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An Application of Course Scheduling in the Brazilian Air Force

Julio C. O. Lopes

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AN APPLICATION OF COURSE SCHEDULING IN THE BRAZILIAN AIR FORCE

GRADUATE THESIS

Julio C O Lopes, Captain, BAF

AFIT-LSCM-ENS-12-09

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio
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AN APPLICATION OF TIMETABLE PROBLEM IN
THE BRAZILIAN AIR FORCE

THESIS

Presented to the Faculty
Graduate School of Logistics and Acquisition Management
Air Force Institute of Technology
Air University
Air Education and Training Command
In Partial Fulfillment of the Requirements for the
Degree of Master of Science

Julio C O Lopes,
Captain, BAF

Mar 2012

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AN APPLICATION OF TIMETABLE PROBLEM IN
THE BRAZILIAN AIR FORCE

Julio C O Lopes, MSC
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Approved:

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Deckro F. Richard , DBA (Chairman)                   Date

____________________________________   _______________
Johnson, Alan W., Ph.D. (Member)  Date
Abstract

The Institute of Logistic of the Brazilian Air Force (ILA) is responsible for executing the technical courses related with acquisition, supply, maintenance, and logistic functions in the Brazilian Air Force. These courses have an average duration of two weeks with approximately thirty students per course. The average number of courses per year is thirty four. In addition to these courses, the Institute also has the responsibility to organize and execute seminars, meetings, lectures and others unscheduled events. These unscheduled events sometimes result in conflicts with the scheduled courses, potentially resulting in a reduced quality in the execution of these activities.

The schedule of courses for one year is prepared in the previous year. The process of developing the class schedule is based in meetings with the experts and managers involved. The final schedule is based on the knowledge of the team involved in this process. As a result of this manual process some conflicts can occur.

This work presents an approach to avoid these conflicts while producing a more smooth utilization of the resources. In addition, an alternative to increase the number of students trained per year is also presented.

With the utilization of mathematical models and software for optimization, it was possible to show various alternatives and interesting results to improve the overall quality of the service in ILA.
Acknowledgments

First of all I would like to thank for God.

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I am grateful to have had the support of my wife and my children.

Julio C. O. Lopes
# Table of Contents

Abstract .............................................................................................................................. iv  
Acknowledgments ............................................................................................................. v  
Table of Contents ............................................................................................................... vi  
List of Figures .................................................................................................................. viii  
List of Tables ................................................................................................................... ix  
List of Abbreviations ....................................................................................................... ix  

I. Introduction ..................................................................................................................1  
   Background ...................................................................................................................3  
   Importance of the problem ..........................................................................................6  
   Scope and overall research question ..........................................................................7  

II. Literature Review .........................................................................................................8  
   1. Purpose ...................................................................................................................8  
   2. Timetabling ............................................................................................................8  
   3. Approaches to timetabling problem ....................................................................12  
      a. Enumerative Search ...........................................................................................12  
         a-1. Mathematical Programming ......................................................................12  
         a-2. Dynamic Programming ...........................................................................12  
         a-3. Branch and Bound ....................................................................................12  
      b. Heuristic Search .............................................................................................13  
         b-1. Tabu Search ..................................................................................................13  
         b-2. Simulated Annealing ..................................................................................15  
         b-3. Ant Colony ..................................................................................................17  
         b-3. Artificial Bee Colony. ................................................................................17  
         b-4. Genetic Algorithm .......................................................................................20  
         b-5. Others heuristics and methods .................................................................24  
         b-6. Comparison about performance for some heuristics ............................25  

III. Methodology ..............................................................................................................29  
   1. Introduction ..........................................................................................................29  
   2. Problem formulation ............................................................................................31  
   3. Decision Variables ..............................................................................................32
4. Constraints .................................................................................................................. 33
5. Objective Function .................................................................................................... 34
6. Mathematical formulation ....................................................................................... 35
7. Methodology issues ................................................................................................. 38
8. Experimental design ............................................................................................... 39

IV. Analysis and Results .............................................................................................. 45
   1. Context of Model results ...................................................................................... 45
   2. Analysis of the first approach .............................................................................. 45
   3. Analysis of the second approach ......................................................................... 47
   4. Analysis of the number of students trained per year ......................................... 53

V. Conclusions and Recommendations ........................................................................ 59

V. Recommendations for future research ..................................................................... 63

Appendix A .................................................................................................................... 64
Appendix B ..................................................................................................................... 70
Appendix C ..................................................................................................................... 72
Appendix D ..................................................................................................................... 74
Bibliography .................................................................................................................. 75
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1 Hotel utilization based on the courses scheduled for 2011 in the TCA 37-11 approved</td>
<td>5</td>
</tr>
<tr>
<td>2-1 Flowchart of a binary genetic algorithm</td>
<td>23</td>
</tr>
<tr>
<td>3-1 Hotel utilization based on makespan approach</td>
<td>40</td>
</tr>
<tr>
<td>3-2 Classroom utilization based on makespan approach</td>
<td>40</td>
</tr>
<tr>
<td>3-3 Hotel utilization based on spreading courses within 40 weeks</td>
<td>43</td>
</tr>
<tr>
<td>4-1 Number of classroom utilized with hotel capacity for 92 students in makespan approach</td>
<td>46</td>
</tr>
<tr>
<td>4-2 Number of students per week based on the approved TCA 37-11</td>
<td>49</td>
</tr>
<tr>
<td>4-3 Number of students per week based on second approach for 38 weeks</td>
<td>50</td>
</tr>
<tr>
<td>4-4 Number of students per week based on second approach for 42 weeks</td>
<td>51</td>
</tr>
<tr>
<td>4-5 Number of students per week based on second approach for 45 weeks</td>
<td>52</td>
</tr>
<tr>
<td>4-6 Number of students per week based on second approach for 38 weeks considering an increasing in number of students</td>
<td>54</td>
</tr>
<tr>
<td>4-7 Number of students per week based on second approach for 42 weeks considering an increasing in number of students</td>
<td>55</td>
</tr>
<tr>
<td>4-8 Number of students per week based on second approach for 45 weeks considering an increasing in number of students</td>
<td>56</td>
</tr>
<tr>
<td>Algorithm 1 – Tabu search</td>
<td>14</td>
</tr>
<tr>
<td>Algorithm 2 – Simulated Annealing</td>
<td>17</td>
</tr>
<tr>
<td>Algorithm 3- Ant Colony System</td>
<td>19</td>
</tr>
<tr>
<td>Algorithm 4 – Artificial Bee Colony</td>
<td>21</td>
</tr>
</tbody>
</table>
List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-1 Courses scheduled for 2011</td>
<td>30</td>
</tr>
<tr>
<td>3-2 Solution for the makespan approach with currently capacities</td>
<td>39</td>
</tr>
<tr>
<td>3-3 Solution for the second approach for 40 weeks</td>
<td>42</td>
</tr>
</tbody>
</table>

List of Abbreviations

ILA – Institute of Logistic of the Brazilian Air Force
COMGAP – Comando Geral de Apoio (General Command of Support)
TCA 37-11 – Tabela do Comando Da Aeronautica (Table of Air Force Command)
B&B – Branch and bound algorithm
TS – Tabu Search Algorithm
SA – Simulated Annealing Algorithm
ACO or ACS – Ant colony Algorithm
ABC – Artificial Bee Colony
GA – Genetic Algorithm
AN APPLICATION OF COURSE SCHEDULING IN THE BRAZILIAN AIR FORCE

I. Introduction

Statement of Problem

The problem addressed in this thesis is to establish a course schedule for one year at one institution in the Brazilian Air Force in an efficient way.

The Instituto de Logística da Aeronáutica (ILA), or in English, the Air Force Institute of Logistics, is responsible for all technical courses with short duration related with maintenance and supply functions. In addition, ILA is also responsible for seminars, lectures, and special events related to maintenance and supply. However, these seminars, lectures and events are not scheduled when class schedules are set. This results in conflicts for resources utilization, poor quality in some activities, and frequently overtime and dissatisfaction among the personnel impacted.

The schedule of the courses for one year is prepared between August and November in the preceding year. The definition of the course itself, objective of the course, the number of students, syllabus, and duration is defined by another unit in Brazilian Air Force, COMGAP, to which ILA is subordinated. ILA negotiates with COMGAP over the schedule of the courses planned for the next year based on the constraints of resources available.

Some courses are hybrid or blended courses, where some disciplines are done by e-learning before the in residence classroom utilization. In the case of the blended courses, there is
little flexibility related with the period of realization, and sometimes there is the involvement of
civilian organizations. Therefore those courses are treated with more attention due the
involvement of third parties.

While the timetabling problem has been shown to be a NP-hard problem (Even, Itai, &
Shamer, 1976), the approach currently utilized in the Brazilian Air Force is based on the
previous year’s schedule, using a subjective decision process. The process for scheduling the
courses at ILA is time consuming, complex, tiring, and sometimes results in conflicts with the
utilization of resources available.

The main resource constraint in ILA for courses realization is the hotel capacity. Due to
economic conditions in Brazil, it is too difficult for a military student to lodge in a civilian hotel,
where the expenses are not compatible with the military salary. Therefore, ILA has a hotel
specifically for students and instructors. This hotel has capacity for eighty students. Besides the
hotel, others constraints are also present. ILA has four classrooms, one with capacity for forty
two students, one for thirty five students, one for thirty three students and a smaller classroom for
only twenty five students. ILA has two laboratories with computers which are used as
classrooms in some courses. Laboratory 1 has eighteen computers, while laboratory 2 has thirty
three computers. The instructors for those courses come from others units in the Brazilian Air
Force for a specific subject, and did not represent any additional constraint for the scheduling.

The absence of a clear method for the scheduling of the courses, and a process based on
subjective decisions often results in poor quality of the service, mainly due the conflicts that
occurs with the unscheduled special events (seminars, meetings, and lectures). The objective of
this work is to develop a tool that could provide support for two different types of schedule
goals. One goal is to concentrate the courses in some periods where the realization of others events are prohibited and providing periods without courses when others events could be scheduled. Another goal is to smooth the utilization of resources based on the schedule, avoiding great fluctuations, then a special event could occurs anytime with reduced impact compared with the current situation.

Background

ILA was created in 1988 with the mission to develop the technical and management capabilities of professionals in the logistics systems of the Air Force, through education and research.

Since 1988 the Institute has been growing in the number of courses offered, number of students taught, it variety of courses offered and the field of operation. Currently, there are twenty seven courses based on face-to-face instruction, seven hybrid courses and eight courses totally computer-based learning. Due to the low impact in the utilization of onsite resources available, the computer based learning is not considered in this work. Despite this low resources utilization, e-learning is rapidly growing in the last few years. The trend is to increase the number of courses, students and coverage offered in this fashion.

The cost involved for traditional classroom instruction is considerably higher than the e-learning courses in the Brazilian Air Force. This fact is driving the creation of new courses and expanding the offering of existing e-learning courses.
The traditional classroom course mode is responsible for the training of almost one thousand students per year with a growth rate of 4%, on average, in the last five years. Hybrid courses are responsible for three hundred twenty students per year with a growth rate higher than 100 % in every year since 2008 when this type of course was first offered. In the case of e-learning courses, the rate of increase in the number of students was greater than 10 % in the last three years with investments being made to allow growth rates even higher than the actual 10%. Except for the e-learning courses, the duration of a course is frequently between one to four weeks. The time slot for scheduling is always one week.

At the end of the previous year, often November, one document (TCA 37-11) is published with the definition of the number of courses, period of realization, number of students allowed per course, the objective of the course, pre-requisites for registration, and the syllabus of the course. This document is approved by COMGAP (General Command of Support) and it is the end product of a process started in August. This process is time consuming and the results are currently not the results of any mathematical optimization. There is no objective function specified as a driver; the result reflects only one possible solution that “seems reasonable”.

As mentioned before, there are some events not scheduled in advance. This fact results in some conflicts in the utilization of the resources available, resulting in a poor quality of the service. One example of this reduced quality is the utilization of the auditorium as a classroom, where the auditorium does not have adequate installation for the students to take notes. Another example is the sharing of computer between two students in the laboratory.

The process of enrollment for each course is under the responsibility of COMGAP.ILA receives as input the names of each student one month before the course starts. Special events
often are demanded between one to two months before they occur. Therefore, the process of planning the TCA 37-11 in the previous year is fundamental for the execution of the courses.

In the actual process of developing TCA 37-11, the unique constraint considered is the capacity of the hotel. As noted before, ILA has its own hotel to provide lodging for the students. While the hotel has a capacity for ninety guests, ten of those ninety spaces are reserved for instructors, general officers and special guests. Figure 1, based on the TCA 37-11 approved in October-26-2010 with the instructions for the courses in 2011, shows the fluctuation in the utilization of the hotel. In some years the schedule has sometimes utilized the rooms reserved for instructors, resulting in possible problems with additional beds in the room and reduced quality of service.

