + X84 + X85 + X100 >=1

190] X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X33 + X69 + X73 + X83 + X84 + X85 >=1

191] X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X69 + X70 + X73 + X83 + X84 + X85 + X86 >=1

192] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X38 + X40 + X41 + X70 + X86 + X87 + X88 >=1

193] X30 + X31 + X32 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X70 + X86 + X87 + X88 >=1

194] X28 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X70 + X86 + X87 + X88 + X89 +X90 >=1

195] X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X70 + X86 + X87 + X88 + X89 >=1

196] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X38 + X40 + X41 + X70 + X73 + X85 + X86 +X87 + X88 >=1

197] X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X61 + X62 + X63 + X65 + X66 + X67 + X71 + X72 + X77 +X78 + X79 >=1

198] X2 + X6 + X5 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 +

X60 + X61 + X62 +X63 + X65 + X66 + X67 + X71 + X72 + X75 + X76 + X77 + X78 + X79 + X80 + X99 >=1

199] X2 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X13 + X61 + X63 + X64 + X65 + X66 + X67 + X71 + X72 + X77 + X76 + X78>=1

200] X2 + X5 + X6 + X7 + X8 + X9 + X10 + X12 + X14 + X15 + X61 + X62 + X63 + X64 + X65 + X67 + X66 + X71 +X72 + X74 + X75 + X76 + X77 + X78 >=1

201] X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X45 + X46 + X47 + X48 + X49 + X50 + X51 + X52 + X53 + X54 + X55 + X56 + X57 + X58 + X59 + X60 + X61 + X62 + X63 + X64 + X65 + X66 + X67 + X68 + X69 + X70 + X71 + X72 + X73 + X74 + X75 + X76 + X77 + X78 + X79 + X80 + X81 + X82 + X83 + X84 + X85 + X86 + X87 + X88 + X89 + X90 + X91 +X92 + X93 + X94 + X95 + X96 + X97 + X98 + X99 + X100 <=6

END

ALL VARIABLES ARE BINARY.

Modified SCLP Model for S-300 PMU-1

MAX

100X1 + 92.75X2 + 85.25X3 + 73.75X4 + 93.05X5 + 100X6 + 97.5X7 + 87.25X8 + 96.25X9 + 92.75X10 + 84.45X11 + 98.8X12 + 77.2X13 + 87.05X14 + 90.05X15 + 86.25X16 + 97.6X17 + 100X18 + 95.8X19 + 98.8X20 + 96.4X21 + 100X22 + 97.3X23 + 91.7X24 + 89.45X25 + 88.6X26 + 90X27 + 98.5X28 + 97.6X29 + 100X30 + 97X31 + 94.85X32 + 84.6X33 + 100X34 + 81.5X35 + 83.65X36 + 78.2X37 + 97X38 + 78.45X39 + 56.75X40 + 56.75X41 + 98.5X42 + 91.5X43 + 91.5X44 + 100X45 + 80.75X46 + 63.45X47 + 79.7X48 + 79.85X49 + 85.85X50 + 92.75X51 + 100X52 + 96.25X53 + 96.25X54 + 79.7X55 + 96.25X56 + 96.25X57 + 82.05X58 + 83.05X59 + 90.5X60 + 90.9X61 + 77.2X62 + 94X63 + 73.75X64 + 82.25X65 + 82.25X66 + 89.85X67 + 91.5X68 + 88.6X69 + 84.6X70 + 85.7X71 + 88.7X72 + 92.2X73 + 92.75X74 + 85.25X75 + 92.6X76 + 83.2X77 + 84.9X78 + 82.75X79 + 94.95X80 + 93.25X81 + 94X82 + 92.75X83 + 82.95X84 +91.75X85 + 86.55X86 + 75.75X87 + 75.75X88 + 75.75X89 + 93.25X90 + 95.2X91 + 95.8X92 + 81.75X93 + 81.75X94 + 81.75X95 + 81.75X96 + 78.75X97 + 100X98 + 86.25X99 + 99.4X100

ST

1] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75>=1

2] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75>=1

3] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 >=1

4] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 >=1

5] X2 + X3 + X4 + X6 + X7 + X64 + X65 + X74 >=1

6] X2 + X3 + X6 + X7 + X64 + X65 >=1

7] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 >=1

8] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 >=1

9] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 >=1

10]X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 >=1

11] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 + X78 >=1

12] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X77 + X78 >=1

13] X2 + X3 + X6 + X7 + X64 + X65 + X77 + X78>=1

14] X2 + X3 + X6 + X7 + X64 + X65 +X66 + X77 + X78 >=1

15] X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X77 + X78 >=1

16] X2 + X3 + X4 + X5 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X76 + X77 + X78 >=1

17] X1 + X2 + X3 + X4 + X5 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X76 + X77 + X78 >=1

18] X1 + X2 + X3 + X4 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X77 >=1

19] X1 + X2 + X3 + X4 + X5 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X76 + X77 + X78 >=1

20] X1 + X2 + X3 + X4 + X5 + X6 + X7 + X64 + X65 +X66 + X74 + X75 + X76 + X77 + X78 >=1

21] X2 + X3 + X4 + X5 + X6 + X7 + X61 + X64 + X65 +X66 + X72 + X74 + X75 +

X76 + X77 + X78 >=1

22] X2 + X3 + X4 + X5 + X6 + X7 + X61 + X64 + X65 +X66 + X72 + X74 + X76 +

X77 + X78 >=1

23] X2 + X3 + X5 + X6 + X7 + X64 + X65 +X66 + X77 + X78 >=1

24] X2 + X3 + X4 + X5 + X6 + X7 + X61 + X64 + X65 +X66 + X71 + X72 + X74 +

X75 +X76 + X77 + X78 >=1

25] X2 + X5 + X6 + X7 + X61 + X64 + X65 + X77 + X78 >=1

26] X2 + X3 + X4 + X5 + X6 + X7 + X8 + X10 + X61 + X64 + X65 +X66 + X67 + X71

+ X72 + X74 + X75 +X76 + X77 + X78 >=1

27] X2 + X3 + X5 + X6 + X7 + X8 + X61 + X64 + X65 +X66 + X71 + X72

+ X74 +X76 + X77 + X78 >=1

28] X5 + X6 + X7 + X61 + X64 + X65 + X77 + X78 >=1

29] X5 + X6 + X7 + X61 + X64 + X65 + X72 + X77 + X78 >=1

30] X2 + X5 + X6 + X7 + X8 + X10 + X61 + X64 + X65 +X66 + X71 + X72

+ X76 + X77 + X78 >=1

31] X5 + X6 + X7 + X8 + X10 + X61 + X72 + X77 + X78 >=1

32] X2 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X61 + X63 + X64 + X65 + X67 +

