The Impact of Atmospheric Fluctuations on Optimal Boost Glide Hypersonic Vehicle Dynamics

Melissa A. Dunkel

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THE IMPACT OF ATMOSPHERIC FLUCTUATIONS ON OPTIMAL BOOST
GLIDE HYPersonic VEHICLE DYNAMICS

THESIS

Melissa A Dunkel, Second Lieutenant, USAF

AFIT-ENY-MS-17-M-257

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

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THE IMPACT OF ATMOSPHERIC FLUCTUATIONS ON OPTIMAL BOOST GLIDE
HYPERSONIC VEHICLE DYNAMICS

THESIS

Presented to the Faculty
Department of Aeronautics and Astronautics
Graduate School of Engineering and Management
Air Force Institute of Technology
Air University
Air Education and Training Command

In Partial Fulfillment of the Requirements for the
Degree of Master of Science in Astronautical Engineering

Melissa A Dunkel, BS
Second Lieutenant, USAF

March 2017

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THE IMPACT OF ATMOSPHERIC FLUCTUATIONS ON OPTIMAL BOOST GLIDE
HYPersonic Vehicle Dynamics

Melissa A. Dunkel, BS
Second Lieutenant, USAF

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Abstract

A project under the management of Air Force Research Laboratory has begun development of a six degree of freedom model for use in hypersonic vehicle development and application. One area of interest is the perturbation of vehicle behavior based on atmospheric fluctuations – how the performance of the vehicle changes with respect to “hot”, “cold” and standard day conditions. The method developed to fill this need uses real-world data from the Global Forecast System to create a “hot” and “cold” day dataset to compare with the standard day model. The key parameter is atmospheric density, a value calculated over a series of given points around the globe for any given dataset on a given day, and which directly impacts the lift and drag acting on the hypersonic vehicle, primarily over its re-entry trajectory. The results from simulations demonstrate trends that contradict expectation – the colder day cases result in a further longitude being achieved on average and yet experience a higher average drag. The optimal solution fluctuated 5-10% of the total range, or approximately 1.5 degrees in longitude, with matching orders of magnitude in fluctuations in the force of drag acting on the vehicle. General trends are stable – the two key trends with respect to longitude and drag remain true overall – the “cold” day cases have both the largest average drag and the longest distance traveled. Some analysis of the results proves these are reasonable results. These results enhance the strategic picture, but more test cases and analysis must be done before this model is ready for use.
Acknowledgments

Above and before all, I must give the glory to the Creator of the vast and marvelous universe we study. This work and everything that proceeded it is all for you. To my husband, thank you for all of the support and understanding as I worked through this research. To the numerous other teachers, friends, and family members who assisted me on everything from code to concepts, I could not have done it without you.

Melissa A Dunkel
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LIST OF ACRONYMS

JFCC–GS - JOINT FUNCTIONAL COMPONENT COMMAND GLOBAL STRIKE

CAV - COMMON AERO VEHICLE

AFRL - AIR FORCE RESEARCH LABORATORY

MATLAB® - MATRIX LABORATORY

NLP - NON-LINEAR PROGRAMMING

GPOPS-II - GENERAL PSUEDOSPECTRAL OPTIMIZATION SOFTWARE II

NCEP - NATIONAL CENTERS FOR ENVIRONMENT PREDICTION

NOAA - NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION

GFS - GLOBAL FORECASTING SYSTEM

AFIT - AIR FORCE INSTITUTE OF TECHNOLOGY

ECMWF - EUROPEAN CENTRE FOR MEDIUM-RANGE FORECASTS

IFS - INTEGRATED FORECAST SYSTEM

ICBM - INTERCONTINENTAL BALLISTIC MISSILE

OIF - OPERATION IRAQI FREEDOM

OEF - OPERATION ENDURING FREEDOM

AFB - AIR FORCE BASE

RK4 - RUNGE-KUTTA 4TH ORDER
I. Introduction

General Issue

The development of accurate modeling programs for hypersonic vehicles and trajectories has become a necessity in today’s strategic climate. As simulations progress, an item of increasing interest is the impact of atmospheric fluctuations on the performance of the vehicle, to include wind, rain, temperature, and density conditions. An analysis must first be conducted in order to determine the relevance of such conditions to the creation of an accurate model. Using available optimization software, weather data, and vehicle data, an accessible first step may be created in the effort to create more realistic vehicle trajectories and thus enhance the strategic picture presented to decision makers.

Motivation

Dynamic capability of assets is a fundamental element of operational performance. The atmosphere has the potential to affect this component through the fluctuation of lift and drag in relation to atmospheric density conditions. For future research and missions, the inclusion of meteorological conditions in pre-mission or developmental models could significantly increase the accuracy of initial estimates for reachability and produce a more accurate time of arrival.
Operational Applications

In early 2016, the Congressional Research Service compiled a report on the issue of Conventional Prompt Global Strike. The research concluded weapons relevant to this goal were to be able to “strike targets anywhere on Earth in as little as an hour;” [1]. The purpose of such a weapon would be to strike targets with a small window of vulnerability or during a time when the enemy considered using high yield weapons of their own [1].

Improving the understanding of atmospheric impacts in these situations would give leadership a more realistic understanding of military strike capability, and therefore improve the ability to make a good decision under the constraint of time. Of the available engineering factors impacting a prompt strike capability, the atmosphere is the least controllable, and therefore has a higher probability of negative influence. Proper mitigation of its effect on the mission begins with a better consideration of the reaction of the solution to the environments the atmospheric conditions create.

Air Force Research Lab Applications

In 2006, U.S. Strategic Command formed a Joint Functional Component Command for Global Strike (JFCC-GS) to plan and execute prompt global strike missions [1]. The conceptual Common Aero Vehicle (CAV) and related research grew from this focus. One key element of the CAV development was a need for segregation from nuclear weapons so that other countries would not believe a conventional weapon to be nuclear and retaliate accordingly. The requirement for segregation led to the expansion of air launch and other mobile launch platforms as possible deployment options [1]. Air Force Research Laboratory (AFRL) began in concert with other government
organizations to create models and practical applications relevant to both the CAV and mobile platforms. The JFCC-GS initiative spurred a need for research in the application of atmospheric-impacted optimal trajectories for these scenarios [2].

A three degree of freedom model is in production for air launch conventional prompt strike scenarios. One aspect of this modeling yet to be explored is sensitivity of the solution to meteorological conditions, a feature in which many customers of the model show interest [3]. Determining the level of impact a real-world atmosphere has on an optimal trajectory is a possible first step to building a better model

**Problem Statement**

The effects of a fluctuating atmosphere, while easy to conceptualize, require more in-depth modeling of environments than typically included in optimal trajectory modeling systems. The objective of the research is to determine the extent to which an optimal air launched hypersonic boost-glide trajectory is impacted by temperature and pressure deviations, a trajectory summarized in Fig 1.

![Air-Launch Hypersonic Boost-Glide Trajectory](image)

**Figure 1** Air-Launch Hypersonic Boost-Glide Trajectory
The methodology requires the calculation of trajectories maximizing range (longitude in degrees) while influenced by meteorological conditions. The deviations will then be compared to the standard day model. This will give an indication of the level of importance of the inclusion of real world atmospheric data in a model, and inform researchers in the process of building an ever-more complex simulation of conventional prompt global strike capability. Additionally, it will provide a methodology of equation development for meteorological impacts.

First, the meteorological conditions must be transformed into a usable quantity for use in the dynamic equations. A usable quantity may be derived from real-world prediction data in conjunction with the Ideal Gas Law for the calculation of a new density model. The inclusion of a series of more accurate density values will impact lift and drag calculations and thus the overall model performance. The model will be evaluated at three separate sets of conditions – standard day, a “warm” atmospheric day, and a “cool” atmospheric day, the parameters of which will later be defined. All of the relevant equations will be run through General Pseudospectral OPtimization Software (GPOPS), an optimization software package built using MATLAB® as its foundation for interfacing [4]. Using an optimizer to model the trajectory allows a more complete picture to be formed of the impact of the atmospheric density through the demonstration of how the answer changes even when control input is involved. It gives the best case of vehicle performance in the worst-case scenario presented using improved atmospheric density modeling.

The problem begins with the creation of a straight-forward air-launch hypersonic scenario. This separates into two phases, the launch and hypersonic glide portions of the flight. Each will have their own equations of motion and controls, while ensuring
continuity of variables to allow the scenario to transition from one phase to the next. The state values from the first phase will be used as the transition to the next phase. Both phases will be calculated in terms of radius, latitude, longitude, velocity, flight path angle, and heading angle, with a flight path control on Phase 1 and a bank angle control on Phase 2 based on the difference in flight dynamics between launch and glide. Each of the phases will include the relevant atmospheric condition equations.

The final step will be to compare trajectories and drag values to determine the impact meteorological conditions have in the dynamic performance of the vehicle in each scenario. This will be done through the evaluation of the difference between standard day and the two other sets of conditions in maximum longitude achieved as well as drag values the vehicle may experience. The objective is to determine the extent to which the atmospheric density impacts capability and whether it merits further investigation. The investigative question thus becomes whether the variable atmospheric conditions alters the answer achieved by the optimization software and if so, how much the solution changes.

Preview

The purpose of this study is to determine the impact of a real-world atmospheric model on an optimal solution using GPOPS-II, a meteorological prediction model, and dynamic equations. Four chapters describe the remainder of the research. The next chapter, Chapter II, provides a literature review and an overview of the background research required to appropriately construct and answer the given problem. It encompasses much of the context needed to understand the techniques and processes
used to obtain a solution. This chapter will include a general description of dynamic optimization, pseudospectral methods, and previous research accomplished with these methods, as well as a description of available atmospheric modeling. A basic outline of air launch initiatives and related vehicles will also be presented, along with possible points of origin. Chapter III outlines the methodology intended to solve the problem, to include the problem statement, the assumptions made, the steps required to achieve a solution, and a description of the solution criteria. Chapter IV describes the results of the research, and implements the analysis outlined in Chapter III. If errors arise, it will also detail their solution as it is relevant to future applications of this research. The research concludes with Chapter V, which will highlight the important results of Chapter IV and give recommendations for future research in the subject.
II. Literature Review

Chapter Overview

The literature review outlines previously employed or related techniques used to examine each aspect of the problem, from atmospheric and gravitational modeling to the types of vehicles included in the analysis, in order of application. While some elements of this study have been achieved in previous research, the key to creating an effective tool is using modeling equations and concepts previously employed to build a larger, more complex system. While many of the theories presented throughout the chapter are useful, others required adjustment or discarding for the creation of the new scenario.

Atmospheric Models

Appropriate atmospheric modeling is essential to the development of this study. Many disciplines use a simplified Standard Day density model to determine atmospheric effects on flight. In most cases, this is sufficient to give an understanding of the aspects of flight.

Standard Day Density Model

In depicting the atmosphere, a series of exponential equations are commonly used in combination with a data set referred to as 1976 Standard Atmosphere [5]. The model is built using an averaged data structure constructed as a function of height above sea level [5]. On non-standard days, this model can be different from the reality of the atmosphere. Equation 1 displays the relationship between height and density in this representation of the atmosphere,
\[ \rho = \rho_0 e^{-\beta(R - R_e)} \]  

(1)

where Table 1 defines each parameter.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
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<tr>
<td>( \rho_0 )</td>
<td>Density at Earth’s Surface</td>
<td>( kg/m^3 )</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Scale Height</td>
<td>( km^{-1} )</td>
</tr>
<tr>
<td>( R )</td>
<td>Radius from Center of Earth</td>
<td>( km )</td>
</tr>
<tr>
<td>( R_e )</td>
<td>Radius of the Earth</td>
<td>( km )</td>
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Figure 2 below is a visualization of this same relationship.

![Figure 2 Exponential Density Model [6]](image)

As Fig 2. depicts, the model is exponential. In the semi-log plot the curves display a nearly linear relationship, pointing to an exponential relationship between the height and density. Standard day density is quickly calculated in a computationally expensive process. The process gives a good estimate for density, but is not sufficient when more exacting requirements for accuracy exist.
Non-Standard Atmospheric Models

The Department of Defense (DOD) offers four models for non-standard day atmospheric calculations. These include “Hot”, “Cold”, “Polar”, and “Tropical,” [7]. Figure 3 displays general trends of the four non-standard models in comparison to the standard model.

![Non-Standard Atmospheres, SI Units](image)

Figure 3 Non-Standard DoD Temperature Models [7]
As Fig. 3 illustrates, a wide variety of temperature models cover this set of Non-Standard atmospheres. The density for this atmosphere is then computed using the Ideal Gas Law [8]. Some of the more easily accessible models only extend to approximately 30.5 km, decreasing accuracy for high altitude calculations [7]. Much like the standard day models, the density may be fit to an exponential curve.

**Real World Hot and Cold Day Considerations**

Definitions abound for “hot” and “cold”, but they typically refer to a fixed data set and a general model. A different definition is required for this study, a new definition that hinges on an old concept – Milankovitch cycles [9].

To begin, a day can be considered based on the primary hemisphere in which the re-entry takes place, as this is the longest time the vehicle spends in atmosphere in the sequence. When the primary hemisphere is closest to the sun (e.g. the northern hemisphere from June to September), this is a “hot” day for the purposes of atmospheric data. Although not all summer days are “hot” in terms of temperature, a large number of data files will average out to an approximation of a “hot” day. A similar definition can be applied to define a “cold” day [9]. A day will be considered “cold” when the primary hemisphere is furthest away from the sun. (e.g. the Northern Hemisphere from November to March). Figure 4 displays the tilt of the earth over the four seasons.
As demonstrated in the image above, the equator remains at approximately the same distance from the sun year-round in comparison with the Northern and Southern Hemispheres [9]. For the purposes of this study, the slight eccentricity of the Earth’s orbit around the sun, and thus the differences in aphelion and perihelion, will not be taken into account.

**The Ideal Gas Law**

Pressure, volume, and temperature are aspects of a gaseous medium that may be related to each other through an equation of state [11]. This relationship was fit to a modeling equation known as the Ideal Gas Equation. Although it is not a perfect representation, in most cases it is considered “good enough”. Equation 2 is its typical form [11]

\[ pV = nRT \]  

(2)

where the parameters are defined in Table 2.
<table>
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<th>Description</th>
<th>Units</th>
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<td>Pa</td>
</tr>
<tr>
<td>(V)</td>
<td>Volume</td>
<td>(m^3)</td>
</tr>
<tr>
<td>(n)</td>
<td>Number of moles of Gas</td>
<td>(Mol)</td>
</tr>
<tr>
<td>(R)</td>
<td>Universal Gas Constant</td>
<td>(J/M*K)</td>
</tr>
<tr>
<td>(T)</td>
<td>Temperature</td>
<td>K</td>
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</tbody>
</table>

Equation 2 does not contain the quantity of interest, density. Equation 3 is a form in which the density is one of the variables, a key parameter for the development of a more accurate model is necessary in the calculation of lift and drag [12]

\[
p = \rho RT
\]  

(3)

where \(\rho\) refers to the density. Equation 3 must be further manipulated to produce

\[
\rho = \frac{p}{RT}
\]  

(4).

While the Ideal Gas Law is still not a perfect representation of atmospheric density in a real-world scenario, it brings the solution closer through the use of real world temperature and pressure to calculate a density value. Because this model includes more available data, it provides a distinct picture of atmospheric fluctuation based on seasonal patterns. The equation allows the perturbations of warmer or colder days to be reflected in the modeling process through the inclusion of actual meteorological data collected at various altitudes and locations and extrapolated to create a complete picture of world-wide atmospheric conditions. The flexibility this provides makes it a valuable improvement on the standard day density model.

The ideal gas law can be expanded to cover more in-depth conditions such as moist air, water vapor, and dry air, but for this study it is sufficient to model the effects of
non-standard temperature and pressure [6]. Coupled with an appropriate model, it provides a method to examine the meteorological influence on flight conditions.

**Weather Models**

The Ideal Gas Law allows the quantification of a relationship between atmospheric data and the atmospheric density, a value essential to modeling launch and reentry. However, the use of this relationship requires accurate atmospheric data. The needed information – pressure and temperature – may be provided from a number of sources. The data extends up to approximately 70 kilometers, and includes pressure and temperature at specific altitudes and latitude/longitude coordinates [13]. There are two primary atmospheric data models in common use.