![Number of students per week based on approved schedule in TCA 37-11](image)

Figure 1-1 – Hotel utilization based on the courses scheduled for 2011 (TCA 37-11/2010)

The hotel capacity is not the only constraint for this scheduling problem, but the others constraints are not directly considered in the manual process due the complexity of the problem and the flexibility involved in these constraints. Others constraints include the laboratory
capacity and the classroom capacity. In both cases if an excess number of students are enrolled this can be dealt with by the inclusion of additional seats in the classroom or the sharing of computers between two students in the lab. These possible solutions results in student dissatisfaction with the quality of the course affected.

ILA has two laboratories with computers. The large laboratory (laboratory 2) has thirty three computers and the smaller laboratory (laboratory 1) has eighteen computers. These laboratories are used mainly for courses related with the Enterprise Resource Planning (ERP) system used in the Brazilian Air Force (SILOMS); however, these courses also demand a classroom for traditional instruction.

ILA has three classrooms, each one with different capacities. Classroom number 1 has a capacity for forty three students, classroom number two has capacity for thirty four students and classroom number 4 has capacity for twenty students. Due to the layout, illumination, and the incidence of sunlight, classroom number four is avoided when possible.

**Importance of Problem**

Given the conditions mentioned, an optimized scheduling of courses could improve the quality of the training provided while minimizing conflicts during the realization of others events, allowing better utilization of the resources available. The utilization of decision support in the scheduling of courses could also reduce the effort spent in the process of elaborating the TCA 37-11. Therefore, this work could develop a useful tool to improve the quality, and reducing the time spent in the process of developing the timetable. A successful implementation in this case could also allow the extension of this tool to others institutions in the Brazilian Air
Scope and Overall Research Question

The scope of this thesis is to develop and apply a tool to optimize the scheduling of courses, based on a predefined objective functions. In this work, two different types of objective function have been used. One objective, minimize makespan, is the reduction of the time employed to perform all courses, allowing some free periods without courses when others events could occur and when the planning and/or evaluation of the courses could be done.

Another objective function is the reduction of resources demanded at any time. Because others events (seminars, lectures, meeting and so forth) are not previously schedule, and can occur at any time, this type of objective function could minimize the interference between the courses and these events.

The next chapter provides a literature review necessary to develop the specific research methodology for this thesis. A summary of the mathematical optimization models and heuristics in the literature provides the base for the algorithm selection in the methodology chapter. Based on the model and a branch and bound algorithm, various scenarios were presented and discussed.
II. Literature Review

Purpose

This chapter reviews the current timetabling literature as well as some methods used to optimize the final schedule. Through the analysis of the specific problem and the previous work in the literature, this chapter will establish the scholarly base for the research methodology used in this thesis. Evaluation of various modeling approaches will assist in the development of a model to determine the best technique to solve the problem presented.

Timetabling

The literature yields various definitions of timetabling: The timetabling problem consists in fixing a sequence of meetings between teachers and students in a fixed period of time, satisfying a set of constraints of various types (Schaerf, 1999). Timetabling is the allocation, subject to constraints, of given resources to objects being placed in space and time, in such a way to satisfy as nearly as possible a set of desirable objectives (Wren, 1996), or according to Collins Concise Dictionary (4th Edition) a timetable is a table of events arranged according to the time when they take place.

Schaerf (1999) classified the timetabling problems in three main classes:

**School timetabling:** The weekly scheduling for all the classes of a high school, avoiding teachers meeting two classes in the same time and vice versa;

**Course timetabling:** The weekly scheduling for all the lectures of a set of university courses, minimizing the overlaps of lectures of course having common students;
**Examination timetabling**: The scheduling for exams of a set of university courses, avoiding overlapping exams of courses having common students, and spreading the time of exams for the students as much as possible.

Based on Schaeff’s classification, the courses at ILA is close to a school timetabling approach, despite the fact that the interference of teachers and students is not a constraint, due to the condition treated in this thesis. In the Brazilian Air Force there is at least three possible teachers (normally five) for any class, therefore when one instructor is not available on a specific day, another instructor can be assigned. Another difference in this case is that for any course, there is no possibility for the student to choose the class to be taken, and the course has fixed students and classes. As a military school, the students are assigned.

The generic definition of a university timetabling problem can be considered the task of assigning a number of events to a limited set of timeslots in accordance with a set of constraints. Corne *et al.* (1995) suggest that the different type of timetabling constraints can be categorized into five main classes:

- **Unary constraints**: that involve just one event, such as the constraint “event $a$ must not take place on Wednesday”, or the constraint “event $b$ must occur in timeslot $c$”.

- **Binary constraints**: these concern pairs of event, such as the constraint “event $a$ must take place before event $b$” or the event clash constraint that specifies if a person (or others resources) is required to be present in a pair of events, then these must not be assigned to the same timeslot.

In ILA there is no sequence constraints for any course. Any course can be placed before or after the others courses. Only students with pre-requisite are assigned by COMGAP.
Capacity constraints: those are governed by room capacities, laboratory capacities, and so forth.

Event spread constraint: that concern requirements such as the “spreading-out” or “clumping-together” of events within the timetables in order to easy student/teacher workload, and/or to agree with a university’s timetabling policy.

Agent constraints: Those are imposed in order to promote the requirements and/or preferences of the people who will use the timetables, such as the constraint “lecturer x likes to teach on Tuesdays” or “lecturer y must have n free afternoons per week”.

In this thesis, the problem presented only has unary and capacity constraints; this problem results in much fewer constraints than the several university problems treated in literature.

These problems are subject to many constraints that are usually divided into two categories: "hard" and "soft" (Burke et al., 1997).

Hard constraints are rigidly enforced. Examples of such constraints are:

- No resource (students or staff) can be demanded to be in more than one place at any one time.

- For each time period there should be sufficient resources available for all the events that have been scheduled for that time period.

Soft constraints are those that are desirable but not absolutely essential. In real-world situations it is often impossible to satisfy all soft constraints. Examples of soft constraints (in both exam and course timetabling) are:

- Time assignment: a course/exam may need to be scheduled in a particular time period.
- Time constraints between events: one course/exam may need to be scheduled before/after the other.

- Spreading events out in time: students should not have exams in consecutive periods or two exams on the same day.

- Coherence: professors may prefer to have all their lectures in a number of days and to have a number of lecture-free days. These constraints conflict with the constraints on spreading events out in time.

- Resource assignment: professors may prefer to teach in a particular room or it may be the case that a particular exam must be scheduled in a certain room.

A more detailed range of exam timetabling constraints that are in use in British universities can be seen in (Burke et al., 1996). More details of examination timetabling constraints and approaches arising in practice can also be seen in (Carter and Laporte, 1996), while the same authors discuss course timetabling in (Carter and Laporte, 1998).

There are several surveys on educational timetabling problems. Schaerf (1999), and Qu et al. (2009) give an overview of the literature on problems that belong to the three categories mentioned. There are also recent overviews on examination timetabling (Carter & Laporte, 1996) and university course timetabling (Bardadym, 1996; Carter & Laporte, 1998; Burke & Petrovic, 2002; Lewis, 2007).
Approaches to timetabling problem

In the next paragraphs an overview of the most common techniques available in the literature for timetabling problem is presented. (Fang, 1994; Doria et al, 2002; Qu et al, 2009)

Enumerative search

Mathematical Programming: Mathematical programming is a family of techniques for optimizing a function constrained by independent variables. However, it is only suitable for small timetabling/scheduling problems (Fang, 1994). Several such approaches in timetabling problem exist, for example, based in linear and integer programming (Tripathy, 1984), or Lagrangean relaxation (Arani et al., 1988; Tripathy, 1984). Tripathy (1984) reaffirmed the potential of Lagrangean relaxation for solving large-scale integer linear programming problems through the proper formulation and suitable partitioning of the timetabling problem.

Dynamic programming: Dynamic programming is a powerful algorithmic paradigm in which a problem is solved by identifying a collection of sub problems and tackling them one by one, smallest first, using the answers to a sequence of small problems to help determines larger ones, until the whole problem is solved. As the divisions are often done by time, we are dynamically moving through a time horizon; hence the name. Often dynamic programs are solved by working backwards from the result we want to obtain the course of action which will cause it to result. This is known as backward recursion. If we build up a solution from a present condition we are doing forward recursion.

Branch and Bound: Branch and Bound (B&B) searches the complete space of solutions for a given problem for the best solution. However, explicit enumeration is normally impractical due to the exponentially increasing number of potential solutions. The use of bounds for the function
to be optimized combined with the value of the current best solution enables the algorithm to implicitly search parts of the solution space.

At any point during the solution process, the status of the solution with respect to the search of the solution space is described by a pool of yet unexplored subset of this and the best solution found so far. Initially only one subset exists, namely the complete solution space and the best solution found so far is infinite. The unexplored subspaces are represented as nodes in a dynamically generated search tree, which initially only contains the root; each iteration of a classical B&B algorithm processes one such node. The iteration has three main components: selection of the node to process, bound calculation, and branching (Clausen 1999). This approach has been considered ineffective for large timetabling problems (Fang, 1994).

The following overview of heuristics searches follow the development given by Rossi-Doria et. al (2003).

**Heuristic Search**

**Tabu Search** - Tabu search (TS) is a local search metaheuristic which relies on specialized memory structures to avoid entrapment in local minima and achieve an effective balance of intensification and diversification. TS has proved remarkably powerful in finding high-quality solutions to computationally difficult combinatorial optimization problems drawn from a wide variety of applications (Aarts and Lenstra 1997, and Glover and Laguna 1998). More precisely, TS allows the search to explore solutions that do not decrease the objective function value, but only in those cases where these solutions are not forbidden.
The TS algorithm is outlined in Algorithm 1 from Rossi-Doria et al. (2003), where \( l \) denotes the tabu list. In summary, it considers a variable set of neighbor’s and performs the best move that improves the best known solution, otherwise it performs the best non-tabu move chosen among those belonging to the current variable neighborhood set.

Algorithm 1 Tabu search (Rossi-Doria et al. 2002) pag 346

**input:** A problem instance \( I \)

- \( s \) ← random initial solution
- \( L \) ← Ø

**while** time limit not reached do
  **for** \( I = 0 \) to \( 10\% \) of the neighbor’s do
  - \( s_i \) ← after \( i \)-th move
    - compute fitness \( f(s_i) \)
  **end for**
  **if** \( 3s_i | f(s_i) < f(s) \) and \( f(s_i) = f(s) \) then
    - \( s \) ← \( s_i \)
    - \( L \) ← \( L \cup E_i \) where \( E_i \) is the set of events moved to get solution \( s_i \)
  **else**
    - \( s \) ← best non-tabu moves between all \( s_i \)
    - \( L \) ← \( L \cup E_b \) where \( E_b \) is the set of events moved by the best non-tabu move
  **end if**
**end while**

**output:** An optimized solution \( s_{best} \) for \( I \)

Paquete and Stutzle (2002) developed a Tabu Search methodology for exam timetabling where ordered priorities were given for the constraints. The constraints were considered in two ways: (1) one constraint at a time from the highest priority, where ties were broken by considering the lower priority constraints; (2) all the constraints at a time, starting from the highest priority. The second strategy obtained better results. The length of the tabu list was
adaptively set by considering the number of violations in the solutions. It was observed that the length of the tabu list needed to be increased with the size of the problems.

Schaerf (1999) emphasized a disadvantage of local search methods in not allowing the user to analyze only partially filled in timetables. Without a review of partial solutions it does not permit one to focus only on a group of lectures which are specifically critical to be scheduled.

Nguyen et al. (2010) applied a Tabu search algorithm in nine real world instances of University timetabling with large sizes. They have found that experimental results were generally better than handmade schedules being used in practice.

**Simulated Annealing** – Kirkpatrick, Gellat and Vecchi (1983) has explained the utilization of Simulated Annealing for optimizations problems. Di Gaspero (2003) has provided a good explanation of Simulated Annealing (SA). Simulated Annealing was proposed by Kirkpatrick et al. (1983) and Cerny (1985). It was extensively studied by Aarts and Korst, Van Laarhoven and Aarts, among other researchers, in various instances. The method got its name as an analogy to controlled cooling of a collection of hot vibrating atoms. The idea is based on accepting non-improving moves with a probability that decreases with time.

The process starts by creating a random initial solution, \( s_0 \). The main procedure consists of a loop that randomly generates at each iteration a neighbor of the current solution.

