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Practical measurement of complexity in dynamic systems

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Abstract

A difficulty in complexity theory is lack of a clear definition for complexity, particularly one that is measurable. Those approaches that provide measurable definitions for the absolute complexity of a system often impose the requirement of perfect or near-perfect knowledge of system structure.

In practice, it is intractable or impossible to measure the complexity of most dynamic systems. However, by measuring behavioral complexity in context with environmental scenarios, it is possible to set bounds on a system's absolute (maximum) complexity and estimate its total complexity. As this paper shows, behavioral complexity can be determined by observing a system's changes in kinetic energy.

This research establishes a methodology for measuring complexity in dynamic systems without the requirement of system structure knowledge. This measurement can be used to compare systems, understand system risks, determine failure dynamics, and guide system architecture.

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1. Introduction

A difficulty in complexity theory is a clear definition for complexity, particularly one that is measurable. In many cases throughout the literature, it seems the authors are speaking of different or even mutually exclusive phenomena. The most apt summary of the issue is the tongue-in-cheek assertion by complexity researcher Seth Lloyd: "I can't define it for you, but I know it when I see it." [5]

1.1. Definitions of Complexity

A root cause in the lack of unified complexity definitions is that there are in fact several types of complexity. The first formal treatment of complexity focused on algorithmic complexity, which reflects the computation requirements for a mathematical process. [5] Senge and Sterman include also dynamic complexity, which is primarily characterized by difficult to discern cause-effect relations. [9] [10]

To further muddy the waters, there is the concept of complex adaptive systems. These are defined by properties such as consisting of agent networks, having multiple levels of organization, development of internal models, exhibiting ‘phase transitions’, and exploitation of niches in the fitness landscape. [13]

One of the most workable definitions is thermodynamic depth, which actually seems to unify the two complexity camps. Thermodynamic depth as complexity asserts that complexity is a “measure of how hard it is to put something together”. [8] [6] There are several variations on this approach, with the commonality that complexity disappears for both ordered and purely random systems. [7] [5]

Bar-Yam defines the complexity of a physical system as the length of the shortest string, s , that can represent its properties. This can be the result of measurements and observations over time. [1]

However, there are some inherent difficulties in using these definitions. For one, it is very difficult to approach a macro-level system, particularly one with hidden structure, and determine the information content. While information and thermodynamic entropy-based measures form a theoretical basis for complexity measurement, there remains a need for a more accessible route.

There are hints that energy plays a role in defining complexity. As a general rule, systems commonly recognized as complex process more energy than less complex ones. For instance, civilizations move more energy than a single town. However, mere energy flow is not a measure of complexity, since a single arbitrary large-bore “pipe” (a relatively simple system) could be constructed that matches the energy flow through any complex system.

A metric that solves the large-bore problem is proposed by Chaisson. By measuring the energy rate density in (1), where E is energy flow through a system, τ is the time epoch, and m is system mass, Chaisson obtains results that correlate well with other notions of complexity. [3]

$$\Phi_m = \frac{E}{\tau * m} \quad (1)$$

However, the energy rate density metric has some drawbacks. By normalizing with respect to mass, this metric produces incorrect results for complexity when comparing some systems. For example, suppose an electronic brain is built to mimic the operation of a human brain. The human brain may process energy at the same rate as a theoretical electronic brain, but due to differences in basic materials (i.e. the weight of neurons vs. semi-conductors), the two systems, which most theorists would recognize as identically complex, could have vastly different Φ_m values. Thus, by normalizing with respect to mass instead of function, the Φ_m metric produces incorrect results for the relative complexity of systems.

A practical difficulty in using the Φ_m metric is determining the appropriate mass and energy to use. In measuring the Φ_m of a civilization, Chaisson uses the mass of humanity and the total energy processed by

the civilization. However, the total energy of a civilization does not flow through only its humans, but also its machinery, beasts of burden, vehicles, etc., the mass of which is a difficult quantity to measure.

All metrics discussed thus far contribute ideas toward a measure that 1) correlates with notions of complexity at all scales; 2) is measurable for any dynamical system; and 3) may be practically employed .