**Global Forecast System**

National Centers for Environmental Prediction (NCEP) built a model for weather forecasting called the Global Forecast System (GFS). GFS is a coupled model which is derived from the data of four other models to include an atmospheric model, an ocean model, a soil model, and a sea ice model [14]. It creates a dataset in grid format for locations around the globe. It has a base horizontal resolution of 18 miles, but this resolution decreases in accuracy over the length of time forecasted [14]. This is the data model recommended for use by the Air Force Institute of Technology (AFIT) for the development of this study due to its wide availability, although accuracy was not as high as other models [15].
European Centre for Medium Range Weather Forecasts

The European Centre for Medium-Range Weather Forecasts (ECMWF) provides member states of the European Union with a prediction model for atmospheric and meteorological conditions. Finite element discretization and Gaussian grid reduction are used to create this model, long considered the most accurate of the models available. ECMWF models were built from the integrated forecast system (IFS) and provide an accurate ten to fifteen-day forecast [16]. The model includes elements to account for atmospheric dynamics and physics, atmospheric composition, marine and land qualities, uncertainty quantification, and forecast evaluation. Data is freely available to member states, but for purchase to outside entities [16].

Gravitational Models

Flight conditions can also depend on the model of the earth used in calculation. Therefore, the accuracy of the optimized scenario relies on the accuracy of the assumptions used to build the scenario. Simple models create an acceptable level of accuracy, but more accurate gravitational models allow the dynamics to behave more accurately. The level of accuracy required by the research determines which model should be used [17]. Earth is not perfectly spherical, nor is it a point mass. Additionally, interaction with solar events and other space weather can create large perturbations in atmospheric behavior [18]. The variability of the earth is difficult to model in its entirety; often models choose to include only the largest scale effects, or choose only a subset of other factors to consider.
Two-Body Model

Two of the more common earth gravitational models are the spherical and oblate earth approximations. For the purposes of many studies, the spherical earth model is sufficient. Its gravitational field can be modeled using the universal law of gravitation equation, Eq. (5),

\[ g = \frac{G m_e m_b}{R^2} \]  

where Table 3 defines the key parameters [19].

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value/ Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G )</td>
<td>Gravitational Constant</td>
<td>( 6.67384 \times 10^{-11} , N )</td>
</tr>
<tr>
<td>( m_e )</td>
<td>Mass of the Earth</td>
<td>( 5.9722 \times 10^{24} , kg )</td>
</tr>
<tr>
<td>( m_b )</td>
<td>Mass of CAV</td>
<td>( 907 , kg )</td>
</tr>
<tr>
<td>( R )</td>
<td>Radius from Center of Earth</td>
<td>Variable</td>
</tr>
</tbody>
</table>

Assuming \( m_b \) is much, much smaller than the mass of the earth, Eq. (5) may be simplified to Eq. (6), using \( \mu \) to approximate the numerator of the universal law of gravitation

\[ g = \frac{\mu}{R^2} \]  

The assumption \( \mu \) represents is in most cases reasonable. For example, the CAV is approximately 907 kg, while the Earth is approximately \( 5.9 \times 10^{24} \, kg \) [20]. The CAV is a mere \( 1.5 \times 10^{-20} \% \) of the Earth’s mass. The validation of this assumption results in a simplified distance squared relationship in the calculation of the gravitational pull, a model often referred to as the square model. Figure 5 presents the relationship between distance and acceleration due to gravity present in the square model.
This provides a gravitational model is more accurate the further away from the Earth the object modeled travels, but is acceptable for general dynamic modeling of gravitational behavior.

**Non-Square Gravitational Models**

One possible method of model improvement would be to use a non-square law formulation of the relationship between gravity, location, and altitude. There are many different methods of application under a non-square model. Inclusion of zonal harmonics, real-world data collections, and third-body perturbations are a few examples of ways to expand the accuracy of a gravitational model [17]. Figure 6 displays an example comparison between square model results and zonal model results.
Although several differences exist between the two models, for the purpose of this study, the square model approximation will be considered sufficient. Because the focus of this study is atmospheric effects, the gravitational model will be kept simple.
**Common Aero Vehicle**

The CAV began as part of an initiative to achieve “near real-time global reach,” [21]. Through study it was determined that the way to accomplish this mission was through a space plane or similar technology. From this discussion, the concept of CAV developed. At its most basic, the CAV describes a re-entry vehicle able to be launched on a typical launch platform into the exosphere and returned to an exact location with the intention of unloading cargo at a specified target [21]. Due to the multi-mission aspect of such a model, it could be applied to a number of military scenarios, particularly in support of Air Force Basic Doctrine (ABD) ideals of global range and flexibility [22].

The CAV employment scheme contains three basic stages, first, the launch on a capable vehicle into Low Earth Orbit (LEO) or a ballistic trajectory. Second, the vehicle either re-enters or follows the related next step in the ballistic trajectory. Third, the vehicle delivers its payload. Fourth and finally, the vehicle is either destroyed or recovered [21]. These stages and their inherent options result in a number of flight trajectories reflected in Figs. 7 and 8.

![Figure 7 Suborbital Ballistic Flight Paths [21]](image)
Two types of launch platforms may be used with the CAV. Either a ground-based rocket or an air-launch booster such as Pegasus will compose the fielded platform for the CAV and vehicles like it [21]. Recommended ground-based rockets include modified intercontinental ballistic missile launchers such as Minuteman and Peacekeeper, converted into a platform called Minotaur [23]. Although Minotaur has been phased out of use, this vehicle can be used as a conceptual platform for scenario creation. Air launch boosters such as Pegasus would be used in concert with large aircraft such as B-52s and KC-10s [1]. The benefit of such a platform would be in the differentiation between conventional prompt strike and nuclear weaponry, as foreign entities would be aware that the location and method of launch would determine the nature of the strike. Additionally, it would provide greater mobility and protection for strike capabilities [1].

Simple model parameters for the CAV are no longer available as an original source. Due to the lack of accessibility, the parameters derived for use in this study come from previous work done by Jorris [24]. Most of the parameters are simple values for use
in modeling software, but the coefficients of lift and drag given by Mach number and angle of attack (AOA) are given in tables, and thus must be curve-fit to ensure a continuous model of AOA, Mach number, and related lift and drag coefficients [25].

Figures 9 and 10 give a visual representation of the continuous model created through curve-fit.

![Figure 9 Coefficient of Lift as a Function of Mach and AOA](image)

**Figure 9 Coefficient of Lift as a Function of Mach and AOA [25]**
Figure 10 Coefficient of Drag as a Function of Mach and AOA [25]

The red dots represent the given data points, and the grid in between represents the extrapolated data for use in the model. These are complex models, good for a higher-fidelity representation of the CAV behavior in flight. In simulations where detailed flight dynamics are important, such a model increases the fidelity of the dynamics. However, when the lift and drag are influenced by the factor of interest in a study, and other datasets have already created a computationally expensive scenario, the use of such a high level of detail in the vehicle dynamics is not as important.
Launch Platforms

Minotaur

Ground-based rocket launches are one possible platform for the CAV. Recommended for such an application are re-purposed Intercontinental Ballistic Missile (ICBM) engines, which form the basis of the Minotaur rocket family [26]. Both Peacekeeper and Minuteman solid rocket engines were used in the family, but the most relevant and useful rocket developed – the Minotaur IV – contained a re-purposed Peacekeeper engine. Interested agencies created Minotaur IV for space lift purposes. It was a four stage solid rocket launch vehicle, able to carry payloads into LEO. Figure 11 represents the launch capability of the platform by launch location, payload mass, and destination altitude:

Figure 11 Minotaur IV Launch Capability [26]
As Fig. 11 demonstrates, the Minotaur family has sufficient payload capacity for the orbital altitudes required for a boost-glide scenario, as it is able to carry between 1100 and 1600 kg of payload. Orbital ATK, the primary contractor involved with the development of the Minotaur family, states that multiple locations are available via portable launch support, a fact also reflected in Fig. 11 [23].

**Pegasus**

Although there is technology to support ground-launched CAV, many organizations have begun to develop air-launch capabilities more diligently for the reasons previously mentioned [1]. Air launch requires an appropriate booster. One recommended booster for use is the Pegasus booster. The Pegasus is a three stage air launch to space booster, flown on both a B-52 and the Orbital Stargazer aircraft (L-1011) [27]. Propulsion values required to model its launch are not readily available, however, given its mass and other relevant numbers, capability may be derived. Depending on the target orbit or trajectory, only one stage of the booster may be used, or all three [27]. Figure 12 represents capability of the booster based on payload mass and inclination of target orbit.
As Fig. 12 demonstrates, the Pegasus booster has a limited payload capacity depending on the inclination of launch and the altitude it is required to attain. For heavier payloads such as larger hypersonic vehicles, this booster is not sufficient. As previously mentioned, the mass of the CAV is approximately 900 kg, a number in excess of the given capacities in Fig. 12.

**Modified Centaur**

Higher efficiency boosters may be required for larger hypersonic vehicles. One particular example is the Martin Marietta Centaur Upper Stage, a heritage propulsion unit from the Titan IVB [28]. Although it is not currently designed for use in an air-launch scenario, it contains the requisite thrust, specific impulse, and mass to carry a hypersonic vehicle to orbit given realistic parameters. Additionally, the general specifications of the Centaur Upper Stage are readily available [28]. Figure 13 outlines the basic structure of the Centaur Upper Stage.
Figure 13 Centaur Upper Stage Structural Layout [28]

The dashed lines represent the portions of the upper stage (e.g. payload fairing) that would typically be put in place for a full scale Titan IV launch. While these would not be necessary for a smaller scale, single stage launch, it provides an understanding of how required modifications could occur.

Aircraft

Two military aircraft have the capability to launch the CAV, the B-52 and the KC-10 [1]. The B-52 has long supported strategic bombing initiatives, from its first flight in 1954 through more recent applications in conflicts such as Operation Iraqi Freedom (OIF) and Enduring Freedom (OEF). Only the H model is still in US Air Force inventory,
assigned to Minot AFB in North Dakota, and Barksdale AFB in Louisiana. The B-52 can fly a 70,000 pound payload, at speeds up to 650 miles per hour and altitudes up to 50,000 feet. They can be modified to carry air launch payloads. The maximum range of the B-52 is 8800 miles. Although old now, the airframe is expected to last past 2040, making it a reasonable platform for developments in the near future [29]. Figure 14 displays a B-52.

The KC-10 is a tanker and cargo aircraft first used in 1981. It has flown thousands of missions in multiple conflicts, ranging from Desert Storm to OIF/OEF. It has a cargo capability of 170,000 pounds at a speed of 619 mph and an altitude of 42000 feet. It has a range of 4400 miles. While not as fast and far-reaching as a B-52, it has a higher cargo capacity. It has not been previously modified for air launch purposes, and would require development prior to deployment for CAV related-purposes. Figure 15 illustrates a typical KC-10 configuration [30].
Optimal Control Theory

An optimal control problem may be posed in a number of mathematical formulations. However, each method contains key elements that remain the same. The goal of the problem posed is to find a control $u$ such that a set of dynamic equations in the form

$$\dot{x} = P(x(t), u(t), t)$$  \hspace{1cm} (7)

will follow an admissible trajectory that minimizes the following “performance measure” typically referred to as a cost function

$$J = h(x(t_f), t_f) + \int_{t_0}^{t_f} g(x(t), u(t), t) dt$$ \hspace{1cm} (8)

where $x$ and $u$ are the optimal trajectory and control, respectively. The cost function takes on a variety of forms based on the variable in need of minimization. The form required develops from which variables are fixed or free in the simulation. Due to the nature of the problem, it is typically solved using discretization and numerical methods [31]. This methodology is also suitable for maximizing the cost function. Maximization may be accomplished through the conversion of the maximization problem to a minimization
problem, a simple process. For example, to maximize a given final state variable component \( x_f \), the cost function takes the form

\[
J = -x_f
\]  

(9).

Using similar techniques, it is possible to achieve a variety of cost functions that allow the minimization or maximization of variables.

The cost function and dynamic equations may be limited by a series of boundary conditions of the form

\[
t_0, x_0, and \psi(x_f, t_f) = 0
\]  

(10)

and equality and inequality constraints structured as

\[
C(x_f, t_f x_0, t_0) \leq 0
\]  

(11)

where the parameters are defined in Table 3.

<table>
<thead>
<tr>
<th>Table 4 Cost Function Boundary Condition Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
</tr>
<tr>
<td>----------</td>
</tr>
<tr>
<td>( t_{o,f} )</td>
</tr>
<tr>
<td>( x_{o,f} )</td>
</tr>
<tr>
<td>( \psi )</td>
</tr>
</tbody>
</table>

These equations and concepts provide the basic outline of an optimization problem, and are essential pieces to any GPOPS-II scenario [32].

**Pseudospectral Methods**

Numerical methods are essential to the development of solutions to the optimal control problem, and pseudospectral methods are a class of numerical methods. Several software packages implement this for use in optimization programs [33]. Non-linear programming (NLP) solvers are typically used to implement pseudospectral methods.
Typically, SNOPT is employed, a solver which uses Sequential Quadratic Programming (SQP) with a quasi-Newton approximation of the Hessian or Lagrangian matrices which are an essential step in finding optimal solutions. NLP solvers use discretized equations to solve the problem at a series of points. Collocation schemes are then used to approximate each of the points [33]. The combination of the collocation scheme and a NLP solver creates a tool for finding an optimal solution to a given problem [33].

**General Pseudospectral Optimization Software**

GPOPS is a program using MATLAB® mathematical capabilities which employs pseudospectral methods and non-linear programming to solve optimal control problems. It allows the inclusions of boundary conditions and constraints. Through a system of structures and arrays, the program can handle multiple phases of dynamics, each with its own set of constraints and boundary conditions. This is done through the inclusion of transition conditions between the phases in order to achieve seamless dynamics. Cost functions may be assigned to the overall scenario, or to the individual phases based on user needs [4]. Although the structures in GPOPS are pre-assigned, these structures must all be filled by the user. A guess must also be input for the initial solution the program will attempt. The quality of the guess often influences whether the solver will be able to achieve an “optimal” answer. Scaling can also be important to the ability to find a solution. Multiple examples of previously achieved optimization problems may be found in the User’s Manual, as well as a full description of the many features that come with the software package [4]. Figure 16 below displays the general outline of the GPOPS-II data structure.
This structure contains many of the values require for a dynamic optimization scenario, from constraints and bounds to control variables and dynamic equations. Each piece has requisite input or output values that provide information vital to the solution. The mesh values have default inputs that may be changed as needed.

**Previous AFIT Research**

Work in trajectory optimization has been completed by a number of previous AFIT students, but of these students, three previous students have contributed work that directly impacted this study.
Yaple pursued a mission planning tool with the objective of combining several phases to create a launch-to-termination optimization sequence. Once the sequence was complete, a tool was to be built around it for the use of decision makers in prompt global strike situations [34]. Her approach to multi-mission phase combination provided the starting point for the multi-phase combination used in this study. Masternak created a re-entry optimization scenario for the purpose of testing a more accurate heating model [35]. Jorris optimized waypoint and no fly zone requirements in a re-entry scenario [24]. Their work in optimization represents essential elements of the approach attempted in this study, as well as informed the research when choosing other options.

Summary

Each of these concepts is important to the development of previous research in the aspects required to build the modeling tool presented in this study. While not all are used, they present information relevant to the development of the system created. They also provide some background to the problem the tool attempts to solve, as well as offer possible avenues of methodology and analysis.
III. Methodology

Chapter Overview

The methodology of calculations is essential to the relevance and future application of this study. A four step process was developed for the accomplishment of the research to ensure that the methodology required would be completed. The first step is to create the atmospheric model. The second step is to build dynamics. The third is to run the relevant simulations through a series of test conditions. The final part of the process is to analyze the data calculated through the simulations and search for trends that shed light on atmospheric impact in the scenario. This chapter steps through how each of these steps will be accomplished. The basic code related to this methodology may be located in Appendix A.

Atmospheric Model Construction

Standard Day Model

The key to this study is the development of a more accurate atmospheric model than the Standard Day density model. This allows a simulation of hot and cold days in addition to a standard day, and a comparison amongst the three. The first model to discuss is the starting model – the standard day density model. This is the model used in the previous studies [5]

\[ \rho = 1.225 e^{-\beta \Delta H} \]  

(12)

where Table 4 defines the key parameters.
Table 5 Standard Day Density Model Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Density</td>
<td>Variable</td>
<td>$kg/m^3$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Scale height</td>
<td>0.14</td>
<td>$km^{-1}$</td>
</tr>
<tr>
<td>$\Delta H$</td>
<td>Altitude above Earth’s surface</td>
<td>Variable</td>
<td>$km$</td>
</tr>
</tbody>
</table>

*Note: This equation is a simplified form of Eq. (1)*

The standard day density model is simple, and allows a very straight-forward calculation to find atmospheric density based upon altitude. It is a line of best fit based on standard day conditions, and used for many applications across disciplines [6].