Johnson et al. (1989, 1991) discussed in detail the application of Simulated Annealing to a wide range of optimization problems and the impact of different design choices in the performance of the algorithms.

Abramson (1991) has done a study for timetabling problem using SA algorithm, and concluded that SA is a viable tool. The speed of the algorithm can be further improved by
implementing a parallel program, and the results showed good speed up until there are too many competing processors.

Duong and Lam (2004) employed Simulated Annealing on the initial solutions generated by constraint programming for the exam timetabling problem at HMCM University of Technology. The authors noted that when limited time is given, it is crucial to tune the components in Simulated Annealing to the specific problems to be solved.

Burke et al. (2004) studied a variant of Simulated Annealing, called the Great Deluge algorithm. The search accepts worse moves as long as the decrease in the quality is below a certain level, which is originally set as the quality of the initial solution and gradually lowered by a decay factor. The decay factor and an estimate of desired quality represent the parameters in this approach. The authors noted that such parameters can be pre-defined by the users, who are usually not experts on meta-heuristics. The initial solutions, however, need to be feasible to calculate the decay factor. It was shown to be effective and generated some of the best results in some large course timetabling when compared with other approaches (Carter, Laporte and Lee 1996).

Zhang et al. (2010) solved a timetabling problem using a simulated annealing based algorithm with a new-designed neighborhood structure. Zhang et al. (2010) compared this approach with others effective approaches. Their method was considered competitive.
**Algorithm 2 Simulated annealing (Rossi-Doria et al. 2002) pag 342**

**input:** A problem instance $I$

$s ←$ random initial solution

**{Hard Constraints phase}**

$Th ← Th0;$

**while** time limit not reached and $hcv > 0$ **do**

Update temperature;

$s' ←$ Generate a neighbouring solution of $s$

if $f(s') < f(s)$ then

$s ← s'$;

else

$s ← s'$ with probability $p(T, s, s') = e^{-(f(s') - f(s))/T}$

end if

$sbest ←$ best between $s$ and $sbest$

end while

**{Soft Constraints phase}**

$Ts ← Ts0$

**while** time limit not reached and $scv > 0$ **do**

Update temperature

$s' ←$ Generate a neighbouring solution of $s$

if $hcv = 0$ in $s'$ then

if $f(s') < f(s)$ then

$s ← s'$

else

$s ← s'$ with probability $p(T, s, s') = e^{-(f(s') - f(s))/T}$

end if

$sbest ←$ best between $s$ and $sbest$

end if

end while

**output:** An optimized solution $sbest$ for $I$

---

**Ant Colony** - Ant colony optimization (ACO) is a metaheuristic proposed by Dorigo et al. (1996). The inspiration of ACO is the foraging behavior of real ants. Ants use the environment as a medium of communication. They exchange information indirectly by depositing pheromones, all detailing the status of their "work". The information exchanged has a local scope, only an ant located where the pheromones were left has a notion of them. This system is called "Stigmergy" and occurs in many social animal societies. ACO has been applied successfully to numerous combinatorial optimization problems including the quadratic assignment problem, satisfy ability
problems, scheduling problems and so forth (Rossi Doria et al, 2002). The algorithm presented here is the first implementation of an ACO approach for a timetabling problem. It follows the ACS branch of the ACO metaheuristic, which is described in detail in Bonabeau (1999) and which showed good results for the travelling salesman problem Dorigo (1997).

The basic principle of an ACS for tackling the timetabling problem is outlined in Algorithm 3.

Dowsland and Thompson in 2005 developed Ant Algorithms based on the graph coloring model studied in Costa (1997) for solving a version of the exam timetabling problem without soft constraints (i.e. to find the lowest number of timeslots). Extensive experiments were carried out to measure the performance of the algorithm with different configurations. These include the initialization methods, trail calculations, three variants of fitness functions and different parameter settings. The results obtained were competitive to the others on the same dataset based on the quality of the results and on the speed of convergence. It was also observed that the initialization methods had significant influence on the solution quality. Extensions of the algorithm to incorporate other constraints (i.e. time windows, seating capacities and second-order conflicts) were also discussed.

Socha, Knowles and Sampels (2002) devised a construction graph and a pheromone model appropriate for university course timetabling. Using these they were able to specify the first ACO algorithm for this problem. Compared to a random restart local search, it showed significantly better performance on a set of typical problem instances, indicating that it can guide the local search effectively. Their algorithm underlines the fact that ant systems are able to handle problems with multiple heterogeneous constraints. Even without using problem-specific
heuristic information they found it possible to generate good solutions. With an improved local search, exploiting more problem specific operators, they would expect a further improvement in performance.

Eley (2006) compared two modified ant algorithms based on the Max-Min ant system for course timetabling (Socha, 2002), and the ant colony algorithm for graph coloring problems (Costa and Hertz, 1997). It was observed that the simple ant colony algorithm outperformed the Max-Min ant system when both algorithms were hybridized with a hill climber. Eley also concluded that adjusting parameters can considerably improve the performance of ant systems.

Algorithm 3 provides an example of a ant colony approach.

<table>
<thead>
<tr>
<th>Algorithm 3 Ant Colony System</th>
<th>(Rossi-Doria et al. 2002) pag 336</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau(e, t) \leftarrow 0 \forall (e, t) \in E \times T )</td>
<td></td>
</tr>
<tr>
<td><strong>input:</strong> A problem instance ( I )</td>
<td></td>
</tr>
<tr>
<td>calculate ( c(e, e') \forall (e, e') \in E^2 )</td>
<td></td>
</tr>
<tr>
<td>calculate ( d(e), f(e), s(e) \forall e \in E )</td>
<td></td>
</tr>
<tr>
<td>sort ( E ) according to ( &lt; ), resulting in ( e_1 &lt; e_2 &lt; \ldots &lt; e_n )</td>
<td></td>
</tr>
<tr>
<td>( j \leftarrow 0 )</td>
<td></td>
</tr>
<tr>
<td><strong>while</strong> time limit not reached <strong>do</strong></td>
<td></td>
</tr>
<tr>
<td>( j \leftarrow j + 1 )</td>
<td></td>
</tr>
<tr>
<td><strong>for</strong> ( a = 1 ) <strong>to</strong> ( m ) <strong>do</strong></td>
<td></td>
</tr>
<tr>
<td>{construction process of ant ( a }}</td>
<td></td>
</tr>
<tr>
<td>( A_0 \leftarrow \emptyset )</td>
<td></td>
</tr>
<tr>
<td><strong>for</strong> ( i = 1 ) <strong>to</strong> ( n ) <strong>do</strong></td>
<td></td>
</tr>
<tr>
<td>choose timeslot ( t ) randomly according to probability distribution ( P ) for event ( e_i )</td>
<td></td>
</tr>
<tr>
<td>perform local pheromone update for ( \tau(e_i, t) )</td>
<td></td>
</tr>
<tr>
<td>( A_i \leftarrow A_i^{-1} \bigcup{e_i, t} )</td>
<td></td>
</tr>
<tr>
<td><strong>end for</strong></td>
<td></td>
</tr>
<tr>
<td>( s \leftarrow ) solution after applying matching algorithm to ( A_n )</td>
<td></td>
</tr>
<tr>
<td>( s \leftarrow ) solution after applying local search for ( h(j) ) steps to ( s )</td>
<td></td>
</tr>
<tr>
<td>( s_{best} \leftarrow ) best of ( s ) and ( C_{best} )</td>
<td></td>
</tr>
<tr>
<td><strong>end for</strong></td>
<td></td>
</tr>
<tr>
<td>global pheromone update for ( \tau(e, t) \forall (e, t) \in E \times T ) using ( C_{best} )</td>
<td></td>
</tr>
<tr>
<td><strong>end while</strong></td>
<td></td>
</tr>
<tr>
<td><strong>output:</strong> An optimized candidate solution ( s_{best} ) for ( I )</td>
<td></td>
</tr>
</tbody>
</table>
**Artificial Bee Colony (ABC)** - The ABC algorithm was proposed by Karaboga in 2005 for unconstrained optimization problems. Subsequently, the algorithm has been developed by Karaboga and Basturk (2005, 2006, 2007 and 2008) and extended to constrained optimization problems. The Artificial Bee Colony (ABC) algorithm uses a colony of artificial bees. The bees are classified into three types: 1. Employed bees, 2. Onlooker bees, and 3. Scout bees. Each employed bee is associated with a food source, which it exploits currently. A bee waiting in the hive to choose a food source is an onlooker bee. The employed bees share information about the food sources with onlooker bees in the dance area. A scout bee, on the other hand, carries out a random search to discover new food sources.

In a robust search process, exploration and exploitation must be carried out together. In the ABC algorithm, the scout bees are in charge of the exploration process, while the employed and onlooker bees carry out the exploitation process (Karaboga and Basturk, 2007).

In the algorithm, one half of the population consists of employed bees and the other half consists of onlooker bees. The number of food sources equals the number of employed bees. During each cycle, the employed bees try to improve their food sources. Each onlooker bee then chooses a food source based on the nectar amount available at that food source. An employed bee whose food source is exhausted becomes a scout bee. The scout bee then searches for a new food source.

The position of a food source represents a solution for an optimization problem. The nectar amount of the food source is the fitness of the solution. Each solution is represented using a D-dimensional vector. Here, D is the number of optimization parameters. Initially, SN solutions are generated randomly, where SN equals the number of employed bees.
Sabar, Ayob, and Kendall (2009) proposed the Honey-Bee Mating Optimization algorithm (ETP-HBMO) for solving examination timetabling problems. They concluded that the ETP-HBMO can produce good quality solutions for benchmarks examination timetabling problems.

Khang, Phuc and Nuong (2011) have used the ABC algorithm for timetabling problems. In their work the algorithm was able to find a solution near to optimal solution and have shown to be a promising field of study. Sabar et al.(2011) has studied bee algorithm comparing with some benchmarks in the literature. They conclude the proposed approach obtains the best results compared with other approaches on some instances, indicating that the honey-bee mating optimization algorithm is a promising approach in solving educational timetabling problems. Khang, Phuc and Nuong (2011) is provided as at algorithm 4.

<table>
<thead>
<tr>
<th>Algorithm 4 Artificial Bee Colony (Khang, Phuc and Nuong, 2011)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Initialize population with random solutions</td>
</tr>
<tr>
<td>2. Evaluate fitness of the population</td>
</tr>
<tr>
<td>3. <strong>While</strong> (stopping criterion not met)</td>
</tr>
<tr>
<td>//Forming new population</td>
</tr>
<tr>
<td>4. Select sites for neighborhood search</td>
</tr>
<tr>
<td>5. Recruit bees for selected sites (more bees for best e sites) and evaluate fitnesses.</td>
</tr>
<tr>
<td>6. Select the fittest bee from each patch</td>
</tr>
<tr>
<td>7. Assign remaining bees to search randomly and evaluate their fitnesses.</td>
</tr>
<tr>
<td>8. <strong>End While.</strong></td>
</tr>
</tbody>
</table>

**Genetic Algorithm** - The genetic algorithm (GA) is an optimization and search technique based on the principles of genetics and natural selection. A GA allows a population composed of many individuals to evolve under specified selection rules to a state that maximizes the “fitness”
(i.e., minimizes the cost function). The method was developed by John Holland (1975) over the course of the 1960s and 1970s and popularized by one of his students, David Goldberg, who was able to solve a difficult problem involving the control of gas-pipeline transmission for his dissertation (Goldberg, 1991). He was the first to try to develop a theoretical basis for GAs through his schema theorem. The dissertation of De Jong (1975) showed the usefulness of the GA for function optimization and made the first concerted effort to find optimized GA parameters. Since then, many versions of evolutionary programming have been tried with varying degrees of success.

Haupt R. L., and S. E. (1998) on page 23 have listed some of the advantages of a GA:

- Optimizes with continuous or discrete variables,
- Does not require derivative information,
- Simultaneously searches from a wide sampling of the cost surface,
- Deals with a large number of variables,
- Is well suited for parallel computers,
- Optimizes variables with extremely complex cost surfaces (they can jump out of a local minimum),
- Provides a list of optimum variables, not just a single solution,
- May encode the variables so that the optimization is done with the encoded variables, and
- Works with numerically generated data, or experimental data.

The GA begins, like any other optimization algorithm, by defining the optimization variables, the cost function, and the cost. It ends like other optimization algorithms, by testing for
convergence. In between, however, this algorithm is quite different. A path through the components of the GA is shown as a flowchart in Figure 2-1.