X66 + X71 +X72 + X76 + X77 + X78 >=1

33] X2 + X3 + X5 + X6 + X7 + X8 + X9 + X10 + X61 + X63 + X64 + X65 + X67

+ X66 + X71 +X72 + X74 + X75 + X76 + X77 + X78 >=1

34] X2 + X3 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X13 + X61 + X63 + X64 +

X65 + X67 + X66 + X71 +X72 + X74 + X75 + X76 + X77 + X78 + X79 >=1

35] X5 + X6 + X7 + X8 + X9 + X10 + X11 + X61 + X63 +X67 + X71 + X72 + X77 +

X76 + X78 >=1

36] X2 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X61 + X63 + X65 + X66 + X67 + X71 + X72 + X77 + X76 + X78 + X79 >=1

37] X5 + X8 + X9 + X10 + X11 + X12 + X13 + X61 + X62 + X63 + X67 + X71 + X72 + X78 + X79 >=1

38] X5 + X6 + X8 + X9 + X10 + X11 + X12 + X13 + X61 + X62 + X63 + X66 + X67 + X71 + X72 + X76 + X77 + X78 + X79 >=1

39] X5 + X8 + X9 + X10 + X11 + X12 + X13 + X61 + X62 + X63 + X67 + X71 + X72 + X78 + X79 >=1

40] X5 + X6 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X60 + X61 + X62 + X63 + X67 + X71 + X72 + X76 + X77 + X78 + X79 >=1

41] X8 + X9 + X10 + X11 + X12 + X13 + X15 + X60 + X61 + X62 + X63 + X67 + X72 + X79 >=1

42] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X17 + X20 + X60 + X61 + X62 + X63 + X67 + X79 >=1

43] X5 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X60 + X61 + X62 + X63 + X67 + X71 + X72 + X76 + X78 + X79 + X80 + X99 >=1

44] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X59 + X60 + X61 + X62 + X63 + X67 + X71 + X72 + X79 + X80 + X99 + X100>=1

45] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X17 + X60 + X61 + X62 + X63 + X67 + X71 + X72 + X79 >=1

46] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20

+ X21 + X22 + X23 + X24 + X25 + X59 + X60 + X62 + X63 + X67 + X79 + X81 + X80 + X82 + X99 + X100 >=1

47] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X59 + X60 +X61 + X62 + X63 + X67 + X79 + X80 + X99 + X100 >=1

48] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X24 + X59 + X60 + X62 + X63 + X79 + X99 >=1

49] X8 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X23 + X22 + X24 + X25 + X59 + X60 + X62 + X63 + X79 + X99 >=1

50] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X23 + X22 + X24 + X25 + X59 + X60 + X62 + X63 + X79 + X99 + X100 >=1

51] X8 + X9 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X23 + X22 + X24 + X25 + X59 + X60 + X62 + X63 + X69 + X79 + X80 + X81 + X82 + X99 + X100 >=1

52] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X23 + X22 + X24 + X25 + X59 + X60 + X62 + X63 + X67 + X69 + X79 + X80 + X81 + X82 + X99 + X100 >=1

53] X9 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X23 + X22 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X63 + X69 + X79 + X80 + X81 + X82 + X83 +X99 + X100 >=1

54] X11 + X12 + X13 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X59 + X60 + X62 + X69 >=1

55] X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 +X24 + X25 + X59 + X60 + X62 + X63 + X69 + X79 + X82 + X99 + X100>=1

56] X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X63 + X69 + X79 + X80 + X81 + X82 + X83 + X99 + X100>=1

57] X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X69 + X73 + X79 + X80 + X81 + X82 + X83 + X84 + X85 + X99 + X100 >=1

58] X12 + X15 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X69 >=1

59] X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 +X24 + X25 + X26 + X27 + X59 + X60 + X62 + X69 + X82 + X83 + X99 + X100>=1

60] X12 + X14 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X69 + X73 + X81 + X82 + X83 + X84 + X85 + X99 + X100 >=1

61] X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X69 >=1

62] X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X69 + X82 + X83 + X73 + X100 >=1

63] X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X69 + X73 >=1

64] X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X29 +

X59 + X69 + X73 + X82 + X83 + X84 >=1

65] X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 +

X26 + X27 + X28 + X29 + X59 + X60 + X69 + X73 + X81 + X82 + X83 + X84 +

X85 + X100 >=1

66] X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 +

X29 + X33 + X59 + X69 + X73 + X82 + X83 + X84 + X85 >=1

67] X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 +

X29 + X69 + X73 >=1

68] X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 +

X28 + X29 + X30 + X33 + X59 + X60 + X69 + X73 + X82 + X83 + X84 + X85 +

X100 >=1

69] X20 + X21 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X69 + X73 +

X85 >=1

70] X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 +

X29 + X30 + X33 + X59 + X69 + X73 + X82 + X83 + X84 + X85 >=1

71] X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 +

X29 + X30 + X33 + X59 + X69 + X73 + X82 + X83 + X84 + X85 >=1

72] X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 +

X29 + X30 + X33 + X59 + X69 + X73 + X82 + X83 + X84 + X85 >=1

73] X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 +

X31 + X32 + X33 + X59 + X69 + X70 + X73 + X82 + X83 + X84 + X85>=1

74] X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 +

X33 + X69 + X73 + X83 + X84 + X85 >=1

75] X21 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X33 + X69 + X73 + X84 + X85 >=1