**Improved Model**

The first step to building an improved model is to find large datasets of real world atmospheric data to use in its creation. The best data available comes from the National Oceanic and Atmospheric Administration (NOAA) via a modeling system called Global Forecast System (GFS). The data is stored online in archives, and may be downloaded free of charge by any organization or individual. The most difficult portion of the process is translating the data to usable format. It begins in a “.grb2” format and must be read into a MATLAB® “.mat” file for use by the optimizer.

The file conversion process was developed by the AFIT Physics Department. The programming required falls into a subset of functions called “nctoolbox.” For this study, the “.grb2” conversion functions were modified from the existing “nctoolbox” functions. This allows data sets to be collected and converted [36].

For this study, a total of 160 test cases were collected and converted to a “.mat” usable file containing a $\beta$ and $\rho_0$ value for a 0.5 degree grid around the globe. These
represent two years of “hot” data from the months of July and August of 2015 and 2016, and one year of “cold” data from December and January of 2016. They were then refined to a smaller dataset that allowed the scenarios to run on all of the same conditions save the parameter being changed for the test cases. It is to be expected that not all of the test cases would be able to run using the same parameters, due to the wide variety of conditions these scenarios represent. These data files cover the globe with a grid defined in 0.5 degree increments, and at varying sampled altitudes at each of these increments [14]. It provides temperature and pressure data at each of these points. From this information, a model may be extrapolated using latitude and longitude coordinates. It is a sparse matrix in that not every point is covered and some points are more heavily sampled then others in the global grid. However, it represents an increased focus on the impact of global changes from location to location on the flight path of a sub-orbital hypersonic vehicle, as opposed to the classic exponential model which only models the changes based on altitude and on a standard day. As atmospheric density appears explicitly in the equations of motion through the lift and drag equations, it is the easiest value to change. The GFS does not give density information, and so the given temperature and pressure data must be used to calculate density [12]. This also avoids the challenges of appropriately modeling temperature, as its trend lines are unique in shape.

The Ideal Gas Law equation was used in this calculation. It maintains the exponential nature of the density data, a feature modeled by the standard day, but allows departures from standard temperature and pressure to impact the density distribution, as demonstrated in Eq. (4) from Chapter II:

\[ \rho = \frac{p}{RT} \quad (4) \]
Once each point on the grid has been run using the Ideal Gas Law to convert pressure and temperature to density, the densities are then used to create an exponential model. Figure 17 shows a sample data point with its related altitudes and density values in comparison to the exponential model.

Figure 17 Sample Density at 0 degrees Latitude and Longitude

The error for this sample set displays best the difference between the two models in terms of density, as shown in Fig. 18:
Once each grid point has been calculated, *fminsearch*, a function available as part of the MATLAB® optimization toolbox, is run to find a $\beta$ value (scale height) that best models the densities at that grid point. The $\beta$ value is saved along with the density at Earth’s surface $\rho_0$ so that for any data point on the grid, an exponential model may be used to represent the relevant data. This new model with every grid point is then used in the optimizer in the place of the standard day exponential model for every test case downloaded and converted from GFS. Each dataset is run separately in the optimizer, meaning that every test case subject to the same weather patterns throughout that test case. All 102 test cases are run for five different sets of initial conditions. The framework of this model is meant to capture the stochastic nature of the atmosphere. Figure 19 provides a summary of the process used to create the weather model.
As Fig 19 demonstrates, the process is multi-step, and yields hundreds of optimal results, as well as millions of related density models. With the atmospheric model built, the next step is to build the requisite dynamic models.

**Vehicle Dynamic Model**

The vehicle dynamic model may be split into two phases, launch and re-entry. Figure 1 previously displayed the key features of the phases. Phase 1 begins at the initial time, and ends at burnout, while Phase 2 begins at burnout and ends at a terminal window defined using final state boundaries. Each phase has a unique control variable and different aspects to their dynamic equations. However, both phases use the same coordinate system as may be referenced in Fig 20.
Each of the red circled symbols represents one of the six coordinate and state values used in both phases of flight.

**Phase 1: Launch**

The launch equations developed from two different sources. The first step was to develop the primary dynamic equations. One possible set of equations originated from example problems given for the GPOPS-II program. They are modeled for Cartesian Coordinates and lack a way to identify directly the location of the vehicle with respect to the Earth’s surface. However, they were the equations used for the first iterations of the
program, with appropriate conversion equations for use in transferring data between the phases. Three key equations formed this approach:

\[ \ddot{r} = v \quad \text{ (13)} \]

\[ \dot{v} = \frac{\tau u + D}{m} + g \quad \text{ (14)} \]

\[ D = \frac{1}{2} \rho v^2 C_D A \quad \text{ (15)} \]

and

\[ T = -\dot{m} I_{sp} g_0 \quad \text{ (16)} \]

where Table 5 defines the key parameters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{r} )</td>
<td>Change in position</td>
<td>( km/s )</td>
</tr>
<tr>
<td>( \dot{v} )</td>
<td>Change in velocity</td>
<td>( km/s^2 )</td>
</tr>
<tr>
<td>( T )</td>
<td>Thrust</td>
<td>( N )</td>
</tr>
<tr>
<td>( D )</td>
<td>Drag</td>
<td>( N )</td>
</tr>
<tr>
<td>( m )</td>
<td>Mass of the vehicle</td>
<td>( kg )</td>
</tr>
<tr>
<td>( g, g_0 )</td>
<td>Pull of gravity</td>
<td>( m/s )</td>
</tr>
<tr>
<td>( u )</td>
<td>Control input</td>
<td>( rad )</td>
</tr>
<tr>
<td>( v )</td>
<td>Velocity</td>
<td>( km/s )</td>
</tr>
<tr>
<td>( C_D )</td>
<td>Coefficient of drag,</td>
<td>unitless</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Atmospheric density</td>
<td>( kg/km^3 )</td>
</tr>
<tr>
<td>( A )</td>
<td>Area of Vehicle exposed to drag</td>
<td>( m^2 )</td>
</tr>
<tr>
<td>( \dot{m} )</td>
<td>Mass flow rate of rocket</td>
<td>( kg/s )</td>
</tr>
<tr>
<td>( I_{sp} )</td>
<td>Specific Impulse</td>
<td>( s )</td>
</tr>
</tbody>
</table>

These equations were then split into the necessary \( x, y, \) and \( z \) Cartesian coordinates for use in GPOPS. However, this methodology made the transition between the two phases difficult due to a lack of explicit relationships between the two coordinate sets. A new set of equations were required to ensure the dynamics were appropriately modeled. The new
set of equations follow the derivation presented in [37]. It begins with a set of kinematic and force equations:

\[ \dot{r} = V \sin \gamma \]  
\[ \dot{\theta} = \frac{V \cos \gamma \cos \psi}{r \cos \phi} \]  
\[ \dot{\phi} = \frac{V \cos \gamma \sin \psi}{r} \]  

\[ \dot{V} = \frac{T}{m} (\cos \zeta \cos \epsilon) - \frac{D}{m} - g \sin \gamma + r \omega_e^2 \cos \phi (\cos \phi \sin \gamma - \sin \phi \sin \psi \cos \gamma) \]  

\[ \dot{\psi} = \frac{1}{m \cos \gamma V} \left[ \frac{T}{V_m} (\sin \zeta \sin \sigma + \cos \zeta \sin \epsilon \cos \sigma) + \frac{L}{V_m} \cos \sigma \right] - \frac{g}{V} \cos \gamma \cos \psi \tan \phi + \frac{2 \omega_e \cos \phi \cos \psi}{V} \]  

where Table 6 highlights the included variables.

**Table 7 Initial Equation of Motion Parameters**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description/Units</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{V} )</td>
<td>Change in inertial velocity</td>
<td>( km/s^2 )</td>
</tr>
<tr>
<td>( V )</td>
<td>Inertial velocity</td>
<td>( km/s )</td>
</tr>
<tr>
<td>( T )</td>
<td>Thrust</td>
<td>( N )</td>
</tr>
<tr>
<td>( m )</td>
<td>Mass of vehicle</td>
<td>( kg )</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Angle between thrust and velocity on pitch axis and velocity plane</td>
<td>( Rad )</td>
</tr>
<tr>
<td>( \epsilon )</td>
<td>Angle between thrust and velocity on lift axis and velocity plane</td>
<td>( Rad )</td>
</tr>
<tr>
<td>( D )</td>
<td>Drag</td>
<td>( N )</td>
</tr>
<tr>
<td>( g )</td>
<td>Acceleration due to gravity</td>
<td>( km/s^2 )</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Flight path angle</td>
<td>( Rad )</td>
</tr>
<tr>
<td>( r )</td>
<td>Radius from the center of the earth</td>
<td>( Km )</td>
</tr>
<tr>
<td>( \omega_e )</td>
<td>Angular velocity of the earth</td>
<td>( Rad/s )</td>
</tr>
<tr>
<td>( \phi )</td>
<td>Latitude of the vehicle with respect to Earth’s latitude coordinates</td>
<td>( Rad )</td>
</tr>
<tr>
<td>( \psi )</td>
<td>Heading angle of the vehicle</td>
<td>( Rad )</td>
</tr>
<tr>
<td>( L )</td>
<td>Lift</td>
<td>( N )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Bank angle of the vehicle</td>
<td>( Radians )</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Longitude of the vehicle with respect to Earth’s longitude coordinates</td>
<td>( Radians )</td>
</tr>
</tbody>
</table>
Once these equations have been defined, they must be simplified for two reasons – first, not all of the conditions presented here are necessary for a general model of launch. Second, while additional terms increase the accuracy of model, they also increase the non-linear complexity and thus the time required to calculate the model, a key factor when running a high volume of test cases due to the time required to reach a solution.

The first step of simplification is done through the assumption that the velocity and the thrust of the rocket are in the same direction. This is not always the case, and decreases vehicle maneuverability in the scenario, but it simplifies the calculations required. This means

$$\zeta = \epsilon = 0 \text{ radians} \quad (23).$$

Another simplification is to consider the rotation of the Earth negligible. If the goal of the scenario was to determine the true maximum range, this would not be an acceptable simplification. Additionally, this assumption becomes unacceptable in polar regions, where the rotation of the earth would be a much more significant quantity. To avoid this complication, scenarios will be designed which will not reach the polar region. Over the course of the scenario, approximately 1200 seconds, using Eq. (24), Earth would rotate approximately five degrees at a rotation velocity of $7.3 \times 10^{-5} \frac{rad}{s}$, as demonstrated in the equation below

$$\text{rotation} = \omega_e \ast \text{time}_{scenario} \quad (24).$$

However, considering the complexity of these terms, and that the goal of this scenario is to evaluate the difference in optimal scenarios, not the numeric value of the objective
itself, it is an acceptable loss of accuracy to consider the rotation of the Earth negligible. Therefore

\[ \omega_e \approx 0 \]  

(25).

The next simplification is to modify the flight path angle equation. This is because the flight path angle is the control variable in this scenario, and is no longer governed only by the dynamics of flight but also by control input.

Finally, bank angle is not a relevant quantity for a launching rocket, as banking a rocket does not change the effective surface area (lift) of the vehicle, meaning that

\[ \sigma \approx 0 \]  

(26)

These simplifications yield the following seven equations to define the motion of air launch:

\[ \gamma = \gamma_{input} + \gamma_{control} \]  

(27)

\[ \dot{r} = V \sin \gamma \]  

(28)

\[ \dot{\theta} = \frac{V \cos \gamma \cos \psi}{r \cos \phi} \]  

(29)

\[ \dot{\phi} = \frac{V \cos \gamma \sin \psi}{r} \]  

(30)

\[ \dot{V} = \frac{T}{m} - \frac{D}{m} - g \times \sin \gamma \]  

(31)

\[ \dot{\psi} = \frac{T}{V m} - \frac{g}{V} \cos \gamma + \frac{V}{r} \cos \gamma + \]  

(32)

\[ \psi = -\frac{R_V}{r} \cos \gamma \times \cos \psi \times \tan \phi \]  

(33)

where the variables have been previously defined in Table 6. With the main dynamics equations outlined, the additional equations required to calculate certain variables must
also be defined. As previously mentioned, the Earth’s gravitational pull \( g \) is defined using a spherical earth and uniform gravitational pull as defined previously in Eq. (6). Thrust is a function of mass flow rate, specific impulse, and initial gravitational pull, as defined in Eq. (16) and Table 8.

### Table 8 Thrust Equation Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition/Units</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dot{m} )</td>
<td>Mass flow rate</td>
<td>33.74</td>
<td>( kg/s )</td>
</tr>
<tr>
<td>( m_{fuel} )</td>
<td>Mass of Fuel</td>
<td>5259.17</td>
<td>( kg )</td>
</tr>
<tr>
<td>( I_{sp} )</td>
<td>Specific Impulse</td>
<td>440</td>
<td>( s )</td>
</tr>
<tr>
<td>( g_0 )</td>
<td>Initial gravitational pull</td>
<td>0.00981</td>
<td>( km/s^2 )</td>
</tr>
</tbody>
</table>

These values come from [28], a modeling of the Centaur Upper Stage for the Titan IV rocket. Previous tests have used the Pegasus for air launch scenarios. However, due to the desire to keep the launch stage a simple single-stage-to-orbit scenario, a stronger booster is required because Pegasus cannot reach the needed altitudes with a single stage. The Centaur Upper Stage, if assumed to be ideally expanded throughout its burn [28], has enough thrust to reach appropriate altitudes in a single stage, and a mass the B-52 can manage. These variables will be discussed in more detail later in Chapter III in the initial conditions.

Drag has been previously defined in Eq. (15), and its definition and variables remain the same for this formulation. The variable \( \rho \) was previously defined when density modeling was discussed in more detail in the atmospheric modeling methodology section. There is no cost function associated with the first phase; only the second phase has a cost function which is applied to the overall optimization scenario, due to the manner in which GPOPS-II solves two-phase scenarios.
To ensure the optimization tool can find an appropriate solution, it must be given a “good” initial guess. An initial guess maybe considered “good” if it gives a good approximation of the dynamic features of the trajectory, an approximation accomplished using the exact same dynamics as were given to the optimizer in combination with a fixed step solver. This develops an initial set of states which help the optimizer choose an appropriate starting point from which to iterate. The initial guess gives an indication of the shape and order of magnitude of the optimal solution, and the closer this matches to actual behavior, the “better” the initial guess may be considered to be. Building the initial guess requires the use of the same dynamic equations as well as the construction of a fixed step solver. It also uses the same constants and initial conditions as the optimizer. The fixed step solver was constructed using a Runge-Kutta 4th order method (RK4) [38]

\[ y_{n+1} = y_n + h(k_1 + 2k_2 + 2k_3 + k_4) \]  
(34)

Where

\[ k_1 = f(x_n, y_n) \]  
(35)
\[ k_2 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_1) \]  
(36)
\[ k_3 = f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hk_2) \]  
(37)
\[ k_4 = f(x_n + h, y_n + hk_3) \]  
(38)

And

\[ h = \Delta t \]  
(39)

This solver is then applied to the given dynamics with the initial condition sets given in Table 9.
Table 9 Runge-Kutta Fixed Step Solver Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_0$</td>
<td>Initial time</td>
<td>0</td>
<td>$s$</td>
</tr>
<tr>
<td>$r_0$</td>
<td>Initial Radius</td>
<td>$50,000$</td>
<td>$ft$</td>
</tr>
<tr>
<td></td>
<td>($40,000$)</td>
<td>$+20.9\times10^6$</td>
<td></td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>Initial Longitude</td>
<td>0, 35, $-130$</td>
<td>$deg$</td>
</tr>
<tr>
<td>$\phi_0$</td>
<td>Initial Latitude</td>
<td>0, 35, 35</td>
<td>$deg$</td>
</tr>
<tr>
<td>$V_0$</td>
<td>Initial Velocity</td>
<td>0.29</td>
<td>$km/s$</td>
</tr>
<tr>
<td>$\psi_0$</td>
<td>Initial Heading Angle</td>
<td>0</td>
<td>$deg$</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>Time Step</td>
<td>1</td>
<td>$s$</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>Guess Control Input</td>
<td>40</td>
<td>$deg$</td>
</tr>
<tr>
<td>$\gamma_0$</td>
<td>Initial Flight Path Angle</td>
<td>40</td>
<td>$deg$</td>
</tr>
</tbody>
</table>

These initial conditions are largely based on two different sets of parameters. The first set of parameters are general test cases. This parameter governs the initial latitude and longitude sets shown in Table 9, as well as the initial radius. These are parameters varied for data analysis purposes, comparisons done to determine the holistic impact of the new atmosphere model. It is important to note that in the initial guess solution, the guess control input is then added to over the course of the iteration to make the rocket trajectory realistic and feasible. The second set of parameters stem from the capability of the B-52 “Stratofortress”, the launch platform chosen for this study based on its global reach capability and total mass and altitude capacity. It can handle a greater mass when range and altitude are considered. According to [29], the B-52 may manage up to the following:
Table 10 B-52 Flight Initial Condition Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$</td>
<td>Velocity of platform</td>
<td>650</td>
<td>mph</td>
</tr>
<tr>
<td>$alt$</td>
<td>Altitude</td>
<td>50,000</td>
<td>ft</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of Cargo</td>
<td>31,500</td>
<td>kg</td>
</tr>
<tr>
<td>$D$</td>
<td>Total range</td>
<td>8,800</td>
<td>mi</td>
</tr>
</tbody>
</table>

The parameters governed by the platform include initial radius and velocity. The radius is varied between the highest altitude maintainable by the platform and a reasonable altitude for the launch [29], [21].