2. Energetic Complexity

Dynamic systems, by definition, yield changes in their kinetic energy over time. Using this fact, we can define an **Activated Energy Transfer** as *an instance where, for a discrete element of a system, its kinetic energy in the direction of a generalized coordinate transitions from a non-zero value to a zero value, or from a zero to a non-zero value.*

As an example, given a particle enclosed in a container, every collision with the sides of the container is considered an activated energy transfer. The definition of activated energy transfers allows the following definition of **Energetic Complexity**: *the number of activated energy transfers for a given system within a particular epoch and above a particular functional level. It is denoted by ζ .*

This is, in essence, complexity by demonstration. A static system may have *potential* for complex behavior, but its complexity cannot be measured until it is active. Depending on the scenario, a system may respond with differing amounts of complexity. A plane flying through clear skies may stay straight and level, but be required to make complex maneuvers when flying through a storm.

A key to the utility of this definition is the ease with which it may be applied. For instance, if we choose an epoch size larger than the time required for basic cell functions, a human's internal structure is dependent on exercising a finite number of energy transfers. That is, it must ensure a constant flow of energy within the body, and perform work using that energy, in order to remain alive. If energetic complexity is reduced, it means that part of the body is processing less energy – the tissue is either reduced in its function (e.g. a sleeping brain), is dead, or is removed.

Important to this definition is 'functional level'. Specifying the functional level (the threshold above which a system function can be realized) relates the measurement to a particular level in the hierarchy of system functions. For instance, a human's complexity may be measured at the cellular, molecular, or atomic levels, and given the same epoch size, the ζ value increases for each progressively smaller scale. The functional level of interest might be different between generalized coordinates and is analogous to energy levels.

It should be noted that other researchers have alluded to both the multi-scale and functional natures of complexity, but there is no apparent prior synthesis of these ideas. Bar-Yam has written extensively on multi-scale complexity and what he terms the "complexity profile", applying it in particular to warfare. As he writes, "complexity at a particular scale includes all possible force actions at or above this scale." [2] In his book on complexity, systems engineer Suh argues that complexity can only be defined in the functional domain vs. the physical domain, and relates complexity to the ability to satisfy a functional requirement. [11]

3. Consistency with the Field and Further Extension

While the concept of energetic complexity represents a new approach in looking at complexity, in order for it to be shown valid, there must be coherence and consistency with the accepted definitions of complexity in literature. Note that energetic complexity measures the dynamic complexity of a system.

As explored earlier, complexity is often defined as a deep property of a system that is in many ways intangible. However, as pointed out by Corning, such definitions “exclude the extremes associated with highly ordered or strictly random phenomena, even though there can be more or less complex patterns or order and more or less complex patterns of disorder.” [5] In short, these definitions do not allow for graduations in complexity. In rejecting “simpler” systems as not being complex, these definitions eliminate the very basis for which to determine whether a system is complex or not!

“Deep-property” definitions of complexity reflect a combination of energetic complexity and system structure. For example, a linear dynamic system (e.g. where cause and effect are apparent) can have the same ζ value as a network (Fig. 1). However, complexity theorists are more likely to describe a network as complex than a linear system. The main difference between these systems is the path in the linear system is predictable, whereas the network path is dependent on additional factors whose properties must be known to make predictions. It is the difference between the *absolute complexity* or the *total complexity* of the two systems.



Fig. 1. Two systems with identical energetic complexity. Which one is more complex? (a) Linear; (b) Network

Absolute complexity is defined here as the maximum possible complexity for any given scenario over the range of all possible scenarios, S_m , as in (2). Energetic complexity may be used to *measure* absolute complexity. If it was possible to observe a system’s response to all possible scenarios and measure the energetic complexity, this information could be used to construct a description of the absolute complexity of the system. As the certainty of a system’s structure and function decrease the certainty in (and ability to measure) absolute complexity is also reduced.

$$\zeta_A = \max(\zeta | S_m), m \in [1, \infty) \quad (2)$$

Energetic complexity, as defined above, is a single observation of system responding to a particular scenario. **Total complexity** may thus be written as (3) where a particular scenario S_m is defined as a unique set of system conditions (exogenous and internal), and Ψ is defined as the set of all possible scenarios driving a unique system response. F is the cumulative density function (CDF) of Ψ .

$$\zeta_T^{\text{sys}} \equiv E[\zeta(\text{sys}, \Psi)] = \int_{\Psi} \zeta(\text{sys}, S_m) dF(S_m) \quad (3)$$

For a set of finite, discrete, and independent scenarios, total complexity may be written as in (4) where $p(S_m)$ is the probability mass function over Ψ .

$$\zeta_T^{\text{sys}} = \sum_{\Psi} \zeta(\text{sys}, S_m) p(S_m) \quad (4)$$

For a given system, one scenario may cause it to respond with a particular behavior, whereas another may produce a completely different behavior. As defined here, scenarios which produce identical behavior in the system(s) under study are indistinguishable from one another. This enables a framework for system study where the infinite possibilities of system scenarios may be parsed to a workable, finite set. For most systems of interest, this finite set may still be too large for practical analysis, but the solution to this problem will be addressed later.