76] X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X69 + X70 + X73 + X83 + X84 + X85 >=1

77] X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X69 + X70 + X73 + X82 + X83 + X84 + X85 + X86 >=1

78] X21 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X33 + X69 + X73 + X83 + X84 + X85 >=1

79] X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X33 + X69 + X73 + X84 + X85 >=1

80] X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X69 + X70 + X73 + X84 + X85 >=1

81] X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X69 + X70 + X73 + X83 + X84 + X85 >=1

82] X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X70 + X73 + X84 + X85 >=1

83] X21 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X69 + X70 + X73 + X83 +X84 + X85>=1

84] X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X69 + X70 + X73 + X83 +X84 + X85 >=1

85] X26 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X70 + X73 + X84 + X85 >=1

86] X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X69 + X70 + X73 + X83 + X84 + X85 + X86 >=1

87] X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X69 + X70 +

X73 + X83 + X84 + X85 + X86 >=1

88] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X70 + X73 + X85 + X86>=1

89] X26 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X70 + X73 +

X84 + X85 + X86 >=1

90] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X70 + X73 + X85 +

X86 >=1

91] X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X40 +

X70 + X73 + X83 + X84 + X85 + X86 >=1

92] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X40 + X70 + X73 +

X85 + X86 +X87 >=1

93] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X38 + X40 + X41 +

X70 + X73 + X84 + X85 + X86 +X87 >=1

94] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X40 +

X41 + X70 + X73 + X84 + X85 + X86 +X87 >=1

95] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X70 + X86 + X87 >=1

96] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X38 + X40 + X70 +

X86 + X87 >=1

97] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X40 +

X41 + X70 + X85 + X86 + X87 + X88 >=1

98] X30 + X31 + X32 + X34 + X35 + X36 + X38 + X40 + X70 + X86 + X87 >=1

99] X28 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X40 + X41 +

X55 + X70 + X86 + X87 + X88 + X89 + X90 >=1

100] X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X70 +

X86 + X87 + X88 >=1

101] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X87 + X88 >=1

102] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X70 + X86 + X87 + X88 + X89 + X90 >=1

103] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X55 + X70 + X86 + X87 + X88 + X89 >=1

104] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X70 + X86 + X87 + X88 + X89 + X90 >=1

105] X31 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X55 + X87 + X88 >=1

106] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X86 + X87 + X88 + X89 +X90 >=1

107] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X55 + X68 + X86 + X87 + X88 + X89 +X90 >=1

108] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X55 + X86 + X87 + X88 + X89 +X90>=1

109] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X87 + X88 + X89 >=1

110] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X87 + X88 + X89 + X90 >=1

111] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X87 + X88 + X89 + X90 >=1

112] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X55 + X68 +

X87 + X88 + X89 +X90 +X91 >=1

113] X34 + X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X55 + X87 + X88 + X89 + X90 >=1

114] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X55 + X68 + X87 + X88 + X89 + X90 >=1

115] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X55 + X87 + X88 + X89 + X90 >=1

116] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X55 + X68 + X87 + X88 + X89 + X90 >=1

117] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X55 + X87 + X88 + X89 + X90 >=1

118] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 >=1

119] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X55 + X68 + X87 + X88 + X89 +X90 + X91 >=1

120] X35 + X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X55 + X68 + X87 + X88 + X89 +X90 + X91 >=1

121] X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X55 + X68 + X87 + X88 + X89 +X90 + X91>=1

122] X36 + X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 >=1

123] X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X47 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 +X93 >=1

124] X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X47 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 +X93 >=1

125] X37 + X38 + X39 + X40 + X41 + X42 + X43 + X44 + X46 + X47 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 +X93 >=1