Given these initial conditions, the dynamics are then propagated forward over a given stretch of time. For this phase, the time was determined by the burn time of the rocket. The burn time was calculated using the following relationship

$$ t_{burn} = \frac{m_{fuel}}{\dot{m}} \quad (40). $$

Using the values from Table 8

$$ m_{fuel} = 5259.17 \text{ kg}; \quad \dot{m} = 33.74 \text{ kg/s} \quad (41) $$

in combination with Eq. (40), this yields a time of burn out of

$$ t_{burn} = 155 \text{ seconds} \quad (42). $$

It is important to note that this mass of fuel is not the total capacity of the Centaur Upper Stage. It can manage up to 21036.707 kilograms of fuel [28]. However, it was necessary to decrease available fuel in order to appropriately limit the scenario. With the full fuel capacity at ideal expansion, the Centaur Upper Stage would carry the hypersonic vehicle into full orbit, thus allowing a near infinite number of trips around the globe due to the limited orbital modeling provided. At 25% of its full fuel capacity, it can only achieve sub-orbital flight, thus allowing a reasonable answer to be achieved. This
assumption is reasonable because this launch system has a liquid fuel engine, which may be appropriately fueled without major redesign requirements, as may be required on a solid rocket motor [39]. The final development is the determination of the coefficient of drag, $C_D$. Using [40] and [28], as well as some worst case and average values, the booster has a $C_D$ of 0.2. One constraints was placed on Phase 1, other than the dynamics constraints – a minimum fuel constraint. The minimum fuel constraint is a simple constraint based on the mass flow rate of the rocket engine and the amount of fuel present, a calculation which yields a total maximum burn time of 155 seconds for the rocket engine. With each of the variables defined, the initial guess may be created.

Figures 21 through 25 display the results of this simulation for the initial guess of the first phase. This also gives a general idea of what the dynamic solution should look like in the optimal scenario.

![Figure 21 Launch Initial Guess, Conditions 0 Deg Latitude and Longitude](image)

*Figure 21 Launch Initial Guess, Conditions 0 Deg Latitude and Longitude*
Figure 22 Launch Initial Guess, Conditions 35 Deg Latitude and Longitude

Figure 23 Launch Initial Guess, Conditions 35,-130 Deg Latitude and Longitude
Each of the graphs represent simple, uncontrolled dynamics with limited control input possibilities. Due to the lack of control, they will not perfectly resemble the optimal answers retrieved using GPOPS-II. The initial guess graphs demonstrate some initial variation in answers, as demonstrated in Table 10.
Table 11 Scenario Change for Phase 1 Initial Guess

<table>
<thead>
<tr>
<th>Conditions</th>
<th>$\phi, \theta, \text{Altitude}$</th>
<th>$\Delta \theta$ in degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude/Longitude</td>
<td>$0^\circ, 0^\circ, 50,000 \text{ ft}$</td>
<td>0.63</td>
</tr>
<tr>
<td>Latitude/Longitude</td>
<td>$35^\circ, 35^\circ, 50,000 \text{ ft}$</td>
<td>0.773</td>
</tr>
<tr>
<td>Latitude/Longitude</td>
<td>$35^\circ, -130^\circ, 50,000 \text{ ft}$</td>
<td>0.773</td>
</tr>
<tr>
<td>Initial Altitude</td>
<td>$0^\circ, 0^\circ, 40,000 \text{ ft}$</td>
<td>0.64</td>
</tr>
<tr>
<td>Initial Altitude</td>
<td>$0^\circ, 0^\circ, 30,000 \text{ ft}$</td>
<td>0.65</td>
</tr>
</tbody>
</table>

This demonstrates that before the optimization is applied, the difference in answer for the scenario can vary up to 0.143 degrees. Using Eq. (43), the haversine formula [40]:

$$d = 2r \sin^{-1}\left(\sqrt{\sin^2 \frac{\Delta \phi}{2} + \cos \phi_0 \cos \phi_f \sin^2 \frac{\Delta \theta}{2}}\right)$$ (43)

where Table 11 defines the variables used in Eq. (43).

Table 12 Haversine Formula Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>Distance traveled over Earth’s surface</td>
<td>km</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius of the Earth</td>
<td>km</td>
</tr>
<tr>
<td>$\phi_0, f$</td>
<td>Initial and Final Latitude</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Longitude</td>
<td>rad</td>
</tr>
<tr>
<td>$\Delta \phi$</td>
<td>Change in Latitude</td>
<td>rad</td>
</tr>
<tr>
<td>$\Delta \theta$</td>
<td>Change in Longitude</td>
<td>rad</td>
</tr>
</tbody>
</table>

This difference in longitudinal degrees can be a difference of approximately 16 kilometers in the answer for the maximum variation between test conditions. This small value is to be expected with little to no control input available.

The initial guess and the dynamics for Phase 1 of the optimization scenario having been established, the inter-phase conditions must then be defined.
Interphase Development

The key variables between Phases 1 and 2 are fundamentally the same. There are only three transitions which necessarily occur between Phases 1 and 2 for appropriate modeling purposes. First and most importantly, the flight path angle $\gamma$ shifts from a control variable to only a state variable. $\gamma$ is now completely a function of dynamic relationships. The second transition is the inclusion of bank angle terms in the equations of motion. This means an initial value for bank angle must be established. Due to the previously mentioned aspects of flight dynamics – primarily that a launch vehicle does not have a true bank angle quantity – the initial bank angle may be considered to be zero. Thus:

$$\sigma_0 = 0 \text{ degrees}$$

(44)

This assumption has additional validity in that there is very little atmosphere at the termination of Phase 1, meaning that lift, the primary dynamic factor impacted by bank angle, is very close to zero itself, rendering the lift-bank term in the equations inconsequential. Therefore, it may initially be considered zero. The bank term now becomes the control term. The third transition is the removal of the thrust terms. The vehicle is a hypersonic glide vehicle, meaning that there will be no engine input to the scenario. The lack of “airbreathing” engine additionally means that the impact of atmospheric conditions on the engine will not be analyzed. To complete the transition, each end state of Phase 1 is set equal to the initial state of Phase 2. Phase 2 will be re-entry, as the vehicle is only capable of achieving sub-orbital flight with its given constraints.
Phase 2: Re-Entry

Once the launch equations had been developed, the next step was to tie them to the re-entry equations. The re-entry equations follow from the same initial equations as the launch equations, but with several different assumptions made. Starting again from Eqs. (18) through (23), the first assumption to be made is

\[ T = 0 \text{ Newtons} \] (45)

This assumption may be made due to the lack of propulsion in use on the hypersonic glide vehicle. Therefore, any terms containing thrust are removed from the equations. Additionally, when thrust is set equal to zero, any terms with the variables \( \zeta \), \( \varepsilon \) are removed, thus making these values irrelevant as well. Finally, the rotation of the earth may be considered to be approximately zero. Although the use of Eq. (24) once again demonstrates this is not strictly true, again presenting a several degree difference in answer, some assumptions may be made pertaining to the relevance of its inclusion. From previous studies, [42], this assumption has further been demonstrated.

As was demonstrated previously [42], the rotation of the earth may be considered negligible for this study. Thus, the final equations become:

\[ \dot{r} = RV \sin \gamma \] (46)

\[ \dot{\theta} = \frac{RV \cos \gamma \cos \psi}{r \cos \phi} \] (47)

\[ \dot{\phi} = \frac{RV \cos \gamma \sin \psi}{r} \] (48)

\[ \dot{V} = -\frac{D}{m} - g \ast \sin \gamma \] (49)

\[ \dot{\gamma} = \frac{L}{V m} \cos \sigma - \frac{g}{V} \cos \gamma + \frac{RV}{r} \cos \gamma \] (50)
\[
\dot{\psi} = \frac{L \sin \sigma}{m \cos \gamma V} - \frac{R \gamma}{r} \cos \gamma \cos \psi \tan \phi
\]  
(51)

Where the variable definitions presented in Table 7 still apply. In this case, all initial conditions are derived from the end state of the previous phase. Therefore, they may not be a given constant, particularly as the test cases change. The only change not related to the previous phase end state values is the change in vehicle dynamic parameters for the hypersonic vehicle. In particular, this means lift and drag coefficients, mass, and effective area. From previous research done by Jorris, [25], these characteristics have either been given or derived for the CAV, the hypersonic vehicle being modeled for this study. In particular, Table 13 outlines these critical values.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>Vehicle mass</td>
<td>907</td>
<td>( kg )</td>
</tr>
<tr>
<td>( S (or A) )</td>
<td>Effective area</td>
<td>750</td>
<td>( in^2 )</td>
</tr>
<tr>
<td>( C_D )</td>
<td>Coefficient of drag</td>
<td>0.192</td>
<td>( unitless )</td>
</tr>
<tr>
<td>( C_L )</td>
<td>Coefficient of lift</td>
<td>0.557</td>
<td>( unitless )</td>
</tr>
</tbody>
</table>

In Chapter II, it was mentioned that Jorris developed a linear fit model for the coefficients of lift and drag based on given data for the purpose of creating an AOA profile [25]. The AOA was then used as a control variable. However, the use of this model was avoided in this study primarily due to issues of model complexity and test variable isolation. In order to calculate the appropriate coefficients, the model developed by Jorris requires a Mach number and angle of attack value. The Mach number calculation must be done using temperature. Temperature is one of the key variables in creating the atmospheric density model. It is also very difficult to create a line of best fit to ensure the proper temperature value has been found for the given location and altitude. To reduce complexity, a fixed
lift and drag coefficient were created from the data in Jorris’ model [25]. Additionally, variable isolation is much more difficult when two particular variables are now involved in the scenario improvements. Future work may more effectively solve this problem, but for this study, the lift and drag coefficients remain constant in the effort to determine how a simple shift in density model can impact the overall solution. The control variable is bank angle in this dynamic formulation, and was given a range of

$$-60\, \text{degrees} \leq \sigma \leq 60\, \text{degrees}$$  \hspace{1cm} (52)

The final developmental element is the cost function. This equation is very important to the study because it establishes a metric by which the performance of the vehicle dynamics in the more accurate atmospheric model may be measured. It is important to note that the vehicle dynamic metric being measured here is overall performance rather than aspects based on individual qualities of the performance (e.g. specific lift/drag requirements, etc). Therefore, the cost function takes the form

$$J = -\theta_f + \alpha_1 \int_{t_{1i}}^{t_{1f}} \left( \frac{u_1}{u_{\text{maximum}_1}} \right)^2 dt + \alpha_2 \int_{t_{2i}}^{t_{2f}} \left( \frac{u_2}{u_{\text{maximum}_2}} \right)^2 dt \hspace{1cm} \text{rad}$$  \hspace{1cm} (53)

and Table 14 defines the parameters.
Table 14 Cost Function Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta_f )</td>
<td>Final Longitude</td>
<td>( Rad )</td>
</tr>
<tr>
<td>( u_{1,2} )</td>
<td>Control Input by Phase (1,2)</td>
<td>( Rad )</td>
</tr>
<tr>
<td>( u_{\text{maximum1,2}} )</td>
<td>Maximum Allowable Control Input by Phase (1,2)</td>
<td>( Rad )</td>
</tr>
<tr>
<td>( \alpha_{1,2} )</td>
<td>Integral Constraint Cost Function Weight by Phase (1,2)</td>
<td>0.009</td>
</tr>
<tr>
<td>( t_{1,1f,2i,2f} )</td>
<td>Initial Time, Final Time by Phase (1,2)</td>
<td>( Sec )</td>
</tr>
</tbody>
</table>

The most important term in the cost function is the final longitude. This is the metric by which overall vehicle performance will be measured, a concept developed further in later sections. The two control terms are meant to inhibit the change in control input and create more realistic control profiles. Also, in the case of a highly non-linear optimization scenario, it was demonstrated over the development of this study to improve the ability of the optimizer to reach an optimal solution. The essential element of the addition of these two terms was to ensure that they did not overwhelm the longitude in the cost function calculation. The way to achieve this was to give each integral term appropriate coefficients, denoted in Table 14 as \( \alpha_1 \) and \( \alpha_2 \). Through testing and observation of the control variables in Phase 1 and 2, a value of 0.009 was assigned to both coefficients. This allowed a reasonable control profile while ensuring that the final longitude would be the dominant variable in the cost function.

Once the appropriate dynamics have been constructed, an initial guess must again be created, as was done for Phase 1. The methodology is same, but the initial conditions
are different. For the initial guess of Phase 2, Table 15 displays the relevant initial conditions.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_0 )</td>
<td>Initial time</td>
<td>( t_{f_{\text{inal}1}} )</td>
<td>sec</td>
</tr>
<tr>
<td>( r_0 )</td>
<td>Initial Radius</td>
<td>( r_{f_{\text{inal}1}} )</td>
<td>ft</td>
</tr>
<tr>
<td>( \theta_0 )</td>
<td>Initial Longitude</td>
<td>( \theta_{f_{\text{inal}1}} )</td>
<td>deg</td>
</tr>
<tr>
<td>( \phi_0 )</td>
<td>Initial Latitude</td>
<td>( \phi_{f_{\text{inal}1}} )</td>
<td>deg</td>
</tr>
<tr>
<td>( V_0 )</td>
<td>Initial Velocity</td>
<td>( V_{f_{\text{inal}1}} )</td>
<td>( \text{km/s} )</td>
</tr>
<tr>
<td>( \psi_0 )</td>
<td>Initial Heading Angle</td>
<td>( \psi_{f_{\text{inal}1}} )</td>
<td>deg</td>
</tr>
<tr>
<td>( \Delta t )</td>
<td>Time Step</td>
<td>1</td>
<td>sec</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>Initial Flight Path Angle</td>
<td>( \gamma_{f_{\text{inal}1}} )</td>
<td>deg</td>
</tr>
<tr>
<td>( \sigma_0 )</td>
<td>Guess Control Input</td>
<td>0</td>
<td>deg</td>
</tr>
</tbody>
</table>

As shown in Table 15, with the exception of the control input, each of the variables is set equal to the conditions at the end of the first phase. This is true both for the initial guess and overall optimization scenario, just as this is true for the first phase in the overall scenario and the first phase initial guess. The initial conditions and dynamics established, the dynamics must then be run through the fixed step solver, both with the standard day and real-world atmospheric models, dependent on the scenario being run in the optimizer – the model used must be the same in both. Figures 26 through 30 display a summary of the results.
Figure 26 Initial Guess Phase 2 Conditions 0 Degrees Latitude and Longitude

Figure 27 Initial Guess Phase 2 Conditions 35 Degrees Latitude and Longitude
Figure 28 Initial Guess Phase 2 Conditions 35 -130 Degrees Latitude and Longitude

Figure 29 Initial Guess Phase 2 Conditions 30,000 ft Initial Altitude
Figures 26 through 30 demonstrate an appropriate model for the flight dynamics, a fact confirmed through comparison with modeling done in [42]. Given these baseline results from the initial guess, an initial difference between the scenario arises in terms of total longitude achieved, as demonstrated in Table 16.