![Flowchart of a binary GA](image)

**COMPONENTS OF A BINARY GENETIC ALGORITHM**

- Define cost function, cost, and variables
- Select GA parameters
- Generate initial population
- Decode chromosomes
- Find cost for each chromosome
- Select mates
- Mating
- Mutation
- Convergence Check
- done

Figure 2-1 Flowchart of a binary GA. (Haupt and Haupt 2004)

A cost function generates an output from a set of input variables (a chromosome).

The number of generations that evolve depends on whether an acceptable solution is reached or a set number of iterations is exceeded. After a while all the chromosomes and associated costs would become the same if it were not for mutations. At this point the algorithm
should be stopped. Most GAs keep track of the population statistics in the form of population mean and minimum cost.

**Others heuristics and methods**

Pongcharoen *et al.* (2008) have described the Stochastic Optimization Timetabling Tool (SOTT) where genetic algorithms, simulated annealing, and random search are embedded. The algorithms include a repair process which ensures that all infeasible timetables are rectified. The tool has showed a better performance related with velocity when using Genetic algorithm, but the better fitness was achieved using simulated annealing.

The memetic algorithm attempts to improve the performance of a genetic algorithm by incorporating local neighborhood search (Moscato and Norman, 1992). Paechter *et al.* (1996) have used this approach for timetabling problem.

The Graph Based Sequential Techniques is described in the paper by Welsh and Powell in 1967 and represented an important contribution to the timetabling literature. In exam timetabling problems, the exams can be represented by vertices in a graph, and the hard constraint between exams is represented by the edges between the vertices. The graph coloring problem of assigning colors to vertices, so that no adjacent vertices have the same color, then corresponds to the problem of assigning timeslots to exams.

Qu *et al.* (2009) have emphasized that hybridizations of different techniques have been very widely investigated in recent exam timetabling research. Although different authors have favored
different approaches, it has been observed that hybrid approaches are usually superior to pure algorithms. Zhang et al. (2010) have used a simulated annealing with a neighborhood structure for timetabling problems. Burke et al. (1996) have used Memetic Algorithm with hill climbing and light and heavy mutation. Caramia et al. (2001) used iterated algorithm with novel improving factors. Merlot et al. (2003) used constraint programming as initialization for Simulated Annealing and hill climbing algorithm. Eley (2006) has used ant algorithms with hill climbing operators among others researchers. For example, in the “Hybrid Metaheuristic: Proceedings of 4th International Workshop, HM 2007, Dortmund, Germany, October 8-9, 2007” there are fourteen papers about hybrids metaheuristics.

Comparison about performance for some heuristics

Colorni and Dorigo (1998) have compared the results obtained by simulated annealing, tabu search and two versions, with and without local search, of the genetic algorithm for timetabling problems. Their results have showed that a GA with local search and a tabu search based on temporary problem relaxations both outperform simulated annealing and handmade timetables within the same time of processing. In their work the timetabling problem has a list of $m$ teachers ($20–24$); a list of $p$ classes involved (10 for the two paired sections); and a list of $n$ weekly teaching hours for each class ($30$). The SA parameters were Cooling rate $0.95$; Max cycles without improvement $50$; Problem reinitialization No; Problem relaxation Yes; and numbers of swaps with relaxed constraints $5$. The TS parameters were Max tabu list length $200$; Min tabu list length $30$; Max cycles without improvement $30$; Problem reinitialization No;
Problem relaxation Yes; numbers of swaps with relaxed constraints 3. The GA parameters were probability of first mutation 0.30; probability of mutation k 0.01; probability of crossover 0.80; and N (population number)= 15.

Wilke and Ostler (2008) have compared school timetabling problem using Tabu Search, Simulated Annealing, Genetic and Branch & Bound Algorithms. Tabu Search (TS) improvements were found in about 6 minutes, which was equivalent to approximately 6,000 iterations. They performed 200 moves with 1 iteration and the best regular tabu list length seemed to be approximately 40 elements. For Simulated Annealing, they decided to use a reduction factor of 0.9 and an initial acceptance probability of 0.8 to cool down quite slow, and the cooling factor decrease Factor set to 0.9. The computation ends after a maximum 20 Million iterations or after 2 hours or if a plan with zero costs is found. The Genetic Algorithm uses 30 individuals and runs at most 2 hours i.e. 7,200 seconds, but no information about others parameters was found. Wilke and Ostler found that the Tabu Search, compared to the other algorithm, took only a short time to find good solutions. Simulated Annealing was slower than Tabu Search. Its solution becomes better after approximately 450 seconds. Simulated Annealing outperformed Tabu Search in best and average performance. The Genetic Algorithm was able to improve the best found solution during the whole 2 hours run time. There is a chance that this improvement would continue if more computing time were used. In their experiments all generated time tables violate one or more hard constraints, e.g. no valid time table was produced. Branch & Bound required the longest computation time, although no information was given about the initial solution used to start the algorithm. After 8 hours the best solution objective of 3300 penalty points, which is quite expensive, the improvement was 79%, which is in some
degree is good. They recommend Simulated Annealing to generate the School Time tables, because it is the best tradeoff between execution time and quality of result.

Azimi (2004) has compared SA, TS, GA and ACS for Examination Timetabling Problem. His results have showed that ACS and then TS algorithm worked better in comparison with others algorithms, but ACS starts with better initial solution than TS and thus, it has less reduction in cost of solution. He concluded that ACS works better on these problems.

Rossi-Doria et al. (2002) have compared five heuristics for timetabling problem (GA, TS, ACS, SA and interacted local search). They believe that it will be very difficult to design a metaheuristic that can tackle general instances, even from the restricted class of problems provided by their generator. Additionally, they confirmed that knowing certain aspects of an instance does not guarantee that we will know about the structure of the search space, nor does it suggest a priori that we will know which metaheuristic will be best.

Corne et al. (1994) emphasized that a GA can handle hard and soft constraints in the same way and find a good trade-off between them. It will do the best it can even on unsolvable problems. Additionally, a GA can be quite easily and effectively induced to evolve multiple distinct optima.

According to Gyori et al. (2001), GA was applied in this optimization problem because it is robust enough in such a large problem space. They introduced a new set representation, which meets the demands better than previous cases. Hard and soft constraints to be satisfied by the timetables have been defined. The method proved to be efficient in a real life application of a secondary school. The set representation meets the demands better than former ones.
Therefore, there is no algorithm that could be considered superior others. Any algorithm has advantages and disadvantages. The performance of the algorithm depends on the parameters used and on the type of problem. Normally for large problems, the heuristic method would provide a good relation quality/time. In this thesis, the schedule is considered a small timetabling problem; the branch and bound algorithm can be used to find the optimal solution. While classic branch and bound has been used in this effort, the review of the current heuristic based timetabling literature is provided both for completeness and a reference. A future analyst should be confronted with large timetabling problem.

Chapter II describes the methodology used in the thesis.
III. Methodology

Introduction

This study analyzes the timetabling problem at Instituto de Logística da Aeronáutica (ILA). The objective of this work is to develop a tool to optimize the course scheduling and, therefore, reduce the time spent during the process to develop this schedule.

The objective function to be optimized (minimized) is “makespan” in the first scenario and the leveling of the number of students in the second scenario. These two scenarios will be compared with the actual process where the course scheduling is prepared “by hand”.

Yang and Jat (2011) have defined three groups of problems instance (small, medium and large). In this thesis, the problem can be considered small, with 34 courses, 4 classrooms, 2 laboratories, a maximum number of students of 80 at any time, and a maximum student per event of 50 students.

The time interval in this study is one week. The duration of each course is always a multiple of one week as showed in the Table 3-1. Seven of these 34 courses are hybrid courses. Due to the specific characteristics in the admission/enrolment process, and the contract with private institutions for these seven courses, a negotiation might be done when scheduling these courses.
<table>
<thead>
<tr>
<th>course</th>
<th>duration (weeks)</th>
<th>n° students (max)</th>
<th>type of course</th>
</tr>
</thead>
<tbody>
<tr>
<td>caalf</td>
<td>1</td>
<td>20</td>
<td>in class</td>
</tr>
<tr>
<td>cac</td>
<td>2</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>cacfo</td>
<td>1</td>
<td>20</td>
<td>in class</td>
</tr>
<tr>
<td>cadi</td>
<td>1</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>cadti/2</td>
<td>1</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>cambel</td>
<td>2</td>
<td>20</td>
<td>in class</td>
</tr>
<tr>
<td>capsa</td>
<td>3</td>
<td>10</td>
<td>in class</td>
</tr>
<tr>
<td>casup</td>
<td>2</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>catcis</td>
<td>2</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>cbmo</td>
<td>3</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>ccmnp</td>
<td>2</td>
<td>15</td>
<td>in class</td>
</tr>
<tr>
<td>cenm</td>
<td>2</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>cfms</td>
<td>2</td>
<td>20</td>
<td>in class</td>
</tr>
<tr>
<td>cgps</td>
<td>2</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>cgup</td>
<td>3</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>cidmat</td>
<td>2</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>cimbe</td>
<td>2</td>
<td>20</td>
<td>in class</td>
</tr>
<tr>
<td>cins</td>
<td>3</td>
<td>15</td>
<td>in class</td>
</tr>
<tr>
<td>cneg 1</td>
<td>3</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>cneg 2</td>
<td>3</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>codmat</td>
<td>2</td>
<td>20</td>
<td>in class</td>
</tr>
<tr>
<td>cpac</td>
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<td>in class</td>
</tr>
<tr>
<td>cpat</td>
<td>1</td>
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<td>in class</td>
</tr>
<tr>
<td>cpnp</td>
<td>3</td>
<td>15</td>
<td>in class</td>
</tr>
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<td>cpoa 1</td>
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<td>12</td>
<td>in class</td>
</tr>
<tr>
<td>cpoa 2</td>
<td>2</td>
<td>12</td>
<td>in class</td>
</tr>
<tr>
<td>cst</td>
<td>1</td>
<td>30</td>
<td>in class</td>
</tr>
<tr>
<td>cbmo (s)</td>
<td>1</td>
<td>40</td>
<td>hybrid</td>
</tr>
<tr>
<td>ceslog (s)</td>
<td>6</td>
<td>30</td>
<td>hybrid</td>
</tr>
<tr>
<td>cfacc 1 (s)</td>
<td>1</td>
<td>42</td>
<td>hybrid</td>
</tr>
<tr>
<td>cfacc 2 (s)</td>
<td>1</td>
<td>42</td>
<td>hybrid</td>
</tr>
<tr>
<td>cima 1 (s)</td>
<td>1</td>
<td>42</td>
<td>hybrid</td>
</tr>
<tr>
<td>cima 2 (s)</td>
<td>1</td>
<td>42</td>
<td>hybrid</td>
</tr>
</tbody>
</table>

Table 3-1 – courses scheduled for 2011
As previously mentioned, the actual process of course scheduling starts in August and finishes in November, when the approval and publication of the document TCA37-11 (Aeronautic Command Table number 37-11) occurs. This process is based on a series of meetings, normally one meeting per month during the planning period of two days of duration. During these meetings, the demand for each course is discussed. This discussion can result in a change in the number of students per course or the creation/deletion of others courses.

Each new alternative of the course schedule is analyzed with the existing constraints by hand. Therefore this process is arduous and time consuming. As the actual process is not based in any stated objective function, a large number of alternatives are possible, personal feelings are often predominant, and conflicts of opinions can occur.

The constraints are important for the final course schedule, since no violation is allowed. These constraints include the total number of student per week, the number of students allowed per room, number of students allowed per laboratory, and the continuity of the course (no preemption is allowed).

**Problem formulation**

According to Tripathy (1984), the process of producing a timetabling has three main phases:

(i) Deciding the group of students who are to attend a particular subject.

(ii) Specifying the facilities required by the subject.

(iii) Determining when each subject is to take place.
The first two phases involve certain major decisions to be taken by the academic staff and the administrator. These often do not follow any specific mathematical rules and hence are done manually. The first two phases, when completed, completely define the subjects, and these form the data for use in the third phase.

**Decision Variables**

Given a predefined number of courses per year ($n$ courses), the index $i$ represents a specific course in the model where $i = 1, \ldots, n$. Each course $i$ has a specific duration of weeks ($Ti$) and a number of students ($Ni$).

The time interval in this problem is one week, because any course duration is a multiple of one week. As the month of January is reserved for vacation, the week of carnival and the last two weeks of the year are reserved due to holidays, the maximum number of weeks available are 45 ($m$). The index $j$ represents each specific week available for one course. Therefore $j = 1, \ldots, m$ or $j = 1, \ldots, 45$ in this case.

Based on these definitions, the decision variable for this problem can be expressed as $x_{ij}$. $\ldots x_{ij}$ is a binary number equal to one if the course $i$ is allocated at time $j$ and 0 otherwise.