126] X37 + X38 + X39 + X41 + X42 + X43 + X44 + X46 + X55 + X68 + X87 + X88 + X89 +X90 + X91 + X92 >=1

127] X37 + X38 + X39 + X41 + X42 + X43 + X44 + X46 + X47 + X55 + X68 + X88 + X89 +X90 + X91 + X92 +X93 >=1

128] X37 + X38 + X39 + X41 + X42 + X43 + X44 + X45 + X46 + X47 + X55 + X68 + X88 + X89 +X90 + X91 + X92 +X93 + X96 >=1

129] X37 + X38 + X39 + X41 + X42 + X43 + X44 + X45 + X46 + X47 + X55 + X68 + X88 + X89 +X90 + X91 + X92 +X93 + X96 >=1

130] X37 + X39 + X41 + X42 + X43 + X44 + X45 + X46 + X47 + X55 + X68 + X88 + X89 +X90 + X91 + X92 +X93 + X94 + X96 >=1

131] X37 + X39 + X42 + X43 + X44 + X45 + X46 + X47 + X49 + X55 + X68 + X89 +X90 + X91 + X92 +X93 + X94 + X96 >=1

132] X37 + X39 + X42 + X43 + X44 + X45 + X46 + X47 + X55 + X68 + X88 + X89 +X90 + X91 + X92 +X93 + X94 + X96 >=1

133] X38 + X37 + X39 + X42 + X43 + X44 + X45 + X46 + X47 + X55 + X68 + X88 + X89 + X90 + X91 + X92 +X93 >=1

134] X37 + X39 + X42 + X43 + X44 + X45 + X46 + X47 + X49 + X55 + X68 + X89 +X90 + X91 + X92 +X93 + X94 + X96 >=1

135] X39 + X42 + X43 + X44 + X45 + X46 + X47 + X49 + X68 + X91 + X92 +X93 +

X94 +X95 + X96 >=1

136] X43 + X44 + X45 + X46 + X47 + X49 + X68 + X91 + X92 +X93 + X94 + X95 + X96 >=1

137] X39 +X43 + X44 + X45 + X46 + X47 + X55 + X68 + X89 +X91 + X92 +X93 + X94 + X96 >=1

138] X43 + X44 + X45 + X46 + X47 + X49 + X68 + X91 + X92 +X93 + X94 + X96 >=1

139] X43 + X44 + X45 + X46 + X47 + X48 + X49 + X91 + X92 +X93 + X94 + X95 + X96 + X97 >=1

140] X43 + X44 + X45 + X46 + X47 + X48 + X49 + X91 + X92 +X93 + X94 + X95 + X96 >=1

141] X43 + X44 + X45 + X46 + X47 + X49 + X91 + X92 +X93 + X94 + X96>=1

142] X44 + X45 + X46 + X47 + X48 + X49 + X91 + X92 +X93 + X94 + X95 + X96 >=1

143] X44 + X45 + X46 + X47 + X48 + X49 + X91 + X92 +X93 + X94 + X95 + X96 + X97 >=1

144] X44 + X45 + X46 + X47 + X48 + X49 + X92 +X93 + X94 + X95 + X96 + X97>=1

145] X44 + X45 + X46 + X47 + X48 + X49 + X50 + X91 + X92 +X93 + X94 + X95 + X96 + X97 >=1

146] X44 + X45 + X46 + X47 + X48 + X49 + X50 + X92 +X93 + X94 + X95 + X96 + X97 >=1

147] X44 + X45 + X46 + X47 + X48 + X49 + X50 + X92 +X93 + X94 + X95 + X96 + X97 >=1

148] X44 + X45 + X46 + X47 + X48 + X49 + X50 + X51 + X92 +X93 + X94 + X95 + X96 + X97>=1

149] X45 + X46 + X47 + X48 + X49 + X50 + X92 +X93 + X94 + X95 + X96 + X97>=1

150] X45 + X46 + X47 + X48 + X49 + X50 + X51 + X93 + X94 + X95 + X96 + X97 + X98>=1

151] X45 + X46 + X47 + X48 + X49 + X50 + X51 + X53 + X93 + X94 + X95 + X96 + X97 + X98>=1

152] X45 + X46 + X47 + X48 + X49 + X50 +X93 + X94 + X95 + X96 + X97 >=1

153] X45 + X47 + X48 + X49 + X50 + X51 + X93 + X94 + X95 + X96 + X97 >=1

154] X45 + X47 + X48 + X49 + X50 + X51 + X52 + X53 + X94 + X95 + X96 + X97 + X98 >=1

155] X48 + X49 + X50 + X51 + X52 + X53 + X56 + X57 + X58 + X94 + X95 + X96 + X97 + X98 >=1

156] X48 + X49 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

157] X45 + X47 + X48 + X49 + X50 + X51 + X53 + X93 + X94 + X95 + X96 + X97 + X98 >=1

158] X45 + X48 + X49 + X50 + X51 + X53 + X94 + X95 + X96 + X97 + X98 >=1

159] X45 + X48 + X49 + X50 + X51 + X53 + X94 + X95 + X96 + X97 + X98 >=1

160] X45 + X47 + X48 + X49 + X50 + X51 + X52 + X53 + X94 + X95 + X96 + X97 + X98 >=1

161] X48 + X49 + X50 + X51 + X52 + X53 + X56 + X57 + X58 + X94 + X95 + X96 +

X97 + X98 >=1

162] X48 + X49 + X50 + X51 + X52 + X53 + X57 + X58 + X94 + X95 + X97 + X98>=1

163] X45 + X48 + X49 + X50 + X51 + X53 + X94 + X95 + X97 + X98 >=1

164] X48 + X49 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

165] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

166] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

167] X48 + X49 + X50 + X51 + X52 + X53 + X57 + X58 + X95 + X97 + X98 >=1

168] X48 + X50 + X51 + X53 + X58 + X95 + X97 + X98 >=1

169] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X97 + X98 >=1

170] X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X97 + X98 >=1

171] X51 + X52 + X53 + X54 + X56 + X57 + X58 + X98 >=1

172] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X95 + X97 + X98 >=1

173] X48 + X50 + X51 + X52 + X53 + X57 + X58 + X97 + X98 >=1

174] X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X98 >=1

175] X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X97 + X98 >=1

176] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X97 + X98 >=1

177] X50 + X51 + X52 + X53 + X54 + X57 + X58 + X98 >=1

178] X48 + X50 + X51 + X52 + X53 + X54 + X56 + X57 + X58 + X97 + X98 >=1

179] X51 + X52 + X53 + X54 + X56 + X57 + X58 + X98 >=1

180] X52 + X53 + X54 + X56 + X57 + X58 + X98 >=1

181] X6 + X5 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X60 + X61 + X62 +X63 + X66 + X67 + X71 + X72 + X76 + X77 + X78 + X79 + X80 + X99 >=1

182] X2 + X6 + X7 + X5 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X61 + X62 +X63 + X65 + X66 + X67 + X71 + X72 + X74 + X75 + X76 + X77 + X78 + X79 >=1

183] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X18 + X20 + X21 + X23 + X22 + X24 + X59 + X60 + X61 + X62 + X63 + X67 + X69 + X71 + X72 + X79 + X80 + X81 + X82 + X99 + X100 >=1

184] X8 + X9 + X10 + X11 + X12 + X13 + X15 + X60 + X61 + X62 + X63 + X67 + X71 + X72 + X76 + X78 + X79 >=1

185] X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X16 + X17 + X19 + X21 + X20 + X59 + X60 + X61 + X62 + X63 + X67 + X79 + X99 >=1

186] X25 + X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X69 + X70 + X73 + X83 + X84 + X85 + X86 >=1

187] X12 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X59 + X60 + X62 + X69 + X82 + X100 >=1

188] X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X59 + X69 + X73 + X82 + X83 + X84 + X85 >=1

189] X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X33 + X59 + X60 + X69 + X73 + X82 + X83 + X84 + X85 + X100

$>=1$

190] X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 + X28 + X29 + X30 + X33 + X69 + X73 + X83 + X84 + X85 >=1

191] X26 + X27 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X70 + X73 + X84 + X85 + X86 >=1

192] X30 + X31 + X32 + X33 + X34 + X35 + X36 + X38 + X40 + X41 + X70 + X86 + X87 >=1

193] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X70 + X86 + X87 + X88 >=1

194] X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X55 + X70 + X86 + X87 + X88 + X89 >=1