Total complexity is therefore the collection of every possible pairing of a scenario with the complexity response for a given system. This is, in fact, isomorphic with the statistical complexity calculated by entropic methods. Yet, we reached the definition using two types of data: the energetic complexity values and the set of all possible scenarios.

With this new insight, it is clear that energetic complexity is not inconsistent with complexity as defined in the literature. Systems which consist of networks, have multiple levels of organization, anticipate the future, and have many ‘niches’ (per Waldrop [11]) are more absolutely complex than systems that don’t; systems with non-linear responses, disproportionate effects, and non-obvious cause-effect linkages (per Senge [9]) are more absolutely complex than systems without.

3.1. Complexity, Chaos, and Randomness

As discussed, it is assumed complexity exists on a spectrum of behavior, that progresses from static (or linear) to complex to chaotic. A chaotic system is one whose state at time $t > t_0$ cannot be calculated beyond an arbitrary precision α . That is, an arbitrarily small error in initial conditions (at time t_0) can yield large errors in the calculation of the final state at time t . An energetically complex system may exhibit chaotic dynamics and therefore also be a chaotic system. The two concepts are not mutually exclusive.

Energetic complexity is a physical phenomenon, whereas chaos is a mathematical one. In this view, ζ serves as the backbone of the complexity spectrum of behavior, and can be used as a tool to determine the class of behavior. What is important is the relationship between ζ and highly chaotic systems.

One reason that entropy-based complexity metrics are so appealing is they approach zero for random processes as well as static ones. Most working in the complexity field accept this as a requirement for a good metric, although what happens between those limiting cases has little bearing. [7]

At first glance, it appears the ζ measure increases without bound as energy increases. However, recall that ζ is based on physical processes and is subject to physical limits; there is a limit to the amount of energy that physical systems may contain (e.g. particles in a box, stars, life forms). At the high entropy (disordered) limit, as energy of the particles increases (e.g. isochoric temperature increase), the particles accumulate so much kinetic energy that their collisions degrade their physical structure (e.g. molecular disruption). At this point, complexity falls. For a macroscopic “particle”, as it degrades into smaller and

less ordered components with the energy increase, the ability of this system to perform a function degrades. This degradation results in the activated energy transfers falling below the functional level. Completing the relationship, as energy is removed from a system, the kinetic energy of particles is reduced, and gradually drops below the functional threshold.

It should be noted that the exact relationship between complexity and entropy is system dependent and dependent on the entropy-changing process itself (which, per Li [7], is acceptable). For an isothermal volume-changing process at the high-entropy limit, as volume is increased, ζ decreases and approaches zero. Conversely, as volume is decreased, ζ increases until it reaches a point where the particles are contained so tightly that the kinetic energy of the particles falls below the functional (energy) threshold. Similar arguments can be made for other basic entropy-changing processes.

It is important to distinguish truly random processes from apparently random ones. A vehicle driving erratically might appear to exhibit random behavior to the outside observer; but what appears as random might in fact be very causal. What if the vehicle is under fire? Erratic motion (being *unpredictable* to the *enemy*), is in that case a highly effective strategy for survival, and is in fact a very complex behavior.

3.2. Agents and Energetic Complexity

Agent-based models are of great importance to complexity theory, and measures of complexity should reflect increasing complexity with increasing activity of agents.

“Boids” (Bird-androids) are a type of agent based model using rule sets based on the motions of animal flocks, herds, and swarms. [4] Compare a 3-boid flock against 3 equivalent non-interacting aircraft. In going from point A to point B, which takes place over a particular epoch in both cases, non-interacting aircraft will make flight path adjustments due to atmospheric disturbances. A boid-aircraft will make these adjustments as well, but also make adjustments due to each of the boid guidance rules. Therefore the boid flock has higher energetic complexity than the set of non-interacting aircraft.

Recalling the fact that non-cooperative flight is a subset of cooperative flight (equivalent to setting certain rule weightings to zero), if no boid rules are invoked, both systems have equivalent complexity.

Note also that human-piloted aircraft in formation interact with each other in similar fashion to the boid rules. Adjustments are made based on the relative positions and velocities of the other aircraft. Therefore, such a set of interacting aircraft can have the same (or greater) ζ than the boid flock.