**Table 16 Scenario Change for Phase 2 Initial Guess**

<table>
<thead>
<tr>
<th>Initial Conditions</th>
<th>Variable Values</th>
<th>Change in Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude/Longitude</td>
<td>0°, 0°</td>
<td>11.534°</td>
</tr>
<tr>
<td>Latitude/Longitude</td>
<td>35°, 35°</td>
<td>14.48°</td>
</tr>
<tr>
<td>Latitude/Longitude</td>
<td>35°, −130°</td>
<td>14.48°</td>
</tr>
<tr>
<td>Initial Altitude</td>
<td>40,000 ft</td>
<td>11.511°</td>
</tr>
<tr>
<td>Initial Altitude</td>
<td>30,000 ft</td>
<td>11.476°</td>
</tr>
</tbody>
</table>

Using the haversine formula again, the variation between the phases at maximum is approximately 3.004 degrees. This is a difference in distance across the Earth’s surface of approximately 350 km. In addition to providing the optimizer with the initial guess, this solver also provides a baseline by which the quality of the optimal dynamic solution may
be evaluated. As later results demonstrate, the optimal solution is very similar to the
initial guess. This completes the development of the dynamic model.

**Simulations**

With the models built, the next step is to run the simulation in the optimizer. The
optimizer is included rather than a simple trajectory generator because it clarifies the
picture presented by the data. If a simple trajectory generation were used, the differences
between trajectories would be purely dynamic, with little realism. The inclusion of
control variables ensures that any aspects of the new density model that may be easily
overcome by control input are mitigated, so that the best case of the worst case scenarios
may be presented.

Beyond building the dynamics, this also requires each of the states and controls to
be given a reasonable range within which the optimizer may perturb values. Because
many of the variables are the same between the two scenarios, the limits are the same.
The limits must be developed using values reasonable to the scenario, where these were
developed in [42]. In some cases, this means that they have been determined through trial
and error in the initial guess creation. For example, heading angle is always represented
as a value between 0 and 360 degrees. It therefore makes sense to give heading angle
bounds between 0 and 360 degrees. In contrast, the bank angle limits, as previously
mentioned, were determined through previously testing [42]. Following similar logic:
Table 17 Scenario Boundaries

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Bounds</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝑟</td>
<td>Radius from center of earth to vehicle</td>
<td>[6378, 7378]</td>
<td>km</td>
</tr>
<tr>
<td>𝜃</td>
<td>Longitude</td>
<td>[0 360] *</td>
<td>deg</td>
</tr>
<tr>
<td>𝜙</td>
<td>Latitude</td>
<td>[0 90]</td>
<td>deg</td>
</tr>
<tr>
<td>𝑉</td>
<td>Velocity of Vehicle</td>
<td>[0 8]</td>
<td>km/𝑠</td>
</tr>
<tr>
<td>𝛾</td>
<td>Flight Path Angle</td>
<td>[−90 90]</td>
<td>deg</td>
</tr>
<tr>
<td>𝜨</td>
<td>Heading Angle</td>
<td>[0 360]</td>
<td>deg</td>
</tr>
<tr>
<td>𝜎</td>
<td>Bank Angle</td>
<td>[-60 60]</td>
<td>deg</td>
</tr>
<tr>
<td>𝑚_{fuel}</td>
<td>Vehicle fuel mass</td>
<td>[0 21036.707]</td>
<td>km</td>
</tr>
</tbody>
</table>

** This has been converted from -180 to 180 degrees due to angle conversion errors in GPOPS II

Velocity is one term that has not been developed. The velocity limit of the vehicle was calculated with the general circular orbit equation to represent the upper limit, orbital velocity

\[ V = \sqrt{\frac{\mu}{R}} \]  

(54)

where Table 18 defines the required variables.

Table 18 Orbital Velocity Calculation Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝜇</td>
<td>Earth gravitational constant</td>
<td>398600.5</td>
<td>km³/s²</td>
</tr>
<tr>
<td>𝑅</td>
<td>Radius of vehicle from center of Earth</td>
<td>6478</td>
<td>km</td>
</tr>
<tr>
<td>𝑉</td>
<td>Orbital velocity of vehicle</td>
<td>7.8442</td>
<td>km/𝑠</td>
</tr>
</tbody>
</table>

The value was then rounded up. Due to the sub-orbital nature of the flight path, the object should never reach this velocity, but the limit was kept higher to allow an easier solution convergence, with the understanding that this velocity would never been achieved. Once again, the true set limits were determined using trial and error. The dynamics serve as the necessary constraining factor.
Once the bounds have been established, the simulations may be run, using the eight sets of changing initial conditions previously noted but summarized in Table 18:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test Conditions 1</td>
<td>Latitude</td>
<td>0</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Longitude</td>
<td>0</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Alt</td>
<td>50,000</td>
<td>Ft</td>
</tr>
<tr>
<td>Test Conditions 2</td>
<td>Latitude</td>
<td>35</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Longitude</td>
<td>35</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Alt</td>
<td>50,000</td>
<td>Ft</td>
</tr>
<tr>
<td>Test Conditions 3</td>
<td>Latitude</td>
<td>35</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Longitude</td>
<td>−130</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Alt</td>
<td>50,000</td>
<td>Ft</td>
</tr>
<tr>
<td>Test Conditions 4</td>
<td>Latitude</td>
<td>0</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Longitude</td>
<td>0</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Alt</td>
<td>40,000</td>
<td>Ft</td>
</tr>
<tr>
<td>Test Conditions 5</td>
<td>Latitude</td>
<td>0</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Longitude</td>
<td>0</td>
<td>Deg</td>
</tr>
<tr>
<td></td>
<td>Alt</td>
<td>30,000</td>
<td>Ft</td>
</tr>
</tbody>
</table>

Each set of conditions is then run using 102 streamlined test cases including “hot” and “cold” day conditions developed from the GFS data as well as with the original standard day model to create several datasets for comparison. The final step is to analyze the data using a set of metrics.

**Data Analysis**

The final step in the study is to analyze the data to determine the impact of the new model on the answer in comparison with the standard day model, as well as the difference between “hot” and “cold” day answers as previously limited in this study. In addition to the comparison of longitude, a comparison between average drag values and their standard deviations by scenario will be used as a parameter for the analysis.
Comparisons between mean values and standard deviations assuming a normal distribution will quantify the difference between the parameters.

Each of these metrics together create a picture of the impact of the changing density on the scenario, and allows estimations of additional impact in more complex scenarios.

Summary

There are four steps to the completion of this study. First, the atmospheric model must be completed. Second, the dynamics must be built. Third, the simulations must be run. Finally, the data obtained must be analyzed in order to determine the validity of the hypothesis. With the appropriate methodology outlined, the next step is to run the simulations and analyze the data obtained.
IV. Analysis and Results

Chapter Overview

With the methodology fully established, the test conditions must be run and analyzed. First, some simple steps must be taken to ensure the quality of data being input into the atmospheric model, as well as the validity of the algorithm in use for the atmospheric data conversion. Once this is complete, the test conditions may be run and compared to each other in order to establish trends and draw conclusions.

Results of Simulation Scenarios

The first step in the analysis is to do some examination and validation of the real world weather model. The most accurate and variable value in this model is the temperature data used to construct it. An easy check to ensure that the temperature data displays accurate ranges, and also to confirm the previously given definitions of a “hot” day and “cold” day is simply to create contour plots using the temperature data from the downloaded GFS files. The outlines of Northern Africa, North America, and some of Asia are all highlighted in this temperature distribution, demonstrating the proof of concept that the Northern Hemisphere during summer does increase in temperature with respect to the Southern Hemisphere. Additionally, cooler ocean waters allow the outlines of the continents to be clearer, further confirming the real-world aspect of the GFS model data. The outlines of Central and South America, Southern Africa, and Australia are all clearly visible in the temperature distribution. The contours have been set to the same scale of temperatures, using the maximum and minimum temperatures recorded from [43].
Table 20 Maximum and Minimum Temperatures Recorded on the Earth

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Temperature</td>
<td>331</td>
<td>K</td>
</tr>
<tr>
<td>Minimum Temperature</td>
<td>185</td>
<td>K</td>
</tr>
</tbody>
</table>

This allows a good comparison between the two. With this temperature scaling, continents are evident due to their difference in temperature with the surrounding oceans and bodies of water. In the Northern Hemisphere summer, the Northern Hemisphere continents are clearer and warmer, while in the Northern Hemisphere winter, the Southern Hemisphere continents are clearer and warmer due to their temperature differentials. This is precisely what would be expected for an accurate temperature model, thus confirming its validity. Figures 31 and 32 display a visualization of this check.
These graphs plainly display the features previously described.

**Real-World Density Model Check**

Another important evaluation that must be made is the performance of the real-world data model. Using a minimization of the RMS error between the real-world model and the real world data to find an appropriate scale height factor, $\beta$, the scale height and related initial density value are then used to build an exponential model of the real-world data in the same form as Eq. (1). The most important aspect of this model is that it must be closer to the real-world data than the standard day model, or it represents a decrease rather than an increase in accuracy. The first step was to perform a visual check. Table 21 describes the test case used for this analysis.
Table 21 Density Model Comparison Variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Standard Day Model</th>
<th>Real-World Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.14 $km^{-1}$</td>
<td>0.1211 $km^{-1}$</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>1.225 $kg/m^3$</td>
<td>1.3684 $kg/m^3$</td>
</tr>
<tr>
<td>$RMS error$</td>
<td>0.1367</td>
<td>0.0243</td>
</tr>
</tbody>
</table>

Figure 33 displays a visual comparison between the three datasets – the standard day and real world models, and the actual density data from GFS. These were all calculated using the same altitude.

Figure 33 Comparison Between Density Models and Density Data

It is difficult to tell which is closest overall, but the visual check does reveal that the real-world model is less accurate at higher altitudes. Figure 34 addresses the difficulty in
understanding the difference by providing a plot of the error in density value over the altitude.

![Standard Day Model Error versus Real-World Model Error](image)

**Figure 34 Error Between Real World and Standard Day Models and Density Data**

Figure 34 demonstrates a consistently larger error between the standard day model and the real-world data than between the real-world model and the real-world data, until nearer to the maximum altitude displayed. This is consistent with features noted in Fig. 33. This indicates a higher accuracy in the real-world model closer to the ground. However, the differences between models at the higher altitudes is nearly negligible in terms of the drag experienced, thus preserving the additional accuracy where the impact is highest. Table 22 provides a summary of the key data points for this analysis.

**Table 22 Density Error Between Models**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Density in $kg/m^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Density Error for Real-World Model</td>
<td>0.005</td>
</tr>
<tr>
<td>Mean Density Error for Standard Day Model</td>
<td>0.02472</td>
</tr>
</tbody>
</table>
Table 22 shows that the average drag error between the Standard Day model and the real world data is nearly a full order of magnitude higher than the average error between the real-world data and the real-world model. This confirms that the real-world model is a suitable match for the real-world data given from GFS files.

**Optimal Solution Analysis**

With the temperature model and initial guesses confirmed, the five sets of initial conditions must be run and analyzed. There are several parameters that will be used to measure each set of conditions. First, the distribution of final longitudes, the primary objective, will be analyzed. Amongst these values, the mean, minimum, and maximum final longitude for each type of dataset (hot, cold, standard) must be examined. This gives an understanding of the range of differences between each solution set. Additionally, mean, and standard deviation in drag values for the “hot”, “cold” and standard day cases must be presented for a full understanding of the impact this represents on the vehicle dynamics. While these datasets represent a wide variety of situations, they may be compared for a summary of the overarching impact of the real-world versus exponential density model.

**Conditions Set 0 Degrees Latitude and Longitude**

This subset of conditions begins at 0 degrees latitude, 0 degrees longitude, 50,000 feet in altitude. Figure 35 displays this coordinate set with respect to its location on a map of the globe [43]:
As previously mentioned, the coordinates have no locational significance. They were chosen to represent a breadth of different kinds of coordinate locations on the globe. The location thus defined, the results may be analyzed.

**Longitude Comparisons Conditions (0,0)**

The first set of test conditions yielded a wide distribution of final longitude values. This is indicative of a wide variety in atmospheric scenarios encountered over the course of the 102 test cases. Figure 36 summarizes the resultant maximized longitude of each test case, and includes a reference line for the result of the standard day model.
While Fig. 36 provides a summary of the longitudes achieved by the various test cases, the more important analysis is of the key parameters that represent this data set – the mean values of each, and the associated standard deviation. Table 23 contains a short summary of these parameters.

**Table 23 Longitudinal Parameters for Conditions 0, 0**

<table>
<thead>
<tr>
<th>Quantity Measured</th>
<th>Longitude in Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Hot Range</td>
<td>18.5868</td>
</tr>
<tr>
<td>Average Cold Range</td>
<td>18.7873</td>
</tr>
<tr>
<td>Standard Day Range</td>
<td>17.9049</td>
</tr>
<tr>
<td>Error between Hot and Standard</td>
<td>0.6819</td>
</tr>
<tr>
<td>Error between Cold and Standard</td>
<td>0.8824</td>
</tr>
<tr>
<td>Standard Deviation of Hot</td>
<td>1.0733</td>
</tr>
<tr>
<td>Standard Deviation of Cold</td>
<td>1.2308</td>
</tr>
</tbody>
</table>

As Table 23 illustrates, the error between the average of the hot and cold cases with respect to the standard day model falls well within one standard deviation of the hot and cold cases. However, one standard deviation of either the hot or the cold cases
exceeds one degree of longitude. Using Eq. (43), this equates to a standard deviation of 136 kilometers for the standard deviation of the “cold” cases, and 119 kilometers for the “hot” cases. This represents the opportunity for a substantial difference between days. However, drag data and the additional test conditions must first be analyzed for the development of trends or the possible surfacing of elements which may contradict these initial conclusions.

**Drag Comparisons Conditions (0,0)**

The differences in drag are another way to confirm the difference in vehicle performance, a parameter which allows close examination of the changes between cases. The drag profiles of all 102 test cases for each set of test conditions are unique. For reference and analysis, Fig. 37 displays a sample drag profile that has been plotted from the first test case data for this set of test conditions.

![Example Drag Over Time Conditions 0 0](image)

**Figure 37 Sample Drag Profile Conditions (0,0)**
An analysis of Fig. 37 highlights three large spikes in drag, approximately aligned with a similar spike in velocity and a similar position profile. These state profiles may be seen in Appendix B. Another clear trend is the extreme nature of the standard day drag model spike with respect to the spike in drag of the hot and cold test cases. This seems to indicate a larger average drag for the exponential model than for the hot and cold test cases, a phenomenon which explains the difference in longitude previously noted. Table 24 summarizes values, allowing the quick analysis of the single test case graph to be expanded over the entire data set.

Table 24 Drag Comparison Parameters

<table>
<thead>
<tr>
<th>Quantity Measured</th>
<th>Drag in kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Drag Hot Model</td>
<td>2.6844</td>
</tr>
<tr>
<td>Mean Drag Cold Model</td>
<td>3.4517</td>
</tr>
<tr>
<td>Mean Drag Standard Day Model</td>
<td>2.7445</td>
</tr>
<tr>
<td>Error between Average Hot and Standard</td>
<td>0.0601</td>
</tr>
<tr>
<td>Error between Average Cold and Standard</td>
<td>0.7072</td>
</tr>
<tr>
<td>Standard Deviation of Average Drag Hot</td>
<td>0.3497</td>
</tr>
<tr>
<td>Standard Deviation of Average Drag Cold</td>
<td>5.1398</td>
</tr>
</tbody>
</table>

The trends presented by these values match exactly the expected dynamic performance of the vehicle, and highlight some important aspects of the real-world data model. First, the cold model experiences the largest amount of drag of the models, which follows the expected trend exactly. Additionally, the standard day model falls in between the hot and cold, as it should if it may be truly considered a “standard” day. This is an encouraging validation of the values presented. However, it points to a possible issue with the longitudinal values, and the basis for this analysis. This seems to imply that the launch phase may have more to do with the reachable longitudes than initially theorized, meaning that the “hot” and “cold” day distances traveled may not followed expected
trends. The standard deviations in drag are large, which easily accounts for the differences in distance.

**Conditions Set 35 Degrees Latitude and Longitude**

These datasets were run using 35 degrees latitude, 35 degrees longitude as the initial location, with an initial altitude of 50,000 feet. Figure 38 displays this coordinate location on the surface of the earth:

![Figure 38 Coordinate Set (35, 35) [44]](image)

This coordinate set is even further from US launch locations that the first set. However, due to the scenario flexibility lent by the use of the B-52 for modeling purposes, this coordinate set remains a reasonable initial location.