Another decision variable in this problem is the laboratory ($l$) to be used. As ILA has two laboratories, the index $k$ is used to designate if the laboratory to be used is laboratory 1 or laboratory 2. Therefore, the variable $l_{ijk}$ is a binary variable that is equal to 1 when the laboratory $k$ is used for course $i$ at time $j$, and 0 otherwise.

The last decision variable in this problem is the room to be used ($r$). As ILA has four rooms, the index $w$ is used to define if the room to be used is the room 1, 2, 3 or room 4.
Therefore the variable $r_{ijw}$ is a binary variable that’s equal to 1 when the room $w$ is used for course $i$ at time $j$, and 0 otherwise.

**Constraints**

Based on the decision variables, the constraints in this problem can be developed.

The first constraint considered is related with the objective function $C_{\text{max}}$. $C_{\text{max}}$ represents the maximum number of weeks used in the schedule. Minimizing the maximum completing time of all activities, $C_{\text{max}}$, is a classic scheduling objective. It processes the classes as quickly as possible, leaving available capacity for others events. Therefore, when any course is placed at time $j$, $C_{\text{max}}$ have to be greater than or equal to the week $j$. This constraint is expressed by:

$$C_{\text{max}} \geq x_{ij} \cdot j \quad \forall i, \forall j$$

The second constraint considered is the hotel capacity. Due to the capacity of the hotel, the total number of students per week should be no more than 80 students. Therefore, for each timeslot $j$, the sum of students should be less than or equal to 80. Each course, within the $n$ courses, has a defined number of student, therefore the parameter $N_i$ represents the maximum number of student for course $i$. When a course $i$ is allocated at timeslot $j$ ($x_{ij} = 1$) the sum of the products $x_{ij} \cdot N_i$ must be less than 80. This constraint is expressed by :

$$\sum_{i=1}^{n} x_{ij} \cdot N_i \leq 80 \quad \forall j.$$

The third constraint considered is the laboratory capacity. As laboratory 1 has a limit of 18 students, the product $l_{ij} \cdot N_i$ must be less than or equal to 18, and the laboratory 2 has a limit of
33 students, therefore the product $l_{ij_2} \times N_i$ must be less than or equal to 33

$$\sum_{i=1}^{n} l_{ij_1} \times N_i \leq 18 \quad \forall j, \quad \sum_{i=1}^{n} l_{ij_2} \times N_i \leq 33 \quad \forall j \quad \text{labatory capacity}.$$

Of course, at time $j$ only one course must be taken for each laboratory (fourth constraint). The binary variable $l_{ci}$ indicates the necessity for the utilization of any laboratory for course $i$, is expressed by

$$x_{ij} \times l_{ci} = \sum_{k=1}^{2} l_{ijk} \quad \forall i, \forall j.$$

The fifth constraint considered is the room capacity. Each course needs a room even when the laboratory is used. As room 1 has a limit of 35 students, the product $r_{ij_1} \times N_i$ must be less than or equal to 35, and the others rooms has a limit of 42 for room 2, 33 for room 3 and only 25 for room 4, expressed by:

$$\sum_{i=1}^{n} r_{ij_1} \times N_i \leq 35 \quad \forall j, \quad \sum_{i=1}^{n} r_{ij_2} \times N_i \leq 42 \quad \forall j, \quad \text{capacity of classroom 1 and 2}$$

$$\sum_{i=1}^{n} r_{ij_3} \times N_i \leq 33 \quad \forall j, \quad \sum_{i=1}^{n} r_{ij_4} \times N_i \leq 25 \quad \forall j \quad \text{capacity of classroom 3 and 4}$$

Of course, at time $j$ only one course may be taken for each room (sixth constraint), and the course must be taken in one classroom (seventh constraint).

$$\sum_{i=1}^{n} r_{ijw} \leq 1 \quad \forall j, \forall w$$

$$x_{ij} = \sum_{w=1}^{4} r_{ijw} \quad \forall i, \forall j$$

The eighth constraint is the duration of the course. The sum of the weeks scheduled for one course must be equal to the total duration of the course according to the TCA 37-11.
\[
\sum_{j=1}^{m} x_{ij} = T_i \quad \forall i
\]

The last constraint is the continuity of the course. Once a course has started, it is not possible to interrupt the course to be finish later.

In this thesis, there is no constraints related with the sequence of the courses, and no precedence exists between these courses.

\[
N_i * x_{ij} - N_i * x_{i,j+1} + \sum_{j=i+2}^{n} x_{ij} * N_i \leq N_i \quad \forall i
\]

**Objective function**

In this work, there are two different approaches and therefore two different objective functions. The first approach is to minimize the “makespan” or $C_{\text{max}}$. With this approach, the objective is to allow free timeslots without courses that could be used for the others activities as evaluation and planning of courses or seminars, meetings, lectures and so forth.

Another approach is to minimize the maximum number of students per week. With this approach the idea is to have a constant utilization of resources at a minimum required level. When another activity is placed at any time, there is a minimum interference between this extra-activity and the courses and, therefore, minimizing the problems with resources available.

**Decision variables:**

Let $x_{ij}$ (binary) = 1 if course $i$ is placed at time $j$. Let $r_{ijw}$ (binary) =1 if room $w$ is placed for course $i$ at time $j$. Let $l_{ijk}$ (binary) =1 if lab $k$ is placed at time $j$ for course $i$.

$C_i = 1$ if course $i$ needs a lab. $N_i =$ number students in course $i$. $T_i =$ duration of course $i$. 

---

35
Mathematical formulation

In the first instance (makespan) the problem is formulated as:

Minimize \( C_{\text{max}} \)

Subject to:

\[
C_{\text{max}} \geq x_{ij} \quad \forall i, \forall j
\]

\[
\sum_{i=1}^{n} x_{ij} \cdot N_{i} \leq 80 \quad \forall j \quad \text{hotel capacity}
\]

\[
\sum_{i=1}^{n} l_{ij1} \cdot N_{i} \leq 18 \quad \forall j, \sum_{i=1}^{n} l_{ij2} \cdot N_{i} \leq 33 \quad \forall j \quad \text{laboratory capacity}
\]

\[
\sum_{i=1}^{n} r_{ij1} \cdot N_{i} \leq 35 \quad \forall j, \sum_{i=1}^{n} r_{ij2} \cdot N_{i} \leq 42 \quad \forall j \quad \text{capacity of classroom 1 and 2}
\]

\[
\sum_{i=1}^{n} r_{ij3} \cdot N_{i} \leq 33 \quad \forall j, \sum_{i=1}^{n} r_{ij4} \cdot N_{i} \leq 25 \quad \forall j \quad \text{capacity of classroom 3 and 4}
\]

\[
\sum_{j=1}^{m} x_{ij} = T_{i} \quad \forall i \quad \text{Duration of the courses}
\]

\[
\sum_{i=1}^{n} r_{ijw} \leq 1 \quad \forall j, \forall w \quad \text{one course per classroom}
\]

\[
x_{ij} \cdot l_{ci} = \sum_{k=1}^{2} l_{ijk} \quad \forall i, \forall j \quad \text{utilization of laboratory when necessary}
\]

\[
x_{ij} = \sum_{w=1}^{4} r_{ijw} \quad \forall i, \forall j \quad \text{utilization of room}
\]

\[
N_{i} \cdot x_{ij} - N_{i} \cdot x_{i,j+1} + \sum_{j=i+2}^{n} x_{ij} \cdot N_{i} \leq N_{i} \quad \forall i \quad \text{no splitting courses}
\]

\[
x_{ij}, r_{ijw}, l_{ijk} \text{ are binary} \quad \forall i, \forall j, \forall w, \forall k
\]

In the second instance (minimum number of students per week) the problem is formulated as:
Minimize : \( A_{max} \)

Subject to:

\[
\sum_{j=1}^{n} x_{ij} \cdot N_i \leq A_{max}
\]

\[
\sum_{i=1}^{n} x_{ij} \cdot N_i \leq 80 \quad \forall j
\]

\[
\sum_{i=1}^{n} l_{ij1} \cdot N_i \leq 18 \quad \forall j , \sum_{i=1}^{n} l_{ij2} \cdot N_i \leq 33 \quad \forall j
\]

\[
\sum_{i=1}^{n} r_{ij1} \cdot N_i \leq 35 \quad \forall j , \sum_{i=1}^{n} r_{ij2} \cdot N_i \leq 42 \quad \forall j ,
\]

\[
\sum_{i=1}^{n} r_{ij3} \cdot N_i \leq 33 \quad \forall j , \sum_{i=1}^{n} r_{ij4} \cdot N_i \leq 25 \quad \forall j
\]

\[
\sum_{j=1}^{m} x_{ij} = T_i \quad \forall i
\]

\[
\sum_{i=1}^{34} r_{ijw} \leq 1 \quad \forall j , \forall w
\]

\[
x_{ij} \cdot l_{ci} = \sum_{k=1}^{2} l_{ijk} \quad \forall i , \forall j
\]

\[
x_{ij} = \sum_{w=1}^{4} r_{ijw} \quad \forall i , \forall j
\]

\[
N_i \cdot x_{ij} - N_i \cdot x_{i,j+1} + \sum_{j=t+2}^{n} x_{ij} \cdot N_i \leq N_i \quad \forall i
\]

\(x_{ij}, r_{ijw}, l_{ijk}\) are binary \(\forall i , \forall j, \forall w, \forall k\)
Methodology Issues

While there are potentially many factors to consider when scheduling activities, as courses, here the hotel capacity, room availability, and laboratory availability are considered. Considering the year of 2010, 34 courses were scheduled to be done at ILA. The schedule generated by hand is compared with the solutions presented in this work.

In this model, as there are 34 courses and 40 time slots, resulting in 1360 binaries variables related with the courses.

The number of constraints in this problem is 40 for the hotel capacity, 40 for each room (there are 4 rooms available), 40 for each laboratory (there are two laboratories), 34 to constrain the duration of the course, 1360 constraints to force the use of one room, 1360 constraints to force the use of laboratory when necessary and the last constraint type has 1292 constraints to avoid any splitting between the beginning and the end of course. Therefore the total number of constraint is 4326. While large, this model can be solved to optimality.
Investigation Design

The makespan problem was solved to find the optimal solution using the Branch and Bound algorithm. The software used for this purpose was LINGO from Lingo Systems Inc. using the default parameters. The solution for the makespan objective function was 23 weeks with a total time to find the optimal solution was 72 hours using a computer with 4 Gb RAM, and 2.7 GHz processor. The solution is showed in the Table 3-2. A Gantt Chart, using the software Gantt project, is available for this solution in appendix B.

<table>
<thead>
<tr>
<th>course</th>
<th>duration</th>
<th>Weeks scheduled for the course</th>
<th>course</th>
<th>duration</th>
<th>Weeks scheduled for the course</th>
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<td>3</td>
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</tr>
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<td>1, 2 and 3</td>
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<td>7, 8 and 9</td>
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<td>12, 13 and 14</td>
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</tr>
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<td>16 and 17</td>
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<td>19 and 20</td>
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<td>16 and 17</td>
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<td>4 and 5</td>
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<td>2</td>
<td>10 and 11</td>
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</tr>
</tbody>
</table>

Table 3-2 – solution for makespan approach

The utilization of the hotel is showed in the Figure 3-1. Figure 3-1 shows that hotel utilization is close to or at capacity in these first 23 weeks if the Cmax schedule is used.
The utilization of the classrooms is often low, and even when there is 4 rooms in use with courses in any week, these courses had fewer students due to the hotel capacity. This fact results in available seats in the classrooms. The number of rooms utilized in this solution is showed below in Figure 3-2.
The laboratory utilization was very low, with the utilization often zero; the slack was always greater than 50% of the capacity.

This solution seems to show a large excess of capacity and might allow more courses than the plan for 2011. However, a high concentration of courses could result in problems with others resources not considered in this thesis. For example, the human resource availability can be a constraint that does not allow this type of approach. The formulation for availability of human resource is complex and is out of scope for this work. Another consideration would be having time to undertake required maintenance. If used to capacity, there is no “stand by” resources for critical repair or scheduled maintenance.

These results illustrate the hotel capacity is the main constraint in this compressed schedule problem. Increasing the room capacity, either by increasing the number of room or the students per classroom, would not dramatically change the result due the hotel limit.

