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Abstract: The index of refraction structure constant, C_n^2 indicates how strongly the index of refraction varies in a region of the atmosphere. These variations usually arise through turbulent motions, creating an inhomogeneous distribution of species, density, temperature and pressure. Because the index of refraction also depends on wavelength, the measured value of C_n^2 will depend on wavelength. This C_n^2 difference generally becomes more pronounced as the difference in wavelength increases. This paper describes a technique for converting between measurements of C_n^2 at different wavelengths, and gives an example for converting from centimeter to visible and near IR wavelengths.

OCIS codes: (010.1330) Atmospheric turbulence; (030.7060) Turbulence; (120.0280) Remote sensing and sensors; (120.3930) Metrological instrumentation; (280.7060) Turbulence.

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1. Background

The wavelength correction for C_n^2 presented herein is based on work by Fiorino and others [1–3]. C_n^2 is used to quantify the variation of the index of refraction, n, within a given region. It shows up as a coefficient for the structure function,

$$D_n(r) = C_n^2 r^{2/3}, \tag{1}$$

which describes the probable variation in n for two points separated by a given distance, r. [4] In the atmosphere, the index at a given point will depend primarily on temperature (T), pressure (P), the chemical species which are present (e.g. water vapour, e_v) and the wavelength of the light. If it were possible to know the structure functions of T,P, and e_v, denoted as C_T^2, C_P^2, and $C_{e_v}^2$ respectively, then it would be straightforward to define C_n^2 at a given wavelength. Unfortunately, it is problematic to measure or predict C_T^2, C_P^2, and $C_{e_v}^2$ on fine scales of time and distance. However, it is possible to estimate C_n^2 from RF or optical measurements.

If one were to measure C_n^2 in a region using instruments operating at dissimilar wavelengths, then it would be found that despite the fact that the underlying T, P, and e_v structures are the same, the resulting C_n^2 values may differ by orders of magnitude. The method presented provides for a way to convert C_n^2 values taken at one wavelength to another. As an example, measurements from C_n^2 measured using Next-Generation Weather Radar (NEXRAD) weather radar are compared to C_n^2 measured by an IR (880 nm) scintillometer. This technique is developed under the assumption that the scintillometer and radar measurements are of the same air volume and that differences in C_n^2 are due to the response of n to T,P, and e_v at each wavelength. For the bands discussed below, water is the only species whose effect on n is significant enough that it will be accounted for individually. All other species are lumped into the 'dry air' response of the atmosphere. This method has the advantage that it uses the average T, P, and e_v along with a value for the refractivity, $N(T,P,e_v)$ at each wavelength. Thus removing the need to know or measure the fine-scale structure of T, P, or e_v.

The conversion between wavelengths is based on a relationship between turbulence parameters and the vertical index variation presented by Tatarskii [5] (see section 15)

$$C_n^2 = a^2 \alpha' L_0^{3/4} \left(\frac{dn}{dz}\right)^2. \tag{2}$$

For constants we follow Tatarskii's conventions. This expression derives from the assumption that turbulence advects air parcels to new locations. These parcels carry with them the temperature and pressure from their original location, and they adiabatically expand or contract to match the pressure of their new location. This expression ties the resulting statistics of the index distribution, C_n^2, to turbulence parameters, $a^2\alpha' L_0^{3/4}$, and the local vertical variation in index, dn/dz. None of the terms except for dn/dz are wavelength dependent. For this reason, if one measures the same atmospheric sample then C_n^2 measurements taken at two different wavelengths, λ_1 and λ_2, should scale according to dn/dz for each wavelength via

$$\frac{C_{n,\lambda 1}^2}{C_{n,\lambda 2}^2} = \left(\frac{dn_{\lambda 1}/dz}{dn_{\lambda 2}/dz}\right)^2. \tag{3}$$

This 'wavelength correction' was first presented by Fiorino and others [1] and is summarized here. Their initial development was based on K-theory, which parametrizes turbulent mixing based on the vertical gradient of potential temperature. While useful in some regimes, K theory is constrained by assumptions about the generators of turbulent motion. The dn_λ was found by taking an appropriate empirical equation for index of refraction as a function of the conservative additives potential temperature, θ and specific humidity, q. This is repeated for each wavelength, and the partial derivatives $\partial n/\partial \theta$ and $\partial n/\partial q$ are then multiplied by the appropriate finite gradients $\Delta\theta/\Delta z$, $\Delta q/\Delta z$, which are found from radiosonde, ground observation, or forecast data. This gives an approximate dn_λ/dz:

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$$\frac{dn_\lambda}{dz} \approx \frac{\partial}{\partial \theta} n(\theta, p, q : \lambda) \frac{\Delta\theta}{\Delta z} + \frac{\partial}{\partial q} n(\theta, p, q : \lambda) \frac{\Delta q}{\Delta z} \tag{4}$$

This method is difficult to apply in practice because it requires high resolution measurements of $\Delta\theta/\Delta z$ and $\Delta q/\Delta z$. Furthermore, it is limited to regimes where K-theory is applicable. That is, when the structure functions of the atmosphere can be estimated from measurements of $\Delta\theta/\Delta z$ and $\Delta q/\Delta z$. Large scale variations in $\Delta\theta/\Delta z$ and $\Delta q/\Delta z$ often go to zero in well-mixed layers of the atmosphere. While K-theory necessarily predicts that turbulence production (and C_n^2 must also go to zero), measurements of C_n^2 do not always support this conclusion.

This work improves upon Fiorino's method by eliminating the limitations imposed by making assumptions about the generators of variations in T, P, and e_v. Instead, it is assumed that C_n^2 measurements which are taken at approximately the same time and location respond to the same structures of T, P, and e_v. Furthermore it is assumed that locally advected air undergoes adiabatic expansion or compression, and that the e_v is dependent on the average e_v modified by temperature and pressure fluctuations only. These assumptions about the relationships among T, P, and e_v are used to create a C_n^2 scaling function that only depends on the averages of T, P, and e_v which are much easier to obtain.

2. The wavelength correction technique

As mentioned, we would like to compute the derivative dn/dz based on the ambient temperature, pressure, and vapor pressure (T, P, e_v). It is important to note that the systematic variation with height is not of interest, only the local variation due to turbulent mixing. For this development it becomes convenient to define Refractivity, $N = (n-1) \times 10^6$, so that $dn/dz = dN/dz \times 10^6$.

Very accurate estimates of wavelength dependent N are available using absorption coefficients from propagation modeling software such as LEEDR or MODTRAN [1,6]. The methods below are derived based on the assumption that wavelength dependent N has been computed using these codes (or some other method) and values of the local T, P, and e_v. Using absorption codes allows for extension of this method to portions of the spectrum where absorption lines are strong, and C_n^2 may be higher due to an increased sensitivity of N to a particular species concentration. In this case, the radar has increased sensitivity to water vapor.

For use in Eq. (3), we would like to cast dN/dz in the form:

$$\frac{dN}{dz} \approx f(T, P, e_v : N) \frac{dT}{dz}. \tag{5}$$

This form has the advantage that, when used, the dT/dz cancels leaving a scaling factor based only on the mean temperature, pressure and humidity:

$$\frac{C^2_{n,\lambda 1}}{C^2_{n,\lambda 2}} = \left(\frac{f_{\lambda 1}(T, P, e_v)}{f_{\lambda 2}(T, P, e_v)} \right)^2. \tag{6}$$

The variations in temperature are tied to changes in pressure and humidity via the adiabatic relationship. The adiabatic assumption is reasonable due to the source of the random component of index structure; T, P, and e_v variations due to the circulatory nature of turbulent eddies. Water content (molar or mass mixing ratio) is relatively homogeneous on the length scales of interest, $10^2 \leftrightarrow 10^3$ m, and pressure diffuses quickly compared to temperature. Note that conservative additives are not needed here because the interdependence of temperature and pressure is captured in the development of $f(T, P, e_v : N)$.

To find $f(T, P, e_v : N)$, first take the total derivative of variation of N, then arrange it into the desired form:

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$$\frac{dN}{dz} = \frac{\partial N}{\partial P}\frac{dP}{dz} + \frac{\partial N}{\partial T}\frac{dT}{dz} + \frac{\partial N}{\partial h}\frac{de_v}{dz}, \tag{7}$$

$$\frac{dN}{dz} = \frac{\partial N}{\partial P}\frac{dP}{dT}\frac{dT}{dz} + \frac{\partial N}{\partial T}\frac{dT}{dz} + \frac{\partial N}{\partial e_v}\frac{\partial e_v}{\partial T}\frac{dT}{dz}, \tag{8}$$

$$\frac{dN}{dz} = \left(\frac{\partial N}{\partial P}\frac{dP}{dT} + \frac{\partial N}{\partial T} + \frac{\partial N}{\partial e_v}\frac{\partial e_v}{\partial T}\right)\frac{dT}{dz} \equiv f(T,P,e_v)\frac{dT}{dz}. \tag{9}$$