3.3. Combat System Example: Tank

Consider a tank system designed to strike a fixed target with four degrees of freedom (DoF): x , y , ϕ , and θ , where ϕ is gun elevation, θ is turret azimuth, and x and y are Cartesian position coordinates. A strike may be achieved by adjusting the state space $\mathbf{x}' = [x \ y \ \phi \ \theta]$ through a control vector \mathbf{u} .

First, consider the case where all degrees of freedom are enabled. $\mathbf{u}' = \begin{bmatrix} a & b & c & d \\ t & t & t & t \end{bmatrix}$, where a, b, c, and d are constants (i.e. constant rate of motion from start to stop). If each DoF is exercised by the control vector, $\zeta(f(x,u)) = 8$. This sets a lower bound on ζ_A .

In the next case, the same tank system is restricted to move in only 3 DoF, having lost the ability to rotate the turret due to damage or energy rationing. The vector $\mathbf{u}' = \begin{bmatrix} a & b & c \\ t & t & t \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix}$ reflects the loss of control. If each available DoF is exercised by \mathbf{u} , $\zeta(f(x,u))=6$, a lower bound for ζ_A of this new system. This result is intuitive, since the loss of a degree of freedom should reflect a loss in the complexity of the system. The same trend is evident if we restrict the motion still further, such that $\mathbf{u}' = \begin{bmatrix} 0 & 0 & c \\ t & t & 0 \end{bmatrix}$. In this case, $\zeta = 2$.

Now consider different control policies, where the components of \mathbf{u} do not drive the system at a constant rate. Suppose $c(t)$ is chosen to drive the tank gun elevation at a non-linear rate according to Figure 2(a). Here, $\zeta = 2$ again. Has the metric failed to capture system complexity?

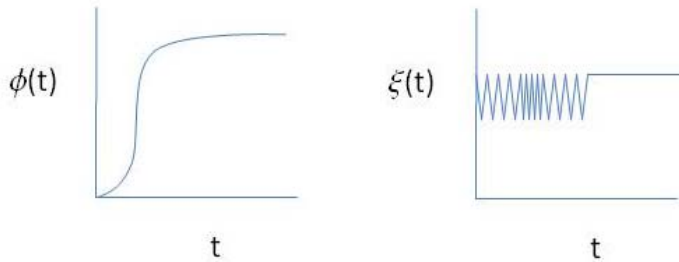


Fig. 2. (a) Non-linear function of $\phi(t)$; (b) Motor brush position $\xi(t)$

This example illustrates why ζ is evaluated at a particular functional level; it prevents double-counting complexity. At the tank system level, this change in control function has no effect on ζ . To discover the effect, it is necessary to explore one functional level deeper: the gun elevation motor subsystem.

Suppose this is a standard DC motor. Every time the brushes in the motor make or break contact (switching polarity and keeping the armature moving), kinetic energy goes through zero, generating ζ counts. Figure 2(b) illustrates this motion. The “missing” complexity exists at this lower subsystem. Extending the tank example still further, suppose that the full-authority system is able to satisfy the range-to-target equation by adjusting the value of only one DoF. However, the controller opts to make movements in all 4 DoF’s to reach the target. This results in an “excess” complexity, behavior which is not necessary to reach the goal.

This example demonstrates why it is important to keep in mind the goals and limitations of a complexity measure. Complexity is a reflection of a system’s potential, not of efficient behaviors. The choice of control vector, given a system’s complexity, is what controls efficiency. Irrational actors can implement control policies that produce highly complex behaviors, yet yield no benefit.

The effectiveness of a complex *adaptive* system can be measured by the choice of control vector against some objective function. That effectiveness, however, is not a measure of its complexity but what it does with that complexity.

In this tank example, total complexity may be calculated by determining a probability mass function for the scenario set and summing the product of that function with the complexity response for each scenario (as in (4)). Practically, the scenario set will consist of the most likely operating conditions. The constructed measure of total complexity may then be used to compare systems and determine suitability. Similarly, absolute complexity (as in (3)), provides additional insight into the complexity potential of the system, but total complexity gives a more complete picture for guiding system design.

4. Conclusions and Future Work

This research demonstrates the validity and use of a new measure of complexity in dynamic systems that may be more practically employed than prior measures. Energetic complexity can be used to compare systems in identical scenario sets, measure historical complexity, or set bounds on the absolute complexity.

As this is only an initial exploratory work, further validation is required. The behavior of this metric under scenarios of system collapse [12] or general evolution could yield insight into developing robust system architectures that are resistant to failure in varied and uncertain fitness landscapes.

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