195] X31 + X32 + X34 + X35 + X36 + X37 + X38 + X40 + X41 + X55 + X70 + X86 + X87 + X88 + X89 >=1

196] X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X38 + X40 + X41 + X70 + X73 + X85 + X86 +X87 >=1

197] X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X61 + X62 + X63 + X67 + X71 + X72 + X77 +X78 + X79 >=1

198] X6 + X5 + X8 + X9 + X10 + X11 + X12 + X13 + X14 + X15 + X60 + X61 + X62 +X63 + X66 + X67 + X71 + X72 + X76 + X77 + X78 + X79 + X80 + X99 >=1

199] X5 + X6 + X7 + X8 + X9 + X10 + X11 + X61 + X63 + X65 + X66 + X67 + X71 + X72 + X77 + X76 + X78>=1

200] X2 + X5 + X6 + X7 + X8 + X9 + X10 + X12 + X61 + X62 + X63 + X64 + X65 + X67 + X66 + X71 +X72 + X74 + X75 + X76 + X77 + X78 >=1

201] X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X14

+ X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27

+ X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X39 + X40

+ X41 + X42 + X43 + X44 + X45 + X46 + X47 + X48 + X49 + X50 + X51 + X52 + X53

+ X54 + X55 + X56 + X57 + X58 + X59 + X60 + X61 + X62 + X63 + X64 + X65 + X66

+ X67 + X68 + X69 + X70 + X71 + X72 + X73 + X74 + X75 + X76 + X77 + X78 + X79

+ X80 + X81 + X82 + X83 + X84 + X85 + X86 + X87 + X88 + X89 + X90 + X91 + X92

+ X93 + X94 + X95 + X96 + X97 + X98 + X99 + X100 <=7

END

ALL VARIABLES ARE BINARY.