Before finding dN/dz for a particular wavelength we make two assumptions in order to use Eq. (9). The first, is that advected air parcels will adiabatically equalize with the local pressure fields via the ideal gas law, $PV = nRT$ and the isentropic expansion equation $PV^{\gamma} =$ constant. Using this assumption we relate variation in pressure to variation in temperature,

$$\frac{dP}{dT} = \frac{P}{T}\frac{\gamma}{\gamma-1} = \frac{P}{T}\Gamma. \tag{10}$$

Our second assumption allows us to simplify variation in water content. We make the simplifying assumption that variations in the vapor pressure is tied predominantly to temperature variation, and not to water density advection or phase changes. The vapor pressure depends on the relative humidity, h, and the saturation vapor pressure, e_s [7], which can be expressed using a common parametrization [8] that is valid in typical boundary-layer conditions

$$e_v = he_s = h6.122\exp\left(17.67\frac{T-273.15}{T-29.65}\right). \tag{11}$$

Taking the derivative with respect to temperature gives

$$\frac{de_v}{dT} = e_v\frac{4355.655}{(T-29.65)^2} = e_v\beta(T). \tag{12}$$

Eq. (9), evaluated using these assumptions is

$$\frac{dN}{dz} = \frac{dT}{dz}\left(\frac{\partial N}{\partial T} + \Gamma\frac{P}{T}\frac{\partial N}{\partial P} + \beta\frac{\partial N}{\partial e_v}\right). \tag{13}$$

The next step is to apply this derivative to expressions for refractivity at a various wavelengths. For conversion from the RF part of the spectrum to the visible and near IR (Optical) portion, empirical expressions defined by Tatarskii and Ciddor [5, 9] will be used. It will be seen that the impact of the humidity is different in each regime. For Ciddor's expressions $\partial N/\partial e_v \approx 0$ because it depends on the locally constant water vapor mixing ratio. In Tatarskii's expression, the water vapor dependence of N is tied to the vapor pressure, e_v. Since e_v varies with T and P, $\partial N/\partial e_v$ will be included in the RF C_n^2 dependence. The next two sections will derive expressions for $f(T,P,e_v)$ in the RF and optical regimes. Care must be taken in application because the Tatarskii's equation has pressure units in millibars while Ciddor's uses Pascals.

3. RF refractivity

The calculation of $f(T,P,e_v)$ for the RF case is based on Tatarskii's expression for refractivity [5] (see section 15),

$$N = \frac{79}{T}\left(P + \frac{4800e_v}{T}\right), \tag{14}$$

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Here the humidity is not given explicitly. It is contained in the vapor pressure, e_v. As stated above, pressure is in millibars and temperature is in Kelvin. Manipulation Eq. (14) evaluated using (13) is straightforward, and yields the expression

$$\frac{dN}{dz} = \frac{dT}{dz}\left[\frac{N}{T}(\beta-2) - \frac{79P}{T}\beta + \frac{79P}{T^2}(\Gamma+1)\right], \tag{15}$$

which has the form required by Eq. (6). The variable N has been retained in the right side of the equation because Tatarskii's expression has no wavelength dependent information.

4. Visible and IR refractivity

In the visible and IR regime, the Ciddor equations for refractivity should be used. Ciddor gives the refractivity as

$$N = \rho_\alpha \frac{N_{\alpha s}}{\rho_{\alpha s}} + \rho_w \frac{N_{ws}}{\rho_{ws}}. \tag{16}$$

Where $N_{\alpha s}$ and N_{ws} are the wavelength-dependent reactivities of dry air and water vapor under standard conditions. Also, $\rho_{\alpha s}$ and ρ_{ws} are the respective densities under the same conditions. The dependence of N on z is contained in the densities of dry air and water vapor in the air, ρ_α and ρ_w. Inserting the expressions for the density of dry air and water vapor gives,

$$N = \frac{PM_\alpha}{ZRT}(1-x_w)\frac{N_{\alpha s}}{\rho_{\alpha s}} + \frac{PM_w}{ZRT}x_w\frac{N_{ws}}{\rho_{ws}}. \tag{17}$$

Which can be expressed as

$$N = \frac{P}{ZT}(A + x_w B). \tag{18}$$

Where Z is the compressibility of moist air; x_w is the molar fraction of water vapor in the air; and A and B are wavelength dependent constants. Here T is temperature in Kelvin and P is pressure in Pascals.