**Longitude Comparisons Condition (35,35)**

This coordinate set was chosen primarily due to the greater fluctuations seen in terms of hot and cold days in this zone on the earth. A large percent of the earth in this zone around thirty-five degrees longitude experiences substantial temperature fluctuation from season to season. Figure 39 displays the distribution of maximum longitude data.
Figure 39 represents the distribution of test case results for the 35-degree longitude and latitude test case. The data set appears to have at least one outlier which could impact the overall data, but due to the large number of test cases here represented, it should not. Table 25 summarizes key points to support this analysis, as well as for comparison with the previous and future test conditions.

<table>
<thead>
<tr>
<th>Quantity Measured</th>
<th>Longitude in Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Hot Range</td>
<td>57.7599</td>
</tr>
<tr>
<td>Average Cold Range</td>
<td>57.6281</td>
</tr>
<tr>
<td>Standard Day Range</td>
<td>57.2156</td>
</tr>
<tr>
<td>Error between Hot and Standard</td>
<td>0.5443</td>
</tr>
<tr>
<td>Error between Cold and Standard</td>
<td>0.4125</td>
</tr>
<tr>
<td>Standard Deviation of Hot</td>
<td>1.3923</td>
</tr>
<tr>
<td>Standard Deviation of Cold</td>
<td>1.4668</td>
</tr>
</tbody>
</table>

This data displays trends that match with the expected dynamics – the vehicle travels furthest on a hot day, shorter on a cold day. Of interest and worth noting is the larger
error between the standard day model and the hot and cold test cases. For this subset, the standard deviation is larger. These standard deviation, using Eq. (43), equates to 255 and 258 km respectively, a large differential. This continues the trend displayed in the first set of test conditions – the standard deviation of these models demonstrates the capacity for a large change in range between scenarios.

**Drag Comparisons Conditions (35,35)**

Given the confirmation developed in the longitudinal analysis above, the next step is to see if the average drag profile supports this behavior. Figure 40 displays a sample drag profile, which, though it appears different from the first drag profile, it is merely a single profile, one set of test cases from a set of 102.

![Example Drag Over Time Conditions 35 35](image)

**Figure 40 Sample Drag Profile for Conditions (35, 35)**

This drag profile appears different from the previous profile in that the “cold” cases match the standard day model much more closely than the “hot” model, a marked
difference from Fig. 37 where the hot and cold profiles were much more closely matched. This may be an outlier case. It must then be left to the data summary to see if the trends noticed in previous data continue. Table 26 gives a summary of the key data points.

<table>
<thead>
<tr>
<th>Table 26 Drag Force Parameters for Conditions 35, 35</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity Measured</td>
</tr>
<tr>
<td>Mean Drag Hot Model</td>
</tr>
<tr>
<td>Mean Drag Cold Model</td>
</tr>
<tr>
<td>Mean Drag Standard Day Model</td>
</tr>
<tr>
<td>Error between Average Hot and Standard</td>
</tr>
<tr>
<td>Error between Average Cold and Standard</td>
</tr>
<tr>
<td>Standard Deviation of Average Drag Hot</td>
</tr>
<tr>
<td>Standard Deviation of Average Drag Cold</td>
</tr>
</tbody>
</table>

Table 26 reflects some trends similar to the first test conditions, but some different. For this subset, the standard day model falls much lower in drag than the hot and cold model, though it remains well within a standard deviation. The standard deviation of the drag is also much smaller than the previous subset. This points to the possibility of a few outliers in the first set of test conditions, or the possibility that the meteorological impact of measuring conditions in a more temperature zone on Earth substantially changes the results. The standard deviation on the drag is smaller, but the order of magnitude in the error is more consistent. This still represents a difference in flight dynamics, beyond the uncertainty created by the real-world model. The hot and cold density trends remain intact, but the standard day trend requires further examination, as it does not match the first set of test conditions.
Conditions Set 35 Degrees Latitude and -130 Degrees Longitude

The next set of initial conditions represents a location much closer to the United States, beginning at 35 degrees latitude, -130 degrees longitude, and 50,000 feet. Figure 41 highlights the approximate location of this coordinate set [44]:

Figure 41 Coordinate Set (35, -130) [44]

The coordinate location shown above allows a calculation set for coordinates west of the Prime Meridian. It is important to note that for solvability purposes, coordinates -180 through 0 degrees were considered positive 180 through 360 degrees. In the solver, this coordinate set was called (35, 230).

Longitude Comparisons Conditions (35, -130)

This set takes place in a zone similar to that of the test conditions (35,35). If the discrepancies previously noted are dependent on this factor, then the results demonstrated by these data sets should be similar in trend and magnitude. Figure 41 displays a summary of the maximum longitude achieved in each test case as it compares to the maximum longitude achieved by the standard day model. It appears to display a wider distribution than the previous two sets of longitudes.
Figure 42 Final Longitude by Test Case and Dataset, Conditions (35, -130)

This set does not appear to have any visible outliers. Table 27 displays a summary of the data values relevant to this test case.

<table>
<thead>
<tr>
<th>Quantity Measured</th>
<th>Longitude in Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Hot Range</td>
<td>252.5082</td>
</tr>
<tr>
<td>Average Cold Range</td>
<td>252.8635</td>
</tr>
<tr>
<td>Standard Day Range</td>
<td>252.2504</td>
</tr>
<tr>
<td>Error between Hot and Standard</td>
<td>0.2578</td>
</tr>
<tr>
<td>Error between Cold and Standard</td>
<td>0.6131</td>
</tr>
<tr>
<td>Standard Deviation of Hot</td>
<td>1.123</td>
</tr>
<tr>
<td>Standard Deviation of Cold</td>
<td>1.272</td>
</tr>
</tbody>
</table>

Table 27 displays a return to some of the trends present by the first set of test conditions. The cold cases have a longer range than the hot, a repeat from the first set of test conditions, but both have a longer range than the standard day, a repeat from the second set of test conditions. The standard deviations here correspond to 245 km and 250 km respectively, a smaller range between standard deviations than in previous test condition
sets. Despite the fluctuations in the range of these values, one trend has become clear—the difference in one standard deviation of test case answers reaches into the hundreds of km, representing the possibility for substantial capability impact.

**Drag Comparisons Conditions (35, -130)**

While certain features are starting to emerge among the longitude results, similar confirmation must be sought from the drag conditions. Figure 43 displays another sample drag profile. This profile retain the same drag spikes presented in each set of test conditions, maintaining similar dynamic performance.

![Example Drag Over Time Conditions 35 -130](image)

**Figure 43 Sample Drag Profile Conditions (35, -130)**

Given the trends noticed above, Table 28 must be analyzed to understand the true extent of the data.
Table 28 Drag Force Parameters for Conditions 35, -130

<table>
<thead>
<tr>
<th>Quantity Measured</th>
<th>Drag in kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Drag Hot Model</td>
<td>2.6754</td>
</tr>
<tr>
<td>Mean Drag Cold Model</td>
<td>2.7292</td>
</tr>
<tr>
<td>Mean Drag Standard Day Model</td>
<td>2.4663</td>
</tr>
<tr>
<td>Error between Average Hot and Standard</td>
<td>0.2091</td>
</tr>
<tr>
<td>Error between Average Cold and Standard</td>
<td>0.2629</td>
</tr>
<tr>
<td>Standard Deviation of Average Drag Hot</td>
<td>0.3833</td>
</tr>
<tr>
<td>Standard Deviation of Average Drag Cold</td>
<td>0.2606</td>
</tr>
</tbody>
</table>

Table 28 displays a similar trend as the longitude in terms of displaying elements of both previous test conditions – the standard day model remains lower in average drag than the hot and cold, but the cold average drag remains larger than the hot. The standard deviation continues to represent smaller drag perturbations, but the longitude confirms that small drag perturbations easily equate to long distances over the surface of the earth.

The most important item of note in this data set is the value of the error between the average cold case values and hot. It exceeds the standard deviation of the average cold case drag, a significant deviation from the other test conditions. This seems to indicate that the cold cases experienced are even more significant for this set of test conditions.

**Conditions Set 0 Degrees Latitude and Longitude, 40,000 feet Altitude**

These conditions represent a change in launch altitude from the maximum 50,000 feet as a measure of performance. The key importance of the altitude change is that the launch vehicle will thus experience more drag due to increased atmospheric density, which has the potential to alter the longitude achievable in Phase 2.
Longitude Comparisons Conditions 40,000 ft

Many of the quantities for this set of test conditions should be similar to the first set of test conditions. Despite the similarities, the final two test condition sets are essential to attempting to establish trends among the data. To begin, Fig. 44 displays very similar distributions to the first set of test conditions. There does appear to be an outlier, but due to the size of the data set, the impact of this point is insignificant.

![Summary of Test Case Solutions, Conditions 40,000 ft](image)

**Figure 44 Final Longitude by Test Case and Dataset, Conditions 40,000 ft**

Table 29 displays key data points to continue establishing or debunking trends in the data. This set of test conditions appears to continue the trend reflected by the vast majority of the data – the cold data set travels furthest, the hot travels less, and the standard day conditions fall the shortest. The standard deviations continue to be large, and using Eq. (43), the ranges may be said to vary at one standard deviation at 136 km and 143 km respectively.
Table 29 Longitudinal Parameters for Conditions 40,000 ft

<table>
<thead>
<tr>
<th>Quantity Measured</th>
<th>Longitude in Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Hot Range</td>
<td>18.1284</td>
</tr>
<tr>
<td>Average Cold Range</td>
<td>18.4551</td>
</tr>
<tr>
<td>Standard Day Range</td>
<td>17.8707</td>
</tr>
<tr>
<td>Error between Hot and Standard</td>
<td>0.2577</td>
</tr>
<tr>
<td>Error between Cold and Standard</td>
<td>0.5844</td>
</tr>
<tr>
<td>Standard Deviation of Hot</td>
<td>1.2262</td>
</tr>
<tr>
<td>Standard Deviation of Cold</td>
<td>1.2949</td>
</tr>
</tbody>
</table>

The drag must now be evaluated to see if similar trends arise.

**Drag Comparisons Conditions 40,000 ft**

Figure 45 resembles Fig. 43 in terms of drag profile trends – consistently, the standard day density peaks at higher values than the hot and cold test cases. Table 30 must be consulted to determine if the key values match similarly.

![Example Drag Over Time Conditions 40,000 ft](image)

**Figure 45 Example of Drag Distribution Over Time for Conditions 40,000**
Table 30 represents the drag parameters. The cold model experiences more drag, which matches the longitudinal results. Additionally, the drag is much less for the standard day model, another quantity which confirms the results of the longitude. Another change to the previously presented trends is that the error for both the hot and cold cases with respect to the standard day exceed the standard deviation value, a trend which could be explained by the lowering of the initial altitude because it lowers the amount of energy the vehicle retains for the second phase, meaning that the vehicle is lower for a longer amount of time, encountering more atmospheric density.

Table 30 Drag Force Parameters for Conditions 40,000 ft

<table>
<thead>
<tr>
<th>Quantity Measured</th>
<th>Drag in kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Drag Hot Model</td>
<td>2.6433</td>
</tr>
<tr>
<td>Mean Drag Cold Model</td>
<td>2.6917</td>
</tr>
<tr>
<td>Mean Drag Standard Day Model</td>
<td>2.4986</td>
</tr>
<tr>
<td>Error between Average Hot and Standard</td>
<td>0.1447</td>
</tr>
<tr>
<td>Error between Average Cold and Standard</td>
<td>0.1931</td>
</tr>
<tr>
<td>Standard Deviation of Average Drag Hot</td>
<td>0.2188</td>
</tr>
<tr>
<td>Standard Deviation of Average Drag Cold</td>
<td>0.4667</td>
</tr>
</tbody>
</table>

This data displays a lower average drag for the standard day model, a feature that would not be expected from this model. However, the key trend of a higher drag for the cold model remains in place.

**Conditions Set 0 Degrees Latitude and Longitude, 30,000 feet Altitude**

The decrease to a 30,000 feet initial altitude for launch should increase the impact of the density model even more so than previously, particularly because the density should be even larger than it was in the 40,000 foot model.
Longitude Comparisons Conditions 30,000 ft

Many of the quantities for this set of test conditions should be similar to the first set of test conditions. The final two test conditions display this similarity to the first set, as well as to each other. The summary of the test case solutions are nearly identical in the longitudinal scatter plots.

Figure 46 Final Longitude by Test Case and Dataset, Conditions 30,000 ft

Table 31 displays key data points to continue establishing or debunking trends in the data. One feature of note is that this is the first case where the standard day range exceeds the average hot and cold ranges. This may be exclusively based on the error trends in the real-world density model, where standard day density deviates more substantially lower in the atmosphere. The standard deviation follows suit in creating a wider standard deviation than in previous cases, exactly what would be expected when lower altitudes are taken into account, because the variations in density are greater closer
to the ground. The standard deviations below represent, according to Eq. (43), differences in distance of 172 km and 145 km respectively.

Table 31 Longitudinal Parameters for Conditions 30,000 ft

<table>
<thead>
<tr>
<th>Quantity Measured</th>
<th>Longitude in Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Hot Range</td>
<td>18.112</td>
</tr>
<tr>
<td>Average Cold Range</td>
<td>18.2881</td>
</tr>
<tr>
<td>Standard Day Range</td>
<td>18.7798</td>
</tr>
<tr>
<td>Error between Hot and Standard</td>
<td>0.6678</td>
</tr>
<tr>
<td>Error between Cold and Standard</td>
<td>0.4917</td>
</tr>
<tr>
<td>Standard Deviation of Hot</td>
<td>1.5527</td>
</tr>
<tr>
<td>Standard Deviation of Cold</td>
<td>1.3006</td>
</tr>
</tbody>
</table>

The drag must now be evaluated to see if similar trends arise.

Drag Comparisons Conditions 30,000 ft

A new behavior arose in the longitude values of this set of test conditions. The standard day model reached a further longitude than the other cases. This may be due to the difference in starting altitude, but the drag must be evaluated to see if a similar trend arises. Figure 47 closely resembles Fig. 45 in terms of drag profile trends. Table 31 must be consulted to determine if the key values match similarly.
Table 32 represents the drag parameters. The hot model experiences more drag, which matches the longitudinal results. Additionally, the drag is much less for the standard day model, another quantity which confirms the results of the longitude, although it must be noted that this is true only for average drag rather than peak drag. Another change to the previously presented trends is that the error for both the hot and cold cases with respect to the standard day exceed the standard deviation value, a trend which could be explained by the lowering of the initial altitude because it lowers the amount of energy the vehicle retains for the second phase, meaning that the vehicle is lower for a longer amount of time, encountering more atmospheric density.
Table 32 Drag Force Parameters for Conditions 30,000 ft

<table>
<thead>
<tr>
<th>Quantity Measured</th>
<th>Drag in kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Drag Hot Model</td>
<td>2.6331</td>
</tr>
<tr>
<td>Mean Drag Cold Model</td>
<td>2.5985</td>
</tr>
<tr>
<td>Mean Drag Standard Day Model</td>
<td>2.3215</td>
</tr>
<tr>
<td>Error between Average Hot and Standard</td>
<td>0.3116</td>
</tr>
<tr>
<td>Error between Average Cold and Standard</td>
<td>0.2770</td>
</tr>
<tr>
<td>Standard Deviation of Average Drag Hot</td>
<td>0.2329</td>
</tr>
<tr>
<td>Standard Deviation of Average Drag Cold</td>
<td>0.1459</td>
</tr>
</tbody>
</table>

In some metrics, this data differs from previous test cases. First, the hot model retains more drag than the cold model. Second, the standard day model has a lower drag than both, a quantity that is reflected in the longitude of this model.