Modified MCLP Model for Arrow-2

MAX

95Y1 + 95Y2 + 95Y3 + 95Y4 + 95Y5 + 95Y6 + 95Y7 + 95Y8 + 95Y9 + 95Y10 +

95Y11 + 95Y12 + 95Y13 + 95Y14 + 95Y15 + 95Y16 + 95Y17 + 95Y18 + 95Y19 +

95Y20 + 95Y21 + 95Y22 + 95Y23 + 95Y24 + 95Y25 + 100Y26 + 100Y27 + 100Y28 +

100Y29 + 100Y30 + 100Y31 + 100Y32 + 100Y33 + 100Y34 + 100Y35 + 100Y36 +

100Y37 + 100Y38 + 100Y39 + 100Y40 + 100Y41 + 100Y42 + 100Y43 + 100Y44 +

100Y45 + 100Y46 + 100Y47 + 100Y48 + 100Y49 + 100Y50 + 100Y51 + 100Y52 +

100Y53 + 100Y54 + 100Y55 + 100Y56 + 100Y57 + 100Y58 + 100Y59 + 100Y60 +

100Y61 + 100Y62 + 100Y63 + 100Y64 + 100Y65 + 100Y66 + 100Y67 + 100Y68 +

100Y69 + 100Y70 + 100Y71 + 100Y72 + 100Y73 + 100Y74 + 100Y75 + 100Y76 +

100Y77 + 100Y78 + 100Y79 + 100Y80 + 100Y81 + 100Y82 + 100Y83 + 100Y84 +

100Y85 + 100Y86 + 100Y87 + 100Y88 + 100Y89 + 100Y90 + 100Y91 + 100Y92 +

100Y93 + 100Y94 + 100Y95 + 100Y96 + 100Y97 + 100Y98 + 100Y99 + 100Y100 +

90Y101 + 90Y102 + 90Y103 + 90Y104 + 90Y105 + 90Y106 + 90Y107 + 90Y108 +

90Y109 + 90Y110 + 90Y111 + 90Y112 + 90Y113 + 90Y114 + 90Y115 + 90Y116 +

90Y117 + 90Y118 + 90Y119 + 90Y120 + 90Y121 + 90Y122 + 90Y123 + 90Y124 +

90Y125 + 90Y126 + 90Y127 + 90Y128 + 90Y129 + 90Y130 + 90Y131 + 90Y132 +

90Y133 + 90Y134 + 90Y135 + 90Y136 + 90Y137 + 90Y138 + 90Y139 + 90Y140 +

93Y141 + 93Y142 + 93Y143 + 93Y144 + 93Y145 + 93Y146 + 93Y147 + 93Y148 +

93Y149 + 93Y150 + 93Y151 + 93Y152 + 93Y153 + 93Y154 + 93Y155 + 93Y156 +

93Y157 + 93Y158 + 93Y159 + 93Y160 + 85Y161 + 85Y162 + 85Y163 + 85Y164 +

85Y165 + 85Y166 + 85Y167 + 85Y168 + 85Y169 + 85Y170 + 85Y171 + 85Y172 +

85Y173 + 85Y174 + 85Y175 + 85Y176 + 85Y177 + 85Y178 + 85Y179 + 85Y180 +

100Y181 + 100Y182 + 100Y183 + 100Y184 + 100Y185 + 100Y186 + 100Y187 +

100Y188 + 100Y189 + 100Y190 + 100Y191 + 100Y192 + 100Y193 + 100Y194 +

100Y195 + 100Y196 + 100Y197 + 100Y198 + 100Y199 + 100Y200

ST

X1 + X2 + X3 + X4 + X5 + X6 + X7 + X8 + X9 + X10 + X11 + X12 + X13 + X14 +

X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X27 +

X28 + X29 + X30 + X31 + X32 + X33 + X34 + X35 + X36 + X37 + X38 + X39 + X40 +

X41 + X42 + X43 + X44 + X45 + X46 + X47 + X48 + X49 + X50 + X51 + X52 + X53 +

X54 + X55 + X56 + X57 + X58 + X59 + X60 + X61 + X62 + X63 + X64 + X65 + X66 +

X67 + X68 + X69 + X70 + X71 + X72 + X73 + X74 + X75 + X76 + X77 + X78 + X79 +

X80 + X81 + X82 + X83 + X84 + X85 + X86 + X87 + X88 + X89 + X90 + X91 + X92 +

X93 + X94 + X95 + X96 + X97 + X98 + X99 + X100 - A = 0

A = 6

1] -Y1 + X3 + X4 >=0

2] -Y2 + X3 + X4 >=0

3] -Y3 = 0

4] -Y4 = 0

5] -Y5 = 0

6] -Y6 = 0

7] -Y7 + X3 + X4 >=0

8] -Y8 + X3 + X4 + X64 >=0

9] -Y9 + X3 + X4 + X64 >=0

10] -Y10 + X3 + X64 >=0

11] -Y11 + X7 + X64 >=0

12] -Y12 + X7 >=0

13] -Y13 = 0

14] -Y14 = 0

15] -Y15 + X7 + X64 >=0

16] -Y16 + X3 + X7 + X64 + X65 >=0

17] -Y17 + X3 + X7 + X64 + X65 >=0

18] -Y18 + X3 + X4 >=0

19] -Y19 + X2 + X3 + X4 + X64 + X65 + X74 >=0

20] -Y20 + X7 + X2 + X3 + X4 + X64 + X65 >=0

21] -Y21 + X7 + X64 + X65 >=0

22] -Y22 + X7 >=0

23] -Y23 = 0

24] -Y23 + X7 >=0

25] -Y25 = 0

26] -Y24 + X6 + X7 + X65 + X78 >=0

27] -Y27 = 0

28] -Y28 = 0

29] -Y29 = 0

30] -Y30 = 0

31] -Y31 = 0

32] -Y32 + X61 >=0

33] -Y33 + X5 + X6 + X7 + X61 + X78 >=0

34] -Y34 + X21 + X24 + X25 + X26 + X27 + X69 + X73 >=0

35] -Y35 = 0

36] -Y36 + X61 + X8 >=0

37] -Y37 + X8 >=0

38] -Y38 + X8 + X10 + X61 + X63 >=0

39] -Y39 + X8 >=0

40] -Y40 + X8 + X9 + X10 + X11 + X61 + X63 >=0

41] -Y41 + X8 + X11 + X63 >=0

42] -Y42 + X11 + X12 >=0

43] -Y43 + X8 + X9 + X10 + X11 + X12 + X13 + X62 + X63 + X79 >=0

44] -Y44 + X8 + X11 + X12 + X13 + X15 + X62 + X63 + X79 >=0

45] -Y45 + X8 + X11 + X12 + X13 + X63 >=0

46] -Y46 + X11 + X12 + X13 + X15 + X17 + X19 + X20 + X60 + X62 >=0

47] -Y47 + X11 + X12 + X13 + X15 + X62 >=0

48] -Y48 + X12 >=0

49] -Y49 = 0

50] -Y50 + X12 >=0

51] -Y51 + X12 + X15 + X17 + X18 + X19 + X20 + X21 + X60 >=0

52] -Y52 + X12 + X14 + X15 + X16 + X17 + X18 + X19 + X20+ X59 + X60 + X62 +

X99 >=0

53] -Y53 + X15 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X59 + X60 + X82 + X100 >=0

54] -Y54 + X20 + X21 >=0

55] -Y55 + X19 + X20 + X21 >=0

56] -Y56 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X59 + X60 >=0

57] -Y67 + X16 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X59 + X69 + X82 >=0

58] -Y58 + X20 + X21 + X24 >=0

59] -Y59 + X19 + X20 + X21 + X24 >=0

60] -Y60 + X17 + X18 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X59 + X69 >=0