For the second instance, the objective is to spread the courses uniformly within a certain number of weeks. Initially the problem was solved for 40 weeks, but in this case there is no interruption within the period with courses, that means, the 40 weeks are continually scheduled for courses. The optimal solution was achieved after 1 hour and 45 minutes. The solution is shown in the Table 3-3, but this result will not be used in further analysis, due the fact that the carnival is a period that must be free of courses, and this fact is not represented in this solution. A Gantt chart for this solution is in appendix C. This result shows that only a maximum of two classrooms were necessary and only one laboratory was necessary. Therefore a reduction on the
number of constraints could be applied, reducing the number of classrooms and laboratories in the model. This reduction in the number of constraints was applied in further investigation.

<table>
<thead>
<tr>
<th>course</th>
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<th>Weeks scheduled</th>
<th>course</th>
<th>duration</th>
<th>Weeks scheduled</th>
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<td>31 and 32</td>
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<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3-3 – solution for second instance

The utilization of the hotel is showed in Figure 3-3, where it is possible to provide a distribution of students across the year.
In this instance the most important factor is the number of weeks over which it is desired to spread all courses. The numbers of rooms utilized in this scenario is always less than or equal to two.

As the time to solve the makespan instance (Cmax) was too long, the configuration of the software was changed from the default. The heuristic level was changed to 5 and the relative optimality tolerance was changed from $5 \times 10^{-8}$ to 0.05. With this configuration only 70 minutes was necessary to find the solution.

As explained by Nauss (1977), parametric integer programming can provide sensitivity analysis, trade-off analysis and contingency analysis. Therefore a parametric integer linear programming (PILP) was implemented to analyze the influence in some constraints, mainly in hotel capacity that is the main factor constraining the objective function.

Figure 3-3 – Hotel utilization in second instance (spreading courses).
In this work, five scenarios were created to analyze the influence of the expansion of hotel capacity. There are four students for each room available in the hotel and each new scenario represents one new room. These new rooms are represented by the variable $g$, where $g$ is the number of new rooms.

For the makespan instance the PILP used is:

For $g = 0, 1, 2, 3$ and 4 solve:

Minimize $C_{\text{max}}$;

Subject to:

$$\sum_{i=1}^{n} x_{ij} \times N_l \leq 80 + g \times 4$$

While others constraints remain the same.

For the second instance where the objective is to spread the courses, it is important to know the influence of the weeks reserved for special events. In these weeks no courses are scheduled in this work. Therefore, when a week is reserved for special events, the variable $x_{ij}$ must be equal to zero for any course.

In the second instance five scenarios were created. One scenario has just the week of carnival without courses, and the others scenarios has others weeks without courses in addition to carnival week. For each scenario the weeks without a course varies from 1 to 7, and when is possible, preferably at the end of the quarter.

The PILP can support a comparison of results and analysis in the next chapter.
IV. Analysis and Results

Context of Model Results

This section details a comparative analysis of the actual timetable using the method developed in this thesis. The first comparison is related with the “makespan” approach, where the objective function is to concentrate the courses to allow free periods for others activities. The second approach is the spread of the courses leveling resources requirements, Various scenarios are compared. Based on the results founded in the second approach, the possibility of increasing the number of students was analyzed.

Analysis of the first approach

The first approach, minimizing makespan, was to concentrate all the courses within a period of time. In scheduling theory minimizing makespan attempts to finish all available jobs (courses in this case) as soon as possible to free up resources for other needs. Therefore, during the period with courses, there is no opportunity to perform any other activity or special events. All available resource, including human resource, should be dedicated to courses in this compressed period period. These others activities (special events) could occur only in the period without courses; all of ILA’s resources should be available for these special events. Therefore, no interferences could occurs between these activities. In addition, the block of “free time”(unscheduled choses) could be available for other activities.

From the result showed in Table 3-2, only 23 weeks are necessary to schedule all the thirty six courses offered during the year of 2011. This result shows that less than half of the year is necessary to execute the courses based in the constraints mentioned in the methodology.
However, others resources not included in the model could not execute all these courses in a short period of time of 23 weeks. For example, the human resources would not be sufficient in this scenario. The availability of personnel is outside of scope in this work. Another example is the maintenance capacity to support the courses, including the maintenance of the facilities and computers. For this reason this approach is not considered practical.

The constraint without slack, in this compressed solution, is the hotel capacity, while the others constraints still have excess capacity. Even in this compressed scenario, there is a excess of classrooms, laboratories, and seats available.

Based in this result it was possible to investigate the increasing in hotel capacity as defined in the PILP described in the chapter III. With a capacity of 92 students in the hotel, it would be possible to reduce the “makespan” of the year’s courses to 20 weeks. Figure 4-1 shows the increase the utilization of classrooms to 93%. However, this result is also not practical and no further analysis is conducted on this issue.

Figure 4-1 – Classrooms utilization in first instance with hotel capacity for 92 students.
If a compressed schedule was possible and/or more space was available at the hotel it would be possible to increase the number of courses offered. Considering the same distribution of the number of students per course, the number of courses could be duplicated without any violation of any constraint considered in the model. Again, however, as mentioned before, there are considerations about others constraints, and the approach to spread the courses within some predefined period of time seems more reasonable.

**Analysis of the second approach**

In this approach the courses are distributed during the year to smooth the utilization of the resources while reserving a few weeks for special events. The unique scenario where there is superimposition of courses and special events, is the last scenario which all courses are distributed within the 45 weeks available.

In this approach five scenarios were created, based on the PILP described previously, to compare with the current schedule. The month of January does not have courses because it is the traditional month for vacations. The week for carnival and Christmas are also not scheduled for courses. Given these restrictions, the first scenario is 38 weeks for courses and 7 weeks for special events; the second scenario is 40 weeks for courses and 5 weeks for special events; the third scenario is 42 weeks for courses and 3 weeks reserved for special events; the fourth scenario is 44 weeks for courses and 1 week for special events; and the last scenario has all 45 weeks with courses without any week specifically reserved for special event. In any scenario, the
weeks without courses are divided between the four quarters and placed at the end of the quarter, except in the last scenario where all 45 weeks available is planned for courses.

Figure 4-2 shows the number of students per week based on the approved schedule in the current TCA 37-11 and the number of classrooms used. In this schedule, the number of students per week varies from 15 to 80 (avg 45, sd 24), with 7 weeks without courses. These 7 weeks are the traditional January, Christmas and carnival holidays. There are 10 weeks when only 1 classroom is used, 15 weeks when 2 classrooms are used and 13 weeks with 3 classrooms.
Figure 4-2 – Hotel utilization based on the courses scheduled for 2011 (TCA 37-11/2010)

Figure 4-3 shows the profile of students per week with 7 weeks reserved for special events. In this scenario the number of students per week varies from 30 to 60 (avg 45, std 7.7) when there is not a free week. In this configuration there is 19 weeks utilizing only one classroom and 29 weeks utilizing 2 classrooms. The total number of students taught is 1718 during the year.
This alternative shows a solution with a smoother utilization of resources than the approved schedule. The standard deviation of the number of students is 24 in the approved TCA 37-11 and just 7.7 in this alternative.

The advantage in this alternative is to plan 7 weeks for special events without conflicts with courses, while the courses is distributed in the others weeks allowing more uniformly demand for ILA resources.

![Figure 4-3](image)

Figure 4-3 – Hotel utilization with courses distributed within 38 weeks.

Figure 4-4 shows the number of students per week in the scenario with 3 weeks reserved for special events. The number of students per week varies from 30 to 50 (avg 41, std 7.4). The number of weeks with one classroom used is seventeen and the number of weeks using two classrooms is 25. Compared with the first scenario which has 38 weeks for courses and 7 weeks for special events, this alternative is significantly different at 5% level (p-value=0.014).
Therefore, the null hypothesis H0: the means in both scenarios are equal, should be rejected at a 5% potential error.

Figure 4-4 – Hotel utilization with courses distributed within 42 weeks.

The last scenario in this approach, without free weeks reserved for special events, results in a solution smoother than the others. The number of students per week varies from 30 to 50 (avg 38, std 6.5). This result is significantly different from the first scenario with 38 weeks at 5% level, but is not significantly different from the second scenario (p-value= 0.07). The number of weeks utilizing only 1 classroom is 22, therefore there is 23 weeks using 2 classrooms. But in this case, when a special event must be placed, there will be potential conflicts with courses in the same week. In addition, any attendees for the special event may need to compete for rooms at the hotel with the assigned students. Figure 4-5 shows the hotel utilization in this last scenario.
Figure 4-5 – Hotel utilization with courses distributed within 45 weeks.

The utilization of the model with only two classrooms and one laboratory was important to reduce the executing times. The higher processing time was found in the last scenario. In this scenario was necessary 5 hours and 31 minutes to run the model, even running the model with only two classrooms and one laboratory.

The period without courses could be used not only for special events, but for others activities such as course evaluation with customers, research, support for process improvement in the Air Force or support for Strategic Acquisition Process like the FX-2 program (acquisition of new fighters for Brazilian Air Force). These activities are currently performed only when required by the high management level for important projects, and could be performed more frequently with this approach to spread the courses.
Analysis of the number of students trained per year

The mathematical model used allows the visualization of the classroom to be used in each course. It is also then possible to analyses the full utilization of the classroom capacity while respecting the others constraints.

In the actual process without any optimization tool, only the schedule of the courses has been done, and the final document (TCA 37-11) does not provide any information about the classroom to be used. Therefore, normally for courses with higher demands only 30 students are planned.

The utilization of optimization tools allows the visualization of the possibility to increase the number of students for some courses. The largest room available at ILA has capacity for 42 students while the second largest has capacity for 35. Therefore, some courses could increase the number of students taught to satisfy the demand without any increase in the number of courses or direct cost with instructors that could be a new constraint for the problem. There would, however, be a potential indirect cost to the instructor who could have larger courses. Therefore, for each scenario presented in the second instance the increase of students is analyzed.

In the case of spreading the courses within 38 weeks with 7 weeks free for special events, it is possible to increase the total number of students per year from 1718 to 2035 or an increase equivalent of 18%. Figure 4-6 shows the hotel utilization in this situation.
In the case where only 3 weeks per year are reserved for special events, the number of total students per year could be increased to 2105. This shows a potential of 22% more students per year than the actual schedule. Again, this alternative is based on the capacity of the classroom, laboratory and the hotel; the demand for each of these courses could be lower than the estimated in this work. A maximum of two classrooms and one laboratory were necessary.

While more students are processed, there is less flexibility for others events because only three weeks are planned for these events. Figure 4-7 shows the hotel utilization in this case.
Figure 4-7 – Hotel utilization with courses distributed within 42 weeks with full utilization in room.

In the extreme case with no week set aside for special events and all 45 weeks are used for courses, it is possible to increase the total number of students per year from 1718 to 2166. That represents an increase of 26%. Again, in this scenario there is no flexibility for other events. There is always a conflict with courses when other event needs to be scheduled. Figure 4-8 exhibits the hotel utilization in this case.
Figure 4-8 – Hotel utilization with courses distributed within 45 weeks with full utilization in room.

Based on these results, the makespan approach needs a more detailed model, including others constraints as personal availability, reprographic service constraints, budget availability and so forth. However, the second approach presents a feasible solution, when these others resources not included in the model do not represent a constraint. This fact is easily proven once the approved document TCA 37-11 has a similar approach, but presenting a large variation of utilization.

In the actual configuration of courses, and students per course, there is 38 weeks with courses and 7 weeks without courses besides the weeks reserved for Carnival, Christmas, and the January month. The approved TCA 37-11 has higher variation in the utilization of resources (standard deviation of 19.3 in number of students per week) than the solution of the mathematical model with the same weeks for courses (standard deviation of 7.7 in the number of student per week).
Due to this fact, the model presents the flexibility to select the desired week reserved for special events where the variable $x_{ij}$ may be fixed to zero. In the approved TCA 37-1, the weeks without course are randomly distributed, only the carnival, Christmas and the January month are previously defined. Thus, this approach could be the first step to use the mathematical model to schedule courses for the next year.

The PILP provided different scenarios in each approach. Once only the second approach presented a reasonable solution, this approach was investigated and served as basis for recommendations in the last chapter of this thesis.

The possibility to increase the number of students taught is also feasible in this second approach. The utilization of the hotel is under the limit, the number of courses per week is always less than or equal to two, and the laboratory capacity is higher than the possible utilization in each solution presented. Increasing the number of students per course should have low impact on the cost of the courses, human resources, maintenance demand, and others constraints not included in the model. The main restriction for increasing the students taught is the capacity of the classrooms and the capacity of the laboratories. Therefore, it is possible to evaluate an alternative to reduce the number of classroom, initially by one, while increasing the capacity in the classrooms remained and expanding the laboratories. In order to perform this evaluation, it is necessary to have more information about strategic goals and perspectives of ILA and BAF in the long term, avoiding a future necessity to return back for the actual configuration.