Comparison Among Test Conditions

In general, the test conditions displayed some similar trends, but not necessarily trends that would be expected. A trend that appeared in four of the five test cases was drag performance – the cold case consistently displayed the highest average drag, while the standard day model consistently represented the lowest. This only changed in the 30,000 ft conditions, a fact that may be expected due to the relative accuracies of the model. For five of the five models, the cold case model allowed the largest longitude value, while in four of them the standard day allowed the shortest. These trends are the opposite of what might be expected, and raises questions about the validity of using average drag as a metric – there may be another more important metric missing, a fact which requires future work to be done before any solid conclusions may be made. Table 33 presents a summary of the key data points from each of the test conditions. Trend-divergent values are highlighted in red.
Table 33 Summary and Comparison of Test Data

<table>
<thead>
<tr>
<th>Test Conditions</th>
<th>$\theta_{\text{hot}}$</th>
<th>$\theta_{\text{cold}}$</th>
<th>$\theta_{\text{standard}}$</th>
<th>$\sigma_{\theta_{\text{hot}}}$</th>
<th>$\sigma_{\theta_{\text{cold}}}$</th>
<th>$D_{\text{hot}}$</th>
<th>$D_{\text{cold}}$</th>
<th>$D_{\text{standard}}$</th>
<th>$\sigma_{D_{\text{hot}}}$</th>
<th>$\sigma_{D_{\text{cold}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50,000 (0,0)</td>
<td>18.5668</td>
<td>18.7873</td>
<td>17.9049</td>
<td>1.0733</td>
<td>1.2308</td>
<td>2.6844</td>
<td>3.4517</td>
<td>2.7445</td>
<td>0.3497</td>
<td>5.1398</td>
</tr>
<tr>
<td>50,000 (35,35)</td>
<td>57.7599</td>
<td>57.6282</td>
<td>57.2156</td>
<td>1.3926</td>
<td>1.4668</td>
<td>2.6458</td>
<td>2.6643</td>
<td>2.5394</td>
<td>0.1702</td>
<td>0.1871</td>
</tr>
<tr>
<td>50,000 (35,-130)</td>
<td>252.5082</td>
<td>252.8635</td>
<td>252.2504</td>
<td>1.123</td>
<td>1.272</td>
<td>2.6754</td>
<td>2.7292</td>
<td>2.4663</td>
<td>0.3883</td>
<td>0.2606</td>
</tr>
<tr>
<td>40,000 (0,0)</td>
<td>18.1284</td>
<td>18.4551</td>
<td>17.8707</td>
<td>1.2262</td>
<td>1.2949</td>
<td>2.6433</td>
<td>2.6917</td>
<td>2.4986</td>
<td>0.2188</td>
<td>0.4667</td>
</tr>
<tr>
<td>30,000 (0,0)</td>
<td>18.112</td>
<td>18.2881</td>
<td>18.7798</td>
<td>1.5527</td>
<td>1.3006</td>
<td>2.6331</td>
<td>2.5985</td>
<td>2.3215</td>
<td>0.2329</td>
<td>0.1459</td>
</tr>
</tbody>
</table>

While these trends require more testing for verification – the inclusion of hundreds more test cases would help solidify the concepts presented here – the trends that do exist may be verified using some simple analysis. First, the longitudinal trends must be examined. It seems contrary that the vehicle would travel further on a cold day, when the atmosphere is at its densest. However, this is only true close to the ground. A cold day also means a contracted atmosphere. On a hot day, in contrast, the air is at its least dense closest to the ground, but the atmosphere has expanded, meaning that a meaningful value (large enough value to create drag) will exist for longer. Figure 48 displays the atmospheric density data for a sample hot day and compares the data with a sample cold day. Both first had the contour maps examined to ensure that representative values were used.
As may be seen in Fig 48, the hot day density begins at a smaller value than the cold, but as the altitude increases, these two values switch, and the hotter atmosphere becomes denser. This would explain why the vehicle does not travel as far on a hot day – in the upper atmosphere where the majority of the travel occurs, it experiences more density than it would on a cold day. The drag difference may be similarly explained. Figure 46 previously demonstrated a sample drag over time plot. Using this same figure, certain features of note lead to some straightforward conclusions, as Fig 49 illustrates.
As Fig 49 highlights, the peaks in the drag distribution are nearly instantaneous at the higher value peaks. However, these large impacts, even a single value, can largely impact the average drag while barely altering flight dynamics. A simple averaging test confirms this – the manipulation of one hot day drag value to one slightly higher changes the average drag by approximately 500 N, while impacting the dynamics of flight for less than a second over a nearly three-quarters of an hour flight. Additionally, there are factors at work not reflected in either the density of the drag comparisons, particularly, the coupled impact of lift that would influence the results in exactly the manner reflected. This also confirms the validity of the trend noticed, while allowing it to be an accurate representation of flight dynamics.

**Chapter Summary**

Through a variety of test cases, the maximum deviation between the standard day model and the hot and cold model was demonstrated to be between 400 and 500
kilometers. The difference in drag, though small, is also substantial when high level fidelity is required for the drag acting on the vehicle. These results indicate that the use of the real-world data merits further investigation, and thus that future work is required.
V. Conclusions and Recommendations

Chapter Overview

The final step is to draw conclusions from the results of the data previously presented. Due to a trend arising amongst the results, the conclusions able to be drawn seem clear, but future work is required to completely validate these discoveries.

Conclusions of Research

As previously highlighted in the analysis of the presented test cases, the difference between the models represents a substantial change in capability, a distance of between 100 and 200 kilometers. Additionally, the difference in drag acting on the vehicle changes as well, with spikes in drag ranging from as small as 10 kN to as high as 60 kN depending on the test case and test conditions. The drag seemed to follow expected behavior for four of the five test conditions, with cold having the highest drag and the highest densities, and the standard day model having the lowest drags and densities. The maximum achievable longitudes varied by a range of approximately 8 degrees in each data set, although some cases retained an outlier. The trends are not static enough yet to draw sincere conclusions about the effectiveness of the real-world density model built from real-world data. More test cases and conditions are required to validate these trends completely. The one factor that remained constant was the relative magnitude of the range impact created by the range of one standard deviation. Even one standard deviation represented at least 100 kilometers difference in range for every set of test conditions, often in a range between one and two degrees of longitude. This hints that a real-world model could have a significant impact on the accuracy of modeling, but more testing is
required to confirm this. The model itself is computationally inexpensive enough to make it worthwhile if its performance is confirmed, as it would then allow developers to use predicted atmospheric conditions, or select a previous day with poor or excellent conditions to do a more specific analysis.

**Impact of Research**

This research cannot prove capability or lack thereof. Rather, it highlights certain aspects of the strategic picture. The improved density model changes the optimal range by 5-10% of the total range. While this percentage is low compared to the capability of the vehicle, it still further develops understanding to the limitations of the vehicle performance. It takes the given conditions and gives a best case answer for the scenario presented. The methodology is demonstrably straight-forward, and provides a method to match the actual behavior of the atmosphere closer than more general models. The quality of the model has room for improvement, but it provides a good starting point for additional testing and further development.

**Recommendations for Future Research**

Future work begins with improving the scale height factor for the real-world model. It fits well at lower altitudes, and performs extremely well at lower altitudes, but deviates more at higher altitudes. Due to the low density present in the upper atmosphere, this results in very small differences in the model, but it merits investigation. In addition to this, more test cases and conditions must be run in the attempt to more firmly establish, or disprove the trends developed in this study. Once these things have been done, more constraints and more accurate modeling of other portions of flight may be added in to
attempt an even more complex analysis of the real-world density model. Other parameters of the vehicle dynamics could also be examined.

**Summary**

The Global Forecast System provides a few years of stored weather data from which an improved atmospheric density model may be constructed. The inclusion of this improved model changes the optimal solution on the order of magnitude of hundreds of kilometers, or approximately 1.5 degrees on the Earth’s surface. This is merely a first step in creating an atmospheric model for use in optimal modeling toolboxes as requested by Air Force Research Labs. These results indicate that the use of the real-world data merits further investigation, and thus future work is required before application. The initial results show a promisingly large amount of change between the standard day models and the hot and cold cases, but only additional analysis may confirm this.
Appendix A: GPOPS and Related Code

% Main Script File for GPOPS-II BOOST-GLIDE SCENARIO
%
%
% Required Files:
% - setAuxdata loads Constants, may easily be modified to add more
% - AscenttestEOM & adjusted_EOMs for Launch Initial Guess data
% - testEOM & EOM_RV for Reentry Initial Guess Data; NOTE: LAUNCH INITIAL GUESS REQUIRED for Reentry Initial Guess to work
% - weathercalc2 calculates density models for given coordinates; NOTE: MUST HAVE WEATHER DATA CONVERTED USING NCTOOLBOX CODE - SAMPLE FILES FROM EACH LEVEL OF PROCESSING HAVE BEEN INCLUDED WITH THIS SET OF FILES
% - Combined_EOM provides the equations of motion for the scenario
% - CombinedEndpoint provides the cost function and inter-phase transition
% - getAllFiles finds and loads in all files from the provided directory;
% this code was built by someone else and I found it open source online
% - RK4U is a fixed step Runge-Kutta solver function
%
% Notes:
% %
%
% Author: 2d Lt Melissa Dunkel (719)482-8576  8 February 2017
%
Because it is a good habit, and clears out any crazy variables you may have hanging about:
clear all; close all; clc;

This establishes the atmospheric model variables from your input weather file as global. This MUST be done for weathercalc2 to work properly:
global B rhoi

This loads all of the weather files (pre-processed - must already be in C structure format, where C.B and C.rhoi are the saved variables for a 361x720 grid) You will need to change the directory to wherever you keep your files:
fileList = getAllFiles('C:\Users\Melissa Dunkel\Documents\IDrive-Sync\Work\Academic\Melissa Thesis\New Combined Phases\Weather Files\GPOPS-Ready\Cold');

% Starts the loop to run all weather files through GPOPS-II:
for b=1:length(fileList)
% Required so that you can load the files found by getAllFiles
    fileList=strjoin(fileList(b,:));
% Load your file
    load(fileList);
% Load the global variables from your weather file:
    B=C.B;
    rhoi=C.rhoi;

% This is your file full of constants
auxdata = setAuxdata;

%----------------------------------------------------------
%----------------------------------------------------------
%----------------------- Boundary Conditions --------------
%----------------------------------------------------------
%----------------------------------------------------------
%Establish initial guess:

run AscenttestEOM;
run testEOM;

%Load resulting data:
load guess;
load guess2;

%----------------------------------------------------------
%----------------------- Limits on Variables ---------------
%----------------------------------------------------------
%Establish boundary conditions. NOTE: In nearly all cases
%except where
%specified, they are the same for both.

%Time guess/limit for launch phase (phase ends at burnout):
t0 = 0;
t0bt = guess.t(end,:);

%Time guess/limit for re-entry phase:
t1 = t0bt;
t1bt = 7000;

%Flight Path Angle Control variable boundaries (radians)
fpaMin1 = -90*pi/180;  fpaMax1 = 90*pi/180;

%Flight Path Angle Dynamic variable boundaries (radians)
fpaMin = -90*pi/180;  fpaMax = 90*pi/180;

%Fuel Constraint boundaries (kg):
fuelMin = 0;  fuelMax = auxdata.initial_fuelmass;

% Position Boundaries (radius from center of Earth) (km):
rادMin = auxdata.Re;  radMax = radMin+500;
%Longitudinal Boundaries (radians):
lonMin    = 0*pi/180;   lonMax    = 360*pi/180;
%Note: 0 to 360 because it does NOT like -180 to 180
degrees...

%Latitudinal Boundaries (radians):
latMin    = 0*pi/180;    latMax    = 90*pi/180;
%Note: Only 0 to 90 because for my purposes I only wanted
my vehicle to
%travel in the Northern Hemisphere

%Velocity Boundaries (km/s):
speedMin  = 0;          speedMax  = 8;

%Bank Angle Control Input Variables (radians):
bankMin   = -60*pi/180;    bankMax   = 60*pi/180;

%Heading Angle Boundaries (radians):
haMin     = 0;             haMax     = 2*pi;

%Final state velocity variable boundaries (km/s):
speedfMin = 0;             speedfMax = 11;

%Final radius variable boundaries (km):
radfMin = auxdata.Re+0;  radfMax = auxdata.Re+40;

%Final time upper boundary (s):
tfMax=10000;

------------------------------------------
--------%
%--------------- Set Up Problem Using Data Provided Above -
--------%
------------------------------------------
--------%
bounds.phase(1).initialtime.lower = t0;
bounds.phase(1).initialtime.upper = t0bt;
bounds.phase(1).finaltime.lower = t1;
bounds.phase(1).finaltime.upper = tfMax;

%State Variable Boundaries estbalished:
%State
Variables:[Radius;Longitude;Latitude;Velocity;FlightPathAngle;HeadingAngle];
bounds.phase(1).initialstate.lower = [9.144+6371, 0*pi/180, 0*pi/180, 0, fpaMin, haMin];
bounds.phase(1).initialstate.upper = [9.144+6371, 0*pi/180, 0*pi/180, 0.29, fpaMax, haMax];
bounds.phase(1).state.lower = [radMin, lonMin, latMin, speedMin, fpaMin, haMin];
bounds.phase(1).state.upper = [radMax, lonMax, latMax, speedMax, fpaMax, haMax];

%Note: State Variable 1 was set to a lower boundary to keep GPOPS from
%ignoring the launch portion of the scenario.
bounds.phase(1).finalstate.lower = [6450, lonMin, latMin, speedfMin, fpaMin, haMin];
bounds.phase(1).finalstate.upper = [radMax, lonMax, latMax, speedfMax, fpaMax, haMax];

%Control variable here is an input to Flight Path Angle.
Note that FPA
%cannot be governed just by a control input, it must
include the coupled
%dynamics as well.
bounds.phase(1).control.lower = fpaMin1;
bounds.phase(1).control.upper = fpaMax1;

%Path Constraint boundaries established:
bounds.phase(1).path.lower = [fuelMin];
bounds.phase(1).path.upper = [fuelMax];

%Set integral constraint boundaries - really here for
%smoothing...it helps you get a better solution:
bounds.phase(1).integral.lower=0;
bounds.phase(1).integral.upper=50000;

%All the same things as above, but Phase 2:
bounds.phase(2).initialtime.lower = t1;
bounds.phase(2).initialtime.upper = t1;
bounds.phase(2).finaltime.lower = t1;
bounds.phase(2).finaltime.upper = tfMax;
bounds.phase(2).initialstate.lower = [radMin, lonMin, latMin, speedMin, fpaMin, haMin];
bounds.phase(2).initialstate.upper = [radMax, lonMax, latMax, speedMax, fpaMax, haMax];
bounds.phase(2).state.lower = [radMin, lonMin, latMin, speedMin, fpaMin, haMin];
bounds.phase(2).state.upper = [radMax, lonMax, latMax, speedMax, fpaMax, haMax];

bounds.phase(2).finalstate.lower = [radfMin, lonMin, latMin, speedMin, fpaMin, haMin];
bounds.phase(2).finalstate.upper = [radfMax, lonMax, latMax, speedMax, fpaMax, haMax];
%Control Variable here is sigma/bank angle:
bounds.phase(2).control.lower = [bankMin];
bounds.phase(2).control.upper = [bankMax];
bounds.phase(2).integral.lower = 0;
bounds.phase(2).integral.upper = 5000000;

%----------------------------------------------------------
---------------%
%---------------------- Provide Guess of Solution ---------
---------------%
%----------------------------------------------------------
---------------%

guess.phase(1).state = guess.s;
guess.phase(1).control = [guess.u];
guess.phase(1).time = guess.t;
guess.phase(1).path = [fuelMin];
guess.phase(1).integral = 0;

guess.phase(2).state = guess2.phase.state;
guess.phase(2).control = [guess2.phase.control];
guess.phase(2).time = guess2.phase.time;
guess.phase(2).integral = 0;
%----------------------------------------------------------
---------------%
%---------------------- Constraints to Link Phase ---------
---------------%
%----------------------------------------------------------
---------------%

%This is required for the inter-phase transition; it is
used in
%CombinedEndpoint:
bounds.eventgroup(1).lower = [zeros(1,6), 0];
bounds.eventgroup(1).upper = [zeros(1,6), 0];

%----------------------------------------------------------
---------------%
%-------------Provide Mesh Refinement Method and Initial Mesh
-------------%

102
% mesh.method            = 'hp-PattersonRao';
% mesh.maxiterations     = 10;
% mesh.colpointsmin      = 3;
% mesh.colpointsmax      = 10;
% mesh.tolerance         = 1e-3;
%^Raised from 1^-6 to 1^-3 because GPOPS took too long to
% get to a
% solution at the smaller tolerance.
% This runs hundreds of test cases, so time was a big
% factor.

nints = 30;
mesh.phase(1).colpoints = 6*ones(1,nints);
mesh.phase(1).fraction  = (1/nints)*ones(1,nints);
mesh.phase(2).colpoints = 6*ones(1,nints);
mesh.phase(2).fraction  = (1/nints)*ones(1,nints);

%----------------------------------------------------------
%------------------- Solve Problem Using GPOPS2 ------------
%----------------------------------------------------------

 tic
 output = gpops2(setup);
%The below code plots a nice summary of the output for every iteration.