Assuming that x_w is constant, and taking the derivative of (18) with respect to z gives

$$\frac{dN}{dz} = N\left[\frac{\Gamma-1}{T} - \frac{1}{Z}\left(\frac{\partial Z}{\partial T} + \Gamma\frac{P}{T}\frac{\partial Z}{\partial P}\right)\right]\frac{dT}{dz}. \tag{19}$$

Z is given by Ciddor [9] as

$$Z = 1 - \frac{P}{T}\left[a_0 + a_1 t + a_2 t^2 + (b_0 + b_1 t)x_w + (c_0 + c_1 t)x_w^2\right] + \left(\frac{P}{T}\right)^2 (d + e x_w^2). \tag{20}$$

With the constants a_i, b_j, c_k, d, and e_v defined in his paper, and t representing temperature in degrees Celsius. The partials of Z are

$$\frac{\partial Z}{\partial P} = \frac{Z-1}{P} + \frac{P}{T^2}\left(d + e x_w^2\right), \tag{21}$$

and

$$\frac{\partial Z}{\partial T} = \frac{1-Z}{T} - \frac{P^2}{T^3}\left(d + e x_w^2\right) + \frac{P}{T}\left[a_1 + 2a_2 t + x_w(b_1 + c_1 x_w)\right]. \tag{22}$$

Substituting (21) and (22) into (19) gives

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$$\frac{dN}{dz} = \frac{N}{TZ}\left\{(\Gamma-1)\left[1+\left(\frac{P}{T}\right)^2\left(d+ex_w^2\right)\right]-P\left[a_1+2a_2t+x_w(b_1+c_1x_w)\right]\right\}\frac{dT}{dz} \tag{23}$$

5. Results

Initial comparisons were done with scintillometer data collected as part of the experiments described in [10]. The measurements were taken during October of 2011. The scintillometer path covered about 7 km from the University of Dayton to the US Department of Veterans Affairs Hospital in Dayton, OH. Reflectivity-based C_n^2 values from the Wilmington, OH (KILN) NEXRAD radar were used to estimate C_n^2 at 880 nm. These were computed using Global Forecast System (GFS) numerical weather prediction (NWP) data, refractive index estimates from LEEDR, and the wavelength correction method described herein. Results have shown good agreement for measurements taken in clear air (Fig. 1). Radar measurements do not agree well when rain or other clutter sources are present. In these cases, returns from the clutter (precipitation or the ground) increase the apparent turbulence. Ground clutter in the radar signal can be especially strong at night, when temperature inversions cause the radar side-lobe beams to refract down more strongly than normal, thus causing return from the ground which can be much stronger than the turbulence return. This clutter has not been removed in the following plots.

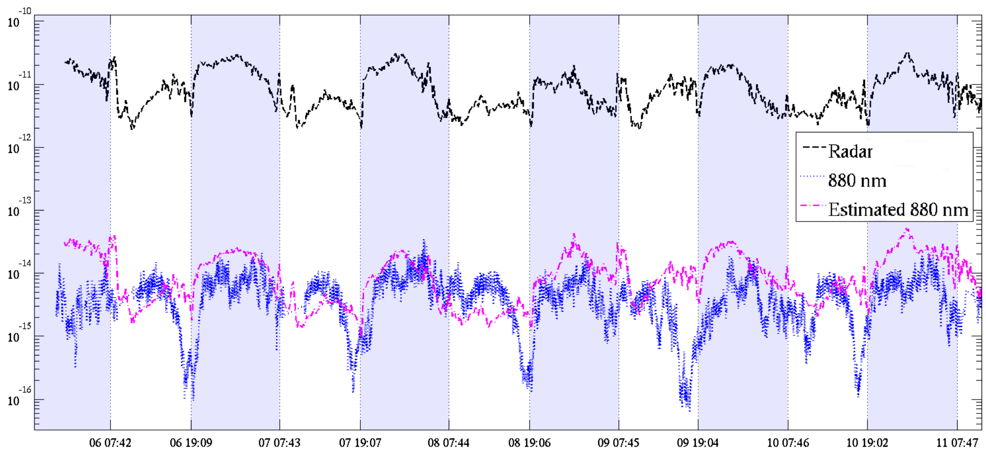

Fig. 1. C_n^2 vs. time. C_n^2 values are from a 10 cm NEXRAD radar (black line), along with scintillometer measurements of the same air volume using an 880 nm scintillometer (blue dotted line), and the estimated measurements of an 880 nm device created from the radar measurements and GFS NWP data (magenta dashed-dotted line). Shaded areas indicate night-time. All times are local Eastern-Daylight Savings Time from October of 2011.