The tool does provide a guide for scheduling that can be relatively rapidly generated to review and adjust to include unquantifiable goals. The analysis provides a base line which can
assist the discussion of the final schedule. Of course it may not be practicable to release more airmen from duty for school in a given week. This would be a consideration for higher leadership to consider.
V. Conclusions and Recommendations

The utilization of an optimization tool supports a faster process to develop scenarios for scheduling the courses at ILA. The time spent for the formulation and for the implementation in software was less than one day. After the implementation, the time spent by the computer for the optimization was an average few hours with the selection of adequate parameters. Heuristic approaches would be even fasting in generating schedules.

The initial approach to concentrate all the courses within a shortest time period was not a good managerial solution due the fact that the courses were concentrated in a 23 week window. This could result in high utilization, with limited slack in the system during these weeks. However, others constraints such as human resources, maintenance capacity, reprographic services and so forth, could make this solution impracticable. A complete analysis considering all resources could be done to evaluate the practicable makespan. In this case, if there is more weeks available than the necessary for special events, the excess capacity could be used for others opportunities, for example be offered for others schools or even rented for private institutions.

The approach to spread the courses within a time windows revealed a more interesting point of view. The comparison of the results in this approach with the approved document based on the actual process (manual) emphasized the possibility leveling the use of the resources.

Based on the decision by the top level leadership about the periods to be reserved for special events, the spreading of the courses can avoid conflicts and defines a constant rhythm of production.
Besides the fact that a more smooth utilization of resources could be possible, it is also possible to increase the number of students taught in this approach. With a definition of the classroom to be used at the end of the computational solution it is also possible to increase the number of students that is planned to use any classroom. Classroom 2, for example, has capacity for 42 students. However, due to the fact that in actual process the classroom to be used is not defined, normally only 30 students is defined for each course. For 2011 only six courses were planned with more than 30 students. The increase in the number of students per year could be at least 18% using the technique of mathematical programming.

The recommendation for a process to be adopted, based on this work, is to use the second approach, spreading the courses. However, the first step is to define the number of weeks to be reserved for others events and when this should occurs. This information, additionally with the courses definitions, could serve as base for a formulation and mathematical optimization in several alternatives to serve as base in a meeting.

While such a planning tool will not supplant the final judgment of the leadership, it does offer the opportunity to investigate various scenarios in a relatively expandable fashion. If proposed requirements were available prior to the first scheduling meeting, a schedule could be developed and presented to assist the initial gathering. In addition, the flexibility of mathematical programming will allow a trained analyst to add requirements and constraints as they develop in the planning process. The utilization of the proposed model might be a first step to more robust scheduling planning.
Implementation

The implementation of a decision support tool should be carefully conducted. The approval of the process should be conducted jointly by ILA and COMGAP.

ILA does not have any optimization software tool. The selection of an appropriate software is important to achieve the desired results. Once it is decided to use a support tool to assist managerial decisions such as the timetabling problem, some precautions should be taken.

The first step is to be sure that all participants understand the reasons for looking for a new system and the expected outcome of the tool. An understanding of the applications and the benefits of the tool could avoid any confusion with the results achieved. This tool can be used to support not only course scheduling problems, but also to support managers in many others decisions.

There are many optimization tools available in the market; LINDO, GAMs, Gurobi, and Mozek among others. A detailed evaluation of the total cost of the software is important for the proper selection and implementation of the tool. This cost must include upfront and ongoing costs since pricing models vary among software vendors. Some vendors license their software by installed workstation, named user, server and/or concurrent user. Costs associated with training, manuals, upgrades, technical support among others must be included.

Since the decision of which tool to be used must include the ability of the vendor to provide training, manuals, upgrades, technical support among others. Therefore, the acquisition and implementation should be conducted by a small cross functional team. This selection and implementation should follow a negotiated schedule. It is important to be able to conduct tests of the software before purchases are made.
The majority of softwares allow the utilization of free period of test or a free demo with some limitations. This fact should be explored to help the decision for the software selection.

After the selection and acquisition of the tool, training of the personnel involved, documentation of manuals and other material in the library, and a test of the software, the desired capability for the analysts should be determined.

Given the analysts are able to perform the utilization of the software, the inputs for the process must be provided. In this case, for course scheduling, the definition of weeks available for courses, period defined for special events, number of courses with specific number of students and duration are the basic input for the problem. The possibility to increase the number of students taught shows the necessity to evaluate the demand for the courses and their importance for the Brazilian Air Force in order to prioritize which course could be planned for classroom 2 with 42 students.

Some inputs can be different from the data used in this thesis. For example, one specific course can be fixed for some time slot based on the knowledge of the managers, or a new situation could imply in a precedence of one course related with another course.

Since these inputs are defined, the model could generate some alternatives for course scheduling. These alternatives should be presented and discussed for the decision makers for evaluation and some modifications can be necessary until the final approval.

It is important to emphasize this is support tool for decision makers. Therefore, it is fundamental the knowledge and expertise of the managers to establish the appropriate schedule.
Recommendations for Future Research

The approach used in this work could also be used in different organizations at Brazilian Air Force. The Brazilian Air Force has other organizations with the mission of education, training, and improvements.

The Air Force Academy has three different undergraduate courses. Each course has four years of duration and approximately 500 students. The BAF also has one high school, located in Barbacena-MG with 700 students. The Air Force University has graduate courses and also could consider mathematical programming for courses and exams scheduling.

Another possibility for further investigation is the further analysis of some heuristics, particularly with some of these larger programs or considering more constraints. These might include human resources, maintenance availability, reprographic services, budget constraints and so forth. The utilization of heuristics could result in “good” solutions in shorter computational time. Any heuristic has parameters that should be defined or analyzed in order to allow faster convergence and/or shorter computational time. This fact demands deeper research and also depend on the software and/or programming knowledge. However, the literature provides an excellent starting point for deciding upon timetabling heuristics and parameters.

Further research, expanding this work, is possible to assist the educational leadership of the Brazilian Air Force in their role of providing educational opportunities to develop airmen skills and their abilities.
Appendix A

1. FORMULATION IN LINGO FOR MAKESPAN APPROACH

!variables

\[ x(i,j) = \text{definition of course } i \text{ at time } j \]
\[ r(i,j,w) = \text{definition of classroom } w \text{ for course } i \text{ at time } j \]
\[ l(i,j,l) = \text{definition of laboratory } l \text{ for course } i \text{ at time } j; \]

Min = Cmax;

SETS:

<table>
<thead>
<tr>
<th>time</th>
<th>1..40/</th>
</tr>
</thead>
<tbody>
<tr>
<td>course</td>
<td>1..34/</td>
</tr>
<tr>
<td>lab</td>
<td>1..2/</td>
</tr>
<tr>
<td>room</td>
<td>1..4/</td>
</tr>
</tbody>
</table>

LINK((course, time): x;
conec((course, time, lab):l;
relat((course, time, room):r;
ENDSETS

!constraints Cmax:

\[ Cmax \geq x(1,18)*1; \]
\[ Cmax \geq x(2,18)*1; \]
\[ Cmax \geq x(3,18)*1; \]
\[ Cmax \geq x(4,18)*1; \]
\[ Cmax \geq x(5,18)*1; \]
\[ Cmax \geq x(6,18)*1; \]
\[ Cmax \geq x(7,18)*1; \]
\[ Cmax \geq x(8,18)*1; \]
\[ Cmax \geq x(9,18)*1; \]
\[ Cmax \geq x(10,18)*1; \]
\[ Cmax \geq x(11,18)*1; \]
\[ Cmax \geq x(12,18)*1; \]
\[ Cmax \geq x(13,18)*1; \]
\[ Cmax \geq x(14,18)*1; \]
\[ Cmax \geq x(15,18)*1; \]
\[ Cmax \geq x(16,18)*1; \]
\[ Cmax \geq x(17,18)*1; \]
\[ Cmax \geq x(18,18)*1; \]
\[ Cmax \geq x(19,18)*1; \]
\[ Cmax \geq x(20,18)*1; \]
\[ Cmax \geq x(21,18)*1; \]
\[ Cmax \geq x(22,18)*1; \]
\[ \text{Cmax} \geq x(23,18) \times 1; \]
\[ \text{Cmax} \geq x(24,18) \times 1; \]
\[ \text{Cmax} \geq x(25,18) \times 1; \]
\[ \text{Cmax} \geq x(26,18) \times 1; \]
\[ \text{Cmax} \geq x(27,18) \times 1; \]
\[ \text{Cmax} \geq x(28,18) \times 1; \]
\[ \text{Cmax} \geq x(29,18) \times 1; \]
\[ \text{Cmax} \geq x(30,18) \times 1; \]
\[ \text{Cmax} \geq x(31,18) \times 1; \]
\[ \text{Cmax} \geq x(32,18) \times 1; \]
\[ \text{Cmax} \geq x(33,18) \times 1; \]
\[ \text{Cmax} \geq x(34,18) \times 1; \]
\[ \ldots \]
\[ \text{Cmax} \geq x(1,40) \times 40; \]
\[ \text{Cmax} \geq x(2,40) \times 40; \]
\[ \text{Cmax} \geq x(3,40) \times 40; \]
\[ \text{Cmax} \geq x(4,40) \times 40; \]
\[ \text{Cmax} \geq x(5,40) \times 40; \]
\[ \text{Cmax} \geq x(6,40) \times 40; \]
\[ \text{Cmax} \geq x(7,40) \times 40; \]
\[ \text{Cmax} \geq x(8,40) \times 40; \]
\[ \text{Cmax} \geq x(9,40) \times 40; \]
\[ \text{Cmax} \geq x(10,40) \times 40; \]
\[ \text{Cmax} \geq x(11,40) \times 40; \]
\[ \text{Cmax} \geq x(12,40) \times 40; \]
\[ \text{Cmax} \geq x(13,40) \times 40; \]
\[ \text{Cmax} \geq x(14,40) \times 40; \]
\[ \text{Cmax} \geq x(15,40) \times 40; \]
\[ \text{Cmax} \geq x(16,40) \times 40; \]
\[ \text{Cmax} \geq x(17,40) \times 40; \]
\[ \text{Cmax} \geq x(18,40) \times 40; \]
\[ \text{Cmax} \geq x(19,40) \times 40; \]
\[ \text{Cmax} \geq x(20,40) \times 40; \]
\[ \text{Cmax} \geq x(21,40) \times 40; \]
\[ \text{Cmax} \geq x(22,40) \times 40; \]
\[ \text{Cmax} \geq x(23,40) \times 40; \]
\[ \text{Cmax} \geq x(24,40) \times 40; \]
\[ \text{Cmax} \geq x(25,40) \times 40; \]
\[ \text{Cmax} \geq x(26,40) \times 40; \]
\[ \text{Cmax} \geq x(27,40) \times 40; \]
\[ \text{Cmax} \geq x(28,40) \times 40; \]
\[ \text{Cmax} \geq x(29,40) \times 40; \]
\[ \text{Cmax} \geq x(30,40) \times 40; \]
\[ \text{Cmax} \geq x(31,40) \times 40; \]
\[ \text{Cmax} \geq x(32,40) \times 40; \]
\[ \text{Cmax} \geq x(33,40) \times 40; \]
\[ \text{Cmax} \geq x(34,40) \times 40; \]
I considered 40 weeks enough - constraint hotel capacity 80;

\[ X(1,1)*20+x(2,1)*30+x(3,1)*20+x(4,1)*30+x(5,1)*30+x(6,1)*20+x(7,1)*10+x(8,1)*30+x(9,1)*30+x(1,1,1)*15+x(12,1,1)*30+x(14,1,1)*30+x(15,1,1)*30+x(16,1,1)*20+x(18,1,1)*15+x(19,1,1)*30+x(20,1,1)*30+x(21,1,1)*20+x(22,1,1)*30+x(23,1,1)*30+x(24,1,1)*15+x(25,1,1)*12+x(26,1,1)*12+x(27,1,1)*30+x(28,1,1)*40+x(29,1,1)*30+x(30,1)*42+x(31,1)*42+x(32,1)*42+x(33,1)*42+x(34,1)*42<=80; \]

... 

\[ X(1,40,1)*20+x(2,40,1)*30+x(3,40,1)*20+x(4,40,1)*30+x(5,40,1)*30+x(6,40,1)*20+x(7,40,1)*10+x(8,40,1)*30+x(9,40,1)*30+x(10,40,1)*30+x(11,40,1)*15+x(12,40,1)*30+x(13,40,1)*20+x(14,40,1)*30+x(15,40,1)*30+x(16,40,1)*30+x(17,40,1)*20+x(18,40,1)*15+x(19,40,1)*30+x(20,40,1)*30+x(21,40,1)*20+x(22,40,1)*30+x(23,40,1)*30+x(24,40,1)*15+x(25,40,1)*12+x(26,40,1)*12+x(27,40,1)*30+x(28,40,1)*40+x(29,40,1)*30+x(30,40,1)*42+x(31,40,1)*42+x(32,40,1)*42+x(33,40,1)*42+x(34,40,1)*42<=80; \]