61] -Y61 + X21 + X24 >=0

62] -Y62 + X20 + X21 + X24 + X25 >=0

63] -Y63 + X21 + X24 >=0

64] -Y64 + X20 + X21 + X24 + X25 >=0

65] -Y65 + X19 + X20 + X21 + X22 + X23 + X24 + X25 + X26 + X69 >=0

66] -Y66 + X21 + X24 + X25 + X26 + X27 + X69 >=0

67] -Y67 = 0

68] -Y68 + X21 + X23 + X24 + X25 + X26 + X27 + X69 + X73 + X83 >=0

69] -Y69 = 0

70] -Y70 + X25 + X26 >=0

71] -Y71 + X24 + X25 + X26 + X27 + X69 + X73 >=0

72] -Y72 + X25 + X26 + X27 + X69 + X73 >=0

73] -Y73 + X25 + X26 + X27 + X28 + X29 + X33 + X69 + X84 + X85 >=0

74] -Y74 + X26 + X29 + X73 >=0

75] -Y75 = 0

76] -Y6 + X26 + X28+ X29 + X30 + X73 >=0

77] -Y77 + X26 + X27 + X28 + X29 + X30 + X33 + X73 + X84 + X85 >=0

78] -Y78 + X26 + X29 + X73 >=0

79] -Y79 + X29 >=0

80] -Y80 + X29 + X30 >=0

81] -Y81 + X26 + X28 + X29 + X30 + X33 + X73 >=0

82] -Y82 + X29 + X30 >=0

83] -Y83 + X29 + X30 + X73 >=0

84] -Y84 + X28 + X29 + X30 + X33 + X73 >=0

85] -Y85 + X29 + X30 + X31 >=0

86] -Y86 + X28 + X29 + X30 + X31 + X32 + X33 + X70 + X73 + X85>=0

87] -Y87 + X28 + X29 + X30 + X31 + X33 + X73 >=0

88] -Y88 + X30+ X31 >=0

89] -Y89 + X29 + X30 + X31 + X32 + X33 >=0

90] -Y90 + X30+ X31 >=0

91] -Y91 + X28 + X29 + X30 + X31 + X32 + X33 + X34 + X70 >=0

92] -Y92 + X30 + X31 + X32 >=0

93] -Y93 + X30 + X31 + X32 + X33 + X34 + X70 + X86 >=0

94] -Y94 + X31 + X32 + X34 + X35 + X70 + X86 >=0

95] -Y95 + X31 >=0

96] -Y69 + X31 + X34 >=0

97] -Y97 + X31 + X32 + X34 + X35 + X36 >=0

98] -Y98 = 0

99] -Y99 + X32 + X34 + X35 + X36 + X40 + X87 >=0

100] -Y100 + X34 + X35 + X36 >=0

101] -Y101 = 0

102] -Y02 + X34 + X35 + X36 + X38 + X40 + X41 + X87 >=0

103] -Y103 + X35 + X36 + X87 >=0

104] -Y104 + X35 + X36 + X37 + X38 + X40 + X41 + X87 + X88 >=0

105] -Y105 = 0

106] -Y106 + X36 + X38 + X87 >=0

107] -Y107 + X36 + X37 + X38 + X40 + X41 + X55 + X87 + X88 + X89 >=0

108] -Y108 + X36 + X37 + X38 + X40 + X41 + X87 + X88 >=0

109] -Y109 + X36 + X38 >=0

110] -Y110 + X36 + X37 + X38 + X87 >=0

111] -Y111 + X38 >=0

112] -Y112 + X37 + X38 + X39 + X40 + X41 + X55 + X87 + X88 + X89 + X90>=0

113] -Y113 + X37 + X38 + X55 + X87 + X88 >=0

114] -Y1114 + X37 + X38 + X39 + X41 + X55 + X87 + X88 + X89 >=0

115] -Y115 + X38 + X37 >=0

116] -Y116 + X37 + X38 + X55 + X88 >=0

117] -Y117 + X37 + X38 + X55 >=0

118] -Y118 + X37 + X39 + X41 + X42 + X55 + X68 + X88 + X89 + X90 >=0

119] -Y119 + X37 + X38 + X39 + X55 + X88 + X89 >=0

120] -Y120 + X37 + X38 + X55 + X88 + X89 >=0

121] -Y121 + X55 >=0

122] -Y122 + X37 + X55 >=0

123] -Y123 + X39 + X42 + X43 + X55 + X68 + X89 + X90 + X91 >=0

124] -Y124 + X39 + X42 + X43 + X55 + X68 + X89 + X90 >=0

125] -Y125 + X55 >=0

126] -Y126 = 0

127] -Y127 = 0

128] -Y128 + X43 + X91 >=0

129] -Y129 + X39 + X42 + X43 + X68 + X89 + X91 >=0

130] -Y130 + X43 + X44 + X46 + X68 + X91 + X92 >=0

131] -Y131 + X43 + X44 + X46 + X91 + X92 + X93 >=0

132] -Y132 = 0

133] -Y133 = 0

134] -Y134 + X44 + X46 + X91 + X92 + X93 >=0

135] -Y135 + X44 + X45 + X46 + X47 + X91 + X92 + X93 + X96 >=0

136] -Y136 + X44 + X45 + X46 + X47 + X92 + X93 + X96 >=0

137] -Y137 = 0

138] -Y138 + X46 + X92 + X93 >=0

139] -Y139 + X45 + X46+ X47 + X49 + X92 + X93 + X94 + X96 >=0

140] -Y140 + X45 + X46+ X47 + X92 + X93 + X94 + X96 >=0

141] $-Y141 + X46+ X93 >=0$

142] $-Y142 + X45 + X46+ X47 + X93 + X94 + X96 >=0$

143] $-Y143 + X45 + X46+ X47 + X49 + X93 + X94 + X96 >=0$

144] $-Y144 + X45 + X49 + X93 + X94 + X96 >=0$

145] $-Y145 + X45 + X47 + X49 + X93 + X94 + X96 >=0$

146] $-Y146 + X45 + X47 + X49 + X93 + X94 + X96 >=0$

147] $-Y147 + X45 + X49 + X94 + X96 >=0$

148] $-Y148 + X45 + X48 + X49 + X94 + X96 + X97 >=0$

149] $-Y149 + X45 + X48 + X49 + X94 + X95 + X96 >=0$

150] $-Y150 + X48 + X49 + X94 + X97 >=0$

151] $-Y151 + X48 + X49 + X50 + X94 + X97 >=0$

152] $-Y152 + X49 + X94 + X95>=0$

153] $-Y153 + X48 + X49 + X95>=0$

154] $-Y154 + X48 + X50 + X51 + X95 + X97 >=0$

155] $-Y155 + X48 + X50 + X51 + X53 + X97 + X98 >=0$

156] $-Y156 + X50 + X51 + X52 + X53 + X97 + X98 >=0$

157] $-Y157 + X48 + X95 + X97 >=0$

158] $-Y158 = 0$

159] $-Y159 + X48 + X97 >=0$

160] $-Y160 + X48 + X50 + X95 + X97>=0$

161] $-Y161 + X48 + X50 + X51 + X53 + X97 + X98 >=0$

162] $-Y162 + X48 + X50 + X97 >=0$

163] $-Y163 = 0$

164] -Y164 + X50 + X51 + X53 + X97 + X98 >=0

165] -Y165 + X51 + X52 + X53 + X58 + X98 >=0

166] -Y166 + X53 + X98 >=0

167] -Y167 = 0

168] -Y168 = 0

169] -Y167 + X53 + X58 + X98 >=0

170] -Y170 + X52 + X53 + X54 + X56 + X57 + X58 + X98 >=0

171] -Y171 + X52 + X53 + X54 + X56 + X57 + X58 >=0

172] -Y72 + X53 >=0

173] -Y173 = 0

174] -Y174 + X58 >=0

175] -Y175 + X53+ X54 + X57 + X58 >=0

176] -Y176 + X53 + X58 >=0

177] -Y177 = 0

178] -Y178 + X53 + X58 >=0

179] -Y179 + X53 + X57 + X58 >=0

180] -Y180 + X58 >=0

181] -Y181 + X8 + X9 + X10 + X11 + X12 + X13 + X61 + X62 + X63 + X67 + X72 + X79 >=0

182] -Y182 + X5 + X8 + X9 + X10 + X11 + X61 + X63 + X67 + X71 + X72 + X76 + X78 >=0

183] -Y183 + X11 + X12+ X13 + X14 + X15 + X16 + X17 + X60+ X62 + X73 + X79 + X99 >=0

184] -Y184 + X8 + X10+ X11 + X12 + X13 + X63 >=0

185] -Y185 + X11 + X12>=0