r = output.result.solution.phase(1).state(:,1);
latf = output.result.solution.phase(1).state(:,3);
lonf = output.result.solution.phase(1).state(:,2);
v = output.result.solution.phase(1).state(:,4);
u = rad2deg(output.result.solution.phase(1).control);
t = output.result.solution.phase(1).time;
t2 = output.result.solution.phase(2).time;
r2 = output.result.solution.phase(2).state(:,1);
u2 = rad2deg(output.result.solution.phase(2).control);
v2 = output.result.solution.phase(2).state(:,4);
lat2 = output.result.solution.phase(2).state(:,3);
lon2 = output.result.solution.phase(2).state(:,2);

figure(1)

subplot(3,2,1)
plot(t,r)
ylabel('Radius From Center of Earth (km)')
xlabel('Time (s)')
title('Location of Vehicle Over Time')
hold on
plot(t2,r2)
legend('Phase 1','Phase 2', 'Location','best')

figure(2)

subplot(3,2,2)
plot(t,u)
ylabel('Normalized Control (FPA then BA)')
xlabel('Time (s)')
title('Control over time')
hold on
plot(t2,u2)
legend('Phase 1 Ux','Phase 1 Uy', 'Phase 1 Uz', 'Phase 2 Bank Angle (normalized)', 'Location','best')

figure(3)

subplot(3,2,3)
plot(t,v)
ylabel('Velocity (km/s)')
xlabel('Time (s)')
title('Change of Vehicle Position over Time')
hold on
plot(t2, v2)
legend('Phase 1', 'Phase 2', 'Location', 'best')

% figure(4)
subplot(3, 2, 4)
plot(lonf*180/pi, latf*180/pi)
ylabel('Latitude (deg)')
xlabel('Longitude (deg)')
title('Latitude vs Longitude')
hold on
plot(lon2*180/pi, lat2*180/pi)
legend('Phase 1', 'Phase 2', 'Location', 'best')

subplot(3, 2, 5)
lati = latf(1);
dlat = latf - lati;
loni = lonf(1);
dlon = lonf - loni;
radius = 6371;
a = (sin(dlat/2).^2) + (cos(lati).*cos(latf).*(sin(dlon/2).^2));
c = 2*atan2(sqrt(a), sqrt(1-a));
d = radius*c;
hold on
dlat = lat2 - lati;
dlon = lon2 - loni;
radius = 6371;
a2 = (sin(dlat/2).^2) + (cos(lati).*cos(lat2).*(sin(dlon/2).^2));
c2 = 2*atan2(sqrt(a2), sqrt(1-a2));
d2 = radius*c2;
plot(t, d)
hold on
dlat = lat2 - lati;
dlon = lon2 - loni;
radius = 6371;
a2 = (sin(dlat/2).^2) + (cos(lati).*cos(lat2).*(sin(dlon/2).^2));
c2 = 2*atan2(sqrt(a2), sqrt(1-a2));
d2 = radius*c2;
plot(t2, d2)
xlabel('Time (s)')
ylabel('Range (km)')
legend('Phase 1', 'Phase 2', 'Location', 'best')

subplot(3, 2, 6)
plot([t; t2], [latf; lat2])
hold on
plot([t; t2], [lonf; lon2])
xlabel('Time (s)')
ylabel('Change (deg)')
legend('Lat', 'Long', 'Location', 'best')
%This plays a nice sound when it finishes, if you want a way to notify yourself...sometimes it'll startle you...

% load Handel
% sound(y(1:20000),Fs)

End

function output = Combined_EOM(input);
dbstop if error

%% Phase 1:

%State variables:
r = input.phase(1).state(:,1);
theta = input.phase(1).state(:,2);
phi = input.phase(1).state(:,3);
velrot = input.phase(1).state(:,4);
gamma = input.phase(1).state(:,5);
psi = input.phase(1).state(:,6);

%Loads in control variable (phase 1):
gammac = input.phase(1).control;

%Loads in time variable (phase 1):
t = input.phase(1).time;

%This adds the control input into the gamma propagated forward using the dynamics:
gamma = gamma +gammac;

%Loads in all the relevant constants:
mdot=input.auxdata.mdot;
Isp=input.auxdata.Isp;
g0=input.auxdata.g0;
Cd=input.auxdata.cd;
A=input.auxdata.A;
mu=input.auxdata.mu;
Re=input.auxdata.Re;

%These present two different density models. If you want the real-world
%density model, use:
[rho, TK] = weathercalc2(input.phase(1).state);

% If you want the standard day density model, use:
% rho = (1.225*1000^3).*exp(-0.14.*(r-Re));

% Calculate thrust and drag:
T = mdot.*Isp.*g0;
D = rho.*velrot.*Cd.*A;

% Positional variables, only change from zero if thrust and velocity vector
% are not aligned:
eta = 0;
eps = 0;

% Distance Squared Gravitational Model
g = (mu./(r.^2));

% Load in rocket mass (changes over time due to burning and expulsion of
% fuel):
m = input.auxdata.mass - (mdot.*t);

% Rotation of the Earth, should you need it:
omegae = input.auxdata.wo;

% Calculate the derivatives of state variables:
rdot = velrot.*sin(gamma);
thetadot = (velrot.*cos(gamma).*cos(psi))./(r.*cos(phi));
phidot = (velrot.*cos(gamma).*sin(psi))./r;
veldot = ((T./m).*(cos(eta).*cos(eps)) - (D./m) -
(g.*sin(gamma));
gammadot = -
((g.*cos(gamma))./velrot)+(velrot.*cos(gamma)./r);
psidot = -((velrot./r).*cos(gamma).*cos(psi).*tan(phi));

% Update the fuel constraint: (Fuel cannot be lower than 0):
fuel_remaining = input.auxdata.initial_fuelmass - (mdot.*t);

% Update the control integral constraint:
int = (gammac./(90*pi/180)).^2;

% Return calculated constraints and state variables to
GPOPS:
output(1).integrand = int;
output(1).path = fuel_remaining;
output(1).dynamics =
[rdot,thetadot,phidot,veldot,gammadot,psidot];

%% Phase 2:

%% Phase 2 state variables:
    r2      = input.phase(2).state(:,1);
    theta2  = input.phase(2).state(:,2);
    phi2    = input.phase(2).state(:,3);
    velrot2 = input.phase(2).state(:,4);
    gamma2  = input.phase(2).state(:,5);
    psi2    = input.phase(2).state(:,6);

%% Input control variables
    interpsigma = input.phase(2).control(:,1);

%Same density calculations available for Phase 1. Choose wisely:
[rho,TK] = weathercalc2(input.phase(2).state);
% rho = 1.225.*(1000^3).*exp(-0.14.*(r2-re));

%Constants:
    Cl= 0.557;
    Cd= 0.192;
    re    = input.auxdata.Re;
    S     = input.auxdata.S;
    gs    = input.auxdata.g0;
    m     = input.auxdata.massi;

% Compute gravity, lift, and drag:
    g = (gs*(re./r2).^2);
    L = (rho.*Cl*S/2).*velrot2.^2;
    D = (rho.*Cd*S/2).*velrot2.^2;

% Calculate dynamic constraints
    rdot2 = velrot2.*sin(gamma2);
    thetadot2 =
        (velrot2.*cos(gamma2).*cos(psi2))./(r2.*cos(phi2));
    phidot2 = velrot2.*cos(gamma2).*sin(psi2)./r2;
    veldot2 = -(D/m)-g.*sin(gamma2);
    gammadot2 = (L./(m*velrot2)).*cos(interpsigma) -
        (g./velrot2).*cos(gamma2)+(velrot2./r2).*cos(gamma2);
    psidot2 = (L.*sin(interpsigma)./(velrot2.*m.*cos(gamma2)))-
        ((velrot2./r2).*cos(gamma2).*cos(psi2).*tan(phi2));
% Update integral constraint:
int2 = (interpsigma./(60*pi/180)).^2;

% Output phase info
output(2).integrand=int2;
output(2).dynamics=[rdot2,thetadot2,phidot2,veldot2,gammadot2,psidot2];
end

function output = CombinedEndpoint(input);
global gamma
% Variables at Start and End of Phase 1:
t01 = input.phase(1).initialtime;
tf1 = input.phase(1).finaltime;
x01 = input.phase(1).initialstate;
xf1 = input.phase(1).finalstate;

% Variables at Start and End of Phase 2:
t02 = input.phase(2).initialtime;
tf2 = input.phase(2).finaltime;
x02 = input.phase(2).initialstate;
xf2 = input.phase(2).finalstate;

% Event Group 1: Lineage Constraints Between Phases 1 and 2
% u=input.phase(1).control(end,:);
eg1f =
[input.phase(1).finalstate(:,1),input.phase(1).finalstate(:,2),input.phase(1).finalstate(:,3),input.phase(1).finalstate(:,4),input.phase(1).finalstate(:,5),input.phase(1).finalstate(:,6)];
output.eventgroup(1).event = [x02-eg1f,t02-tf1];

% Cost Function:

% Establish one of objective variables (longitude):
lon = input.phase(2).finalstate(1,2);

% Create coefficients for integral terms:
 a=0.09;
b=0.09;

% Build integral terms:
bla=(a*input.phase(2).integral)+(b*input.phase(1).integral);

% Create total objective value:
tot = -lon+bla;
% Cost function:
output.objective = tot;
end
function [rho,Bloc,rr,TK]=weathercalc2(input);
% tic
warning('off','all')
global B rhoi
TK=0;
[mrows,ncolumns]=size(input);
for k=1:mrows
    truth= isnan(input(k,:));
end
truth=sum(truth);
if truth>0
    load guess
    rho = zeros(mrows,1);
    for j=1:mrows
        rho(j,:) = 1.225*((1/1000)^3).*exp(-0.14.*50);
    end
    fprintf('First Loop: ')
    toc
else
    rho = zeros(mrows,1);
otherwise
    fprintf('Lon: Min of Input=%.4f,Max of Input=%.4f
     ',min(input(:,2))*180/pi,max(input(:,2)*180/pi))
    fprintf('Lat: Min of Input=%.4f,Max of Input=%.4f
     ',min(input(:,3))*180/pi,max(input(:,3)*180/pi))
    if min(input(:,2))<0
        input(:,2)=input(:,2)+2*pi;
    end
    if min(input(:,3))<0
        input(:,3)=input(:,3)+(3*pi/180);
    end
    latf=floor(input(:,3)*180/pi);
    lonf=floor(input(:,2)*180/pi);
end
for d=1:mrows
    latdec(d,:)=input(d,3)-latf(d,:);
    if latdec(d,:)>0.25 || latdec(d,:)<-0.75
        latf(d,:)=latf(d,:)+0.5;
    elseif latdec<0.25
        latf(d,:)=latf(d,:)-0.5;
    end
end
latf(d,:) = floor(latf(d,:));
elseif latdec(d,:) > 0.75
    latf(d,:) = ceiling(input(d,3)*180/pi);
end
londec(d,:) = (input(d,3)*180/pi) - lonf(d,:);
if londec(d,:) >= 0.25 || londec(d,:) <= 0.75
    lonf(d,:) = lonf(d,:) + 0.5;
elseif londec(d,:) < 0.25
    lonf(d,:) = floor(lonf(d,:));
elseif londec(d,:) > 0.75
    lonf(d,:) = ceiling(input(d,3));
end
end

lonf = (lonf > 360)*-360 + lonf;
latf = (latf > 360)*-360 + latf;
latf = (latf > 180)*-180 + latf;
ll = ([lonf] + 180)*91 + [latf] + 46;
ll = ceil((latf*2) + 180.5);
ll = (ll > 361)*-361 + ll;
ll2 = ceil((lonf*2) + 360.5);
ll2 = (ll2 > 720)*-720 + ll2;
alti = ((input(:,1) - 6371).*(1000/1));
alti = alti.*((1000/1));

for k=1:mrows
    try
        rhoilocal = rhoi(ll(k,:),ll2(k,:));
        rr(k,:) = rhoilocal(1,1);
        Bloc(k,:) = cell2mat(B(ll(k,:),ll2(k,:)));
        rho(k,:) = rhoilocal(1,1)*exp(-Bloc(k,:)*alti(k,:))*(1000^3); % kg/km^3
    catch
        fprintf('Something wrong with , loop %f
',k)
    end
end
end

[mrows, ncolumns] = size(input);
for k=1:mrows
    truth = isnan(input(k,:));
% end
% truth=sum(truth);
% j=1;
% if truth>0
%   
%   load guess
%   rho = zeros(mrows,1);
%   TK = zeros(mrows,1);
%   for j=1:mrows
%       rho(j,:) = 1.0025*((1/1000)^3).*exp(-0.14.*50);
%       TK(j,:) = 300;
%   end
%   fprintf('First Loop: ')
%   toc
%   
% else
%
%   rho = zeros(mrows,1);
%   TK = zeros(mrows,1);
%   
%   latf=floor(input(:,3));
%   lonf=floor(input(:,2));
%   ll=([lonf]+180)*91+[latf]+46;
%   alti= (input(:,1)-6371).* (1000/1);%m
%   fprintf('Second Loop: ')
%   toc
%   
%   for k=1:mrows
%     
%       press=st(ll(k,:)).pressures;
%       alt = st(ll(k,:)).altitude;
%       temp=st(ll(k,:)).temperatures;
%       press = press.*(1/10).*(1000/1); %Pa
%       R = 8.314;
%       
%       rhoi=(press./(R.*temp))*(29/1000); %kg/m^3
%       
%   j=0.001;
%   fun = @(x)objfun(x,rhoi,alt);
%   B=fminsearch(fun,j);
%   
%   rho(k,:) = rhoi(1,:)*exp(-B*alti(k,:))*(1000^3);%kg/km^3
% if alti(k,:)<=47000
% p=polyfit(alt,temp,9);
% TK(k,:)=polyval(p,alti(k,:));
% elseif alti(k,:)>=45000 && alti(k,:)<=86000
% sts =
% [487.17;467.7;436.97;398.57;367.65;336.5];
% altst
% = [160000;180000;200000;225000;250000;278000].*(1/3.2808);
% p=polyfit(altst,sts,15);
% TK(k,:)=polyval(p,alti(k,:));
% elseif alti(k,:)>=86000
% TK(k,:) = 393; % Kelvin
% end
% end
% fprintf('Third Loop: ')
% toc
% if toc>4
% pause()
% end
% end

GFS FILE CONVERSION PROCESS

clear
clc
Start=tic;
Files=dir('./Weather Files/Post-Processed');
Files(1:2)=[];
for b=1:size(Files,1)
    load(['./Weather Files/Post-Processed/'
    Files(b).name]);
    for k=1:size(st,2)
        for count=1:361
            for count2=1:720
                press=st.pressures;
                alt = st.altitude(:,count,count2);
                temp=st.temperatures(:,count,count2);
                press = press.*(1/10).*(10/1); %Pa
                R = 8.314;
                rhoi{count,count2}=(press./(R.*temp))*(29/1000); %kg/m^3
            end
        end
    end

    end
end
end
%     toc(Start)
clear B
j=0.001;
parfor count=1:361
    for count2=1:720
        alt = st.altitude(:,count,count2);
        fun = @(x)objfun(x,rhoi{count,count2},alt);
        B{count,count2}=fminsearch(fun,j);

        if mod(count2,719)==0
            fprintf('%.f done of %s:
',count,Files(b).name)
            toc(Start)
        end
    end
end

%     basepath = 'I:\My Documents\Thesis\Thesis
Code\Combined Phases\Weather Files\GPOPS-Ready\Cold';
    C.rhoi=rhoi;
    C.B=B;
    save(['./Weather Files/GPOPS-Ready/Cold'
Files(b).name],'C')
%     dest = 'I:\My Documents\Thesis\Thesis Code\Combined
Phases\Weather Files\Post-Processed\Completed';
    movefile(['./Weather Files/Post-Processed/
Files(b).name],[ './Weather Files/Processed/
Files(b).name']);
    toc(Start)
end
Appendix B: State Variable Summaries for Test Cases

Test Conditions (0,0):

![Graph of Test Conditions (0,0) State Variables](image)

**Figure 50 Test Conditions (0,0) State Variable Sample Graph**

Test Conditions (35,35)

![Graph of Test Conditions (35,35) State Variables](image)

**Figure 51 Sample State Variables (35,35)**

Test Conditions (35, -130)
Figure 52 Sample State Variables (35,-130)

Test Conditions 40000

Figure 53 Test Conditions 40,000 ft Sample State Variables
Test Conditions 30000

Figure 54 Sample State Variables Test Conditions 30,000 ft
Bibliography


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The Impact of Atmospheric Fluctuations on Optimal Boost Glide Hypersonic Vehicle Dynamics

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A project under the management of Air Force Research Laboratory has begun development of a model for use in hypersonic vehicle development and application. One area of interest is the perturbation of optimal vehicle behavior based on atmospheric fluctuations – how the performance of the vehicle changes with respect to “hot”, “cold” and standard day conditions.