... 

duration of courses;

\[ x(1,1)+x(1,2)+x(1,3)+x(1,4)+x(1,5)+x(1,6)+x(1,7)+x(1,8)+x(1,9)+x(1,10)+x(1,11)+x(1,12)+x(1,13)+x(1,14)+x(1,15)+x(1,16)+x(1,17)+x(1,18)+x(1,19)+x(1,20)+x(1,21)+x(1,22)+x(1,23)+x(1,24)+x(1,25)+x(1,26)+x(1,27)+x(1,28)+x(1,29)+x(1,30)+x(1,31)+x(1,32)+x(1,33)+x(1,34)+x(1,35)+x(1,40)+x(1,37)+x(1,38)+x(1,39)+x(1,40)==1; \]

... 

laboratory 1 capacity;

\[ l(1,1,1)*20+l(2,1,1)*30+l(3,1,1)*20+l(4,1,1)*30+l(5,1,1)*30+l(6,1,1)*20+l(7,1,1)*10+l(8,1,1)*30+l(9,1,1)*30+l(10,1,1)*30+l(11,1,1)*15+l(12,1,1)*30+l(13,1,1)*20+l(14,1,1)*30+l(15,1,1)*30+l(16,1,1)*30+l(17,1,1)*20+l(18,1,1)*15+l(19,1,1)*30+l(20,1,1)*30+l(21,1,1)*20+l(22,1,1)*30+l(23,1,1)*30+l(24,1,1)*15+l(25,1,1)*12+l(26,1,1)*12+l(27,1,1)*30+l(28,1,1)*40+l(29,1,1)*30+l(30,1,1)*42+l(31,1,1)*42+l(32,1,1)*42+l(33,1,1)*42+l(34,1,1)*42<=18; \]

... 

laboratory 2 capacity

\[ l(1,40,1)*20+l(2,40,1)*30+l(3,40,1)*20+l(4,40,1)*30+l(5,40,1)*30+l(6,40,1)*20+l(7,40,1)*10+l(8,40,1)*30+l(9,40,1)*30+l(10,40,1)*30+l(11,40,1)*15+l(12,40,1)*30+l(13,40,1)*20+l(14,40,1)*30+l(15,40,1)*30+l(16,40,1)*30+l(17,40,1)*20+l(18,40,1)*15+l(19,40,1)*30+l(20,40,1)*30+l(21,40,1)*20+l(22,40,1)*30+l(23,40,1)*30+l(24,40,1)*15+l(25,40,1)*12+l(26,40,1)*12+l(27,40,1)*30+l(28,40,1)*40+l(29,40,1)*30+l(30,40,1)*42+l(31,40,1)*42+l(32,40,1)*42+l(33,40,1)*42+l(34,40,1)*42<=18; \]
\[l(1,1,2) * 20 + l(2,1,2) * 30 + l(3,1,2) * 20 + l(4,1,2) * 30 + l(5,1,2) * 30 + l(6,1,2) * 20 + l(7,1,2) * 10 + l(8,1,2) * 30 + l(9,1,2) * 30 + l(10,1,2) * 30 + l(11,1,2) * 15 + l(12,1,2) * 30 + l(13,1,2) * 20 + l(14,1,2) * 30 + l(15,1,2) * 30 + l(16,1,2) * 30 + l(17,1,2) * 20 + l(18,1,2) * 15 + l(19,1,2) * 30 + l(20,1,2) * 30 + l(21,1,2) * 20 + l(22,1,2) * 30 + l(23,1,2) * 30 + l(24,1,2) * 15 + l(25,1,2) * 12 + l(26,1,2) * 12 + l(27,1,2) * 30 + l(28,1,2) * 40 + l(29,1,2) * 30 + l(30,1,2) * 42 + l(31,1,2) * 42 + l(32,1,2) * 42 + l(33,1,2) * 42 + l(34,1,2) * 42 <= 33;\]

...
lclassroom 3 capacity;

\[ r(1,1,3) \times 20 + r(2,1,3) \times 20 + r(3,1,3) \times 20 + r(4,1,3) \times 20 + r(5,1,3) \times 20 + r(6,1,3) \times 10 + r(7,1,3) \times 30 + r(8,1,3) \times 30 + r(9,1,3) \times 30 + r(10,1,3) \times 30 + r(11,1,3) \times 15 + r(12,1,3) \times 30 + r(13,1,3) \times 20 + r(14,1,3) \times 30 + r(15,1,3) \times 30 + r(16,1,3) \times 30 + r(17,1,3) \times 20 + r(18,1,3) \times 15 + r(19,1,3) \times 30 + r(20,1,3) \times 30 + r(21,1,3) \times 20 + r(22,1,3) \times 30 + r(23,1,3) \times 30 + r(24,1,3) \times 15 + r(25,1,3) \times 12 + r(26,1,3) \times 12 + r(27,1,3) \times 30 + r(28,1,3) \times 40 + r(29,1,3) \times 30 + r(30,1,3) \times 42 + r(31,1,3) \times 42 + r(32,1,3) \times 42 + r(33,1,3) \times 42 + r(34,1,3) \times 42 \leq 33; \]

...

lclassroom 4 capacity;

\[ r(1,1,4) \times 20 + r(2,1,4) \times 30 + r(3,1,4) \times 30 + r(4,1,4) \times 30 + r(5,1,4) \times 30 + r(6,1,4) \times 30 + r(7,1,4) \times 20 + r(8,1,4) \times 30 + r(9,1,4) \times 30 + r(10,1,4) \times 30 + r(11,1,4) \times 15 + r(12,1,4) \times 30 + r(13,1,4) \times 20 + r(14,1,4) \times 30 + r(15,1,4) \times 30 + r(16,1,4) \times 30 + r(17,1,4) \times 20 + r(18,1,4) \times 15 + r(19,1,4) \times 30 + r(20,1,4) \times 30 + r(21,1,4) \times 20 + r(22,1,4) \times 30 + r(23,1,4) \times 30 + r(24,1,4) \times 15 + r(25,1,4) \times 12 + r(26,1,4) \times 12 + r(27,1,4) \times 30 + r(28,1,4) \times 40 + r(29,1,4) \times 30 + r(30,1,4) \times 42 + r(31,1,4) \times 42 + r(32,1,4) \times 42 + r(33,1,4) \times 42 + r(34,1,4) \times 42 \leq 25; \]

...

l!imit one course per classroom

@for(time(j): @for (room(w): r(1,j,w)+r(2,j,w)+r(3,j,w)+r(4,j,w)+r(5,j,w)+r(6,j,w)+r(7,j,w)+...r(34,j,w)));
!force the use of lab;

@ for (time(j) : x(4,j) = l(4,j,1) + l(4,j,2));
@ for (time(j) : x(5,j) = l(5,j,1) + l(5,j,2));
@ for (time(j) : x(8,j) = l(8,j,1) + l(8,j,2));
@ for (time(j) : x(11,j) = l(11,j,1) + l(11,j,2));
@ for (time(j) : x(13,j) = l(13,j,1) + l(13,j,2));
@ for (time(j) : x(21,j) = l(21,j,1) + l(21,j,2));
@ for (time(j) : x(24,j) = l(24,j,1) + l(24,j,2));

!force the use of classroom;

@ for (course(i) : @ for (time(j) : x(i,j) = r(i,j,1) + r(i,j,2) + r(i,j,3) + r(i,j,4)));

!constraints of no split courses;

2 * x(2,1) - 2 * x(2,2) + x(2,3) + x(2,4) + x(2,5) + x(2,6) + x(2,7) + x(2,8) + x(2,9) + x(2,10) + x(2,11) + x(2,12) + x(2,13) + x(2,14) + x(2,15) + x(2,16) + x(2,17) + x(2,18) + x(2,19) + x(2,20) + x(2,21) + x(2,22) + x(2,23) + x(2,24) + x(2,25) + x(2,26) + x(2,27) + x(2,28) + x(2,29) + x(2,30) + x(2,31) + x(2,32) + x(2,33) + x(2,34) + x(2,35) + x(2,36) + x(2,37) + x(2,38) + x(2,39) + x(2,40) <= 2;
...
2 * x(2,38) - 2 * x(2,39) + x(2,40) <= 2;
...

!define binary variables
@ for (course(i) : @ for (time(j) : @ bin(x(i,j)));
@ for (course(i) : @ for (time(j) : @ for (lab(l) : @ bin(l(i,j,l)))));
@ for (course(i) : @ for (time(j) : @ for (room(w) : @ bin(r(i,j,w)))));
## Appendix B. Gantt Chart

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*Note: The chart shows the progress and timelines of various tasks across different months.*
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Note: The Gantt chart visually represents the start and end dates for each task, showing the timeline and dependencies.
Appendix D - Quad Chart

**An application of timetabling in BAF**

Cpt Julio Lopes – julio_inb@hotmail.com  
Advisor – Dr Richard F. Deckro  
02/04/2012

### Actual process of course scheduling

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### OBJECTIVES - METHODS

- Develop an optimization process based on objective function
- Analysis and selection of technique available for timetabling problem to be used.
- Two approaches are possible (defined by the Commander):
  - Minimizing makespan - allowing free time for others events
  - Minimizing the utilization of resources
- Inputs are the courses, each course with predefined number of students, duration, and utilization of computers.
  - Constraints are hotel capacity, classroom capacity, laboratory capacity and the time window.

### TECHNICAL APPROACH

- Formulation of the problem
- Analysis of Heuristics used in literature
- Selection of method
- Comparison of different objectives functions and scenarios
- Comparison with actual process

### Schedule

- Develop an useful tool of optimization within 4 months

### Deliverables

- Application of commercial software that allows easily understanding about the tool.
- Technical report describing the method
- Technical report with comparison of different scenarios

### Contact Information

- ILA – Instituto de Logística da Força Aérea Brasileira – Av Monteiro Lobato 5335, Guarulhos – SP, Brazil
- Tel (55) 11-64652109

---

Company: Brazilian Air Force  
Contact: Cpt Julio C O Lopes  
Email: julio_inb@hotmail.com  
Phone: 937 2562702
Bibliography


Duong T.A.,and Lam K.H. (2004).”Combining constraint programming and simulated annealing on university exam timetabling”. In: Proceedings of the 2nd International Conference in


Captain Julio Lopes enlisted in the Brazilian Air Force in 2002 as an engineer. He earned his bachelor degree in Metallurgical Engineer in 1992.

His previous experiences include Gerdau Steel plant, Brazil and the Brazilian Nuclear Industries, Brazil. In Gerdau Group he was the Chief of Electric Arc Furnace, and Process Engineer between May 1992 and May 2002. After Gerdau Group he started to work as Radioprotection Engineer between June 2002 and May 2003.

Currently, his function in Brazilian Air Force is instructor in logistics classes and consultant in logistics acquisitions.
An Application of Course Scheduling in the Brazilian Air Force

Lopes, Julio C. O., Captain, BAF

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Graduate School of Engineering and Management (AFIT/EN)
2950 Hobson Street, Building 642
WPAFB OH 45433-7765

ILA
Av Monteiro Lobato, 6365
Guarulhos – SP – Brazil ZIP CODE 07184-000 tel: (55)11-2465-2009
Email: sec@ila.intraer

DISTRIBUTION STATEMENT A:
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

IL (Institute of logistic in Brazilian Air Force) is responsible for executing 34 courses with an average of two weeks of duration in 2011. Besides these courses, ILA is also responsible for some seminars, meetings, lectures and other events that are not planned during the annual course schedule developing. Therefore these events are considered unscheduled “special events”. The Brazilian Air Force currently utilize a manual process for course scheduling which is time consuming and does not reflect any optimization, because there is no specific objective function. The result of the manual process reflects only one possible solution that “seems reasonable”. The final schedule often results in conflicts with the “special events” and sometimes results in loss of quality due the availability of the resources to cover the course and special events that occurred at same time. This work developed a method that could be used to support the course scheduling process. This project utilizes two different objective functions, and therefore two different mathematical formulations as method to solve the timetabling problem and simulated different scenarios.

DISTRIBUTION STATEMENT A:
APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

Timetabling Problem, Course Scheduling, Scheduling, LP Optimization

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</tr>
</thead>
<tbody>
<tr>
<td>(937) 785-3636 x4325</td>
</tr>
</tbody>
</table>

84