186] -Y186 + X8 + X11 + X12 + X13 + X62 + X63 + X79>=0

187] -Y187 + X20 + X21 + X24 >=0

188] -Y188 + X21+ X24 >=0

189] -Y189 + X20 + X21+ X23 + X24 + X25 + X26+ X27 + X69 >=0

190] -Y190 = 0

191] -Y191 + X29 + X30 + X31 + X33 >=0

192] -Y192 = 0

193] -Y193 + X36 >=0

194] -Y194 + X34 + X35 + X36 + X40 + X87 >=0

195] -Y195 + X35 + X36 + X38 + X87 >=0

196] -Y196 + X31 + X32 + X34 >=0

197] -Y197 + X8 + X61 >=0

198] -Y198 + X5 + X8+ X9 + X10 + X11 + X13 + X61 + X63 + X67 + X71 + X72 >=0

199] -Y199 = 0

200] -Y200 + X5 + X8 + X10 + X61 + X67 + X71 + X72 + X78 >=0

201] 100X1 + 92.75X2 + 85.25X3 + 73.75X4 + 93.05X5 + 100X6 + 97.5X7 + 87.25X8 + 96.25X9 + 92.75X10 + 84.45X11 + 98.8X12 + 77.2X13 + 87.05X14 + 90.05X15 + 86.25X16 + 97.6X17 + 100X18 + 95.8X19 + 98.8X20 + 96.4X21 + 100X22 + 97.3X23 + 91.7X24 + 89.45X25 + 88.6X26 + 90X27 + 98.5X28 + 97.6X29 + 100X30 + 97X31 + 94.85X32 + 84.6X33 + 100X34 + 81.5X35 + 83.65X36 + 78.2X37 + 97X38 + 78.45X39

+ 56.75X40 + 56.75X41 + 98.5X42 + 91.5X43 + 91.5X44 + 100X45 + 80.75X46 + 63.45X47 + 79.7X48 + 79.85X49 + 85.85X50 + 92.75X51 + 100X52 + 96.25X53 + 96.25X54 + 79.7X55 + 96.25X56 + 96.25X57 + 82.05X58 + 83.05X59 + 90.5X60 + 90.9X61 + 77.2X62 + 94X63 + 73.75X64 + 82.25X65 + 82.25X66 + 89.85X67 + 91.5X68 + 88.6X69 + 84.6X70 + 85.7X71 + 88.7X72 + 92.2X73 + 92.75X74 + 85.25X75 + 92.6X76 + 83.2X77 + 84.9X78 + 82.75X79 + 94.95X80 + 93.25X81 + 94X82 + 92.75X83 + 82.95X84 +91.75X85 + 86.55X86 + 75.75X87 + 75.75X88 + 75.75X89 + 93.25X90 + 95.2X91 + 95.8X92 + 81.75X93 + 81.75X94 + 81.75X95 + 81.75X96 + 78.75X97 + 100X98 + 86.25X99 + 99.4X100 - W = 0

W >= 540

END

ALL VARIABLES ARE BINARY.

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Vita

First Lieutenant Ömer Alkanat was born in Istanbul, Türkiye. He graduated from Kuleli Military High School in Istanbul, in 1995. He entered the Turkish Air Force Academy in Istanbul and received the degree of Bachelor of Science in Computer Engineering in 1999. In the same year, he attended undergraduate pilot training course and became a pilot in 2001. He assigned to 192^{nd} Tiger Squadron as an F–16 pilot in 2002 after F–16 training course. Having served two years in 192^{nd} Squadron, he assigned to 191^{st} Cobra Squadron. In 2006 he was chosen for Graduate School of Engineering, Air Force Institute of Technology. Upon graduation, he will be assigned to 9^{th} Main Jet Base in Balikesir as an F–16 pilot.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 074-0188

1. REPORT DATE *(DD-MM-YYYY)*	2. REPORT TYPE	3. DATES COVERED *(From – To)*
20-03-2008	**Master's Thesis**	Sep 2006 - Mar 2008

4. TITLE AND SUBTITLE
DETERMINING THE SURFACE-TO-AIR MISSILE REQUIREMENT FOR WESTERN AND SOUTHERN PART OF THE TURKISH AIR DEFENSE SYSTEM

5a. CONTRACT NUMBER

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER

6. AUTHOR(S)

Alkanat, Ömer., First Lieutenant, TUAF

5d. PROJECT NUMBER

5e. TASK NUMBER

5f. WORK UNIT NUMBER

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(S)

Air Force Institute of Technology
Graduate School of Engineering and Management (AFIT/EN)
2950 Hobson Street, Building 642
WPAFB OH 45433-7765

8. PERFORMING ORGANIZATION REPORT NUMBER

AFIT/GOR/ENS/08-01

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSOR/MONITOR'S ACRONYM(S)

11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

An air defense system is vital for countries to protect their homelands. Today, air defense systems consist of integrated systems such as early warning radars, fighter aircraft, airborne early warning aircraft and surface-to-air (SAM) systems. The Turkish air defense system does not have long range SAM systems. Turkey plans to procure SAM systems to protect her borders. This research develops two location optimization models to optimally locate SAM sites to defend specified areas of the nation. One of the models finds the minimum number of SAM sites to cover the specified area; the other finds the maximum coverage for a given number of SAM sites. The model is formulated as an integer program, and the LINGO 10 software package is used to solve the model. Three candidate SAM systems are examined. All models use the maximum range of each SAM system. Solutions are presented for the decision makers to examine. Sensitivity analysis is used to explore how much the optimal solution(s) change given fluctuations in input values. The main objective of this research is to provide the Turkish Air Force coverage information regarding the three candidate SAM systems. This research also provides a model and an approach that can be used to examine other candidate systems. The results and models presented in this research should facilitate development of a more efficient and effective air defense system to support Turkey's homeland defense.

15. SUBJECT TERMS
Integer Programming, SCLP, MCLP, Location, Surface-to-Air Missiles

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON / 19b. TELEPHONE NUMBER *(Include area code)*
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Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18