A quantitative statistical comparison was performed on the $\log_{10}(C_n^2)$ data. Measurements were taken from 2000 (2400 UTC) hours on 6 October through 0800 (1200 UTC) hours on 11 October. There is an inherent difficulty in synchronizing the data for statistical comparison. Scintillometer data is collected once every minute, and the radar file timestamps have a period that varies from 5 to 10 minutes. Furthermore, the radar could have measured the path at any time within the measurement period. In all cases the scintillometer data is treated as truth. Scintillometer data is linearly interpolated to the radar sample times. This leaves a total of 984 data points.

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Three tests were used to quantify the similarity of the signals. The Root Mean Squared Error (RMSE) gives a measure of the average error (offset and shape error) in the signal. The Pearson correlation test was used to determine if the signals look more similar before or after correction. The Pearson correlation is the zero-mean normalized correlation coefficient at zero-lag. The Pearson p-test is a null hypothesis test that indicates the likely-hood that two independent random signals of the same length would have a correlation greater than or equal to the measured correlation. RMSE before the wavelength correction is 3.4022 after the correction it is 0.6009 which indicates a strong improvement in value agreement. The Pearson correlation coefficient is 0.316 before and 0.335 after indicating a small improvement in the shape agreement. Pearson-P values are on the order of 10^{-27} in both cases indicating that the correlation is significant.

As a check of the algorithm behavior, the radar estimates were corrected to many different wavelengths. The corrections vary from the 10 cm radar wavelength down to a 500 nm wavelength. The corrected C_n^2 value does not monotonically progress toward the scintillometer data as wavelength decreases. Instead, the correction depends on the refractive index at each wavelength, and the empirical relation used. All wavelengths greater than 1 mm used Tatarskii's equation and all others used Ciddor's. Wavelengths that are more strongly absorbed by the atmosphere show higher C_n^2, while more transmissive wavelengths have lower C_n^2 (Fig. 2).

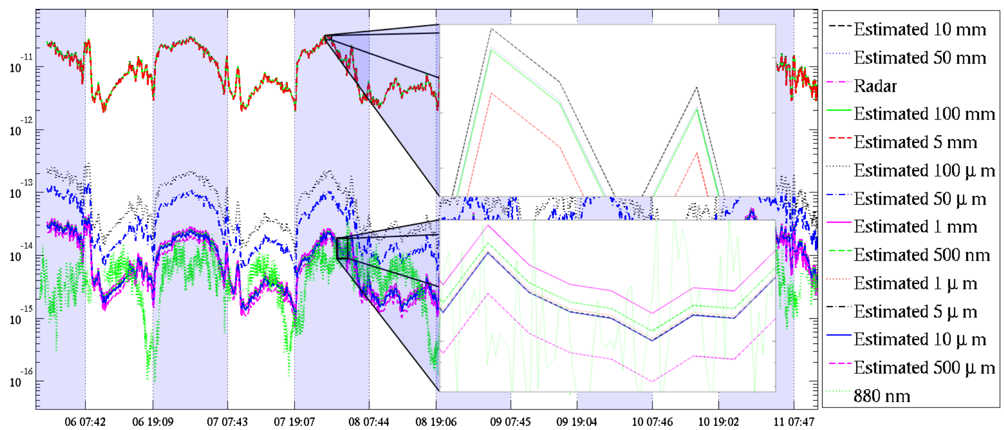

Fig. 2. Computed C_n^2 vs. time at wavelengths decreasing from 100 mm to 500 nm. The order, from top to bottom, in the legend corresponds to the relative C_n^2 of each wavelength. Also included are the measured radar and scintillometer data. Shaded areas are for night, and portions of the plot are magnified so that the fine structure can be seen. Note that the original radar data and the data corrected to 10 cm match, as they should. Times are local EDT from October 2011.

6. Conclusions

Results from the initial comparison appear to validate the method and indicate that the wavelength correction is the largest adjustment that needs to be made when converting radar-derived C_n^2 to the optical regime. It is not surprising that the Pearson correlation does not change much after the wavelength correction, as the over-all shape of the radar data did not change significantly. Results including this wavelength correction plus noise removal have shown strong correlation improvement and will be submitted for publication in the near future.

The method is straightforward to implement, and two common measurement regimes are derived above. It should be possible to extend this technique to other bands, if needed. At all

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wavelengths, the variation is put in terms of refractivity, N. This allows for N to be imported from other models, which can be important near absorption lines whose effects are not captured by the Ciddor or Tatarskii equations. While this method has worked well in the 'wings' of absorption lines, research into index variation within a linewidth of an absorption peak has not been done.

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